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OPTIMAL WELFARE PRICE FOR A ROAD CORRIDOR WITH HETEROGENEOUS USERS

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Abstract. In some countries it is fairly common to see two roads with the same origin and destination competing in the same corridor. One of them is usually a toll highway that offers a better quality to the users compared to its alternative: a free parallel single road, which might be tolled as well. This kind of transport network has been largely studied in the academic literature and particularly the optimal combination of tolls that maximizes economic efficiency. If both roads are tolled the problem is known as the first best, otherwise it is called the "untolled alternative". There is a gap regarding how income distribution affects the optimal toll. The main objective of this paper is to add knowledge in the area by analysing the influence of the distribution of the Values of Travel Time (VTT) of the users of this corridor on the optimal combination of tolls. To solve this problem, the authors define a mathematical model aimed at obtaining the optimal welfare price for this kind of corridor under the hypothesis that drivers decide over the expectation of free flow conditions. The results show that the higher the average VTT the higher the optimal price, and the higher the dispersion (variance) of this VTT the lower the optimal price. It was also found that low income users who are not able to internalize externalities should not travel. Finally, first best pricing and untolled alternative schemes match for high income users.

Keywords: transportation pricing, optimal price, competing roads, social welfare, value of travel time distribution.

Introduction

Transport infrastructure is a necessary requirement for the development of any region. However, to achieve positive economic impact two main additional conditions must be met along with investment in transport infrastructure. Political factors (e.g. funding support) appear as the first condition. For instance, concessions risk allocation in Portugal is shifting from demand risks to availability payment schemes, which is more favourable for the private sector (Cruz, Marques 2013). Economic externalities (e.g. agglomeration) represent the second condition to be met, and transport infrastructure produces many externalities. Increased accessibility and the resultant economic development are among the most notable positive ones. Accidents, air and noise pollution, and other environmental issues, such as impacts on biodiversity, landscape and townscape, are the most important negative ones. In the case of road infrastructure, impacts from congestion have a key effect on net social benefit. Road pricing has been proven a successful means to alleviate congestion (Chung, Recker 2012). Externalities and travel times are reduced due to the toll and at the same time revenues can be generated. However, acceptability towards road pricing is usually low so it may be a serious obstacle to its implementation (Dieplinger, Fürst 2014). In fact, the toll could harm low income people and their opportunity to participate in society might be diminished (Kenyon *et al.* 2003). Depending on the objective function to be optimized (e.g. maximize welfare, maximize social equity, maximize revenues, etc.), the optimal toll might vary substantially (Yin, Yang 2004). It also seems sensible think the richness of a society can influence the optimal price. However, as far as the researchers are concerned, there is a gap in the literature regarding the optimal price in a fully charged corridor for different distributions of the Values of Travel Time (VTT). Particularly they did not find any research estimating the optimal welfare price when varying the conditions of the average VTT and its dispersion. The main contribution of this paper is therefore to add knowledge in the area. In order to do so, a numerical approach will be used in this research since according to the literature review, this

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problem is very complex and it would be harder to obtain a more realistic solution through an analytical model. To carry out such numerical approach some assumptions will be taken.

The remainder of the paper is organized as follows. In Section 1 a literature review on the subject is conducted and the objectives of the paper are defined. Section 2 shows the objective function to be optimized, describes the methodology and explains the adopted assumptions. In Section 3 the parameters and variables to apply to a specific case study are defined. Section 4 shows the results responding to the objectives previously established. Finally, a set of conclusions and suggestions is offered.

1. Literature review

The problem of how to achieve maximum welfare through pricing schemes has been widely studied. Many research pieces have compared first best pricing schemes to second best pricing schemes. For instance, Braid (1996) studied a network with two roads with the total trip demand being fixed, that is price inelastic. If the schedule delay cost is a *V*- shaped function and both roads have the same capacity, then two out of three users will choose the toll highway and the gains of welfare will be two thirds of the hypothetical gains with the first best solution. Liu and McDonald (1998) included the peak and the off-peak hour to a case study consisting of adding two toll lanes to an existing bridge made up of four lanes. Welfare gains were found to be ninety percent greater when the whole bridge was tolled than when only two lanes were tolled. The model was improved (Liu, McDonald 1999) by including a relationship between the peak and off-peak hour periods. The behaviour of a transportation network in three scenarios was tested: without any toll, completely tolled (first best), and tolled only in some stretches (second best). Important differences between the scenario without any toll and the second best were found, such as a decrease of total traffic or a switch from the toll roads to the free roads in the second best scheme. Verhoef (2002) introduced linear functions of cost and demand to the theoretical development of the problem. As in previous studies, a greater welfare was achieved with the first best scheme. Yang and Zhang (2003) found for an urban network with 43 links that the maximum welfare can be achieved through marginal cost pricing, but with the toll in just 10 links welfare was found to be almost identical. Surprisingly, the results show no important difference between the first best and a scenario under the second best pricing scheme. Verhoef *et al.* (2010) also studied generalized costs in congested transport networks and compared first best pricing with second best pricing. They applied the model to Edinburg and found out a greater welfare in the first best scheme.

With respect to the influence of heterogeneity on the optimal toll, Yang *et al.* (2002) noted the advantages of using different VTT depending on the type of traveller. Indeed, VTT was found to be a key factor influencing the response to pricing schemes (Yang, Huang 2004) and user heterogeneity must be taken into account for the path choice model (Zhang *et al.* 2013). Welfare can be greatly influenced by heterogeneity (Van den Berg 2014). In addition, the way in which welfare is measured and whether toll revenue is included as a benefit, influence optimal tolls as well (Mayet, Hansen 2000). With discrete user classes and revenue refunding the pricing scheme can be Pareto-improving only if disutility of travel is reduced (Guo, Yang 2010), although equity concerns of redistribution could turn out to be in inefficient toll levels (Diaz, Proost 2014). Congestion pricing may be progressive in its welfare impact (De Palma, Lindsey 2004), albeit it is noteworthy that the intrinsic difference between VTT distributions influences the fairness of the system (Xu *et al.* 2014). According to Du and Wang (2014) there is little research related to welfare and heterogeneous users.

From the literature review the authors found a gap regarding the optimal welfare price in a fully charged corridor for different VTT distributions of the potential users. Ortega *et al.* (2018) have recently shed light on the problem for the untolled alternative. However, there is undoubtedly room for more analysis on the impact of heterogeneous users, as well as the degree of their heterogeneity. Thus, the objective of the paper is to shed light on this topic by defining a model to obtain the optimal toll price in terms of the VTT distribution for a fully priced interurban corridor with two roads of different quality.

2. Assumptions and methodology

In order to pursue the objective of this paper, an optimization problem has been formulated which is aimed at calculating the optimal combination of tolls that maximizes the social welfare (i.e. minimize the social cost) in an interurban corridor of the above-mentioned characteristics, for different distributions of their VTT. A more detail description of the methodology is available in Ortega *et al.* (2018). Nonetheless, for the sake of clarity a summarized version of the methodology is shown in this paper. The main assumption is that the potential users are supposed not to be familiar with the traffic conditions in the corridor. Therefore, they will decide whether they will travel or not, and through which road they will do it, on the basis of the expected travel time, their VTT, the gasoline cost expected under free flow conditions and the toll in both roads. The extra-costs that the users may end up facing if travel conditions are not as they originally expected cannot be known by them at the time of making the travel decision. This assumption makes sense in interurban corridors since users have habits such as check conditions before the trip (e.g. google maps or traffic GPS systems) and intercity trips can be unexpectedly delayed due to bad weather, road works ahead or an accident to name but three. Indeed, unfamiliar trips were found to be quite different to familiar trips, and there is a gap between expected and actual travel time (Schmitt *et al.* 2015). In other words, an assumption of perfectly well-informed users would be less realistic than the assumption already taken in interurban corridors. Consequently the model is valid only for interurban networks where most users are not familiar with the congestion in the corridor. For the model to be applicable to commuters in an urban transport network, it would be necessary to change this assumption. In the selected transport network the total cost for the society is defined as follows:

$$SC = UC + EC + HOB + GB, \quad (1)$$

where:

- SC is the total social cost [€]; this is the objective function that has to be minimized so the welfare will be maximized;
- UC is the total cost that the users bear per trip [€]; it is divided into four terms, which are travel time, toll, fuel cost and maintenance of the vehicle;
- EC represents the externalities produced by the vehicles [€]; they are the summation of environmental cost – i.e. gas emissions, noise and so on – plus the external costs caused by accidents; the environmental cost is proportional to the fuel consumption, so the more the fuel consumption the more the environmental cost;
- HOB is the net operating balance for the road operator either public or private [€]; it consists of the tolls paid in the highway minus the maintenance cost in the toll highway;
- GB is the result for the government responsible for maintaining the conventional road [€]; it is calculated as the taxes recovered from fuel and tolls paid in the road minus the maintenance cost of the road.

Equation (1) is written in terms of social cost and to avoid double counting tolls and taxes are considered as transfers among society. When the demand increases the key question will be whether the additional users are able to internalize the additional externalities that they do not perceive. Although congestion costs do not appear explicitly, they automatically appear when traffic increases and the speed decreases. The potential users of the corridor are divided into 100 groups according to their income; this is denoted by the index $i = 1, ..., 100$. The index j represents the different roads for the potential users either H (Toll Highway) or R (Conventional Road). The potential users have a daily expenditure limit for transport, which can make them decide not to travel:

$$\begin{aligned} &\text{If } T^j + \phi \cdot GC^j > \Theta \cdot I_i, \ \forall j \\ &\text{users of group } i \text{ do not travel} \\ &N_{i,\exp} = NPU_i \text{ and } N_i = 0; \\ &\text{otherwise } N_{i,\exp} = 0 \text{ and } N_i = NPU_i, \end{aligned} \quad (2)$$

where: T^j is the toll in € in the road j; GC^j is the expected gasoline cost [€] under free flow conditions in the road j; Θ is the limit of expenditure in transport (Litman 2007) in per unit values; I_i is the daily income expressed [€] for group i; NPU_i is the number of potential users of group i; N_i is the number of users of group i who decide to travel; $N_{i,\exp}$ is the number of users of group i who do not travel because the cost they have to bear to travel is too high for them; ϕ is a coefficient that expresses the user's perception with respect to the cost of gasoline (Huang, Burris 2015).

All groups of users decide first whether they travel or not, and if they do it, then they decide through which road they will do it according to the cost they expect to bear in each alternative:

$$DC_i^j = T^j + \phi \cdot GC^j + VTT_i \cdot ETT^j, \quad (3)$$

where: DC_i^j is the cost estimated by the group i for the road j expressed [€]; VTT_i is the values of travel time [€/h] for the group of users i that is strongly related to their income I_i.

In practice, demand elasticity and VTT are only imperfectly correlated (Verhoef, Small 2004), albeit in order to reduce user heterogeneity from two dimensions to one, VTT is assumed to be proportional to income: $VTT_i \cdot 8 = I_i$. For intercity trips (and particularly recreational trips), the VTT may be lower than this because people are not pressed for time, but VTT is used to know differences in the transport policy on the optimal tolls between different societies, thus this proportion will be the same for all VTT curves and will not have any significant influence on the outcome (i.e. the policy lessons will be identical regardless this proportion). Finally, ETT^j is the expected travel time under free-flow traffic conditions in the road j. ETT^j has been set with free flow conditions, but according to the methodology this expectation can be easily changed. However, following findings from the untolled alternative (Ortega *et al.* 2018) these travel expectations would change the value of the optimal tolls but would hardly have any influence on the trends described later. The main difference would be that the longer the expected congestion (and hence the average travel time plus unreliability) the higher the optimal tolls will be since users anticipate the delay. DC_i^j is calculated for each group of potential users who have decided to travel; each of them will travel through the road with the lowest DC. The total cost for the users UC is calculated as the summation of the real cost for each group of users RC_i (Equation (4)), which is in turn calculated from Equations (5)–(8).

$$UC = \sum_i RC_i; \quad (4)$$

$$RC_i = RC_i^H + RC_i^R + RC_{i,\exp}, \ \forall i; \quad (5)$$

$$\begin{aligned} RC_i^H = &\Big(T^H + VTT_i \cdot ETT^H + \\ &(1+\sigma) \cdot VTT_i \cdot \left(RTT^H - ETT^H\right) + \\ &RGC^H + MCV_j\Big) \cdot N_i \cdot (1 - U_i), \ \forall i; \end{aligned} \quad (6)$$

$$\begin{aligned} RC_i^R = &\Big(T^R + VTT_i \cdot ETT^R + \\ &(1+\sigma) \cdot VTT_i \cdot \left(RTT^R - ETT^R\right) + \\ &RGC^R + MCV_j\Big) \cdot N_i \cdot U_i, \ \forall i; \end{aligned} \quad (7)$$

$$RC_{i,\exp} = N_{i,\exp} \cdot \left(DC_{i,\exp} + \frac{1}{2} \cdot \Delta T_{i,\exp}\right), \ \forall i, \quad (8)$$

where: RC_i is the real cost each group of users have to bear [€]; σ is a coefficient, which penalizes the exceeding travel time above free flow (Wardman, Ibáñez 2012); RGC_j is the real gasoline cost in the road j [€]; MCV_j indicates the vehicle maintenance cost in the road j [€], which depends on the length of the road j as well as on the kilometre maintenance cost MCV.

The Equation (8) has been added in order to take into account the social cost for the users who do not travel, that is, the loss of welfare of those users who cannot afford the trip. In other words, this is the so called "rule of a half" in the cost benefit analysis methodology but adapted to users who decide not to travel instead of their usual induced demand. $RC_{i,\exp}$ is the cost to be assigned to the users who do not travel; $DC_{i,\exp}$ is the decision cost that pushes out these users; $\Delta T_{i,\exp}$ is the difference between the toll that makes them not to travel in $DC_{i,\exp}$ and the toll they would pay if they travelled.

Equation (8) can therefore be replaced by benefits from induced demand and the minimum social cost will always be found at the same point. U_i is a binary variable, which takes the value 1 if the users travel through the conventional road or 0 if the users choose the toll highway; in those cases where the users of group i decide not to travel, $RC_i^H = RC_i^R = 0$, since $N_i = 0$, regardless the value of U_i.

RTT^j is the real travel time in the road j, which according to the Bureau of Public Roads is calculated as follows:

$$RTT^j = ETT^j \cdot \left(1 + \alpha^j \cdot \left(\frac{N^j}{CAP^j}\right)^{\beta^j}\right), \tag{9}$$

where: CAP^j is the capacity of the road j; α^j and β^j are necessary coefficients to calculate the real travel time.

The problem of finding the optimal combination of tolls that minimizes SC is nonlinear and nonconvex (Yang, Huang 2005), thus the use of commercial solvers based on mixed integer nonlinear programming does not guarantee that a global optimum can be obtained. For this reason, and considering the moderate size of the problem under study, a simulation-based optimization method is used in this paper. Moreover, and taking into account the large number of variables involved in the problem, in order to analyse the influence of VTT distribution on the optimal combination of tolls we use a realistic case study, which is described in Section 3.

3. Case study

Three different types of data and variables are used in the case study: first, a set of input variables – VTT distribution, tolls, and Total Number of Potential Users (TNPU); second, some parameters that the authors assume constant to facilitate the analysis, e.g. the slope of the roads, fuel taxes, etc.; and third, a set of output variables – minimum SC, optimal tolls, number of users who do not travel and traffic distribution in the corridor, that result as an output of the model. The tolls are considered to be an input because the output variables are influenced by them, that is, SC or traffic distribution will depend on the tolls and not the other way around. The results are obtained for 25 lognormal income distributions. Although the authors have focused the analysis on studying the effects of the VTT distribution and the TNPU on the optimal toll price, the methodology presented in the paper can be easily adapted to study other variables. Table 1 shows the ranges selected for the input variables.

The length of the highway is 90 km and the length of the road is 100 km. The total capacity of the corridor is 6500 veh/h. The capacity share of the conventional road in the corridor is 26.15%. The user's perception with respect to the cost of gasoline φ was set at 0.9 (Huang, Burris 2015), and the limit of expenditure in transport Θ was set at 0.2 (Litman 2007). The ranges for the four variables, VTT_i, T^H, T^R and TNPU have been selected following the recommendations by several researches. The range of tolls has been chosen according to the actual levels in different corridors across Europe, particularly in Spain (Ortega Hortelano 2014). Each VTT distribution is characterized by its average μ, variance σ^2 and shape. The average values were chosen in the following way. The value of time for long distance travellers in the California State Route 91 ranks between 7.09 and 29.42 \$/h (Small *et al.* 2005) and the high variability in the results was explained by the kind of survey conducted, i.e. stated preferences or revealed preferences. For the Interstate 15 in California these values in the peak hour varied from 20 to 40 \$/h (Brownstone, Small 2005) and a high variability was also found. For the Spanish case, De Rus *et al.* (2010) recommend values between 9.18 to 22.34 €/h. Secondly, the variance of the distribution was selected in agreement with Calvo Gonzalez *et al.* (2012), who found that the variance of the distribution for different Spanish regions can increase or decrease

Table 1. Variables selected for the case study

Variable	Definition	Value	Source
VTT_i	distributions of the value of travel time	25 lognormal distributions with 5 different averages (μ = 13, 16, 19, 22 and 25 €/hour); 5 different variances (σ^2 = 84.44, 171.28, 249.28, 324.8 and 391.95)	Small *et al.* (2005); Calvo Gonzalez *et al.* (2012); De Rus *et al.* (2010); Fosgerau (2006); Brownstone, Small (2005); Abrantes, Wardman (2011); Shires, De Jong (2009); Ortiz, Cummins (2011); Nie, Liu (2010)
T^H, T^R	highway toll, road toll	0…8 €	Ortega Hortelano (2014)
TNPU	potential users in the corridor	3000…7000 veh	Kraemer *et al.* (2004)

up to 30% from the average, and with Ortiz and Cummins (2011) who noted that Gini indexes can vary from about 0.20 to almost 0.7 worldwide. Thirdly, with respect to the distribution's shape, it must be a lognormal (Fosgerau 2006), and the variances of this lognormal might range from 10 to 1000 (Nie, Liu 2010). In an update of the valuations conducted by Abrantes and Wardman (2011) and by Shires and De Jong (2009) similar ranges, values and shape are recommended. Both studies conduct a meta-analysis of VTT and the impacts on VTT of GDP, income, travel distance, travel purpose, transport mode and finally survey method are well acknowledged. In other words, VTT is an important parameter in economic appraisal thus it is advisable to place awareness of the case study to avoid errors and misunderstanding in the interpretation of the results. Thus, for this research, the 25 lognormal VTT distributions chosen can be considered a realistic case study since it covers the whole variability of interurban roads in developed regions such as Europe or the United States.

4. Results

This section is divided into five subsections. The first one finds out how the model calculates the optimal tolls, and shows the evolution of social cost for different tolls and traffic levels. The second one studies the effect of the average VTT on the combination of optimal tolls. The third one analyses the effect of the variance of VTT on the optimal tolls. The fourth subsection tests the model with a VTT distribution resembling the income characteristics of Spain. Finally, a comparison between the first best solution (tolling the two roads in the same corridor) and the untolled alternative is offered.

4.1. Social cost, traffic and optimal tolls

In order to simulate a realistic scenario the research will just be focused on lognormal distributions of VTT. Figure 1 depicts the evolution of SC as a function of T^H and T^R for a particular VTT distribution with four different levels of demand (potential users TNPU): from free flow to congestion, where the capacity of the corridor is slightly exceeded. The lognormal VTT distribution chosen for this section does have an average μ = 19 €/h and a variance σ² = 249.28 in the middle of the range previously selected. Obviously if the figure is cut with the plane of $T^R = 0$ we would get the evolution of *SC* under the untolled alternative. The red lines or points show the minimum *SC* and therefore the combination of optimal tolls.

Figure 1 shows that the larger the potential traffic the greater the social cost. This figure also demonstrates that if

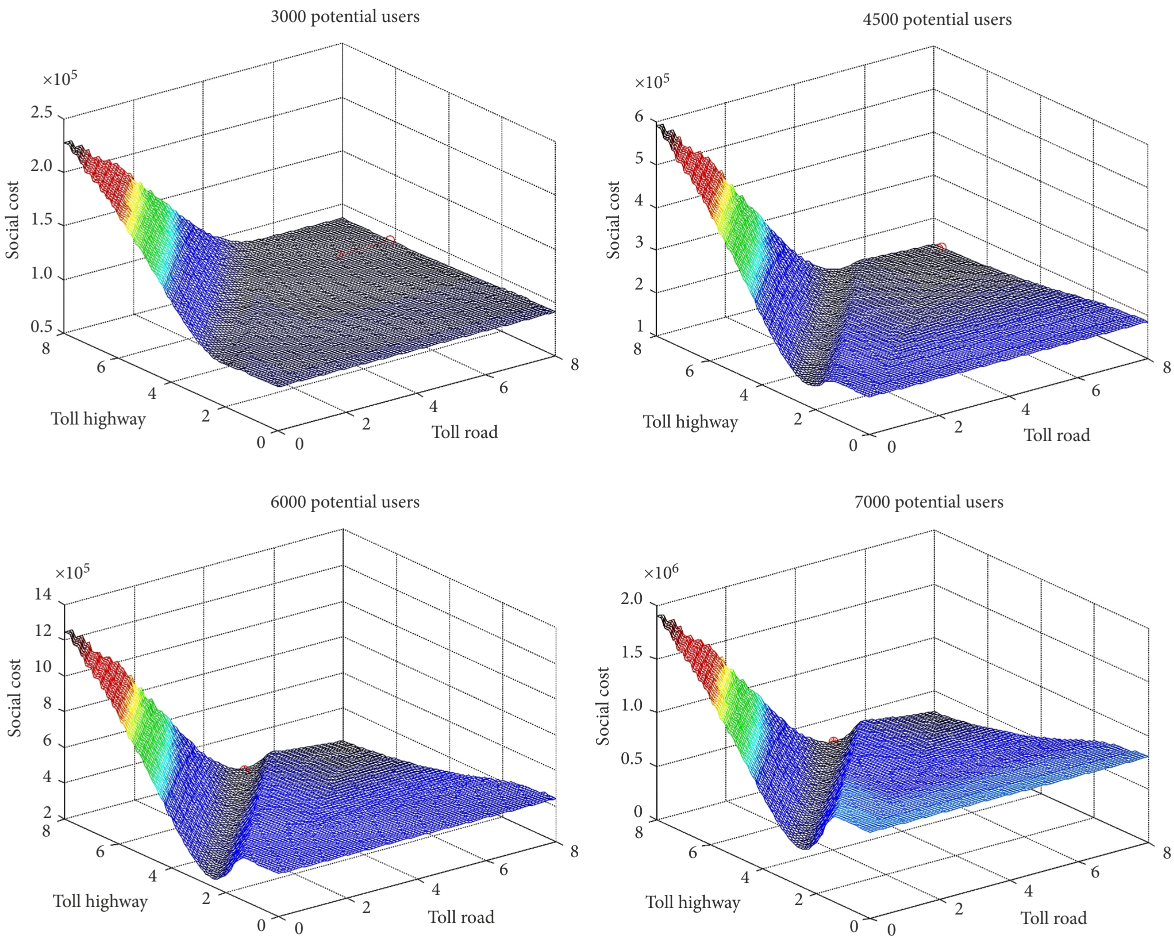

Figure 1. Evolution of social cost *SC* for different toll combinations (T^H and T^R) and levels of potential users TNPU

the combination of tolls is far from the optimal, the social cost will increase hugely. This is particularly true for high tolls in the highway, low tolls in the road and high traffic. Under such scenario the majority of the users would travel through the road and there would be large welfare losses due to the congestion in the road. Two outputs mainly explain the shape of the figure: percentage of users who do not travel and traffic share in the road. For each TNPU the combination of tolls is associated to certain relationships of these two outputs. For instance, the valley that can be seen in the lower graphs indicates similar traffic shares in the road because the difference of the toll between the highway and the conventional road is constant, but the percentage of users who cannot afford the trip is different (i.e. due to the demand elasticity the higher the price the fewer the travellers). There are some small steps that can be seen on the left corner of each graph, and represent changes in these two variables. In fact the continuous distribution of VTT has been implemented as 100 discrete values. The effect stemming from congestion is fully understood in the figure. The tolls for the optimal combination are different depending on the traffic level, as well as the traffic share in the road and the percentage of users who cannot bear the expense of the trip are different too.

For free flow conditions (3000 TNPU) the minimum *SC* is achieved for a highway toll of 6.2 € and a road toll that can vary from 6.5 to 8 €. The tolls for the optimal combination lead to a null traffic share in the road. This way one out of four potential users cannot afford the trip. For the next level of TNPU – 4500 – the toll in the highway should be higher than before (7.7 €) whereas the toll in the road can vary from 7.9 to 8 €. The traffic share in the road would be again null and the percentage of users who do not travel reaches its peak (31%), so the real traffic would be 3105 vehicles.

The trend described above, changes when TNPU is almost at the same level than the capacity of the road (6000). The toll in the highway should be set almost at its maximum (7.9 €) and the toll in the road should decrease down to 5.1 €. The tolls for the optimal combination are unique (that is, they are a point rather than a line in the 3D figure), increase the share of the road (from 0 to 27.71%) and decrease the percentage of users who do not travel (17%). This tendency is similar for a TNPU above the capacity of the corridor. This combination leads to a share of the road slightly above their relative capacity in the corridor (30.12%) and again there are 17% of potential users who do not travel, which is fairly away from the maximum possible.

Thus, for low/intermediate TNPU all users should travel through the highway. In order to avoid an excess of demand and therefore congestion, the higher the traffic, the higher the price of travelling. For intermediate/high TNPU some congestion is unavoidable so it is better to make more users travel and share the traffic between both roads. In other words, with low traffic, low income users trigger more costs than benefits, so it would be better if they do not travel whilst for high traffic the congestion is unavoidable, the benefits of these users would be higher than their costs (i.e. they do not impose any time loss to high income users) and it is better if they travel and the traffic is shared between both roads. There is a percentage of users (17%) who regardless of congestion cannot perceive the externalities through the toll and ultimately would never travel by car. It is remarkable that these results are valid from the point of view of maximization of efficiency, but may arise serious equity concerns.

4.2. Effect of average VTT on the optimal tolls

Figure 2 shows the optimal combination of tolls corresponding to five different VTT distributions – all with the same variance (σ^2 = 249.28) but with different averages μ – as a function of the potential users (Figures 2a and 2b), TNPU and the traffic share in the road and the percentage of users pushed out of the corridor for the optimal combination of tolls (Figures 2c and 2d). Figures 2a and 2c have the same legend and correspond to low and medium averages whereas Figures 2b and 2d have also an identical legend and correspond to high average.

Several conclusions can be drawn from Figure 2. First and broadly speaking, the larger the potential traffic the higher the tolls for the optimal combination. This conclusion is in line with previous findings identified in the literature review. Second, there are two remarkable differences between graphs on the left (low/medium average VTT) and on the right (high average VTT). For high μ distributions when the potential traffic (TNPU) increases there is also a rise in the traffic share of the road. Despite some groups of users could be pushed out, it would be better not to do it because they can afford the trip and internalize the externalities they produce through tolls (Figure 2d). In addition, there are several combinations of optimal prices, but the difference between tolls for each traffic level has to be constant (Figure 2b). For low μ, the share of the road is always null and the greater the traffic the greater the users pushed out (Figure 2c). Basically it proves the fact that users must internalize the externalities they produce but not perceive through a toll. If they cannot afford this toll, then they had better not to travel because they would impose a greater travel time to the "richer" travellers. As we explained in the former subsection, the combination of both price elastic demand and low income users could potentially bring equity issues that would need to be addressed. In this case the optimal combination of tolls consists of a single highway toll and a range of road tolls, among which the government would likely select the minimum one (Figure 2a).

The case for medium average VTT is a transition between both high and low μ (Figures 2a and 2c). Optimal combination of tolls tend to increase with TNPU until reaching the maximum percentage of users who do not travel and the share of the road is null; from then on the tolls are reduced, are unique and the traffic share of

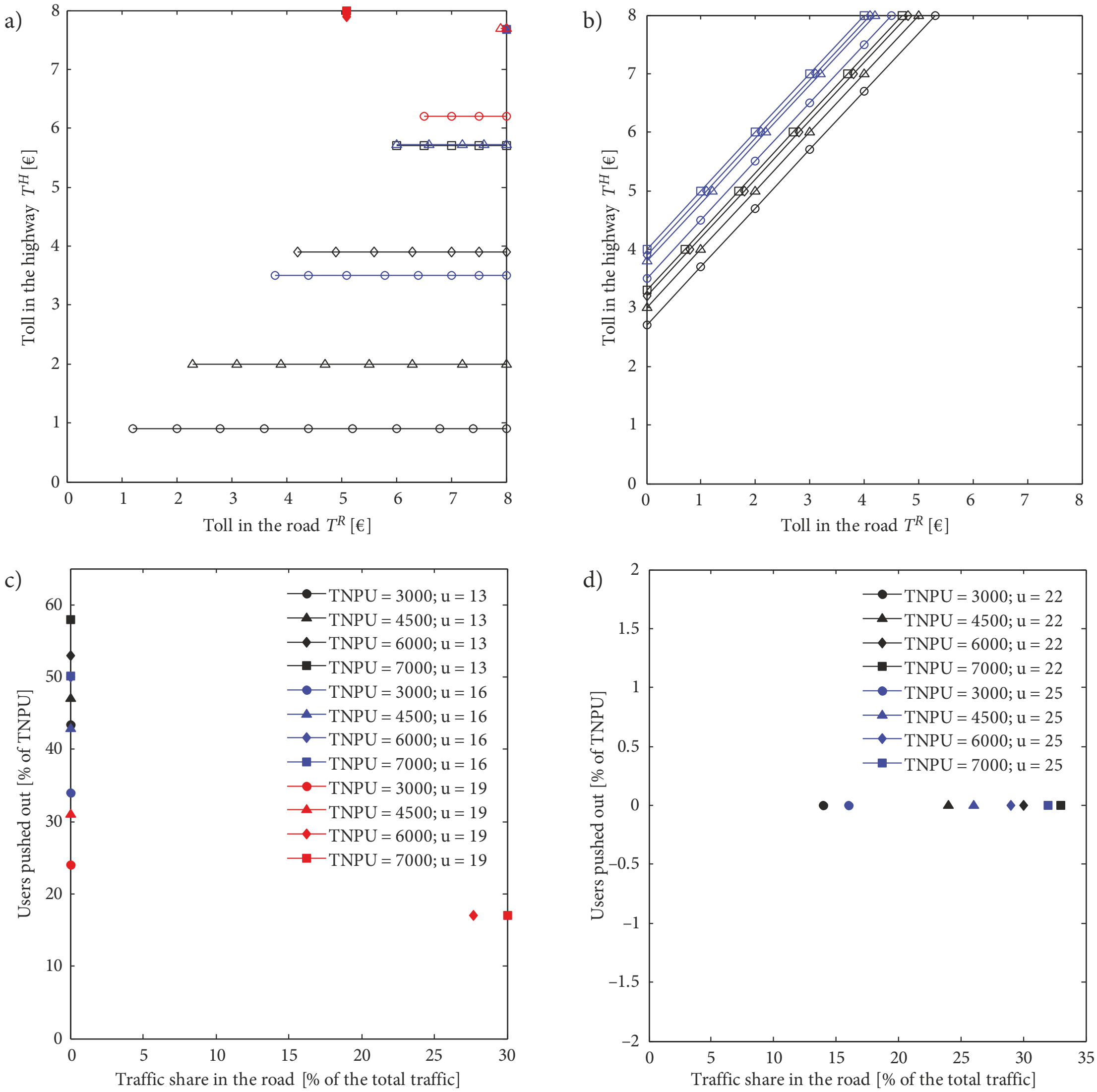

Figure 2. Optimal toll combination (a, b), and Bureau of Public Roads ut in the optimal combination (c, d) for different levels of potential users (TNPU) and different average VTT (μ)

the road increases. The break-even point is related to the TNPU in the corridor, when the congestion is unavoidable. When the users of the corridor are below a certain threshold, there is no congestion (i.e. level of service A or B) and the externalities of using the highway are lower than using the road, it is better that all users drive through the highway. This threshold depends on both the number of potential users and the percentage of users who decide not to travel, so once the level of service is approaching level C, it is better to reduce tolls and make more people travel.

Third, when the traffic is near to the capacity of the corridor, the share of the road is also close to its relative capacity in the corridor although slightly above it. This lies in the fact that, according to Table 1, a higher β has been imposed on the highway. Last, but not the least, *ceteris paribus* the higher the μ the higher the tolls imposed on both roads. This rule applies for low and medium μ distributions and for high μ distributions, but it is not valid for a comparison between them.

4.3. Effect of VTT variance on the optimal tolls

Figure 3 shows the optimal combination of tolls corresponding to five different VTT distributions all with the same average (μ = 19 €/hour), but with a different variance (Figures 3a and 3b) as well as the traffic share in the road and the percentage of users pushed out of the corridor for the optimal combination of tolls (Figures 3c and 3d). Graphs 3a and 3c correspond to high and medium variances whereas Figures 3b and 3d correspond to low variances. As in the previous figure, Figures 3a and 3c on the one hand, and Figures 3b and 3c on the other have respectively the same legend.

Before getting into Figure 3, it is worth noting that these VTT distributions have a medium income μ, thus they are a transition between low income and high income VTT and their behaviour has been already explained. Figure 3 proves that the higher the VTT variance (or dispersion), the lower the optimal toll combination. So, the larger the Gini index the lower the tolls. This means that *ceteris paribus* tolls in regions with a widespread income distribution should be lower than in regions with a concentrated income distribution. With a higher variance there are more "rich" users in the distribution, but there are also more "poor" users, and as a result the peak point of users who cannot afford the trip is achieved for lower combination of tolls. From that point on, the tolls for the optimal combination must be reduced in order to achieve a better trade-off between users who decide not to travel and traffic share between both roads. So, the tolls act as a thermostat that regulates both outputs of the model.

Moreover, the higher the traffic the higher the tolls for the optimal combination. It is also noteworthy that the tolls for the optimal combination are quite similar under congestion. Along with the increase in TNPU there is also an increase in the share of the conventional road. As congestion rises the optimal highway toll goes up whilst the optimal toll in the road goes down, and this encourages travellers to use the road rather than the highway. In fact, under congestion the road is used slightly above than its relative capacity in the corridor. Finally, the break-even point when the tolls must be reduced is reached when the real traffic is around half of the capacity of the corridor, i.e. level of service C. In other words, if the travellers exceed this figure, it would be better to impose lower tolls and share the traffic between both roads rather than both pushing out low income users and having all the traffic in the highway.

4.4. The Spanish case

The optimal combination of tolls for different levels of potential users with a VTT distribution resembling the socioeconomic characteristics of Spain has been calculated. With these results the authors intend to provide insight on the differences between the tolls for the optimal combination according to our methodology and the current toll levels in a particular place. The income distribution has been obtained from the National Bureau of Statistics of Spain and the shape of the distribution is very similar to a lognormal with a low average and high variance. Particularly its average is μ = 13.09 €/h, its variance is σ^2 = 469.92, the Gini index of its VTT distribution would be 0.51 and 31% of potential users would never travel by car. The main results of this analysis are summarized in Table 2.

With the highest tolls in both roads the percentage of users who do not travel is 61%, albeit with the tolls for

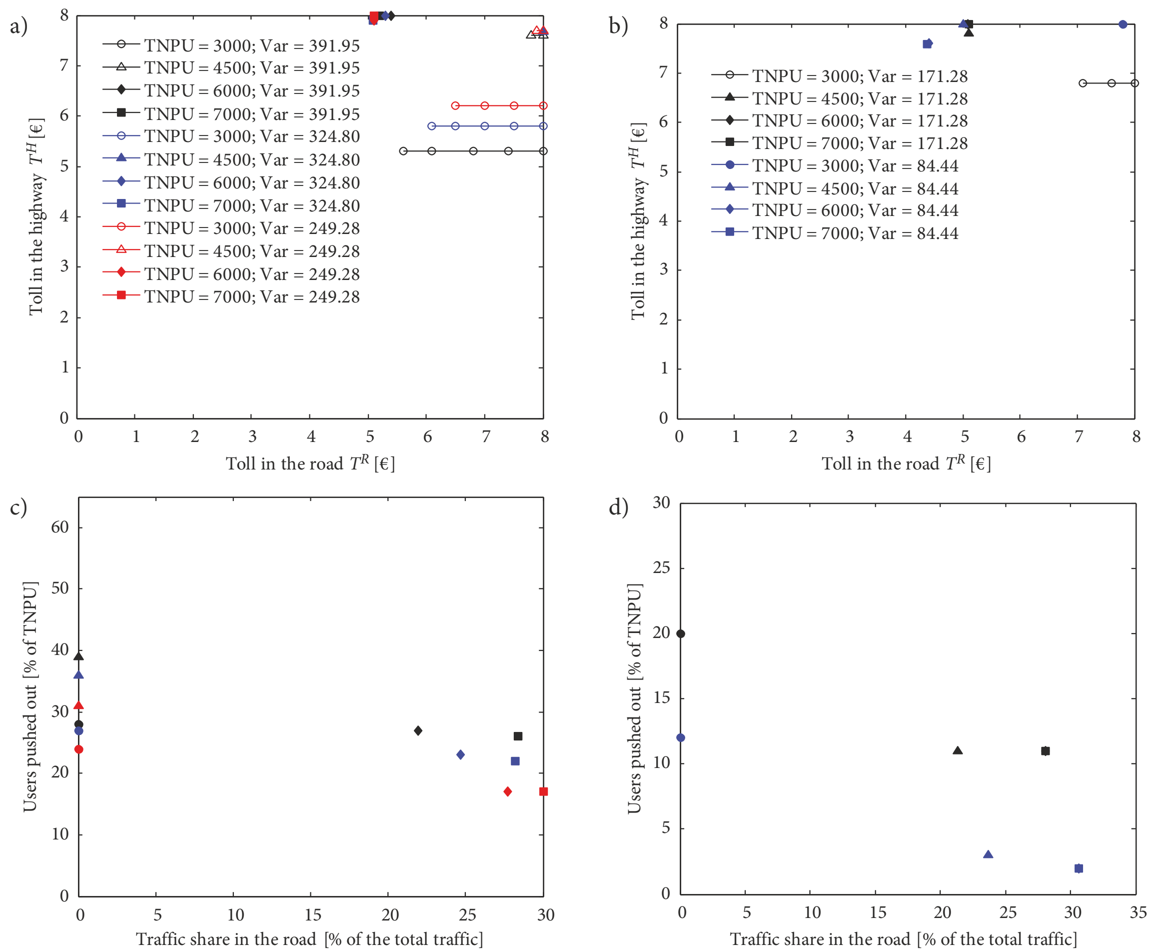

Figure 3. Optimal toll combination (a, b), and traffic share in the road vs users pushed out in the optimal combination (c, d) for different levels of potential users (TNPU) and different variances of VTT (σ^2)

Table 2. Optimal combination of tolls and road share for different levels of potential users (TNPU) considering the income distribution of Spain

Potential traffic (number of users)	Range of optimal price in the toll highway [€], in brackets [€/km]	Range of optimal price in the toll road [€], in brackets [€/km]	Share of traffic in the conventional road [%]	Users pushed out [%]
3000	1.6 (0.018)	1.9…8 (0.019…0.08)	0	40
3500	2.5 (0.028)	2.8…8 (0.028…0.08)	0	44
4000	2.5 (0.028)	2.8…8 (0.028…0.08)	0	44
4500	2.8 (0.031)	3.1…8 (0.031…0.08)	0	45
5000	3.6 (0.04)	3.9…8 (0.039…0.08)	0	48
5500	4.4 (0.049)	4.7…8 (0.047…0.08)	0	51
6000	5.5 (0.061)	5.8…8 (0.058…0.08)	0	55
6500	5.5 (0.061)	5.8…8 (0.058…0.08)	0	55
7000	6.9 (0.077)	7.2…8 (0.072…0.08)	0	59

the optimal combination and 7000 TNPU the number is slightly lower, 59% (i.e. 2870 users in the corridor). The tolls for the optimal combination will always lead on the one hand to a null share of the road and on the other hand to a monotonous increase of the number of users who cannot afford the trip as TNPU increases. In other words, it is more beneficial for society to have potential users out than travelling because they cannot internalize the externalities they produce. Moreover, in order to achieve the optimal welfare solution, the prices should be clearly reduced but a toll in the conventional roads should be imposed (Ortega Hortelano 2014). The main criticism to this transport policy is that a lot of people could not afford the trip and would provoke important inequalities between high and low income people. Whatever the tolls imposed, there are at least 31% of users can not afford the trip. In other words, as seen in the literature review this is an intrinsic difference that can influence the fairness of the system. Moreover, the goal of the developed model is to achieve the maximum welfare without considering equity.

4.5. First best vs untolled alternative

Finally a comparison between the untolled alternative (i.e. the road will remain toll-free) and the fully charged corridor is conducted. This comparison is done for the four extreme cases among the range studied: the highest and lowest average and the highest and lowest variance. Figure 4 shows the optimal combination of tolls for the case of the untolled alternative in red colour and the first best in blue colour. The upper graphs correspond to low average VTT whereas the lower graphs correspond to high average VTT. Furthermore, the left graphs have a high variance whilst the right graphs have a low variance.

A handful of previously explained conclusions arise from Figure 4. Firstly, the larger the traffic the higher the tolls for the optimal combination. Secondly, the higher the variance the lower the tolls for the optimal combination. Thirdly, the higher the average the greater the tolls for the optimal combination. Fourthly, and finally, it is worth explaining the differences between the untolled alternative and the first best. Whereas in the untolled alternative the percentage of users who do not travel for a given VTT distribution remains constant regardless of the TNPU level, this is not the case when the corridor is fully charged. This difference in the percentage of potential users who cannot afford the trip is particularly important for low μ where due to the price elasticity of demand large tolls could push out a high percentage of potential users. It could be thought the optimal solution is reached when there are fewer users in the corridor, but as it has been proved in this paper that is not always true because the optimum will depend on the trade-off between traffic share and the percentage of users who decide not to travel. The capability of the users to internalize the externalities has an important influence on this outcome. Furthermore, only for high μ both approaches match each other and have the same optimal tolls (assuming the combination with the lowest tolls in the optimal range of combinations of the first best solution). Finally, only for low μ the welfare is higher in the first best pricing scenario than in the untolled alternative scenario.

These lessons are more clearly explained in Table 3, which shows the difference of welfare between the untolled alternative and the fully charged corridor as well as the percentage of users who do not travel and the traffic share of the road for two VTT distributions with the middle average and the highest and lowest variance. The tolls for the optimal combination for these two VTT distributions were depicted in Figure 3.

Conclusions

This paper presents a model to obtain the tolls that maximize social welfare in a corridor where a toll highway competes with a parallel conventional road, which could be tolled as well. The model is devised for an interurban corridor with non-recurrent users who take their decisions assuming free-flow traffic conditions. The results of the paper shed additional light on the topic. The tolls for the optimal distribution depend on the trade-off between traffic share and the percentage of user who do not travel.

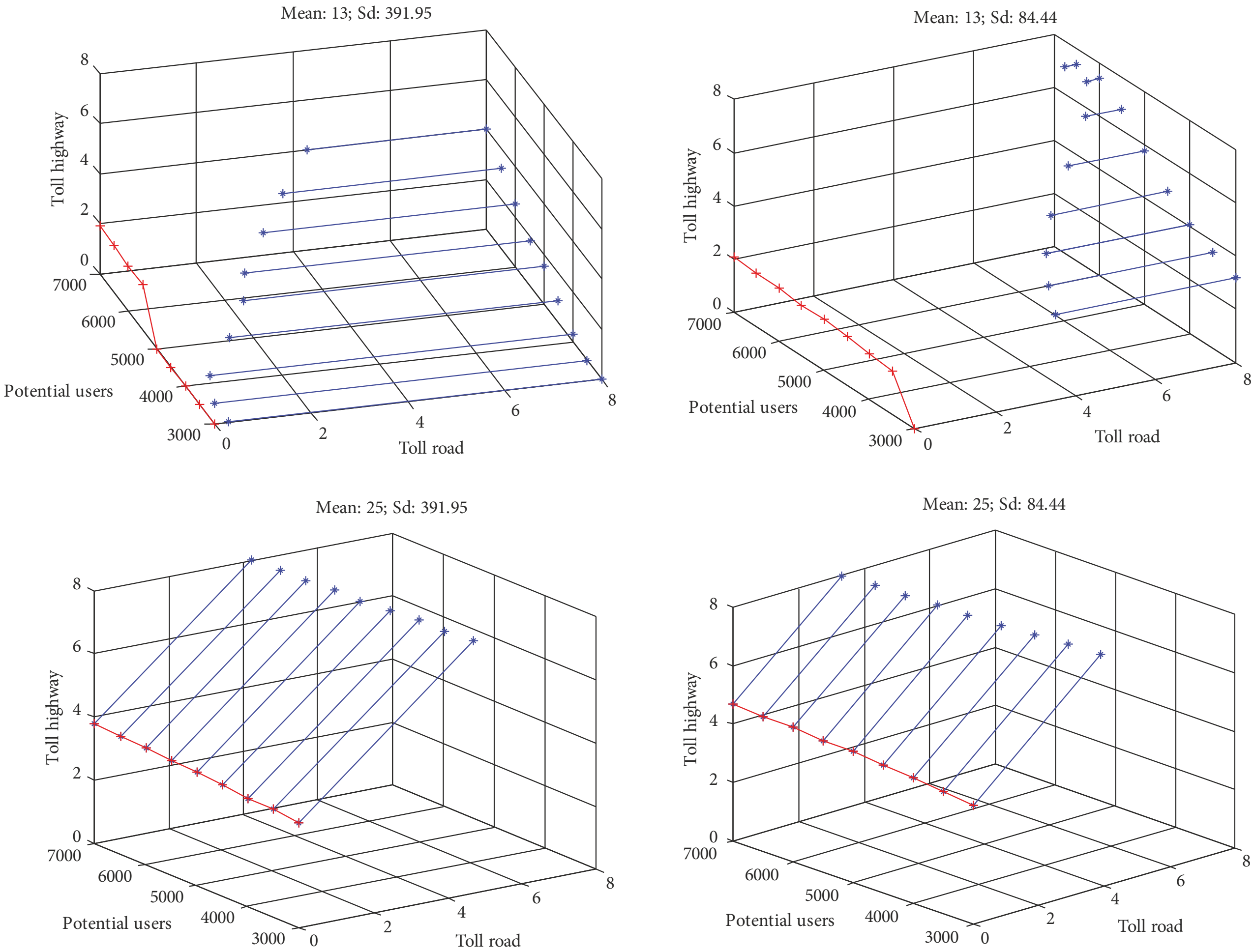

Figure 4. Comparison of optimal combination of tolls between the untolled alternative and the fully charged corridor in the extremes

Table 3. Comparison between the untolled alternative and the fully charged corridor

Distribution VTT	Potential users	3000	5000	7000
μ = 19 €/h; σ^2 = 391.95	% gains of welfare	4.21	13.42	27.91
	% users pushed out (untolled/FCC)	0/28	0/39	0/26
	% road share of traffic (untolled/FCC)	17/0	28/0	34/28.38
μ = 19 €/h; σ^2 = 84.44	% gains of welfare	0.75	1.89	2.84
	% users pushed out (untolled/FCC)	0/12	0/3	0/2
	% road share of traffic (untolled/FCC)	15/0	26/23.71	32/30.61

Changes in VTT distributions affect tolls in a non-linear manner. Under the assumptions made in the paper the following conclusions were obtained: (1) the higher the VTT average (and income) the higher the tolls; (2) the higher the VTT dispersion the lower the tolls; and (3) the larger the number of potential users the higher the tolls. In other words, the tolls that maximize social welfare are an increasing function of the VTT average and a decreasing function of the VTT dispersion. Consequently, optimal tolls should be higher in regions with lower Gini indexes than in regions with higher Gini indexes.

There are some differences in the tolls for the optimal combination depending on the average VTT of the distribution. For low average VTT, tolls in the road have to be higher than in the highway, all the users will travel through the highway and it is not necessary to reach the maximum percentage of potential users who do not travel. For medium average VTT, under free flow conditions the optimal toll in the road has also to be higher than in the highway; once the congestion is unavoidable, even with the tolls at its peak, the tolls must be reduced and the traffic will be shared between both roads. For these two firsts regions, not every potential users are able to internalize the externalities and from the economic efficiency point of view it would be better if they do not travel. Finally, in regions with a high average VTT the optimal welfare obtained with the fully charged corridor is the same as with the untolled alternative.

The model shows that when the demand is higher than half of the capacity of the corridor, the optimal combination of tolls brings about an optimal traffic share according to the capacities of each road. Under free flow conditions the optimal tolls are the ones that make users travel mostly through the highway. As a general rule the greater the potential traffic the higher the traffic share in the conventional road. When the capacity of the corridor is reached, this share is slightly above its relative capacity.

Further research on the topic might include criteria to take into account equity issues, since efficiency maximization does not consider key aspects such as the social exclusion of people who are pushed out. The model could also be improved by substituting the "all or nothing assignment" by a more sophisticated approach based on logistic regressions. Finally, it would be interesting to replicate the model for recurrent users (commuters) who perfectly know the traffic in the corridor, and consequently decide on the basis of both real travel time and monetary cost.

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