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ANALYSIS OF CALCULATION METHODS USED FOR ACCURACY EVALUATION OF THE RESULTS OF ROAD ACCIDENT EXAMINATION

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Abstract. For examination of road accidents often calculations are being performed according to mathematical equations having many variables, real values of which are of stochastic nature and they depend on many accident factors. Therefore it is important to know limits of determination of final values of every being analysed parameter of car movement and their probabilistic characteristics. This paper presents some cases of simulation of some road situations by mathematical methods of calculations, which now are widely used for evaluation of accuracy of values of the calculated parameter (in this paper – the braking distance). Also here is presented some analysis of these methods. The paper presents the following cases of car braking: braking with making attempts to prevent a collision with another car, braking through impact-to-pedestrian prevention and conventional (rectilinear) car braking. The main attention in the analysis of the mentioned road situations is paid to veracity of the car braking distance, i.e. to limits of variation of values of this parameter and also to influence of accidental factors on the final result which is being looked for. Some conclusions on the expediency of application of the described methods of calculations to analysis of road accidents are present.

Keywords: examination of road accidents, method of calculation, error of calculation, car maneuver, probability of impact-to-pedestrian, collision, braking distance.

1. Introduction

In the analysis of road accidents evaluation of values of all parameters of car movement with a desirable accuracy often fails. Road accident is a complex process, predetermined by many factors, different in their nature, intensity and duration.

In analysis of vehicle movement by Lukoševičienė (2001), Mitunevičius (1999) and Noon (1994) deterministic mathematical models are mostly used. However, by using them or especially more simple analytical models it is difficult to determine values of parameters being looked for because of inaccuracies in description of the vehicle parameters and its movement. Mostly those parameters are of accidental nature and more precise methods are necessary for their evaluation.

Usually the greatest attention is paid to car braking which is among the final driver's actions tended to collision prevention, this presented by Broen and Chiang (1996), Evans (1996), Pečeliūnas, Lukoševičienė and Prentkovskis (2003), Pečeliūnas and Prentkovskis (2006), Zebala and Reza (2000). Therefore in this paper, resuming the published scientific works by Guzek and Lozia (1998), Guzek and Lozia (2000), Eubanks (1992), Danner and Halm (1994), Kalinin (Калинин 1984) , we shall present analysis of some dangerous car braking cases applying one of the possible methods of car movement parameters calculations, such as computer experiment, statistical-probabilistic and method of total differentials.

The main attention in this paper is paid to evaluation of the final results of calculations – finding of the veracity limits (scatter) of car movement parameters (braking distance) with simultaneous critical evaluation of methods themselves of calculations mentioned above.

2. Car braking tended to collision prevention

At first let us analyse the method presented by Guzek and Lozia (2000) and based on a computer experiment (programme AUTO.PC3). It can be used for calculation of braking parameters for a car, driver of which has observed an obstacle. This situation is shown in Fig. 1.

Let us say, that the permissible car speed in the road section equals 60 km/h. Driver of the standing car No. 1 is showing the left turn however is waiting for the car No. 2 to pass by in the left lane. Driver of the 3rd car

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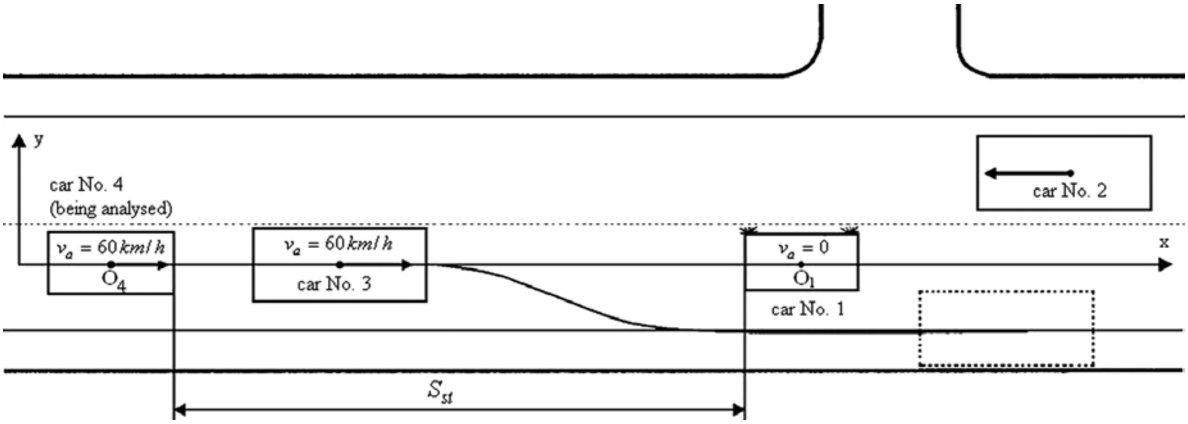

Fig. 1. Diagram of situation before the being analysed accident

approaches dangerously close to the car No. 1 and decides to overtake it from the right side. The 4th car (the examined one) driver sees in front before him a sudden appearance of the standing car No. 1 at the distance of say, 40 m.

Let us analyse behaviour of the 4th car driver. A question arises if the driver running at 60 km/h speed can avoid collision with the car No. 1 and which of the simulation methods can be applied, to what extent their accuracy could be reached.

For simulation the following parameters of car movement are taken with estimation of possible limits of their spread (Guzek and Lozia 2000):

- in the beginning speed of the 4th car: v_a = 16.7 m/s (60 km/h). Distance between the car frontal end and the obstacle in the beginning (e.g. distance to brake the car) would be S_{st} = 40±1 m (± 2.5 %);
- total time of driver's reaction and of response of the braking system: $t_1 + t_2$ = 1±0.2 s (± 20 %);
- time of deceleration increase: t_3 = 0.25 s;
- accepted factor of adhesivity between tyres and road pavement: φ_x = 0.75±0.05 (±6.7 %).

Collision can take place when the least distance between centres of gravity of the 1st and 4th cars $|O_1O_4|$ will be less than sum of distances from the centre of gravity of the 4th car to its frontal left corner and from the centre of gravity of the 1st car to its rear right corner (Fig. 2).

In this case of analysis the limiting value of the safe distance $|O_1O_4|$ equals $|O_1K| + |KO_4|$ = 4.1 m. The following cases of the car maneuver were simulated, such as:

- "Nominal" case of the movement parameters (φ_x = 0.75; t_1 = 1.0 s; S_{st} = 40.0 m);
- "Unsuccessful" case of the movement parameters (φ_x = 0.70; t_1 = 1.2 s; S_{st} = 39.0 m);
- "Successful" case of the movement parameters (φ_x = 0.80; t_1 = 0.80 s; S_{st} = 41.0 m).

Analysis of the obtained results of simulation indicates, that possibility of going round the obstacle (the 1st car) is poor. It is possible only for „successful" or „nominal" combinations of the movement parameters of the 4th car. It is important to mention that simulation by a computer experiment can lead to a set of variants of road situations. It depends on the accepted values of the car movement parameters. For instance, values of time t_1, t_2, t_3, factor φ_x and other quantities for different road situations can obtain different values that mostly are unknown for us. Therefore this method not always leads to high accuracy of calculations.

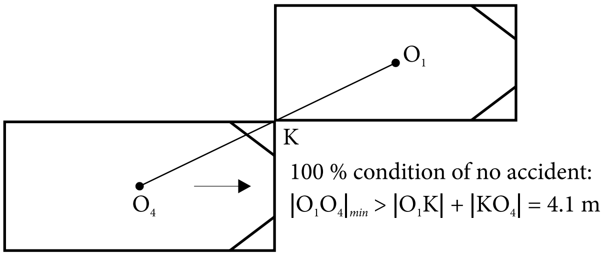

Fig. 2. Diagram of the limiting accident situation of the 4th (O_4) and 1st (O_1) cars

3. Braking when making attempts to prevent a collision with a pedestrian

Besides the method of calculations presented above, now for evaluation of accuracy of road accident calculations a statistical probabilistic method is often used. In this case statistical characteristics of parameters of the being braked car are being evaluated and in accordance with the obtained results possible errors of the calculations are being estimated.

Now we shall briefly discuss the application of this method to simulation of car braking in the case of an unexpected appearance of a pedestrian on the road, see researches by Eubanks (1992), Kalinin (Калинин 1984). The diagram presented in Fig. 3 will illustrate it. Knowing statistical characteristics of the main parameters of movement (for example, the mean value of braking distance $\overline{S}_{st}$, dispersion $D_{S_{st}}$ and the law of distribution of the braking distance S_{st}) we can write the equation for calculation of the braking distance S_{st}:

$$S_{st}^{l,h} = \overline{S}_{st} \pm \arg\varphi(P)\sqrt{D_{S_{st}}}\,, \qquad (1)$$

where: $S_{st}^{l,h}$ – lower ("successful") and higher ("unsuccessful") limits of the obtained values of the braking distance; $arg\varphi(P)$ – coefficient of the integral function

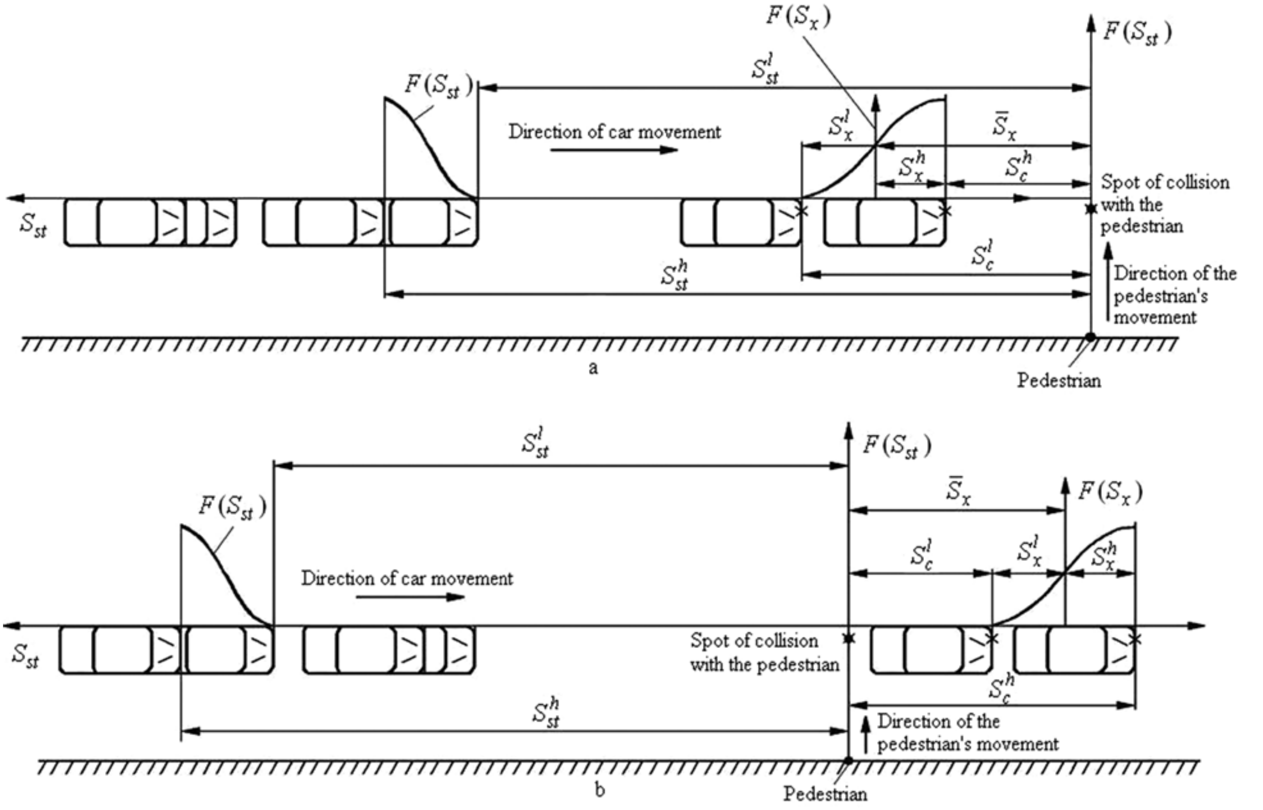

Fig. 3. Diagram of the statistical-probabilistic analysis of collision with the pedestrian: a – before the road accident, b – after the road accident

(dependent on the desirable accuracy).

Based on the existing values of the movement parameters linearization of the data scatter results in the expression (2) of dispersion of the run distance S_x, when the car is being braked, but the full stop occurs after collision with the pedestrian (Fig. 3 a, b):

$$D_{S_x} = \left(\frac{g}{2} \cdot \overline{t}_3 \cdot \overline{\varphi}_x + \sqrt{2 \cdot g \cdot S_{stlock} \cdot \overline{\varphi}_x} \right)^2 \times (D_{t_1} + D_{t_2}) + \left(\frac{S_p \cdot g \cdot \overline{\varphi}_x}{2 \cdot v_p} - \frac{g}{2} \cdot \overline{t}_1 \cdot \overline{\varphi}_x - \frac{g}{2} \cdot \overline{t}_2 \cdot \overline{\varphi}_x - \sqrt{4{,}5 \cdot g \cdot S_{stlock} \cdot \overline{\varphi}_x} \times \sqrt{\frac{g}{2} \cdot S_x^{'} \cdot \overline{\varphi}_x} \right)^2 D_{t_3} + \left(\frac{S_p \cdot g \cdot \overline{t}_3}{2 \cdot v_p} - \frac{g}{2} \cdot \overline{t}_1 \cdot \overline{t}_2 - \frac{g}{2} \cdot \overline{t}_2 \cdot \overline{t}_3 - \frac{3}{4} \cdot g \cdot \overline{t}_3^{\,2} - \overline{t}_3 \cdot \sqrt{\frac{2 \cdot g \cdot S_{stlock}}{\overline{\varphi}_x}} + \overline{t}_3 \cdot \sqrt{\frac{g \cdot S_x^{'}}{2 \cdot \overline{\varphi}_x}} \right)^2 \cdot D_{\varphi_x} + \left(\frac{S_p}{v_p} \cdot \sqrt{\frac{g \cdot S_{stlock}}{2 \cdot \overline{\varphi}_x}} + \frac{g}{4} \cdot \overline{t}_3^{\,2} - \overline{t}_1 \cdot \sqrt{\frac{g \cdot S_{stlock}}{2 \cdot \overline{\varphi}_x}} - \overline{t}_2 \cdot \sqrt{\frac{g \cdot S_{stlock}}{2 \cdot \overline{\varphi}_x}} + \overline{t}_3 \cdot \sqrt{\frac{g \cdot S_{stlock}}{8 \cdot \overline{\varphi}_x}} - \overline{t}_3 \cdot \sqrt{\frac{g \cdot S_x^{'}}{8 \cdot \overline{\varphi}_x}} \right)^2 \cdot \frac{2 \cdot D_{\varphi_x}}{a \cdot S_{stlock}}, \quad (2)$$

where: g – the free fall acceleration: $\overline{\varphi}_x$ – the average value of the factor of adhesivity; S_{stlock} – value of the braking distance, determined in accordance with the track wheel slide (when wheels are locked); $\overline{t}_1$, $\overline{t}_2$, $\overline{t}_3$ – values of mean time of driver's reaction, of response of the braking system and of deceleration increase correspondingly; D_{t_1}, D_{t_2}, D_{t_3}, D_{ϕ_x} – values of dispersion of the corresponding items; S_p – distance gone by the pedestrian till the collision spot; v_p – speed of the pedestrian; S_x' – distance rolled by the car after collision with the pedestrian; a – free term.

In this equation the car is supposed to be slowed down before the collision, however it stops at distance S_x' beyond the line of the pedestrian's movement. Also, the car speed is supposed to be inconstant.

In the situation being analysed many parameters, describing the "driver – car – road (traffic environment)" system are accidental. Therefore we obtain many values of braking distance S_{st} and also those of the being calculated distance S_c, which was run from the collision spot with the pedestrian (Fig. 3 a, b).

If the mean value, dispersion and the law of distribution of quantities S_{st} and S_c are known, then it is possible to describe their lower (i.e. "successful") limits S_{st}^l (S_c^l) as well as their higher ("unsuccessful") limits S_{st}^h (S_c^h); between them all the possible values are scattered (Fig. 3).

If the average value of the run distance $\overline{S}_x$ is positive and exceeds the higher value of S_x^h (Fig. 3 a), not paying attention to the scatter of possible values we can assert that the car will stop before the collision with the pedestrian. If both limits (S_x^l and S_x^h) are located beyond the line of the pedestrian's movement, then even in the case of emergency braking the driver will have no possibility to prevent the collision.

If the car speed is constant, the following simpler equation will be obtained:

$$D_{S_x} = (D_{t_1} + D_{t_2} + 0{,}25 \cdot D_{t_3}) \cdot v_a^2 + \left(\frac{v_a^2}{g \cdot \bar{\varphi}_x} - \sqrt{\frac{v_a^2 \cdot S_x^{'}}{2 \cdot g \cdot \bar{\varphi}_x^{\,3}}} \right)^2 \cdot \frac{4 \cdot g \cdot \bar{\varphi}_x \cdot D_{\varphi_x}}{a \cdot v_a^2}, \qquad (3)$$

where: v_a – initial speed of car movement.

The veracity of parameters of the analysed road accident, as it is seen from the expressions (2) and (3), depends on a set of accidental factors. But having their statistical characteristics (for example, received in experimental research) we can obtain sufficiently accurate results of the analysis of the described road accident (up to ±10 % by Kalinin (Калинин 1984)).

4. The general case of car braking

In the previous sections we analysed fairly complicated road traffic situations (braking before maneuvering, before collision with a pedestrian). Therefore it would be necessary to mention the general (rectilinear) case of car braking, when the braking distance is calculated according to the widely known dependence:

$$S_{st} = (t_1 + t_2 + 0.5 t_3) v_a + \frac{v_a^2}{2 j_x}, \qquad (4)$$

where: j_x – value of the steady longitudinal deceleration.

For analysis of such braking with evaluation of errors of calculations various mathematical methods can be used. In addition to the methods mentioned before, let us examine the method of total differentials as one of the mostly used, see researches by Guzek and Lozia (1998), Danner and Halm (1994).

Description of accuracy of the results of calculations by use of this method enables the evaluation of the main characteristics, determining the length of the braking distance and errors $\Delta(t_1 + t_2)$, Δt_3, Δv_a, Δj_x (see equation 4) of values.

With the help of the method of total differentials we can find the greatest values of relative and absolute errors of the being analysed quantities (here it would be the braking distance S_{st}, when the times t_1, t_2, t_3 are evaluated). In general, the equations used for calculations would be written as follows:

$$\frac{\Delta S_{st}}{S_{st}} = \sum_{i=1}^{r} \left| \frac{\partial S_{st}}{\partial p_i} \cdot \frac{\Delta p_i}{p_{i(0)}} \right|, \qquad (5)$$

$$\Delta S_{st} = \sum_{i=1}^{r} \left| \frac{\partial S_{st}}{\partial p_i} \cdot \Delta p_i \right|, \qquad (6)$$

where: p_i – i-th parameter of the mathematical model, on which the braking distance S_{st} is depending; $p_{i(0)}$ – the nominal value of p_i; Δp_i – the absolute error of p_i.

So, would we want to evaluate the relative error of the car braking distance we would obtain the following equation for calculations:

$$\frac{\Delta S_{st}}{S_{st}} = \left| \frac{\partial S_{st}}{\partial (t_1 + t_2)} \cdot \frac{\Delta (t_1 + t_2)}{S_{st}} \right| + \left| \frac{\partial S_{st}}{\partial t_3} \cdot \frac{\Delta t_3}{S_{st}} \right| + \left| \frac{\partial S_{st}}{\partial v_a} \cdot \frac{\Delta v_a}{S_{st}} \right| + \left| \frac{\partial S_{st}}{\partial j_x} \cdot \frac{\Delta j_x}{S_{st}} \right|. \qquad (7)$$

Let us say, that for calculations of relative and absolute errors of S_{st} parameters of the 7th equation are taken with the following values of errors: $\Delta(t_1 + t_2) = 0.2$ s, $\Delta t_3 = 0.05$ s, $\Delta v_a = 1.0$ m/s^2, $\Delta j_x = 0.5$ m/s^2. Then the nominal values of the parameters will be: $t_1 + t_2 = 1$ s, $t_3 = 0.2$ s, $v_a = 20$ m/s, $j_x = 7.0$ m/s^2.

So we obtain, that the relative error of the braking distance with increase of the initial speed v_a at first quickly falls (to $v_a \approx 10$ m/s) and then descends rather uniformly (Fig. 4).

On the contrary, the absolute error ΔS_{st} of the braking distance constantly and monotonously increases. For instance, at $v_a = 10$ m/s the absolute error equals $\Delta S_{st} \approx \pm 5$ m. But when the initial speed increases, e.g. $v_a = 40$ m/s, it reaches $\Delta S_{st} \approx \pm 25$ m.

It is necessary to mention, that in description of accuracy of the final result of calculations by the method of total differentials there exists a great possibility of wrong choice for values of errors Δ (see equation (7) of the corresponding characteristics t_1, t_2, t_3, v_a, j_x) – like in simulation with computer programmes. It is of especial importance in the case of examination of road accidents under concrete circumstances of car braking.

5. Generalization of analysis of mathematical methods, used for accuracy evaluation of road accident examination

After analysis of the main mathematical methods used for examination of some car braking situations in aspect of evaluation of accuracy of the final results of calculations we can formulate the following generalizations:

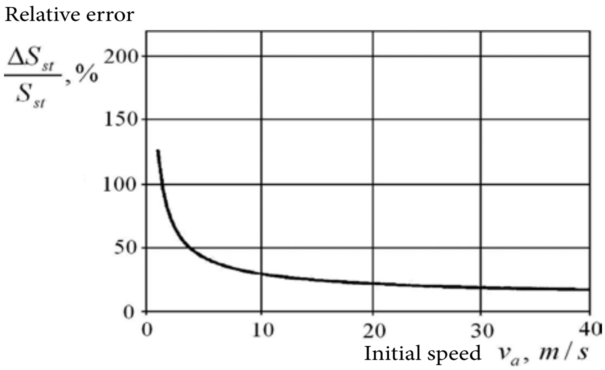

Fig. 4. Dependence of the maximum relative error on the initial speed

- results of other road traffic situations can be described using the presented mathematical methods;
- accuracy of the used mathematical methods is a very important factor of the choice and it is predetermined by the initial data, their reliability and peculiarities of the method choices. For instance, use of computer simulation of the road traffic situation can involve many values of the initial parameters of a car (or other participant of traffic) movement and so it can result in not just one final result;
- use of a statistical-probabilistic method leads to a sufficiently accurate final result of calculations, because the statistical information is collected from experimental results. But often it is complicated or impossible to collect the necessary volume of such data for research of the concrete traffic situation. Thus the accuracy of the mentioned method suffers;
- the method of total differentials can be used for calculations of the greatest relative and absolute values of errors of the analysed parameters, veracity of which often can be determined with difficulty, because the average value of the parameter is unknown. Also there are unknown laws of its scatter.

6. Conclusions

1. For calculations in the stage of restoration of the course of events of a road accident it is important to evaluate the accuracy and error limits of movement parameters of participants of the road accident. Now many mathematical methods are used for this purpose. Among them the following can be marked out: computer simulation, statistical-probabilistic method and method of total differentials.
2. For creation of the reliable list of errors of parameters being calculated for the dangerous road traffic situation it is important to have statistical characteristics (average mean, dispersion, mean square deviation, a.o.) of every analysed parameter. Then based on the presented analysis of accuracy of road accident examination we can suggest that the statistical-probabilistic method of calculations is of sufficient accuracy. Such calculations (with accuracy up to ±10 %) evaluate the scatter of different accidental quantities, characterizing various parameters of car movement.
3. Use of statistical methods of calculations results in a sufficiently accurate description of some typical cases of car movement (e.g., rectilinear braking). But such methods are problematical if applied to analysis of non-typical road accidents (e.g., car overturn), often because of lack of experimental characteristics.

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