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HYDRODYNAMIC COEFFICIENTS AND FORCES ON MULTIHULLS IN SHALLOW WATER WITH CONSTANT OR VARIABLE DEPTH

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Abstract. Numerical and hydrodynamical procedures are developed to compute bidimensional hydrodynamic coefficients and forces on multihulls associated with harmonic oscillations in shallow water with constant or variable depth. The forces are composed of two parts and include the sum of incident and diffracted forces and hydrodynamic reaction. The latter one is used to determinate the hydrodynamic coefficients (added mass and damping). The numerical method used is the Boundary Element Method. We can compute flow around multihulls sections. An application to cylindrical, right triangular and rectangular hull forms is presented.

Keywords: multihulls, hydrodynamic coefficients, helmoltz resonance, shallow water.

1. Introduction

Throughout history, ship designers have sought a hull form for ocean-going vessels that combines high speed with good seakeeping. Traditionally, ships travelling in heavy seas had to be larger in order to achieve stability and high speed. Such vessels are expensive to build and operate; however, they require deep water.

Several kinds of ships have been proposed, especially in the high speed field of application, starting from the traditional catamaran, up to the more recent trimarans, slices and pentamarans. Catamarans have already become a considerable part of the seagoing fleet, particularly in the field of modern sea transport. This is a highly effective method of moving large quantities of non-perishable goods.

The problems of multihull seakeeping have been investigated and discussed by many authors, for instance Wang and Wahab (1971), Lee *et al.* (1973), Van't Veer and Siregar (1995), Van't Veer (1997), Centeno *et al.* (2001).

The theories are usually based on slender body approximation and the problems have been solved in the frequency domain by linear two or three dimensional approaches.

The two-dimensional approaches which are often referred to as the strip theory are the most successful theories to predict the motions of conventional vessels in practical application.

An implicit assumption in the strip theory is that the forward speed should be low, so we could not apply the theory for Froude number higher than approximate 0.5. Much research has been carried out during the last 20 years to develop linear three-dimensional theories of ship motions in the frequency domain, for example Kashiwagi (1993).

The fully three-dimensional analyses are usually time consuming. For high Froude number 2-1/2 D theory may be more robust than 3D potential flow analysis, but further experimental work is needed to validate both of these theoretical approaches.

Most of these theories assume that water is infinitely deep, an assumption justified in part by the fact that conditions producing significant or dangerous motions are perhaps most likely to occur in the open sea. However, there are a number of practical situations where water depth may be an important factor in the ship motions problem (Tuck 1970).

With the growth of multihull ferries along the country coasts, the problem of seakeeping and swash (waves generated by fast ships and onlapping the beach) in shallow water becomes very interesting.

To determinate the motions of ships in finite depth, it is necessary to calculate the hydrodynamic coefficients and forces. The present theory is an application of the

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linearized shallow-water theory to the motions of multihulls (catamaran, trimaran).

2. Mathematical model

We consider a section of a multihull ship with two or three hulls C_1, C_2 and C_3 that oscillate in a domain with finite depth of value h_l at the left and h_r at the right. This multihull ship will undergo three sinusoidal oscillations: a translational along Oy axis (sway), a translation along Oz axis (heave) and a rotational about Ox axis (roll). Under the potential flow theory, the flow is assumed inviscid, incompressible and irrotational and the fluid velocity V is expressed as the velocity $\nabla\Phi$. The potential flow $\Phi(P)$ can be decomposed linearly into separate components due to the incident waves $\Phi_I(P)$, diffraction waves $\Phi_D(P)$ and radiation waves $\Phi_R(P)$:

$$\Phi(P) = \Phi_I(P) + \Phi_D(P) + \Phi_R(P), \tag{1}$$

where $P(y_p, z_p)$ is a point of the domain Ω or on its boundary;

$$\Phi_I(y,z) = -i\frac{gA}{\omega}\frac{\cosh(k(z+h))}{\cosh(kh)}e^{iky}.$$

The frequency ω and the wave number k follow the dispersion relation:

$$\omega^2 = gk\tanh(kh).$$

To solve the problem, the potential has to satisfy Laplace equation in the fluid domain Ω:

$$\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi(y,z)=0 \tag{2}$$

and the following boundaries conditions:

- *linearized free surface conditions:*

$$\frac{\partial\Phi}{\partial n}=\frac{\omega^2}{g}\Phi; \tag{3}$$

- *hull condition:*

$$\frac{\partial\Phi}{\partial n}=\begin{Bmatrix} i\omega n & \text{radiation potential} \\ -\dfrac{\partial\Phi_I}{\partial n} & \text{diffraction potential}\end{Bmatrix}; \tag{4}$$

- *radiation condition:*

$$\frac{\partial\Phi}{\partial n}=ik\Phi \quad \text{on lateral boundaries}; \tag{5}$$

- *bottom condition:*

$$\frac{\partial\Phi}{\partial n}=0 \quad \text{on } z=-h. \tag{6}$$

To calculate forces, first, it is necessary to determinate pressures on any point of multihull bodies which are given by the Bernoulli's equation:

- *radiation pressure:*

$$p_R=-\rho\frac{\partial\Phi_R}{\partial t}; \tag{7}$$

- *wave exciting pressure:*

$$p_W=-\rho\frac{\partial(\Phi_I+\Phi_D)}{\partial t}. \tag{8}$$

Hydrodynamic coefficients. The bidimensional added mass and damping coefficients a_{ij} et b_{ij} are given by:

$$a_{ij}(\omega)=-\int_{(C_1\cup C_2\cup C_3)}\Re(\Phi_{Rj})\frac{\partial\Re(\Phi_{Ri})}{\partial n}dl; \tag{9}$$

$$b_{ij}(\omega)=-\int_{(C_1\cup C_2\cup C_3)}\Im(\Phi_{Rj})\frac{\partial\Re(\Phi_{Ri})}{\partial n}dl, \tag{10}$$

where: $i, j = 1, 3, 4$; dl is the length of a line segment of the boundary.

3. Numerical resolution

By the use of the Boundary Element Method for computational closed domain Ω limited by the surface bodies of catamaran C_1, C_2 and C_3, the free surface S_F, the radiation surface S_R and the bottom surface S_B, the velocity potential can be written under the form:

$$b\Phi(P)=\int_{\Omega}\left[\Phi(Q)\frac{\partial G(P,Q)}{\partial n_Q}-G(P,Q)\frac{\partial\Phi(Q)}{\partial n_Q}\right]ds, \tag{11}$$

$$b=\begin{cases}0 & \text{if } P\in\overline{\Omega} \quad \text{(outside of domain)},\\ \dfrac{1}{2} & \text{if } P\in\partial\Omega \text{ (boundaries of domain)},\\ 1 & \text{if } P\in\Omega \quad \text{(inside of domain)}.\end{cases}$$

The Green function is given by:

$$G(P,Q)=\frac{1}{2\pi}\log(r_{pq}), \tag{12}$$

where:

$$r_{pq}=\sqrt{(y_p-y_q)^2+(z_p-z_q)^2}.$$

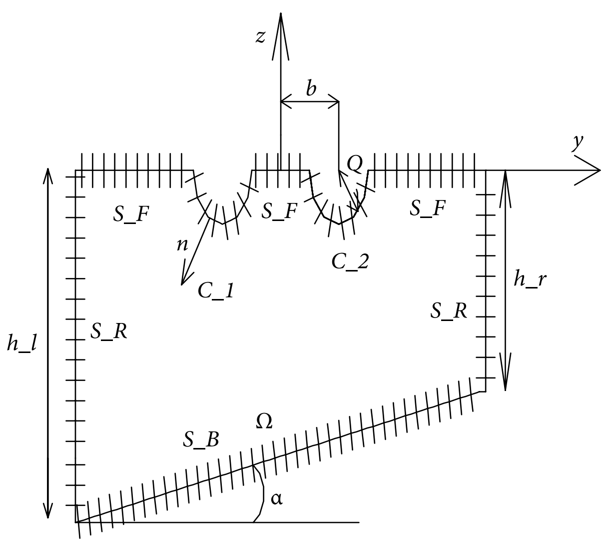

Fig. 1. Coordinate system of a catamaran

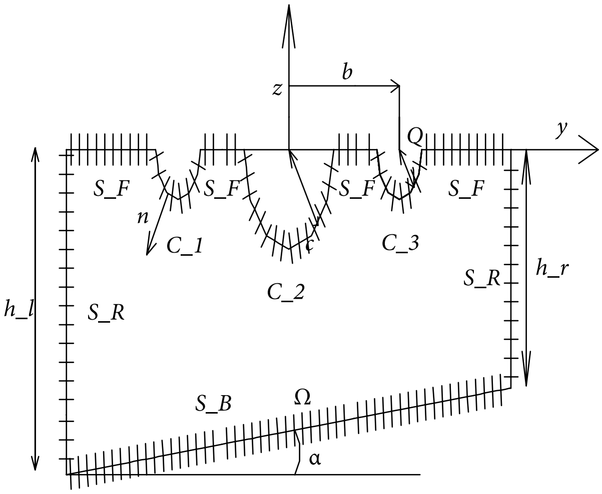

Fig. 2. Coordinate system of a trimaran

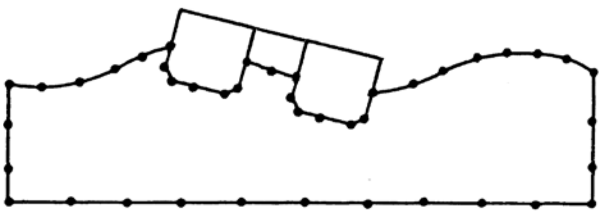

Fig. 3. Approximation of boundary by line segments for a catamaran

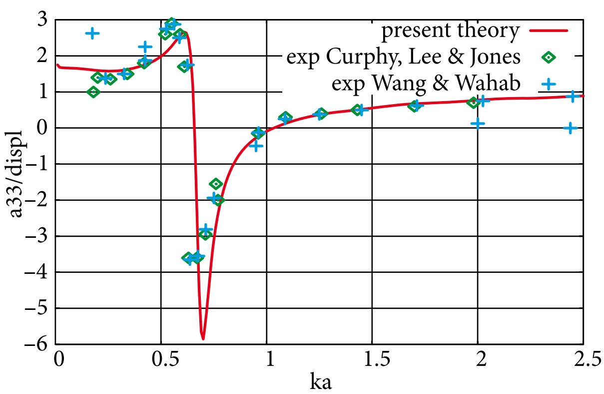

Fig. 4. Heave added mass coefficient versus frequency number for twin semicircular cylinders for b/a=1.5

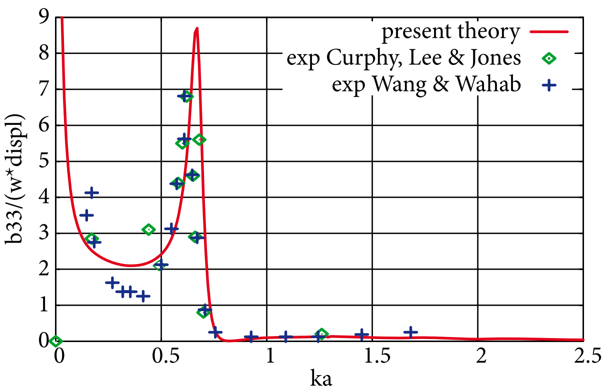

Fig. 5. Heave damping coefficient versus frequency number for twin semicircular cylinders for b/a=1.5

Equation (11) may be solved numerically by dividing all surfaces of boundary of the domain into a number of segments (Fig. 2) and setting $\Phi(P)$ and $\partial\Phi(P)/\partial n$ constants over each segment.

This equation may be written as:

$$\frac{1}{2}\Phi^i + \sum_{j=1}^{N}\left[\int_{\Gamma_j}\frac{\partial G(P,Q)}{\partial nQ}ds\right]\Phi^j =$$

$$\sum_{j=1}^{N}\left[\int_{\Gamma_j} G(P,Q)\right]\frac{\partial \Phi^j}{\partial n_Q}ds \quad i=1,2,...,N, \tag{13}$$

where Γ_j is a segment with index j (Fig. 3).

Using the subdivision in (Fig. 1 or Fig. 2), we can setup the following linear algebraic equations to determine $\Phi(x,y)$:

$$\frac{1}{2}\Phi^i + \sum_{j=1}^{N}\hat{H}_{ij}\Phi^j = \sum_{j=1}^{N}G_{ij}\frac{\partial \Phi^j}{\partial n_Q}. \tag{14}$$

The matrix coefficients can be written in the form:

$$H_{ij} = \begin{cases} \hat{H}_{ij} & \text{if } i \neq j, \\ \hat{H}_{ij} + \frac{1}{2} & \text{if } i = j; \end{cases} \tag{15}$$

$$\hat{H}_{ij} = \int_{\Gamma_j}\frac{\partial G(P,Q)}{\partial n_Q}ds; \tag{16}$$

$$G_{ij} = \int_{\Gamma_j} G(P,Q)ds. \tag{17}$$

4. Results

4.1. Results for a circular and triangular catamaran

Results for two cases of hulls will be demonstrated here. Some of the figures are presented with the experimentally determined results compared with the theoretically ones. Figures 4, 5, 6 and 7 show the added mass and damping coefficients for twin semicircular cylinders for b/a = 1.5 and 4 in constant water depth. The value b is the separation distance between the centerplane of the two hulls and that of one hull and a is the half-beam of one hull. The introduced agreement between theory and experiments is good (Wang and Wahab 1971).

We can observe that the heave added mass coefficient curve (Fig. 4) of a semicircular catamaran becomes negative (for ka ≈0.66). This trough is due to the standing wave generated between the two cylinders as explained by Wang and Wahab (1971). The frequency of wave is a function of the distance between the cylinders. The wider is the distance the smaller is the frequency at which the phenomenon happens. Concerning the heave damping coefficient (Fig. 5), the two-dimensional result agree well with the experimental data. The same figure indicates that the damping coefficient shows a small hump at the symmetric resonance frequency. This hump increases its relative magnitude with the increase of the distance between the cylinders.

This 2D resonance frequency is indeed related to the helmoltz pumping mode or can be seen as the behaviour of a moonpool. It can be approximated using the horizontal watercolumn between the two hulls extended with half a circular cylinder underneath.

We can see added mass and damping coefficients curves that when the frequency approaches zero, the limiting value of added mass becomes infinite as expected in agreement with the two-dimensional results for the case of single cylinder oscillation as shown in Wang and Wahab (1971).

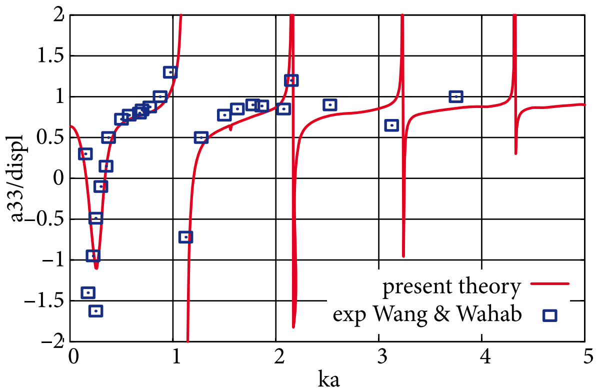

Fig. 6. Heave added mass coefficient versus frequency number for twin semicircular cylinders for b/a = 4

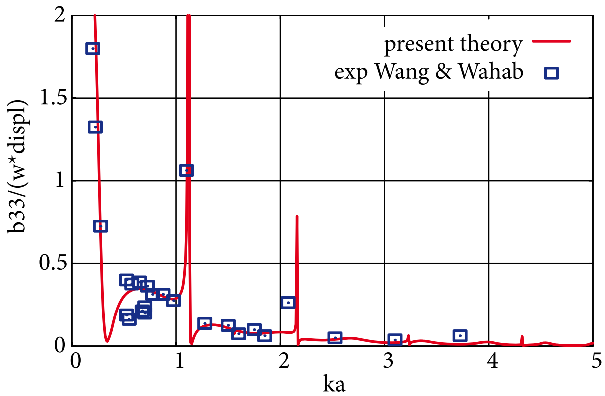

Fig. 7. Heave damping coefficient versus frequency number for twin semicircular cylinders for b/a = 4

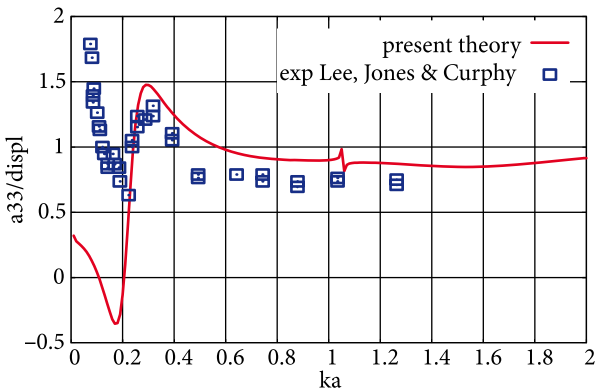

Fig. 8. Heave added mass coefficient versus frequency number for twin right triangles for b/a = 4

Figures 8 and 9 show the added mass and damping coefficients for the twin right triangles for b/a = 4. In this case, large discrepancies can be seen in the agreement between experimental and theoretical results in the low frequency range; however, the agreement still remains good over the remainder of the frequency range tested (Lee *et al.* 1973).

Figures 10 and 11 show the variation of heave added mass and damping coefficients for a twin semicircular cylinders for b/a = 1.5 with the influence of the variation

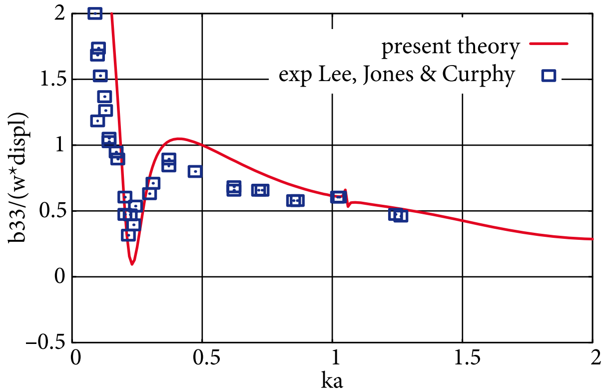

Fig. 9. Heave damping coefficient versus frequency number for twin right triangles for b/a = 4

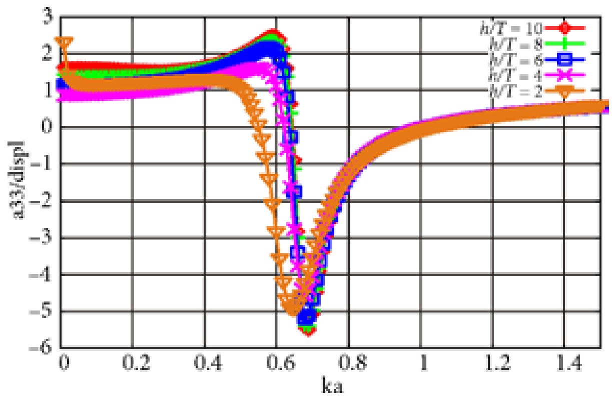

Fig. 10. Heave added mass coefficient versus frequency number for a section of semi-circular catamaran for b/a = 1.5 with the variation of a nondimensional number h/T (depth to draft)

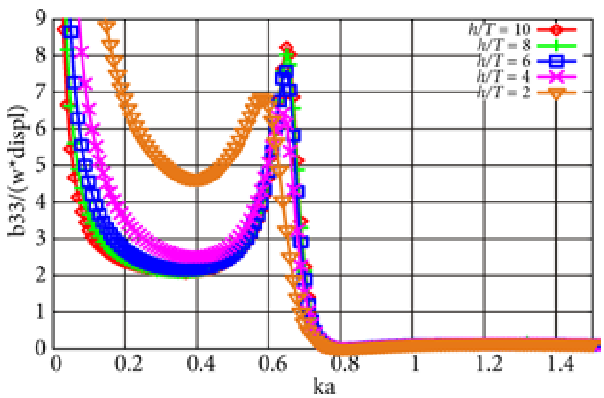

Fig. 11. Heave damping coefficient versus frequency number for a section of semi-circular catamaran for b/a = 1.5 with the variation of a nondimensional number h/T (depth to draft)

of the nondimensional parameter h/T (depth/draft). We can see that the trough and hump on these curves move to the frequencies when depth decreases. As a result the amplitudes of added mass and damping coefficients decrease with the depth. So, for a catamaran that oscillates in shallow water the heave period goes down.

Figures 12 and 13 display the variation of sway added mass and damping coefficients for a twin semicircular cylinders for b/a = 1.5; 2; 3 and 4 compared to the hydrodynamic coefficients of a single cylinder.

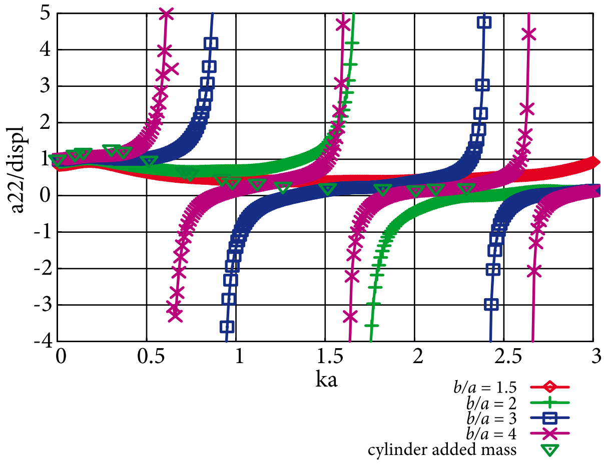

Fig. 12. Sway added mass coefficient versus frequency number for twin semicircular cylinders for b/a = 1.5; 2; 3 and 4

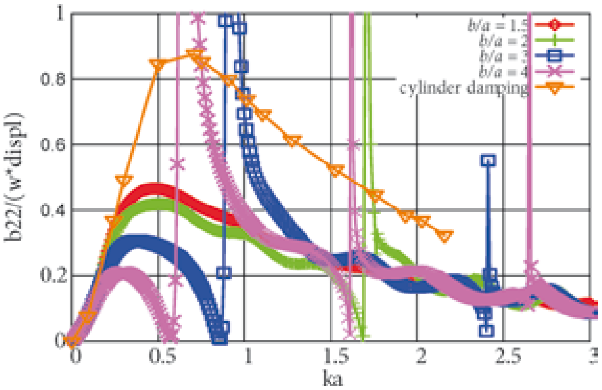

Fig. 13. Sway damping coefficient versus frequency number for twin semicircular cylinders for b/a = 1.5; 2; 3 and 4

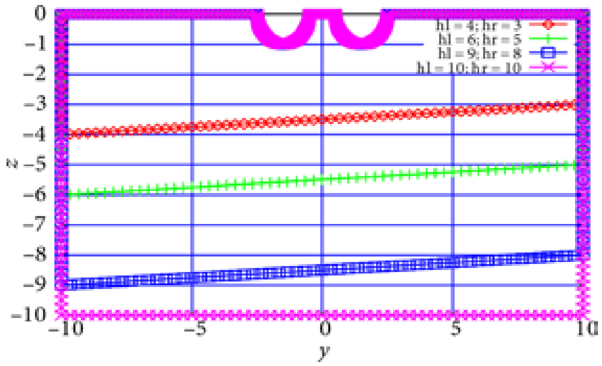

Fig. 14. Domain with variable water depths

The number of characteristic frequencies of sway added mass and damping coefficients curves at which the motion of the fluid between cylinders is strongly excited by a horizontal oscillation of the catamaran is important when b/a increases particularly at low frequencies.

Figures 14, 15, 16 display hydrodynamic coefficients in a domain with variable water depths.

Figures 15 and 16 show the variation of heave added mass and damping coefficients of twin semicircular

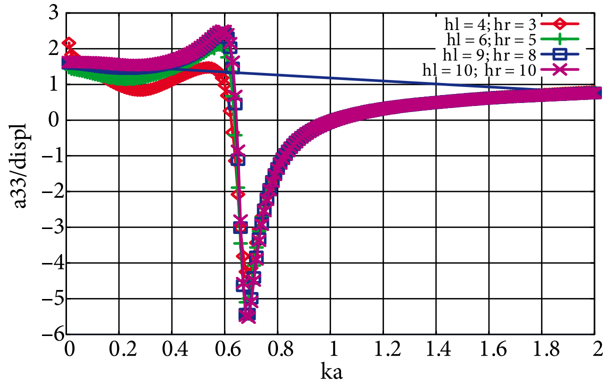

Fig. 15. Heave added mass coefficient of a circular catamaran in variable water depth

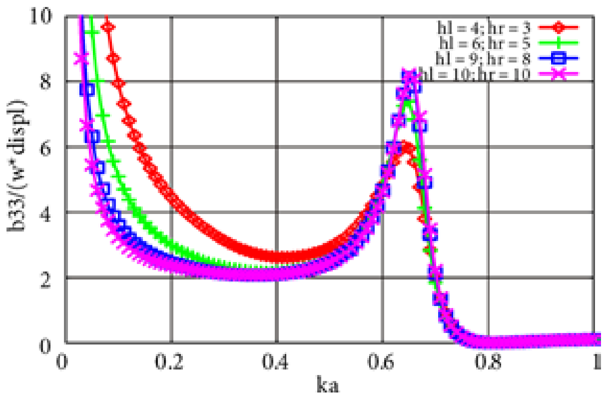

Fig. 16. Heave damping coefficient of a circular catamaran in variable water depth

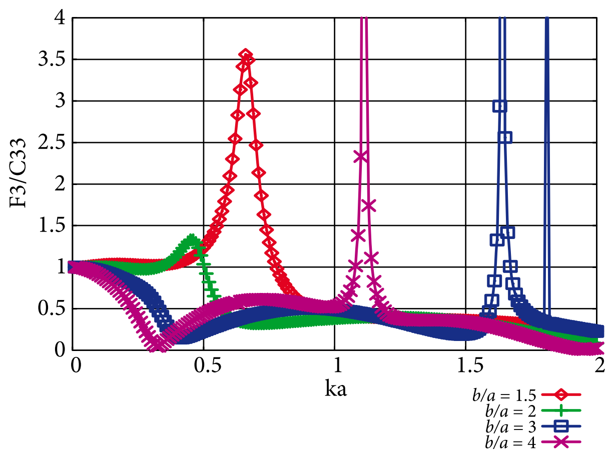

Fig. 17. Heave wave exciting force coefficient versus frequency number for twin semicircular cylinders for b/a = 1.5; 2; 3 and 4

cylinders for $b/a = 1.5$ with variable depth. We can show that when the depth of water decreases, the magnitude of the amplitude of added mass increases while the damping coefficient decreases.

Figures 17, 18 and 19 present the variation of forces of the planar oscillation of twin semicircular cylinders for $b/a = 1.5$; 2; 3 and 4. The analysis of these curves reveals that at low frequencies and for $ka < 1$, the magnitude of heave force becomes preponderant when b/a decreases and inverse will be happening for $ka > 1$. The sway characteristics frequencies are much important at low frequencies as the ratio b/a goes up. For rotational oscillation, the moment of the roll of a catamaran is much important when b/a increases.

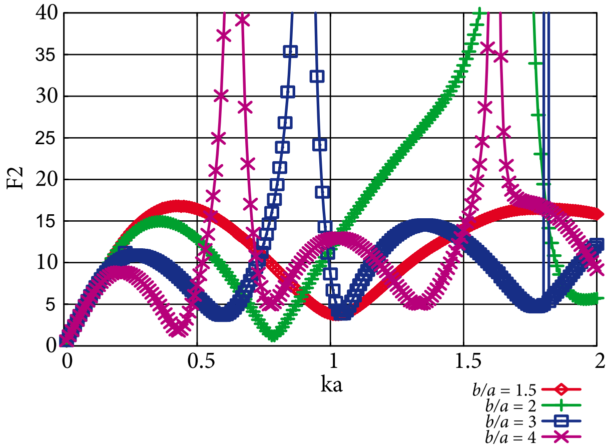

Fig. 18. Sway wave exciting force coefficient versus frequency number for twin semicircular cylinders for $b/a = 1.5$; 2; 3 and 4

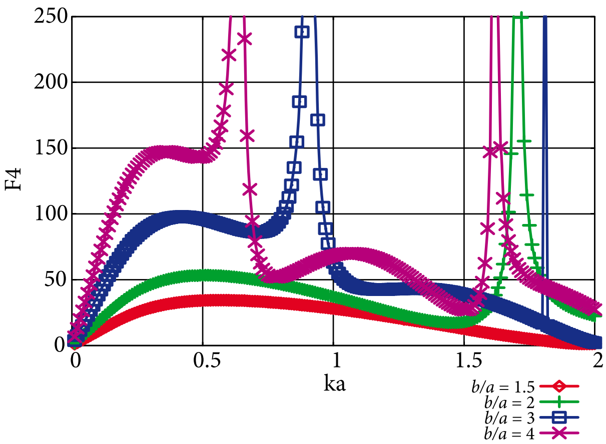

Fig. 19. Roll wave exciting force coefficient versus frequency number for twin semicircular cylinders for $b/a = 1.5$; 2; 3 and 4

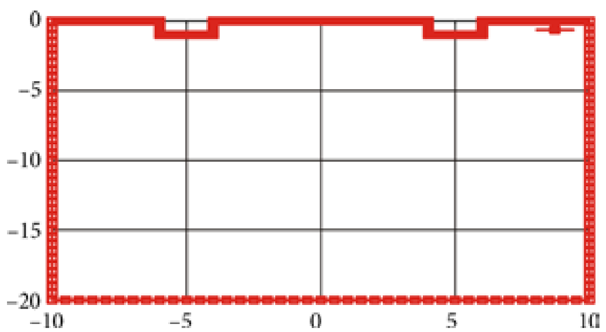

Fig. 20. Domain of a rectangular catamaran

4.2. Results for a rectangular catamaran

Fig. 20 displays domain of a rectangular catamaran.

Figure 21 discloses the variation of heave added mass and damping coefficients of twin rectangular floating bodies for $b/a = 5$. The Helmoltz frequencies can be identified at $(ka)_0 = 0.21$. The two other critical frequencies are $(ka)_1 = 0.82$ and $(ka)_2 = 1.57$.

4.3. Results for a trimaran

Results for a trimaran ship with two cases of hulls – cylindrical and triangular – will be presented next.

The investigated trimaran with cylindrical hulls $b/a = 8$ and $c/a = 2$.

Figs 22 and 23 display heave and sway hydrodynamic coefficients of a semicircular trimaran.

The hump similar to one found in the catamaran can be seen in Fig. 22.

The first frequency at which this hump occurs is for ka between 0.3 and 0.4

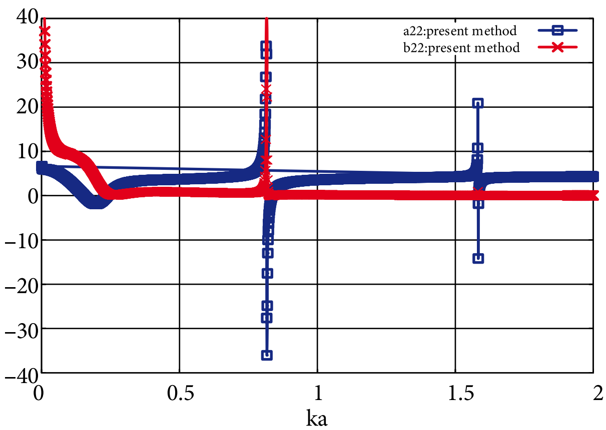

Fig. 21. Hydrodynamic coefficients of a rectangular catamaran

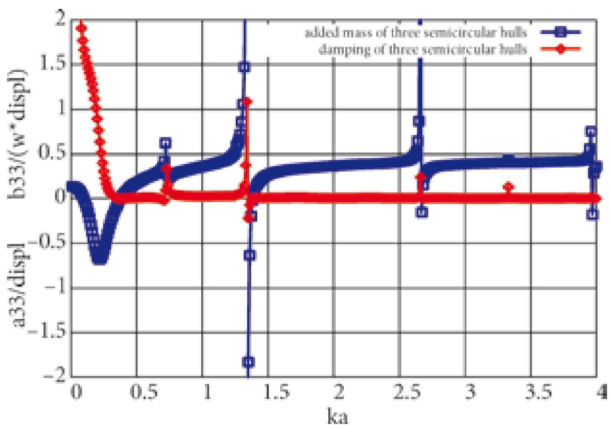

Fig. 22. Heave hydrodynamic coefficients of a semicircular trimaran

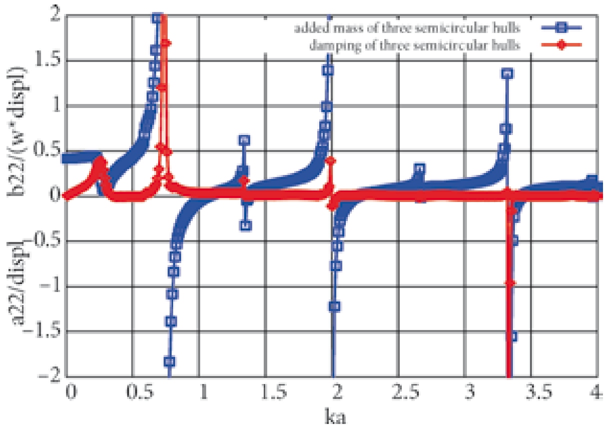

Fig. 23. Sway hydrodynamic coefficients of a semicircular trimaran

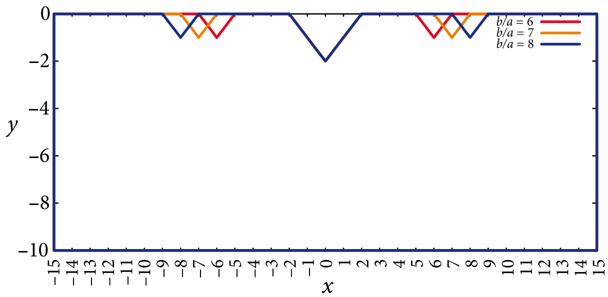

Fig. 24. Triangular trimarans

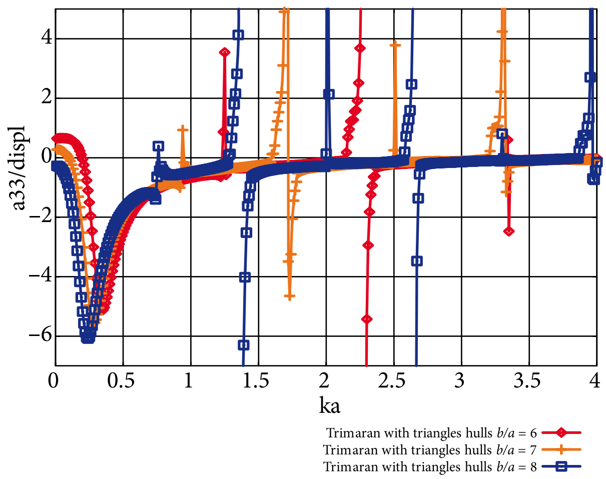

Fig. 25. Heave added mass coefficient

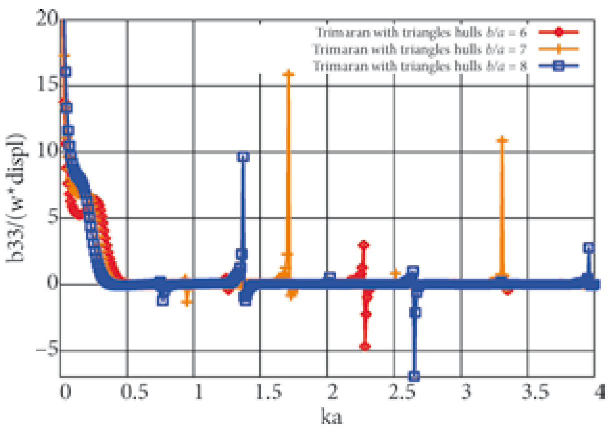

Fig. 26. Heave damping coefficient

Heave, sway and roll hydrodynamic coefficients and forces on a trimaran with triangular hulls b/a = 6, 7, 8 and c/a = 2 are presented in Figs 24–31.

All figures of hydrodynamic coefficients and forces show them growing with the frequency as well as with parameter b/a.

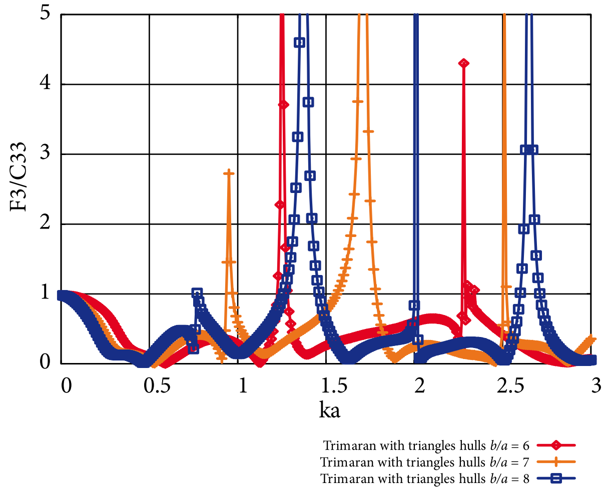

Fig. 27. Heave force

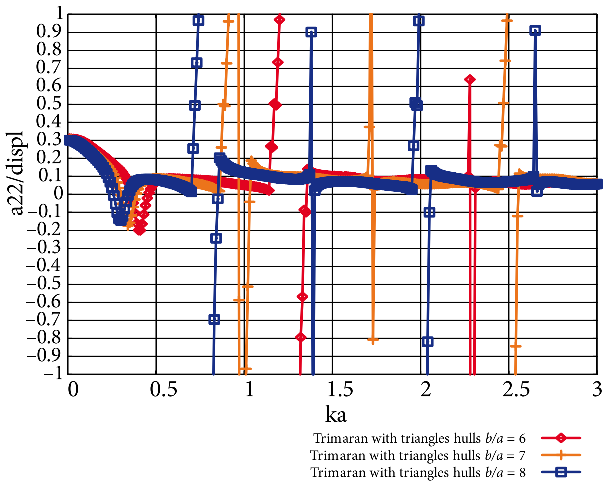

Fig. 28. Sway added mass coefficient

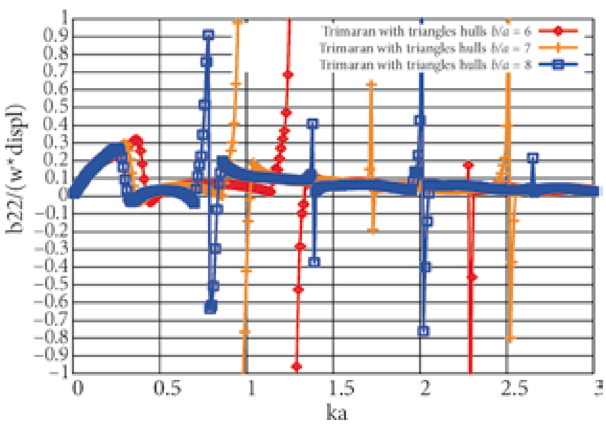

Fig. 29. Sway damping coefficient

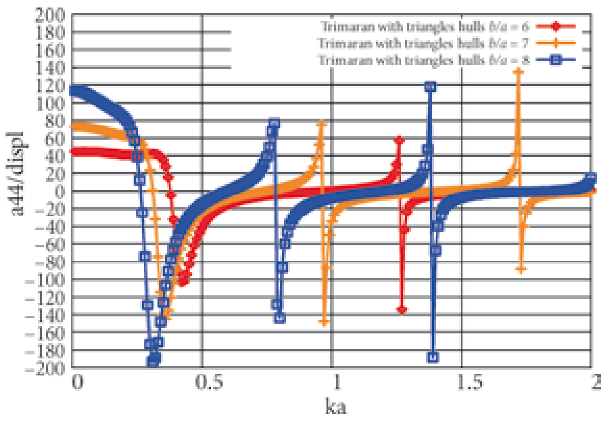

Fig. 30. Roll added mass coefficient

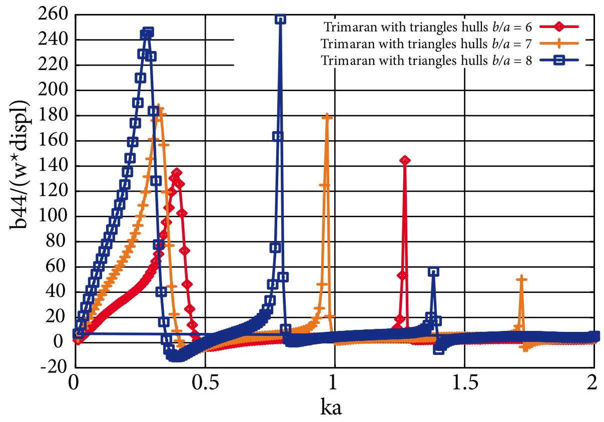

Fig. 31. Roll damping coefficient

5. Conclusions

1. A two-dimensional theory of studying hydrodynamic coefficients and forces for a multihull in shallow water with constant or variable depth water was presented.
2. A numerical method based on the Boundary Element Method was used to compute velocity potential on all the border of the domain.
3. A fairly good agreement between numerical and experimental results is obtained for hydrodynamic coefficients.
4. The validation of the code of calculation of heave hydrodynamic coefficients has permitted to determinate sway and roll hydrodynamic and forces for a multihull in shallow water.
5. This theory can open up the possibility of meshing the free surface between the hulls of a multihull and the bottom of the sea.
6. Moreover, it permits us to calculate the influence of the fluid interaction between the two hulls and also the influence of the variation of depth on hydrodynamic coefficients and forces.
7. The present theory could be extended and employed to calculate multihull motions in shallow water with classical strip method or the 2D ½ theory.

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