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MODELLING THE INTERACTION BETWEEN RAILWAY WHEEL AND RAIL

Stasys Dailydka[1], Leonas Povilas Lingaitis[2], Sergey Myamlin[3], Vladimir Prichodko[4]

[1, 2] Dept of Railway Transport, Vilnius Gediminas Technical University, J. Basanavičiaus g. 28, 03224 Vilnius, Lithuania
[3] Dnepropetrovsk State Technical University of Railway Transport, Lazarian str. 2, 49700 Dnepropetrovsk, Ukraine
[4] JSC "Kriukov Car Building Works", Prikhodko str. 139, 39621 Kremenchug, Ukraine
E-mails: [1] sdailydka@litrail.lt; [2] leonasl@ti.vgtu.lt; [3] mjamlin@nzn.diit.edu.ua; [4] kvsz@kvsz.com

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Abstract. The article presents a mathematical model for assessing the real operating conditions of railway rolling stock, taking into account the situations when the wheel loses contact with rail. The obtained amplitudinal fluctuation characteristics depend on the set roughness function and the running speed of the wheel. When calculating dynamic processes, the contact between wheel and rail should be considered unstable. With the increase of speed, the impact of this instability increases.

Keywords: rail, wheel, contact, dynamics, swinging, modelling.

1. Introduction

The determination of forces acting between wheel and rail and shifts is very important not only for the operational characteristics of rolling stock – traction, vibrations, fuel consumption etc. – but also for the reliability and durability of chassis and the entire rolling stock. Having determined the above introduced parameters, it is possible to more positively solve the tasks of traffic smoothness and comfort and noise reduction. The article presents some examples of a dynamic calculation of the wheel-rail pair.

2. Survey of works

Many researchers, when modelling the movement of railway stock chassis on rails, use the geometric and mechanical properties of contacting bodies and apply various theories to determine the acting forces of slipping bodies by Kalker (1967), Lata (2008), Polách (2002), Czaplicki (2007), Verigo and Kogan (Вериго, Коган 1986). However, considering or ignoring the track roughness, they most frequently examine the models of wheel-to-rail interaction with the assumption that the wheel rolls in an uninterruptible manner. In order to simplify the mathematical models, the authors choose plane-type calculation schemes i.e. on vertical or horizontal planes. Nevertheless, in order to more accurately determine variation parameters, it is expedient to examine a spatial scheme of the model. A number of authors including Danovich (Данович 1981) and Myamlin (Мямлин 2002) have examined mathematical variation models but only some of the works by Myamlin (Мямлин 2003) attempted to mathematically describe the process of the detachment of wheel from rail.

A correct mathematical description of the interaction between wheel and rail when determining the forced vibrations of chassis will be very helpful to more accurately calculate dynamic indices, especially those having a strong influence on traffic safety.

3. Mathematical model

We will examine a mathematical model of wheel-to-rail interaction, taking into consideration the detachment of wheel from rail when the wheel looses contact with rail.

Let's suppose that the wheel is affected by force Q. Then y shall be rail bend, and $R(y)$ – rail reaction to the bend. When there is contact between wheel and rail, we may generate the following differential equation:

$$m \cdot \frac{d^2 y}{dt^2} = -R(y) + Q, \qquad (1)$$

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where: m is the weight of the wheel; t is time; y is vertical displacement (rail bend).

This is the main equation which allowed for Blochin *et al.* (Блохин и др. 1986) to obtain tensions in linear rail and in rail with roughness $\eta(x)$ when the wheel rolls in steady speed v; x is horizontal displacement.

Instead of equation (1), let's examine the equation of the following type:

$$m \cdot \frac{d^2 y}{dt^2} = -R(y) \cdot H(y) + Q, \tag{2}$$

where $H(y)$ is a Heaviside function, i.e..:

$$H(y) = \begin{cases} 0, & when\ y \le 0; \\ 1, & when\ y > 0. \end{cases}$$

In case of the presence of inequalities on rail, then, when selecting under x reference point in the beginning of inequality, the differential equation may be put as follows:

$$m \cdot \frac{d^2 y}{dt^2} = -R(y) \cdot H(y) + Q - m \cdot \frac{d^2 \eta(v \cdot t)}{dt^2}, \tag{3}$$

and it is correct in case of $t \subset [0, \ell / v]$, where ℓ is the length of inequality; $\eta(x) = \eta(v \cdot t)$ is rail roughness.

Further, we will assume that rail reaction to the bend y conforms to the reaction of a resilient – plastic body and may be expressed as follows:

$$R(y) = \beta \cdot \frac{dy}{dt} + k \cdot y, \tag{4}$$

where β is damping coefficient, k is stiffness coefficient.

Strictly speaking, this reaction also depends on the location where the wheel contacts the rail (between ties or at a tie), then instead of (4) we will use the expression:

$$R(y, x) = \left(\beta \cdot \frac{dy}{dt} + k \cdot y \right) \cdot \varphi(x), \tag{5}$$

where $\varphi(x)$ is a periodic function with the period equal to the distance between ties.

In order to make the mathematical model of the wheel – rail interaction that better conforms to real conditions, let's examine the calculation scheme shown in Fig. 1.

Just as previously, y_1 will mean rail bend in the place of contact with the wheel, and $y_2 - h$ will mean the weight centre coordinate of body C, then:

$$y = \frac{m \cdot (y_1 + r) + M \cdot (y_2 - h)}{m + M}, \tag{6}$$

where: m is the weight of the wheel; M is the weight of the body C; r is the wheel radius, the following system of differential equations is made:

$$m \cdot \frac{d^2 y_1}{dt^2} = -R(y_1, x) \cdot H(y_1) + R_1(y_2) + m \cdot g, \tag{7}$$

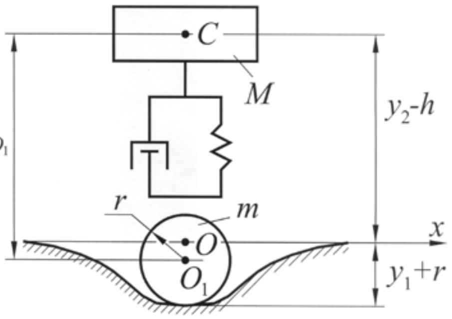

Fig. 1. Wheel and rail contact calculation scheme with resilient suspension

$$(m + M) \cdot \frac{d^2 y}{dt^2} = -R(y_1, x) \cdot H(y_1) + (m + M) \cdot g, \tag{8}$$

where M is the weight of body C, r is wheel radius, $R_1(y_2)$ is the reaction of the resilient suspension that may be expressed as follows:

$$R_1(y_2) = \beta_1 \cdot \frac{dy_2}{dt} + k_1 \cdot y_2. \tag{9}$$

where β_1 is damping coefficient, k_1 is stiffness coefficient.

From (6) we determine the y_2:

$$y_2 = \frac{y - \alpha \cdot (y_1 + r) + \beta \cdot h}{\beta}, \tag{10}$$

where $\alpha = \frac{m}{M + m}$; $\beta = \frac{M}{M + m}$.

Now, after inserting y_2 into equations (7) and (8), we obtain the following system of differential equations:

$$\begin{cases} m \cdot \frac{d^2 y_1}{dt^2} = -R(y_1, x) \cdot H(y_1) + \\ R_1 \cdot \left(y_1 - r - h - \frac{y - \alpha \cdot (y_1 + r)}{\beta} \right) + m \cdot g; \\ (m + M) \cdot \frac{d^2 y}{dt^2} = -R(y_1, x) \cdot H(y_1) + (m + M) \cdot g, \end{cases} \tag{11}$$

which describes the interaction between wheel and rail when there are no inequalities neither on the rail nor on the wheel rolling surface.

After writing down the equation (1) as follows:

$$\frac{d^2 z}{dt^2} + 2n \cdot \frac{dz}{dt} + p_2 \cdot z = g, \tag{12}$$

where: z is transformed vertical displacement; n is damping coefficient; p_2 is stiffness coefficient.

The obtained general solution in analytical forms is as follows:

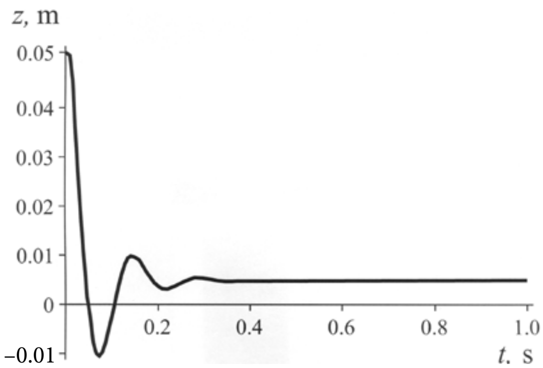

Fig. 2. Chart of the solution of equation (1)

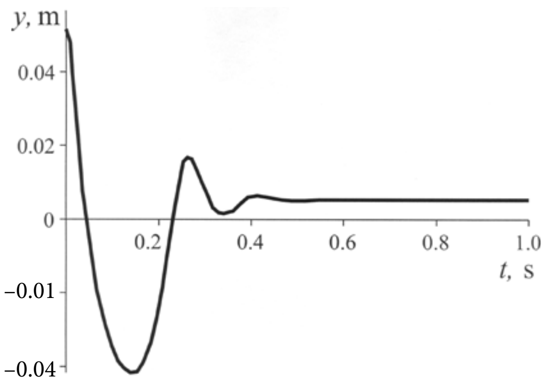

Fig. 3. Chart of the solution of equation (2)

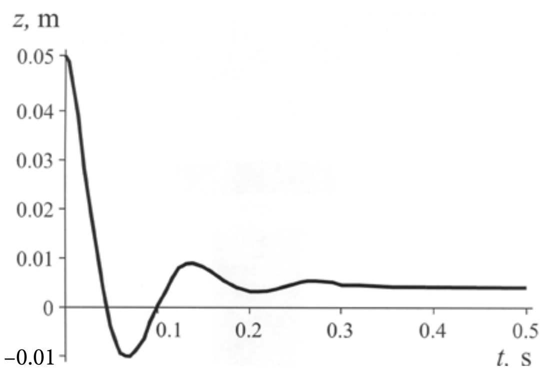

Fig. 4. Chart of the solution of equation (1) (v = 60 m/s)

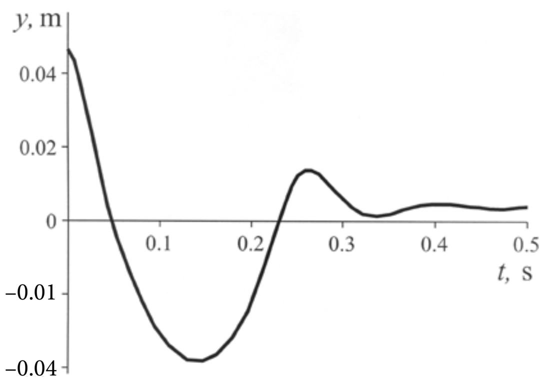

Fig. 5. Chart of the solution of equation (3) (v = 60 m/s)

$$sol := z(t) = \frac{1}{2}\cdot\frac{e^{((-n+a)t)}\cdot\left(-n\cdot g + n\cdot p_2 - a\cdot g + a\cdot p_2\right)}{a\cdot p_2} + \frac{1}{2}\cdot\frac{e^{((-n-a)t)}\cdot\left(n\cdot g - n\cdot p_2 - a\cdot g + a\cdot p_2\right)}{a\cdot p_2} + \frac{g}{p_2}, \quad (13)$$

where $a = \sqrt{n^2 - p_2}$.

When assuming that n = 5 kg/s; p_2 = 500 N/m; g = 9.8 m/s^2 the solution acquires the following form:

$$z(t) = -\frac{1}{475000}\cdot e^{\left(\left(-5+\sqrt{-475}\right)\cdot t\right)} \times \left(2451.0 + 490.2\cdot\sqrt{-475}\right)\times \sqrt{-475} - \frac{1}{475000}\cdot e^{\left(\left(-5-\sqrt{-475}\right)\cdot t\right)} \times \left(-2451.0 + 490.2\cdot\sqrt{-475}\right)\times \sqrt{-475} + 0.0196. \quad (14)$$

A graphic interpretation of this solution is presented in Figure 2.

Under the same assumptions, equation (2) will acquire the following form:

$$dee := \left(\frac{d^2 y(t)}{dt^2}\right) + \left(10\cdot\left(\frac{dy(t)}{dt}\right) + 500\cdot y(t)\right)\times \text{Heaviside}\left(y(t)\right) = 9.8. \quad (15)$$

In general case, there are no possibilities to obtain an analytical solution of this equation, though it could be presented as a solution of equation (1), when $y(t) \geq 0$ and when $y(t) < 0$ we get the following:

$$\frac{d^2 y}{dt^2} = 9.8. \quad (16)$$

When the initial conditions $y(t_*) = 0$ and $y'(t_*) = z'(t_*)$, where t_* is minimum time when $y(t)$ becomes equal to 0. Using the presented solution methods of differential equations, we get the results shown in Fig 3.

When comparing the charts presented in Figs 2 and 3, we can see that $y(t)$ disagrees not only quantitatively but also significantly differs qualitatively.

All calculations were made using the *Maple* bundle by Matrosov (2001) that allows obtaining the analytical solution when it is presented as elementary functions or a numeric solution in a graphic form.

Let's make some calculations for a rolling wheel when inequality:

$$\eta(t) = \frac{f\cdot\left(1-\cos\left(\frac{2\cdot\pi\cdot v\cdot t}{\ell}\right)\right)}{2}, \quad (17)$$

when f is coefficient.

For example, when speed v = 60 m/s (216 km/h) (1), the equation will look as follows:

$$del := \frac{d^2 z(t)}{dt^2} + 10 \cdot \left(\frac{dz(t)}{dt} \right) + 2195 \cdot z(t) =$$
$$9.80 - 3.60 \cdot \cos(12 \cdot \pi \cdot t) \cdot \pi^2, \tag{18}$$

and its solution is shown in Fig. 4.

In this case, equation (3) acquires the following form:

$$dee := \frac{d^2 y(t)}{dt^2} + 10 \cdot \left(\frac{dy(t)}{dt} \right) +$$
$$2195 \cdot y(t) \cdot \text{Heaviside}\big(y(t)\big) =$$
$$9.80 - 3.60 \cdot \cos\big(12 \cdot \pi \cdot t\big) \cdot \pi^2. \tag{19}$$

The solution was made using the Runge-Kutt method a graphic interpretation of which is shown in Fig. 5.

In order to compare the impact of speeds, let's make the solutions of equations (1) and (3) when speed v = 80 m/s (288 km/h) (see Figs 6 and 7).

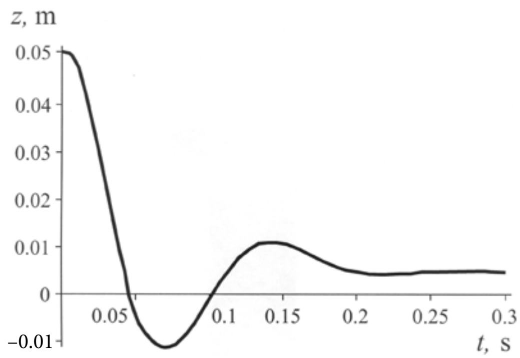

Fig. 6. Chart of the solution of equation (1) (v = 80 m/s)

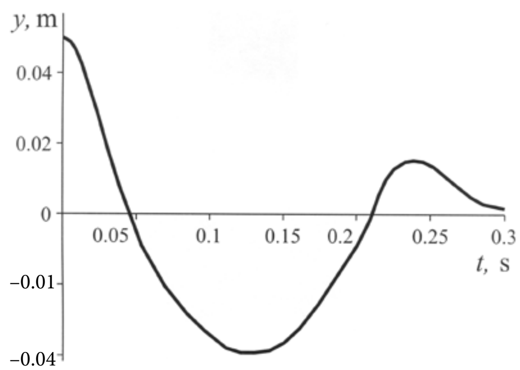

Fig. 7. Chart of the solution of equation (3) (v = 80 m/s)

4. Conclusions

1. The presented numeric calculations should be evaluated as modelling examples which confirm that when calculating the dynamic processes taking place in railway rolling stock, it is necessary to come into the contact between wheel and rail as unstable and having a significant influence on the operational parameters.
2. The higher is the speed, the stronger is the influence.
3. This is very important for selecting chasses for modern rolling stock, since with the increase of speeds higher requirements are put in respect of passenger comfort.

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