EVALUATION OF LEFT TURN CHANNELIZATION AT A SIGNALIZED INTERSECTION UNDER HETEROGENEOUS TRAFFIC CONDITIONS

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Abstract. The behaviour of traffic in the heterogeneous environment of an urban signalized intersection is complex and difficult to model. This paper presents the development of a simulation model to imitate the flow of heterogeneous traffic through a signalized intersection. It discusses the validation of the proposed model in terms of queue density and dissipation of vehicles at an intersection approach and found to be satisfactorily replicating the field conditions. In this study, the model was extended to examine the effects of left turn channelization on vehicle waiting times. Sensitivity analysis was carried out to study the variation of vehicle waiting times. Analysis estimated that vehicle waiting times were reduced if a channelization was provided for a high traffic volume and certain proportions of left turn vehicles in the intersection approach. The length of channelisation has marginal impacts on vehicle waiting times.

Keywords: heterogeneous traffic, signalized intersection, traffic simulation, traffic queue, left turn channelization.

1. Introduction

Intersections are vital nodal points in a transportation network and their efficiency of operation greatly influence the entire road network performance. Homogeneous traffic consists of a stream of identical vehicles (mostly cars) following perfect lane discipline at these intersections. However, under heterogeneous traffic conditions, mix-up of vehicles is high and vehicles do not follow the ordered queue and lane discipline (Fig. 1). Moreover, heterogeneous traffic systems operate differently compared to homogeneous ones due to wide variations in the operating

and performance characteristics of vehicles. Heterogeneous traffic includes motorized two wheelers (MTW), cars (including jeeps and vans), buses, auto-rickshaws (three-wheeled motorized vehicles), light commercial vehicles (LCV) and trucks and bicycles which share the common road space without any physical segregation. At these intersections, smaller vehicles use the lateral gaps between larger vehicles to accommodate the approach space. Due to complex manoeuvres, vehicle interactions and heterogeneity, it is extremely difficult to develop an analytical model for studying such traffic flow characteristics. Hence,

Fig. 1. Homogeneous and heterogeneous traffic characteristics at a signalized intersection

simulation is considered as an effective tool for studying heterogeneous traffic.

Various investigations on signalized intersections have been carried out to determine delays, queue length, platoon dispersion, queue dissipation, driver and vehicular traffic characteristics etc. under homogeneous traffic flow characteristics. These include studies by Kim and Benekohal (2005), Laoufi *et al.* (2004), Clement *et al.* (2004), Mousa (2003), Zuylen and Taale (2001), Addison and Low (1996), Olszewski (1993), Lin and Cooke (1986). Tian and Wu (2005) estimate the capacity at the signalized intersection approach with a short right turn lane based on the length of the short lane, proportion of through and right-turn vehicles and cycle length. Tarawneh, M. S. and Tarawneh, T. M. (2002) studied the effects of auxiliary lane length, right-turn volume and through/right-turn lane group delay on the level of their utilization. Spring and Thomas (1999) developed left turn adjustment factors for double left turn lanes in medium size cities. Hurley (1998) built a mathematical model to estimate the capacity for using exclusive double left turn lanes. Janson and Buchholz (1998) worked out delay equations and saturation flow for both exclusive and shared lanes. Lin (1992) provides the estimates of capacities and left-turn adjustment factors for shared permissive left turn lane.

Only limited research on modelling heterogeneous traffic through signalized intersections has been done. Various simulation models for uncontrolled intersection were designed by Rao and Rengaraju (1998), Agarwal *et al.* (1994), Raghavachari *et al.* (1993) and Popat *et al.* (1989). Marwah *et al.* (2006) created a simulation model for signalized intersection to estimate delay and queue length. Arasan and Kashani (2003) studied the platoon dispersal pattern of heterogeneous traffic at a signalized intersection using a simulation model. Hossain (2001) estimated saturation flow at signalized intersections based on road width, turning proportion and percentage of heavy and non-motorized vehicles using a micro-simulation modelling approach. Maini and Khan (2000) analyzed the discharge characteristics of vehicles and vehicle characteristics at signalized intersections. Arasan and Jagadeesh (1995) considered the effect of vehicle heterogeneity on signalized intersections and proposed a probabilistic approach to estimate saturation flow and delay. While the above studies highlight the importance of addressing traffic flow at signalized intersections, there is a need and further scope for studies on queue formation, queue density, queue accumulation and dissipation at and near signalized intersections areas which have not been fully explored. There is also a need to evolve and study appropriate traffic control and management measures and strategies for better utilization of transport infrastructure and effective traffic regulation and control. This paper presents a methodology and development of a simulation model to study heterogeneous traffic at a signalized intersection. It also explains the collection and analysis of data for calibration and validation of the model. Queue density and dissipation of vehicles are the parameters used to validate the model. Issues related to the applicability of the proposed model were also explained.

Akgüngör, A. P. (2008a and 2008b) analyzed the new delay parameter, which dependent on variable analysis periods at signalized intersections.

2. Development of a simulation model

Modelling vehicular interactions at intersections under heterogeneous traffic conditions in urban roads in India is difficult. Also, due to the stochastic nature of traffic process, simulation is considered as a useful tool to model the traffic flow. The various logics of the simulation model are shown in Figure 2. Vehicles in India generally do not follow lanes and have a tendency to occupy any available space occurring ahead. Hence, the entire road space is considered instead of individual lanes in this model. Seven different types of vehicles and three types of turning movements (left, straight and right) are considered in the model. The simulation model is based on interval scanning technique with fixed increment time advance. The data such as volume, road length and width and initialisation parameters (e.g., headway distribution, acceleration and deceleration characteristics, dissipation rate, lateral and longitudinal spacing at stopped condition, etc.) are given as inputs to the model. The possible model outputs are queue length, the number of vehicles in a queue, queue density and the dissipation of vehicles. The processes such as vehicle generation, vehicle placement, vehicle movement, vehicle accumulation and vehicle dissipation incorporated in the simulation model are explained below.

2.1. Vehicle generation

The vehicle moves from a mid block section of the road stretch to the intersection approach. As the vehicle arrivals at the mid block sections is random, the inter arrival times (headways) are generated by negative exponential distribution using the following expression:

$$H = -(1/\lambda) \ln R,$$

where: H is the headway between the arrival of successive vehicles (s); R is the random number in the range (0 to 1.0); and λ is the mean arrival rate of vehicles (vehicles per second).

The free speeds of vehicles follow normal distribution. The speed of vehicles on the simulation road stretch is based on two assumptions: (a) In the entire simulation stretch of the road, vehicle speed will not be allowed to exceed free speed and (b) the vehicles enter the simulation stretch at its free speed in the mid block section of the road. In this model, the type of the vehicle and turning movement are identified based on the composition of traffic and turning movement, respectively. For example, to identify vehicle type, a random number is generated and checked with the cumulative composition of traffic.

2.2. Vehicle placement

Vehicle placement at mid block sections is based on the availability of transverse and longitudinal spacing.

Non-motorized vehicles are placed near the left edge of the road stretch; as the speeds of these vehicles
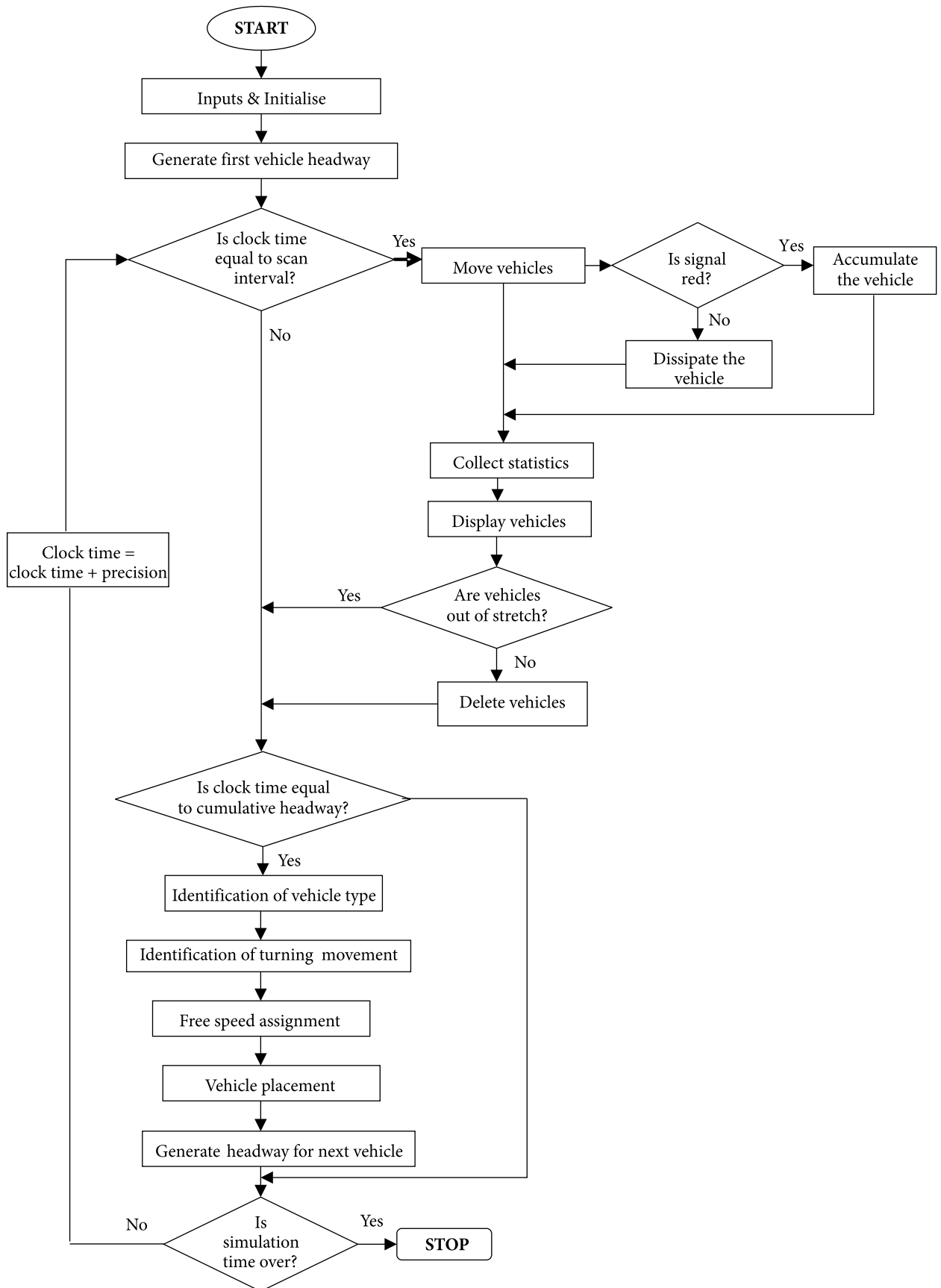
Fig. 2. Framework of the simulation model

are less compared to motorized vehicles, they are expected to use the left edge of the road (it is to be noted that vehicles move on the left side of the road in India; thus, slower vehicles occupy the left most lane). Nevertheless, motorized vehicles can move more freely and faster nearer to the median. So, they are placed from the right to the left edge of the road stretch. Longitudinal and transverse spacings of vehicles are based on their current speeds. The motorized vehicle looks for longitudinal and transverse spaces in the right most section of the road stretch. Fig. 3 depicts typical spacing that may be available at the simulation stretch for the placement of a motorized vehicle. A check is made to determine whether the available longitudinal and transverse spacings on the right extreme side (L_1 and W_1 in Fig. 3) are sufficient to the place entering a vehicle as a follower to this vehicle or not. Thus, if the available transverse and longitudinal spacing are sufficient, the vehicle is placed on the right-most location. If either of them are inadequate, the available longitudinal and transverse spacing with respect to the next-to-be-considered vehicle (L_2 and W_2 in Fig. 3) on the left side are compared with the respective values required for the entering vehicle to enable its placement behind it. Thus, all the longitudinal and transverse spacings are checked progressively along the width of the road toward the left end of the roadway, until the vehicle is placed at the simulation road stretch. Thus, vehicles check the longitudinal and lateral spacings progressively from the right to the left edge of the road stretch. If such spaces are insufficient, subject vehicle reduces its speed to that of its leader (car following rule). Again, similar
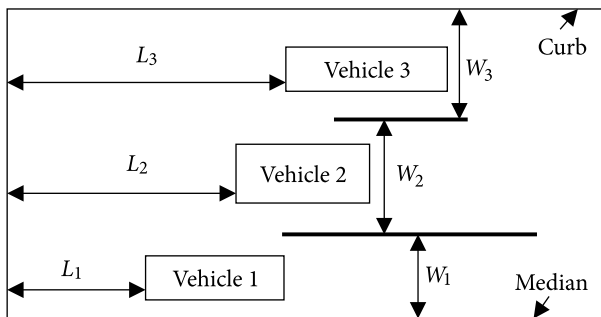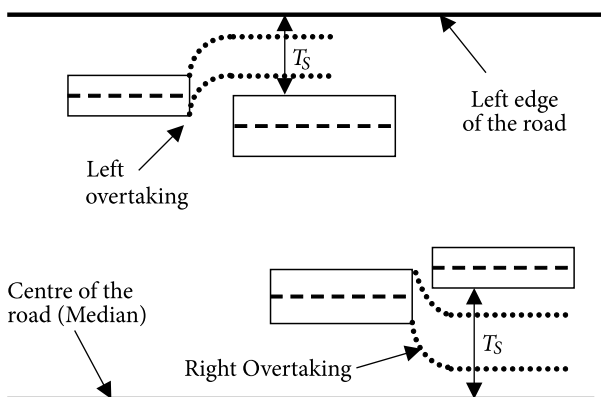

Fig. 3. Vehicle placement logic

Fig. 4. Left and right overtaking manoeuvre

checks for spaces are made starting from the right most edge. For non-motorised vehicles, similar checks are made from the left edge of the road to the right edge (restricted to 2 m based on field observation).

2.3. Vehicle movement

In this simulation model, the vehicle accelerates up to its free speed if there is no slow vehicle in front of it. The position of the vehicle is updated based on the equations of motion:

$$S = ut + \frac{1}{2}at^2;$$
$$v = u + at,$$

where: S is the distance moved by the vehicle (*m*); u is the initial speed of the vehicle (*m/s*); a is the acceleration of the vehicle (*m/s^2*); t is the scan interval (*s*) and v is the speed of the vehicle (*m/s*) at the end of the scan interval.

When there is slow moving vehicle in front of the subject vehicle, overtaking logic is invoked. Left or right overtaking is performed based on the position of the centre line of the overtaking vehicle (Fig. 4). If the centre line of the overtaking vehicle is on the right side of the centre line of the overtaken vehicle, then the overtaking vehicle looks for the availability of lateral and longitudinal spaces on the right side of the overtaken vehicle. If spaces are adequate on the right side, right overtaking is performed; if not, the overtaking vehicle looks for the availability of such spaces on the left side, and if available, left overtaking is performed. A similar overtaking process is applied for the vehicle which centre line is on the left side of the centre line of the overtaken vehicle. (Left or right overtaking manoeuvre is performed based on the availability of longitudinal and transverse spacing of the vehicle in front of the slow vehicle). The minimum required transverse spacing (T_s) for the overtaking vehicle is the sum of 1) width of the overtaking vehicle and 2) left and right side clearances of the overtaking vehicle. If lateral spacing is inadequate on both sides, overtaking is not performed and car following logic is invoked. In car following logic, the speed of the subject vehicle is reduced to the speed of the lead vehicle maintaining a safe spacing from it.

After a reference point (150 m before a stop line based on field observation), vehicles reach the intersection approach. The vehicles change its position after this reference point based on their type of turning movement. For example, if the left turning vehicle moves in the right

Table 1. Observed deceleration rate of different types of vehicles

Vehicle Type	Deceleration Rate (m/s^2)
MTW	1.49
Car	1.18
Bus	0.80
Truck	0.79
LCV	0.91
Auto	1.18
Bicycle	1.0

Table 2. Longitudinal spacing of vehicles at stopped condition (m)

Subject Vehicle Type	Front Vehicle Type						
	MTW	Car	Bus	Truck	LCV	Auto	Bicycle
MTW	0.53	0.67	0.71	0.57	0.66	0.65	0.5
Car	1.22	1.10	1.00	1.00	1.35	1.04	1.0
Bus	1.61	1.8	1.5	1.5	1.35	2.0	2.0
Truck	2.0	2.5	2.5	2.5	2.5	2.5	2.0
LCV	0.71	1.24	1.5	1.5	1.5	0.71	1.0
Auto	0.87	0.87	1.7	1.7	1.63	1.03	1.0
Bicycle	1.5	1.5	2.0	2.0	1.5	1.5	0.8

lane before the reference line, it changes its position to the left lane. When amber starts, the vehicles in the intersection area will accelerate and clear the intersection. The vehicles before the stop line will decelerate and stop at the approaches. Table 1 shows the deceleration rates for different types of vehicles. MTW has a maximum deceleration rate of 1.49 m/s^2 and Truck has a minimum deceleration rate of 0.79 m/s^2.

2.4. Vehicle accumulation

The vehicles arriving near the intersection accumulate on the road when the signal changes to red. The accumulation of vehicles is based on the availability of spacings and type of turning movement. These vehicles occupy positions as close to the stop line as possible. The slow moving vehicles (e.g. bicycles) tend to orient towards the left side of the road (based on field observations). The longitudinal clearances between adjacent vehicles were obtained from the videographic survey of the study area. The longitudinal spacing of subject vehicles at stopped condition depends on the type of the front vehicle. For example, if the subject vehicle to accommodate on the intersection approach is a bus and the front vehicle at stopped condition is a car, the subject vehicle maintains a longitudinal clearance of 1.8 m. Table 2 shows the observed longitudinal clearances used for simulation. The lateral clearances used in the model were obtained from the earlier study (Arasan and Kashani 2003).

2.5. Vehicle dissipation

The vehicles waiting at the stop line start dissipating as soon as the signal phase changes from red to green, after the reaction time of two seconds. The position of the dissipating vehicles is updated for each scan interval using equations of motions. The dissipation rates for different types of vehicles are given in Table 3. Three manoeuvres are possible for individual vehicles when the vehicles clear the intersection area: free movement, overtaking and following. The simulation output will provide queue length, the number of vehicles in a queue, queue accumulation and dissipation.

The logics of vehicle generation, vehicle placement, vehicle movement, vehicle accumulation and vehicle dissipation are incorporated in the model. The simulation model has been programmed using the object oriented approach and implemented in C++ language.

Table 3. Observed dissipation rate of different vehicle categories

Vehicle Type	Dissipation Rate (m/s^2)
MTW	1.58
Car	0.84
Bus	0.29
Truck	0.29
LCV	0.83
Auto	0.8
Bicycle	1.37

3. Model validation

To validate the simulation model, the following two criteria were considered:
1. Validation based on density (the number of vehicles in a queue in 150 square queue meter of the intersection area, PCU/150 sq. m).
2. Validation based on the number of vehicles dissipated (number of vehicles crossing the stop line, PCU/cycle).

As traffic in India is heterogeneous, it was decided to find the proportion of PCU (Passenger Car Unit) instead of the number of vehicles. To validate the model, the signalized intersection at Kotturpuram, Chennai City, India was selected. This intersection is a four-legged right angled intersection each individual approach of which is 7.5 m wide. The layout of the intersection is depicted in Fig. 5.
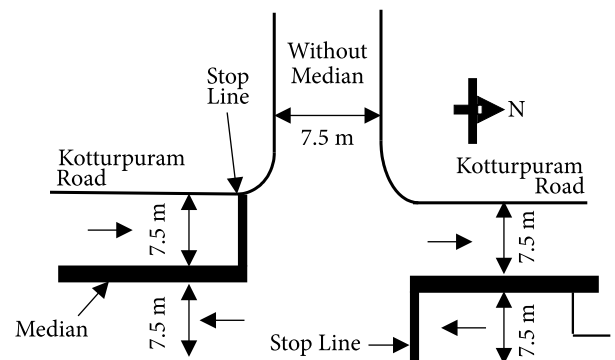

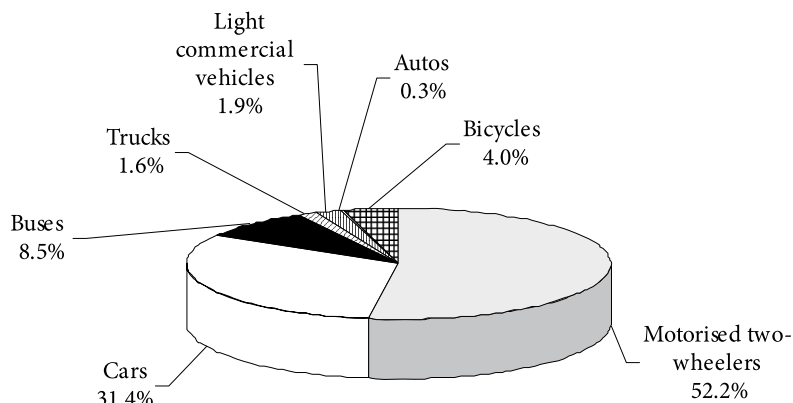**Fig. 5.** Layout of the study intersection

Fig. 6. Traffic composition at a study intersection approach

3.1. Data collection

Traffic movements were captured using videography during traffic peak hours. The data for one approach of the intersection (Kotturpuram road approach) was captured using a video camera. The camera was placed at an elevated position to capture the queue, dissipation of vehicles and longitudinal spacing of vehicles in stopped condition. The input parameters for the simulation model, such as traffic volume, proportion of turning movements, vehicle composition, longitudinal spacing and dissipation rate were extracted from the video data. A total traffic volume at the intersection approach was 3067 veh/h. Vehicular composition of traffic at a study stretch is shown in Fig. 6. There are totally 30 signal cycles for one hour.

3.2. Validation based on queue density

To collect queue density at an intersection approach, 150 square meters (limited by video camera's field of view) of area was considered in the model. The number of vehicles that occupied this area during red time was obtained and queue density (PCU/150 sq. m) was determined. On the basis of hypothesis test (0.05 level of significance), it is seen that the observed and simulated values are not statistically different (Table 4).

3.3. Validation based on the dissipation of vehicles

The model validation based on the dissipation of vehicles was done by determining the dissipation of vehicles per signal cycle. The observed and simulated numbers of the dissipated vehicles were converted into PCU. The statistical hypothesis test revealed that there was no significant difference (0.05 level of significance) between the observed and simulated values.

Thus, the developed simulation model is believed to represent the observed traffic characteristics.

4. Application of the model

Due to unique driver's behaviour in India, queue and lane discipline is disregarded near intersection. This often leads to the situations where left turning vehicles block the straight through vehicles by occupying road space in the approach. This phenomenon can be mitigated by the provision of left turn channelisation at the intersection

approach; there by regulating the left turning vehicles (Fig. 7). This will enhance flow efficiency. The interest in this study was to evaluate the efficiency of left turn channelisation. The developed simulation model was used for this purpose. Channelization can be provided to segregate the left turn vehicles to make the vehicles to quickly

Table 4. Comparison of observed and simulated queue density

Red Phase	Simulated queue density (PCU/150 sq.m)	Observed queue density (PCU/150 sq.m)	Difference	Squared deviation
1	10.5	9.0	−1.5	2.25
2	13.0	9.5	−3.5	12.25
3	9.5	8.0	−1.5	2.25
4	12.5	10.0	−2.5	6.25
5	13.5	11.5	−2.0	4.00
6	11.5	9.5	−2.0	4.00
7	6.0	8.0	2.0	4.00
8	9.5	11.5	2.0	4.00
9	10.0	11.5	1.5	2.25
10	14.5	12.5	−2.0	4.00
11	15.0	13.0	−2.0	4.00
12	9.5	10.0	0.5	0.25
13	13.0	10.5	−2.5	6.25
14	11.0	9.0	−2.0	4.00
15	10.0	12.0	2.0	4.00
16	13.5	14.0	0.5	0.25
17	13.5	12.0	−1.5	2.25
18	12.5	11.0	−1.5	2.25
19	9.5	11.0	1.5	2.25
20	15.0	13.0	−2.0	4.00
21	10.0	12.0	2.0	4.00
22	11.5	9.5	−2.0	4.00
23	13.5	13.5	0.0	0.00
24	10.5	11.5	1.0	1.00
25	12.0	12.0	0.0	0.00
			sum = −15.5	sum = 83.75

a) Left Turn Channelisation

b) Without Channelisation

Channelization

Length 10–50 m

Width 3.5 m

Stop Line

Stop Line

Left Turning Vehicles

Right Turning and
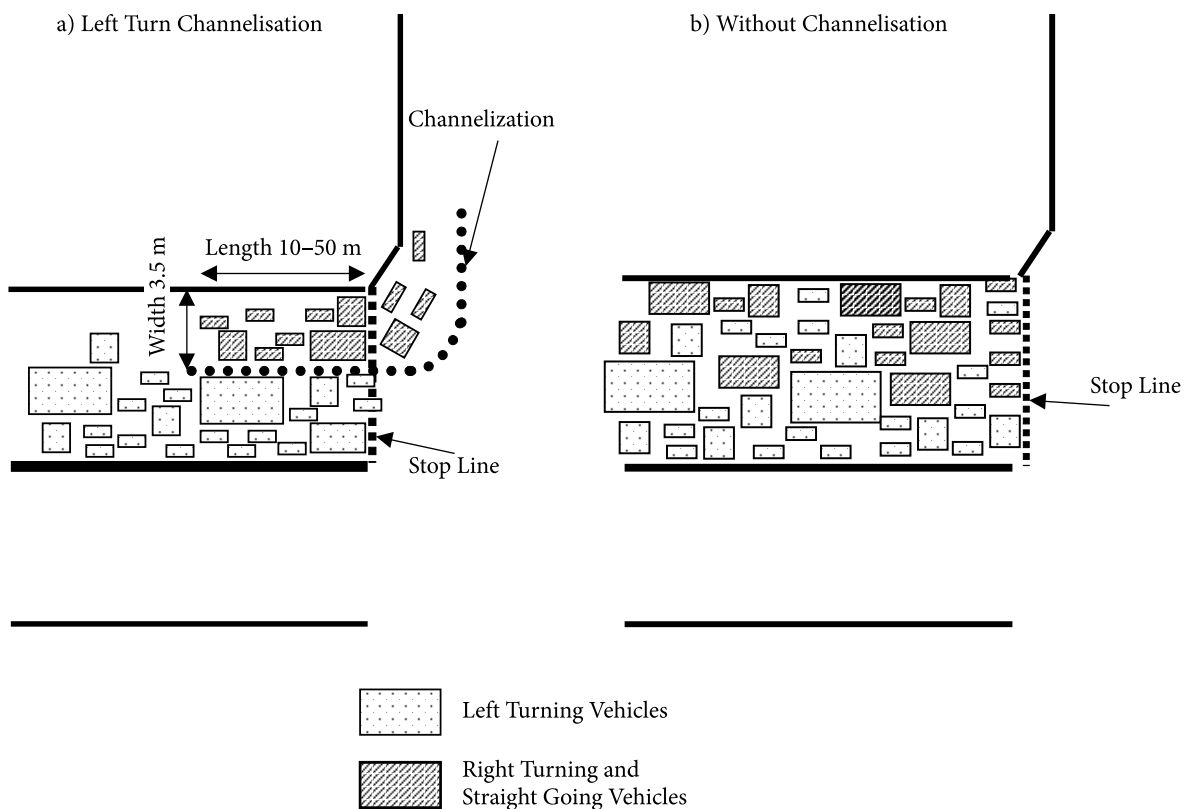Straight Going Vehicles

Fig. 7. Illustration of queuing up of vehicles at a signalized intersection approach with and without channelisation

clear the intersection approach. Segregation can be up to a width of 3.5 m from the left side of the lane by a separation of 30 cm width. The length of the segregation lane may vary from 10 to 50 m. A sensitivity analysis was carried out by varying the total volume, left lane length, left turn volume and a green phase of the signal to study the impacts of channelisation on average waiting time of straight going and right turning vehicles. The total volume of the intersection approach was varied from 1 000 to 3 000 veh/h in increments of 500 vehicles. The channelisation length can be varied from 10 m to 50 m in increments of 10 m. The proportion of left turn vehicles was varied from 5% to 50% in increments of 5%. The simulation model was run for various combinations of the above parameters to perform the sensitivity analysis. As a sample, the values of average waiting time reductions for various combinations of the total volume and left turn channelisation length for 25% left turn volume are shown in Table 5.

Figs from 8 to 10 depict the trend in average waiting times of turning vehicles for various combinations of considered parameters. These figures are indicative of the beneficial effects of left turn channelisation on the average waiting times of vehicles. In particular, it is observed that the highest reduction in average waiting time per vehicle (3.86 s/veh) is achieved for a total volume level of 3 000 veh/h (Fig. 10), indicating an optimal combination of parameters for the case study. Generally, if the left turn volume is less than 20%, channelisation has no impact on the average waiting times of vehicles. If channelisation is not provided at an approach for low volume level, all ve-

Table 5. Reduction in waiting times of vehicles for left turn channelization (25% left turn volume)

Total Volume Level (Veh/h)	Left Turn Channelisation Length (m)	Average Waiting Time Reduction (s/veh) for straight and right turn vehicles
1000	10	2.82
	20	2.65
	30	2.82
	40	2.87
	50	2.89
2000	10	3.08
	20	3.1
	30	3.13
	40	3.05
	50	3.22
3000	10	3.67
	20	3.67
	30	3.68
	40	3.86
	50	3.8

hicles utilizing the road space are optimally based on the turning movement. But, the road space is not optimally utilized if channelisation is provided for low volume level which increases the average waiting times of vehicles. Thus, it can be learned that only for higher proportion of left turn volumes, channelisation has high impacts. The channelisation length has only a marginal impact on the average waiting times of vehicles.
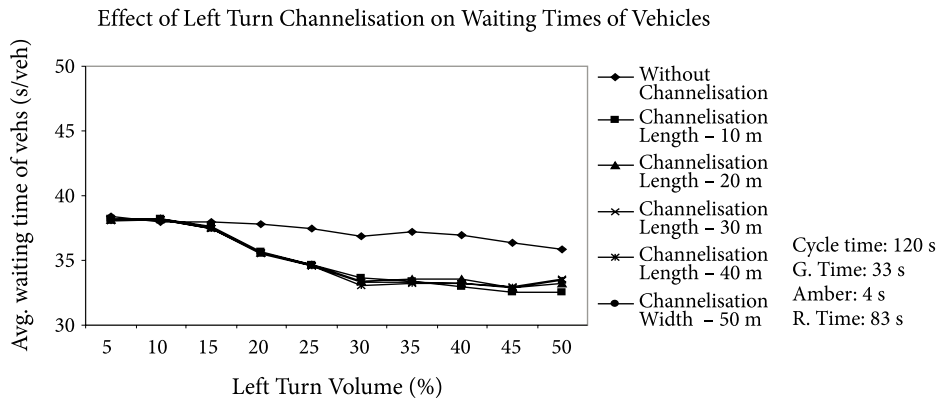
Effect of Left Turn Channelisation on Waiting Times of Vehicles
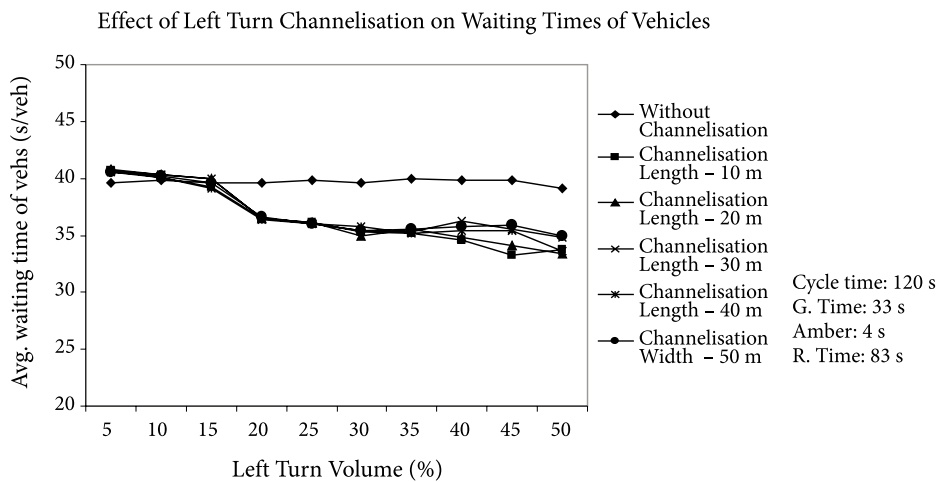

Fig. 8. Impact of left turn channelisation (1 000 veh/h)

Effect of Left Turn Channelisation on Waiting Times of Vehicles

Fig. 9. Impact of left turn channelisation (2 000 veh/h)

Effect of Left Turn Channelisation on Waiting Times of Vehicles
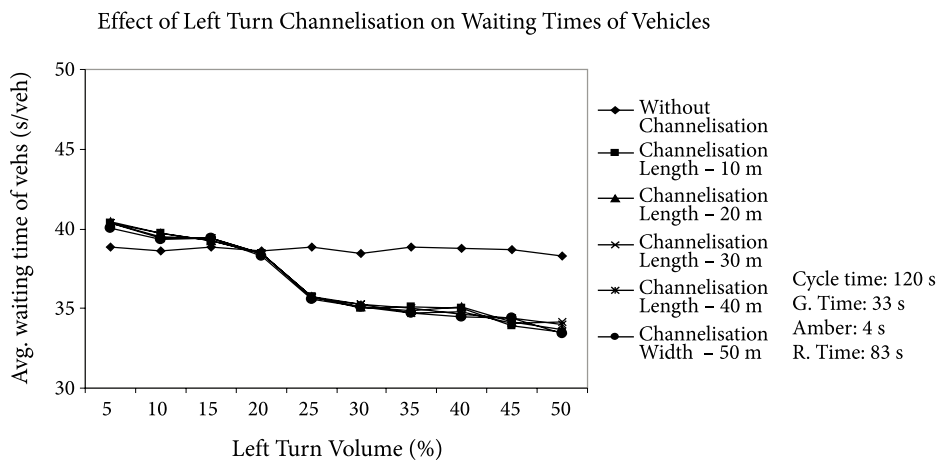

Fig. 10. Impact of left turn channelisation (3 000 veh/h)

5. Conclusions

The traffic simulation model was developed to simulate the flow of heterogeneous traffic through a signalized intersection. The various logics of the simulation model were explained and implemented in the C++ language using the object oriented approach. The model was validated with respect to the dissipation of vehicles and queue density at an intersection approach and found to be satisfactorily replicating the field conditions. The model was applied to study the impacts of left turn channelization at a signalized intersection on average waiting times of vehicles. The channelization is beneficial for high volume levels and certain proportions of left turn volume. The carried out analysis shows that the efficacy of this traffic management measure could be evaluated and its benefits quantified for a case study intersection.

Such models can be employed to assist traffic engineers in taking decisions on traffic management measures under heterogeneous traffic conditions and unique driver behaviour as prevailing in India.

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