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ANALYSIS OF MAIN DYNAMIC PARAMETERS OF SPLIT POWER TRANSMISSION

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Abstract. The review carried out had shown one basic approach of split power transmission to the organization of drive which is applied to stepless transmissions of tractors and parallel hybrid cars. In the split power transmission the power split device uses a planetary gear. Tractor engine power in the split power transmission is transmitted to the drive shaft via a mechanical and hydraulic path. The theoretical analysis of main parameters of the split power transmission of the tractor is presented. The angular velocity of sun and coronary gears of the differential set is estimated by solution of the system of equations in which one equation is made for planetary differential gear, and another – for hydrostatic drive. The analysis of the transmission gear-ratio dependencies on the ratio of hydraulic machines capacities is carried out. Dependence of the variation of angular velocity of the coronary and the sun gears on the ground speed of the tractor is presented. Dependence of sum shaft torque and its constituents, carried by mechanical and hydraulic lines, on sum shaft angular velocity and ground speed of tractor and engine speed is also presented.

Keywords: tractor, split power transmission, gear-ratio, planetary gear, hydrostatic gear, sum shaft, angular velocity, engine speed, torque.

1. Introduction

The stepless transmission is user-friendly, because it has the possibility to select any desirable automobile or tractor speed, particularly, on purpose to slightly increase or decrease driving speed or pulling force.

Stepless transmission is capable to transmit engine torque by two flows enlarged in update tractors. Analogical principle in transmissions of hybrid automobiles is used. Essential feature of such transmissions is that to divide the torque into two parts or for connection of two flows into one, various planet reducers are used. Two split power supply types of transmissions in tractors are essentially distinguished:

1) hydrostatic gears for use of greater power without additional gearshift on the move. The speed diapasons are selected before the move off;
2) use of the lesser hydraulic engine and 4–8 gearshifts after the move off, see researches by Bea (1997), Dziuba and Honzek (1997).

Hybrid automobiles in classical representation are vehicles with two propulsion systems: internal combustion engine and electric motor. Basically, there are two main ways of realizing hybrid drive: parallel and series, see works: German (2005), Hybrids… (2003), Kliauzovich (2007) and High-Performance… (2004). In series drive the engine rotates only generators, operating thus in stationary mode and wheels have a mechanical connection only with electric motor. In parallel drive wheels are driven by both internal combustion engine and electric motor. However, usual transmission is required in that case and the engine has to perform in uneconomical acceleration modes, presented in works: German (2005), Kliauzovich (2007), Johansson and Rantzer (2003). The parallel hybrid vehicle transmissions consist of the power split device, the generator, electrical motor, and reduction gears. The power split device divides the power from the engine, sending one portion to the drive shaft, and the other to the generator. In other words, engine power is transmitted to the drive shaft via a mechanical path and electrical path. The power split device uses a planetary gear, see Kliauzovich (2007), Johansson and Rantzer (2003), Han *et al.* (2004).

Split power transmission construction of the references is analogical and their kinematic and dynamic parameters can be set by the same methods.

On purpose to exploit tractor economically (to use less of money, fuel and the time) the load, i.e. the traction force must be properly selected. Composing the tractor aggregates and carrying out scientific researches it is necessary to know transmission gear ratio and the highest possible torque on the driving wheels and its changing pattern. To count the gear ratio of this transmission

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it is inapplicable to the usual methods of mechanical and hydrostatic transmission, which are presented by Bea (1997), Dziuba and Honzek (1997), High-Performance… (2004), Johansson and Rantzer (2003), Han *et al.* (2004), Neunaber (1996). So here it is difficult to analyze how the gear ratio of stepless split power tractor transmission changes depending on the ratio of working values of hydraulic machines and how the torque transferred hydraulically and mechanically changes depending on tractor driving speed and engine rotation.

The aim of this work is to create the method for determination of the gear ratio of split power transmission, to make the analysis of the gear ratio dependence on the ratio of working volumes of hydraulic machines and to make the dependence analysis of torque transferred mechanically and hydraulically on tractor driving speed within the transport range and the engine rotations.

2. Determination of gear ratio of split power tractor transmission

A firm Fendt set on the tractor split power transmission, in which one part of power is transferred by mechanical gear and another – by hydrostatic gear (Fig. 1), see works of Bea (1997), Dziuba and Honzek (1997). The rotating shaft of the planetary carrier is connected to the engine, and uses a pinion gear to transmit power to the outer ring gear and the inner sun gear. The shaft of the ring gear 2 connects to the summarized shaft "c" through reduction gears 4–5 and hydrostatic gear (hydraulic pump "a" and two motors "b"). The shaft of the sun gear 1 connects to the summerized shaft "c" through reduction gears 6–7. The summarized shaft "c" connects to the drive shaft through a two-step mechanical reducer 8, driving the axles.

The torque from tractor engine is carried to the carrier of the planetary gear set. In the planetary gear set the torque of tractor engine is divided into two flows – hydraulic and mechanical. The both flows are transferred to the sum shaft by hydrostatic and mechanical drives. The total torque is transferred to the driving axles and wheels through a two step mechanical reducer 8. The efficiency of such transmission is greater than efficiency of hydrostatic transmission, see works of Bea (1997), Dziuba and Honzek (1997), Kirka (2002) and Neunaber (1996). Tractor with such transmission begins moving from stationary position by using approximately hydrostatic gear only. At the moment of increasing driving speed the gear ratio and torque are diminished by hydrostatic gear (by diminishing ratio of working volumes of hydraulic engines and pump V_v/V_s). When driving at the highest speed the torque transferred by hydrostatic gear is coming to zero.

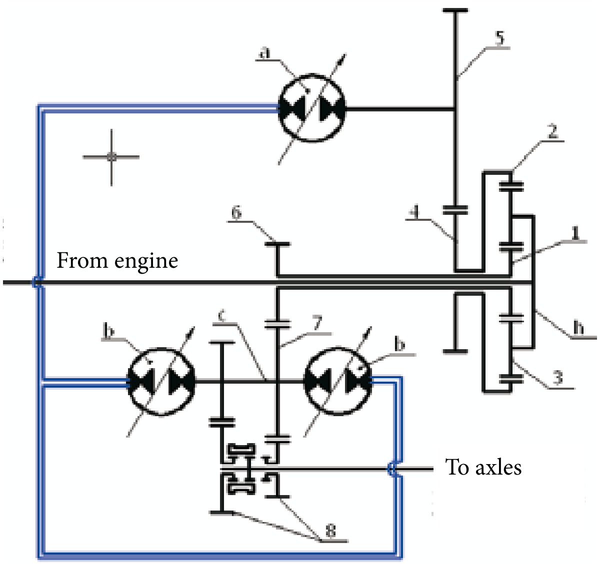

Fig. 1. Kinematic scheme of split power transmission (Fendt Vario): a and b – adjustable hydraulic pump and motors; c – sum. shaft; h – carrier; 1 and 2 – sun and coronary gears; 3 – satellite; 4, 5, 6 and 7 – gears; 8 – two-step reducer

For determination of the angular speed of the sun 1 and the coronary 2 gears ω_1 and ω_2 we make two equations. The first equation is made using Villis formula, presented in works of Kristi and Krasnenko (Кристи и Красненько 1967) for planet set:

$$\frac{\omega_1 - \omega_h}{\omega_2 - \omega_h} = p.$$

From here:

$$\omega_1 - p\,\omega_2 = (1 - p)\omega_h, \tag{1}$$

where: ω_1 and ω_2 – angular speed of the sun and the coronary gears; ω_h – angular speed of the carrier; p – internal gear ratio of planetary set when carrier is stationary.

When rotation direction of sun and coronary gears are opposite the number p is negative:

$$p = -\frac{z_2}{z_1}, \tag{2}$$

where z_1 and z_2 – number of teeth of sun and coronary gears.

Second equation (by Bea 1997) can be expressed using ratio of hydraulic pump and motor angular speed and capacity:

$$\frac{\omega_v}{\omega_s} = \eta_t \frac{V_s}{V_v}, \tag{3}$$

where ω_v and ω_s – angular speed of the hydraulic engines and pump: $\omega_v = \omega_1 \frac{z_6}{z_7}$, $\omega_s = \omega_2 \frac{z_4}{z_5}$; z_4, z_5, z_6 and z_7 – number of gear teeth; V_s and V_v – capacity of hydraulic pump and motor; η_t – coefficient of efficiency of hydrostatic gear.

As volumetric coefficient of efficiency has influence on kinematics parameters of hydrostatic gear and it depends on speed rate and load, so its characteristics can be expressed by equation, which is presented in researches by Kirka (1998), Kirka (2002), Pettersson and Lennartson (2003):

$$\eta_t = 1 - (1 - \eta_t')\left(\frac{n'}{n}\right)^{a_1}\left(\frac{p}{p'}\right)^{a_2}, \tag{4}$$

where: η_t' – nominal value of volumetric coefficient of efficiency; n' and n – nominal and moment cal revolution frequencies of hydraulic machines; p' and p – nominal and moment cal pressures at entrance to hydraulic machine; a_1 and a_2 – coefficients depending on type of hydraulic machines.

From equations (1) and (3) we make a system of equations:

$$\begin{cases} \dfrac{\omega_1}{\omega_2} = \eta_t \left(\dfrac{V_s}{V_v} \right) \cdot \dfrac{z_4\, z_7}{z_5\, z_6}, \\ \omega_1 - p\,\omega_2 = (1-p)\,\omega_h\,. \end{cases} \tag{5}$$

From every equation of system (5) we express ω_1:

$$\begin{cases} \omega_1 = \omega_2 \eta_t \left(\dfrac{V_s}{V_v} \right) \cdot \dfrac{z_4 z_7}{z_5 z_6}, \\ \omega_1 = (1-p)\omega_h + p\omega_2. \end{cases} \tag{6}$$

As left sides of equations are equal, right sides must be equal as well.

$$\eta_t \omega_2 \left(\frac{V_s}{V_v} \right) \cdot \frac{z_4 z_7}{z_5 z_6} = (1-p)\omega_h + p\omega_2\,. \tag{7}$$

From equation (7) we can express ω_2:

$$\omega_2 = \frac{\dfrac{1}{\eta_t}\dfrac{V_v}{V_s}\,\omega_h\,(z_1+z_2)\,z_5\,z_6}{z_1\,z_4 z_7 + \dfrac{1}{\eta_t}\dfrac{V_v}{V_s}\,z_2 z_5 z_6}. \tag{8}$$

From every equation of system (5) we express ω_2:

$$\begin{cases} \omega_2 = \omega_1 \dfrac{1}{\eta_t} \left(\dfrac{V_v}{V_s} \right) \cdot \dfrac{z_5 z_6}{z_4 z_7}, \\ \omega_2 = \dfrac{\omega_1 - (1-p)\,\omega_h}{p}. \end{cases} \tag{9}$$

From equations of system (9) we can express ω_1:

$$\omega_1 = \frac{\omega_h\,(z_1+z_2)\,z_4 z_7}{z_1 z_4 z_7 + \dfrac{1}{\eta_t}\dfrac{V_v}{V_s}\,z_2 z_5 z_6}. \tag{10}$$

Gear ratio from the carrier to the summarized shaft of the gear box:

$$i = \frac{\omega_h}{\omega_{sum}}, \tag{11}$$

where: ω_{sum} – angular speed of summarized shaft.

$$\omega_{sum} = \frac{\omega_h\,(z_1+z_2)\,z_4 z_6}{z_1 z_4 z_7 + \dfrac{1}{\eta_t}\dfrac{V_v}{V_s}\,z_2 z_5 z_6}. \tag{12}$$

So we get the equation of gear ratio from the carrier to the summarized shaft:

$$i = \frac{z_1}{(z_1+z_2)}\frac{z_7}{z_6} + \frac{1}{\eta_t}\frac{V_v}{V_s}\frac{z_2}{(z_1+z_2)}\frac{z_5}{z_4}. \tag{13}$$

3. Analysis of main parameters of split power transmission

Dependence of hydraulic pump and motor capacity, gear ratio of gearbox and hydraulic motor and pump

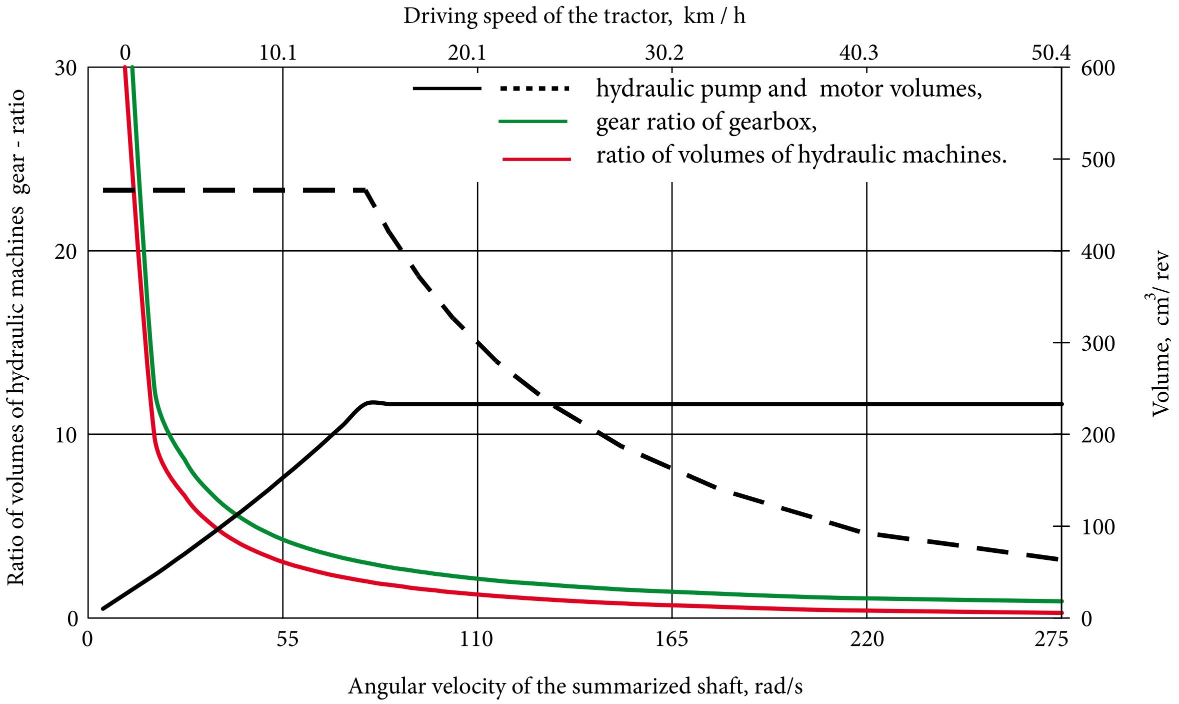

Fig 2. Dependence of hydraulic pump and motor capacity, gear ratio and hydraulic motor and pump capacity ratio on angular velocity of the summarized shaft and driving speed of the tractor

capacity ratio on angular velocity of the sum shaft (transmission series ML – 200) and driving speed of the tractor in the range v = 0.02–50 km/h is shown in Fig. 2.

Analysis of gear ratio changing of the gearbox will be carried out accepting that there are no or are equal oil losses in the line between the hydraulic pump and motors and volumetric coefficient of efficiency of hydrostatic gear η_t is equal to one.

The ratio of capacities of hydraulic machines $(\frac{V_v}{V_s})$ alternate by changing the capacities of hydraulic motors and pump. At the start of moving the capacities of hydraulic motors are greatest (the bend angles – 45 °) while capacity of pump is increased from 0 up to greatest (bend from 0 up to 45 °). Here $(\frac{V_v}{V_s})$ is varying from ∞ up

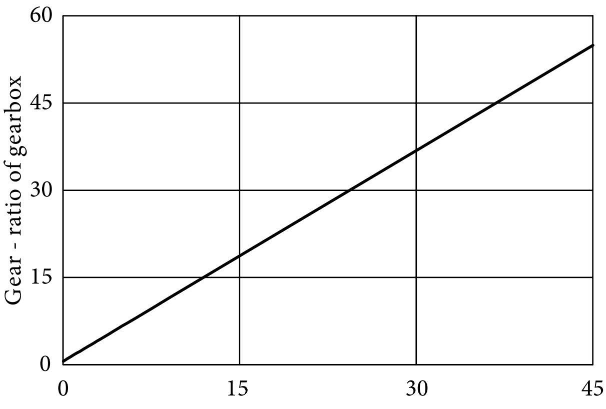

Fig. 3. Dependence of gearbox gear ratio on ratio of angular velocity of hydraulic pump and motor

to 2 while gear ratio of gearbox varies from ∞ (i > 500) to 3.0. After the pump bends up to 45 ° straightening of the hydraulic motors gradually begins (a capacity of hydraulic motors is decreased). Here ratio $(\frac{V_v}{V_s})$ from 2 is diminished further while the gear ratio of gearbox (from planetary set to summarized shaft) from 3.0 is approached to the minimal gear ratio (i = 0.595) when coronary gear of the planetary set is stationary: $i = \frac{z_1}{(z_1 + z_2)} \frac{z_7}{z_6}$ and to the greatest driving speed of the tractor.

As seen from the equation 13, the gear ratio of stepless power split transmission consists of the sum of two equations. The first equation evaluates gear ratio carried from sun gear of the planetary differential set to the summarized shaft. The second – evaluates gear ratio from coronary gear to the summarized shaft.

From the analysis it is possible to make the conclusion that common gear ratio of stepless split power transmission is equal to gear ratios of separate power flows.

Hydraulic motor and pump capacity ratio and gear ratio of gearbox are subjected to summarized shaft angular speed or tractor-driving speed varies according to much the same hyperbolas. Dependence of gearbox gear ratio on ratio of angular velocity of hydraulic pump and motor is shown in Fig. 3.

Dependence of gearbox gear ratio, speed of sun and coronary gears of planetary set on summarized shaft angular speed (tractor driving speed) is shown in Fig. 4.

The angular speed of sun and coronary gears of planetary set ω_1 and ω_2 depending on angular speed of summarized shaft ω_{sum} changes into inverse ratio at the given constant angular speed of the carrier ω_h (the en-

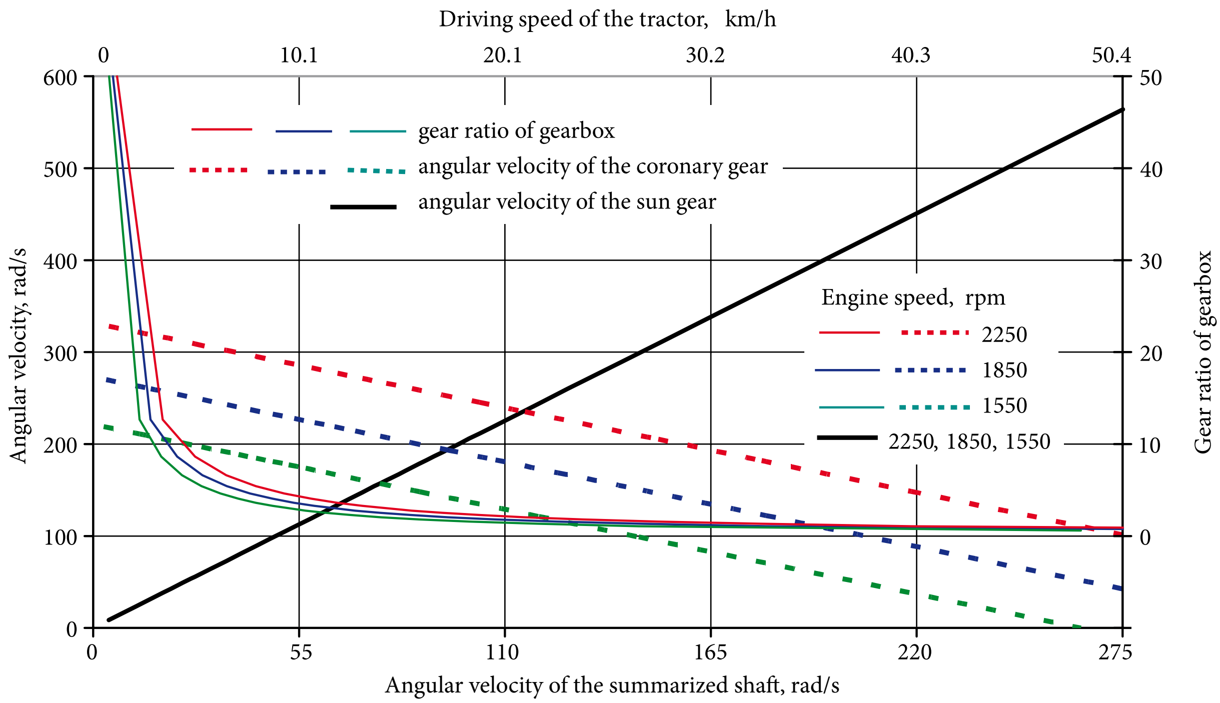

Fig. 4. Dependence of gearbox gear ratio, speed of sun and coronary gears of planetary set on summarized shaft angular velocity (tractor driving speed)

gine speed). At the diminishing of engine speed to maintain the same tractor speed is possible by diminishing the gear ratio. To this end the ratio of capacities of the hydraulic motor and the pump ($\frac{V_v}{V_s}$) are diminished. In this case the sun gear speed of planetary set will be left constant (ω_1 = const) but speed of the coronary gear will diminish (Fig. 4).

The gear ratio changes most intensively at the beginning of the driving. Then coronary gear of the planetary set is rotating at the greatest speed while sun gear has the slowest speed (ω_1 close to zero). The speed of coronary gear ω_2 decreases and rotation of the sun gear increases by increasing driving speed of the tractor (speed of the summarized shaft ω_{sum}).

As it is seen from Fig. 4 the greatest angular speed of the summarized shaft is available when the coronary shaft is fully stopped. In that case, at the nominal rotation of the engine, driving speed of the tractor would be about 70 km/h. Because the greatest driving speed of the tractor is limited up to 50 km/h therefore transfer of the torque by mechanical way when the engine rotation is nominal is impossible. The torque transfer by mechanical way would only be possible by driving at 50 km/h and decreasing engine rotation up to $n \approx 1600$ min^{-1}.

Torque of the engine M_e in the planetary differential set is distributed to the sun and coronary gears in such ratio:

$$\frac{M_2}{M_1} = \frac{z_2}{z_1}, \tag{14}$$

where: M_1 and M_2 – torques of sun and coronary gears.

Torque from sun gear is carried to summarized shaft by mechanical mode while from coronary gear – hydraulically. The summary torque of the summarized shaft is equal to the sum of these torques:

$$M_{s\,sum} = M_{sumM} + M_{sumH}. \tag{15}$$

Torque carried to summarized shaft mechanically will be:

$$M_{sumM} = M_e\,\eta_{mech}\, i_{h\to s}\, i_{red1},$$

where: η_{mech} – coefficient of efficiency of mechanical contour; $i_{h\to s}$ – gear ratio from the carrier to the sun gear: $i_{h\to s} = \frac{z_1}{z_1+z_2}$ and i_{red1} – gear ratio of gears 6–7: $i_{red1} = \frac{z_7}{z_6}$.

Then:

$$M_{sumM} = M_e \eta_{mech} \frac{z_1}{z_1+z_2}\frac{z_7}{z_6}. \tag{16}$$

Torque carried to summarized shaft hydraulically will be:

$$M_{sumH} = M_e \eta_{hm} i_{h\to k}\, i_{red2}\, i_{hs},$$

where: η_{hm} – hydro mechanical coefficient of efficiency of hydraulic contour; $i_{h\to k}$ – gear ratio from the carrier to the coronary gear: $i_{h\to k} = \frac{z_2}{z_1+z_2}$; gear ratio of gears 4–5: $i_{red1} = \frac{z_5}{z_4}$ and i_{hs} – gear ratio of hydrostatic gear:

$$i_{hs} = \frac{1}{\eta_t}\frac{V_v}{V_s}.$$

Then:

$$M_{sumH} = M_e \frac{\eta_{hm}}{\eta_t}\frac{V_v}{V_s}\frac{z_2}{z_1+z_2}\frac{z_5}{z_4}. \tag{17}$$

The summary torque of the summarized shaft is equal to:

$$M_{s\,sum} = M_e\left(\eta_{mech}\frac{z_1}{z_1+z_2}\frac{z_7}{z_6} + \frac{\eta_{hm}}{\eta_t}\frac{V_v}{V_s}\frac{z_2}{z_1+z_2}\frac{z_5}{z_4}\right). \tag{18}$$

The analysis of dependence of mechanical and hydrostatic flows of torque on engine speed and tractor driving speed has been carried out on constant power (P_e = const = 221, kW) regime of the tractor FENDT 930 VARIO^{TMS}, from Tractor... (2006).

Torque M_e of the engine is:

$$M_e = \frac{P_e}{\omega}. \tag{19}$$

The analysis of torques in the gearbox is carried out accepting that all coefficients of efficiency are equal to one. Dependence of torques on the angular speed of summarized shaft ω_{sum} is presented in Fig. 5.

As it is seen in Fig. 5, by varying speed of the tractor and by keeping constant speed of the engine, the torque carried mechanically remains constant while torque carried hydraulically changes according to hyperbola. When tractor is driving (with nominal engine rotations and nominal load) at the speed of less than 30 km/h, the torque in the gearbox is increased hydraulically (by hydrostatic gear).

When speed of the tractor is $v > 32$ km/h hydrostatic gear decreases the torque. The torque carried by mechanical contour is constant. When speed of the tractor approaches to 50 km/h the torque carried by mechanical contour becomes more significant. When driving speed is 50 km/h at nominal engine rotations and load, about 70 % of torque is carried by mechanical contour and about 30 % by hydrostatic contour. It is possible to carry 100 % of torque mechanically at less engine rotations ($n \approx 1600$ min^{-1}).

Dependence of mechanical and hydrostatic flows of torque on engine speed and tractor driving speed is presented in Fig. 6.

As it is seen in Fig. 6, by accelerating speed of the tractor and by keeping constant speed of the engine the proportion of torque carried by mechanical contour increases while proportion of torque carried by hydraulic contour decreases. When decelerating speed of the engine and by keeping constant speed of the tractor, the torque carried by mechanical contour increases while torque carried by hydraulic contour decreases.

Besides previously mentioned characteristics from curves in Fig. 5 and Fig. 6 it is possible to draw the conclusion that at constant tractor working speed regime and constant engine power working regime by decreas-

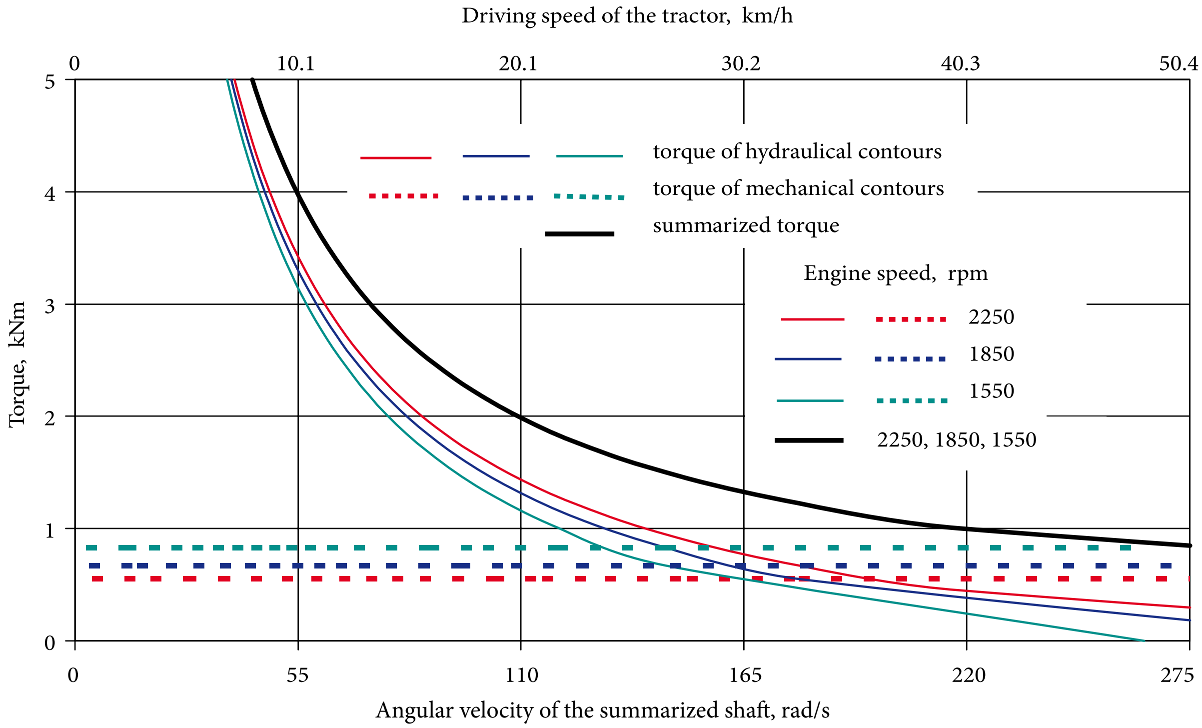

Fig. 5. Dependence of torques transferred to summarized shaft on angular velocity of the summarized shaft at the constant power mode of the engine

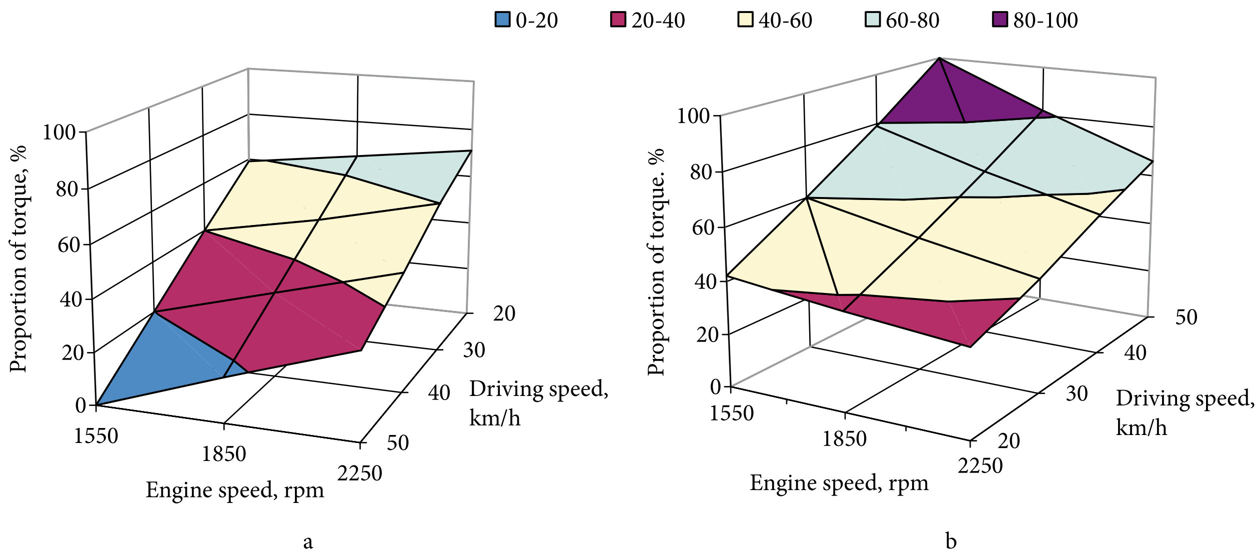

Fig 6. Dependence of hydrostatic and mechanical flows on engine speed and tractor driving speed:
a – dependence of torque carried by hydrostatic flow on engine speed and tractor driving speed;
b – dependence of torque carried by mechanical flow on engine speed and tractor driving speed

ing tractor engine rotations the torque carried by mechanical contour increases while torque carried by hydraulic contour decreases. This feature is purposeful to use in tractor exploitation.

4. Conclusions

1. Carried out review and research had shown one basic approach of stepless split power transmission to the organization of drive which is applied to stepless transmissions of tractors and parallel hybrid cars.
2. In the stepless split power transmission differential planetary gears are used as power split devices.
3. The angular velocity of sun and coronary gears of the differential set is estimated by solution of the system of equations in which one equation is made for planetary differential gear, and another – for hydrostatic drive.
4. The gear ratio of stepless split power transmission consists of the gear ratios sum of separate power flows.

5. The gear ratio of gearbox, hydraulic motor and pump capacity ratio subjected to tractor driving speed varies according to the same hyperbolas.
6. The engine torque transfer by mechanical way only at the nominal engine rotation is impossible, because the greatest driving speed of the tractor is limited up to 50 km/h. The torque transfer by mechanical way would only be possible by driving at 50 km/h and decreasing engine rotation up to ≈ 1600 rpm.
7. At the constant tractor working speed regime and constant engine power working regime by decreasing tractor engine rotations the torque carried by mechanical contour increases while torque carried by hydraulic contour decreases. This feature is purposeful to use in tractor exploitation.

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