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MODELS OF FREIGHT TRANSPORT SYSTEM DEVELOPMENT

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Abstract. When modelling a transport system and evaluating the expenditure for the sustainability of its efficiency, a dynamic model was prepared for developing the capacity of freight transportation within the common transport system. The model of developing transport modes has been elaborated applying the principles of reproduction and break-up theory.

Keywords: transport modes, models, transport system development, reproduction and break-up theory.

1. Introduction

While analysing the literature of the last 10 years, one may notice that principal theoretical research on a transport system concerns the interface of different transport modes, for instance, railway and road transport, railway and maritime transport etc., i.e. combined freight transport (Guélat *et al*. 1990).

The attention of specialists forecasting and elaborating different projects in this field is mainly focused on the systemic aspect of the efficient use of different transport modes and the most economical transport systems. Besides, there were investigated the opportunities of employing the new types of transport equipment as well as various modern means in the interface of transport systems were analysed including buildings and structures necessary for handling and warehousing related to the transportation process and reloading freight from one mode of transport to another (Baublys 2007a, 2007b and 2009).

Lithuanian transport system development strategy foresees investigation into the interface between all transport modes, namely road, railway, maritime, inland waterways, pipeline and in part, air transport (freight transportation by air is of the lowest proportions compared to other transport modes). For this reason, the whole number of models was adapted and principally changed (the criteria of socialist commanding economy were not adequate). With the help of the above introduced models, calculations and investigations on the elaboration of Lithuanian National Transport Development Programme were carried out enabling the specification of its present and future provisions (Baublys 2003).

2. Models of Transport Mode Interface

For modelling a transport system and assessing the costs of its efficient maintenance of operability, a dynamic model serving the development of general transport system capacities was elaborated. The following denoted values have been accepted in the model:
$G(t)$ – general costs for period t; $V(t)$ – transportation capacity of the transport system for period t; m – comparative costs of a transportation capacity unit, costs necessary for the uninterrupted functioning of the transport system; f – comparable costs of a transportation capacity unit necessary for the renewal (replacement of the present ones) of transport means; r – comparative costs of a transportation capacity unit necessary for an increase in transportation capacities; k – the coefficient of proportionality; K_1, K_2 – constants of integration.

The general costs of transport system functioning related to transportation capacity:

$$G(t) = kV(t),$$

where $kV(t) = cV(t) + rV'(t)$,

$$c = m + f.$$

Having solved the differential equation in $V(t)$ attitude, we will obtain:

$$V(t) = K_1 \frac{k-c}{r} t + K_2 .$$

The latter model may be generalised considering three cases of assessing costs necessary for the maintenance of transport system's working ability.

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Case I. First generalisation – presenting general costs in the form of the linear equation:

$$K_1(t) = a_i + b_i t\,,\ i = 1, \ldots, (l)N\,,$$

where: a – initial costs; c_i – the coefficient of cost increase in a time unit.

Coefficients $K_i(t)$ may be interpreted as linear functions for the maintenance of system's working ability.

The function of general costs is put down as follows:

$$G(t) = \sum_{i=l}^{N} K_i(t)\,,\ i = 1, \ldots, (l)N\,.$$

When $G(t) = a = \text{const}$, we shall obtain:

$$V'(t) = \frac{a}{r} - \frac{c}{r}V(t)\,.$$

Consequently,

$$V(t) = \frac{a}{c}\left(l - c^{-\frac{c}{r}t}\right) + e^{-\frac{c}{r}t}\,,$$

$$c_1 = \frac{a}{c} - K\,.$$

The first member used in the expression assesses the influence of external factors on the operation of the transport system (capital input into the development of system's main funds etc.).

The second member characterises the influence of factors (the ageing of the main transport means, the structure of the main transport means fleet etc.).

If $i = 3$, we shall obtain:

$$V(t) = i^{-\frac{c}{r}t}\left[c + A(t)\right],$$

where $A(t) = \dfrac{K_1(t) + K_2(t) + K_3(t)}{r} \cdot e^{\frac{c}{r}t} dt\,.$

Case II. The second generalisation is the coefficients of comparative costs divided into separate components for the sake of assessing separate factors.

In this case, the model of general costs is put down in the following way:

$$G(t) = \sum_{i}^{N} c_i V(t) + \sum_{i}^{M} r_j V'(t)\,,\ i = 1(i)N\,,\ j = 1(l)N.$$

Computer technology enables us to analyse the real levels of specification when processing different forecasted data.

Case III. The third generalisation is the assessment of the reversible impact of transportation capacities on general costs.

The expansion of transportation capacities enhances the realisation of transport services and preconditions the growth of system functioning costs as well as its work ability maintenance and modernisation.

Therefore, the function of general costs is as follows:

$$G(t) = \sum_{i=1}^{S} g_i V(t)\,,\ i = 1(l)S\,.$$

Coefficients g_i may be analysed as the coefficients of proportionality between the costs of transportation capacity and separate elements of the system. In turn, these coefficients may act as transportation capacity functions:

$$g_i = u_i + z_i V(t)\,,\ i = 1(l)S\,,$$

where: u_i – initial costs for the maintenance of system's working ability; z_i – the coefficient of cost increase in a time unit.

For assessing the efficiency of separate transport modes and for their mutual comparison, it is necessary to have a system of indices characterising the operation of the transport system, the coefficients of the value of these indices and the rules of their aggregation into a uniform quantity.

From the whole set of indices there are selected the principal ones characterising the efficiency of transport system operation:

- cost of transportation;
- time consumption for transportation (in monetary expression);
- time consumption for waiting (in monetary expression);
- cost of transportation unit.

The operational efficiency of a transport mode is assessed as follows:

$$I_{EM} = (\alpha d_1) + (\beta d_2) + (\gamma d_3) + (\delta d_4)\,, \tag{1}$$

or, if detailed more specifically according to transport modes, route and freight types:

$$I_{EM_{i,r,c}} = \left(\alpha_{i,r_i c_i} d_{1,i,r,c}\right) + \left(\beta_{i,r,c} d_{2,i,r,c}\right) + \left(y_{i,r,c} d_{3,i,r,c}\right) + \left(\delta_{i,r,c} d_{4,i,r,c}\right), \tag{2}$$

where: i – the index of a transport mode; r – the index of a route; c – freight type index; α – freight category; d_1 – transportation cost; β – time consumption, h; d_2 – recalculation of time consumption for freight unit transportation (into the cost in monetary expression); y – waiting time, h; d_3 – recalculation of freight unit waiting time into the cost (in monetary expression); δ – probability of freight non-delivery, its loss or damage; d_4 – cost of a freight unit.

To assess the operational efficiency of the transport system in general, it is necessary to sum up the meanings of the separate transport modes obtained.

$$I_{EM} = M_1 I_{EM_1} + M_2 I_{EM_2} + \ldots + M_i I_{EM_i} \to \min\,, \tag{3}$$

where: $M_i = \sum_{g=1}^{G} m_i, c$; $m_{i,c}$ – c $(c = 1, \ldots, C)$ a type of freight volume transported by the i-th transport mode; M_i – the general amount of freight transported by the i-th transport mode.

After having solved (1)–(3), it is possible to identify an efficient combination of transport modes. Whereas the assessment of transport modes is exercised as that of the whole transport network in general, the applied data is much aggregated. Such level of aggregation in transportation planning is not always necessary. It usually suffices to select the most efficient transport modes in one corridor existing in the transport system. In such a case, the problem is solved as:

$$I_{EM} = M_{12} + M_2 I_{EM_{22}} + \ ... \ + M_1 I_{EM_1} \to \min , \quad (4)$$

when

$$M_{i,\,r} = \sum_{g=1}^{G} m_{i,\,r,\,c} \ , \quad (5)$$

where: $M_{i,\,r}$ – a general amount of transportation performed by the *i*-th transport mode in corridor *r*; – *c* mode freight transportation performed by the *i*-th transport mode in corridor *r*.

Whereas all indices are expressed by the cost, the application of models does not cause difficulties. Models may also be used for achieving a solution to complex problems, for instance, assessing the influence of efficiency on the functioning of separate transport means.

3. A model for Developing Freight Transportation Modes

Freight transport should be under constant development and meet public freight transportation needs. Development trends shall be anticipated by forecasting freight flows using different transportation modes (roads, railways, waterways, air). Non-stationary processes are usually characteristic of the development models of freight transport modes.

The principles of the reproduction theory and break-up process have been suggested for establishing the development model of transport modes. These are thoroughly investigated in relevant literature (Malek-Zavarei 1982; Nucho 1981). A typical process of reproduction and break-up is characterised by a set of states $E_i\left(i = 0,\, 1,\, 2, \ldots\right)$ and probabilities of shifting to the adjacent state:

$$P_{i,\,i+1}\left(dt\right) = \lambda_i dt + o\left(dt\right),$$
$$P_{i,\,i-1}\left(dt\right) = \mu_i dt + o\left(dt\right),$$
$$P_{ij}\left(dt\right) = o\left(dt\right), \text{ if } \left|j - i\right| > 1.$$

One of the most realistic and common tasks within the process of analysis is the elaboration of the characteristics of using resources for transport modes. Thus, the following model of the task will be prepared.

From the point of view of the process of proliferation and fall, such a task corresponds to calculating probability that time needed for the process to reach the required state *n* (if the process starts from the state *i*) exceeds the defined time *T*. According to Chabischev's inequality:

$$P_i\left\{\tau_n \geq T\right\} = P_i\left\{\tau_n - M_i\tau_n > T - M_i\tau_n\right\} \leq \frac{D_i\tau_n}{\left(T - M_i\tau_n\right)^2} ,$$

where: $M_i\tau_n$ – mathematical probability of time through which state *n* is reached starting from state *i* and $D_i\tau_n = M_i\tau_n^l - \left(M_i\tau_n\right)^2$ is the dispersion of this time.

To calculate $M_i\tau_n$ and $D_i\tau_n$, we shall deduce the equation satisfying the function $m\left(x\right) = M_x\tau_n$ and $d\left(x\right) = M_x\tau_n^2$. Through time *dt* in the trajectory of the process being at point *x*, there may emerge one of three situations (in case we do not count situations the probability of which is $O\left(dt\right)$.

***Situation A*₁.** The process remains unchanged when probability $1 - \left(\lambda_x + \mu_x\right)dt + O\left(dt\right)$.

***Situation A*₂.** The process passes into a neighbouring state when probability $\lambda_x dt + O\left(dt\right)$.

***Situation A*₃.** The process passes into the former neighbouring state when probability $M_x dt + O\left(dt\right)$.

According to the formula of total mathematical probability we shall find:

$$m_i = P_i\left(A_1\right)M_i\left(\tau_n / A_i\right) + P_i\left(A_2\right)M_i\left(\tau_n / A_2\right) +$$
$$P_i\left(A_3\right)M_i\left(\tau_n / A_3\right) + O\left(dt\right) =$$
$$1 - \left(\lambda_i - M_i\right)dt + O\left(dt\right)M_i\left(\tau_n + dt\right) +$$
$$\lambda_i dtM_{\ i+1}\left(\tau_n + dt\right) + \mu_i dtM_{i-1}\left(\tau_n + dt\right).$$

If we do not assess succession by amount $O\left(dt\right)$ and perform an abbreviation, we shall obtain the equation system:

$$\lambda_i m_{i+1} + M_i m_{i1} - \left(\lambda_i + \mu_i\right)m_i = -1 , \quad (6)$$

where $I = 1,\, 2,\, ...,\, n-1,\, mn = 0$.

Analogically, the equation system is obtained

$$\lambda_{i_{id_{i+1}}} + M_{id_{i-1}} - \left(\lambda_i + \mu_i\right)d_i = -2m_i , \quad (7)$$

where: $i = 1,\, 2,\, ...,\, n-1,\, dn = 0$.

When $i = 0$ condition, $\mu_0 = 0$, the above lines acquire the shape:

$$\lambda_0\left(m_1 - m_0\right) = -1,\, \lambda_0\left(d_1 - d_0\right) - 2m_0 .$$

As equations (6) and (7) are uniform, first, we have to solve the equation by the following general shape:

$$\lambda_{iw_{i+1}} + \mu_{w_{i-1}} - \left(\lambda_i + \mu_i\right)\omega_i = -f_i , \quad (8)$$

where: $i = 0,\, 1,\, ...,\, n-1,\, \omega_n = 0$, not specifying the shape of member f_i. Having introduced the note:

$$U_i = \omega_i - \omega_{r+1} . \quad (9)$$

From (8), when $i = 0$, we will obtain $\lambda_0 U_0 = f_0$ and when $i > 0$ $M_i U_{i-1} - \lambda_i U_i = -f_i$, from which $U_i = \left(\mu_i U_{i-1} + f_i\right) / \lambda_i$. Thus,

$$U_0 = f_0 / \lambda_0 ,$$
$$U_1 = \mu_1 / \lambda_1\lambda_0 + f_1\lambda_1 ,$$
$$U_2 = f_0 M_1 M_2 / \lambda_0\lambda_1\lambda_2 + f_1 M_2 / \lambda_1\lambda_2 + f_2 / \lambda_2 ,$$
$$U_1 = P_i \sum_{j=0}^{i} \frac{f_i}{\lambda_j P_j} , \quad (10)$$

where: $P_0 = 1$, $P_i = \mu_1,\, ...,\, M_i / \lambda_1,\, ...,\, \lambda_I$.

Having put the equations together (9), starting with meaning $U_i\left(\omega_i\right)$ until meaning $U_{n-1}\left(\omega_{n-1}\right)$ and having assessed that $\omega_n = 0$, we shall obtain:

$$\omega_i - \omega_n = \sum_{j=1}^{n-1} U_j = \sum_{j=1}^{n-1} P_j \sum_{k=0}^{j} \frac{f_k}{\lambda_k P_k} ,$$

$$\omega_i = \sum_{j=i}^{n-1} P_j \sum_{k=0}^{j} \frac{f_k}{\lambda_k P_k}. \quad (11)$$

With the help of formula (11), it is possible to find meanings m_i and d_i. Having inscribed $f_i = 1$, we find:

$$m_i = \sum_{j=1}^{n-1} P_j \sum_{k=0}^{j} \frac{1}{\lambda_k P_k}, \quad (12)$$

and having inscribed $f_i = 2m_i$, we find:

$$d_i = 2\sum_{j=1}^{n-1} P_j \sum_{k=0}^{j} \frac{m_k}{\lambda_k P_k} =$$

$$2\sum_{j=1}^{n-1} P_j \sum_{k_0}^{j} \frac{1}{\lambda_k P_k} \sum_{l=k}^{n-1} P_l \sum_{m=0}^{l} \frac{1}{\lambda_m P_m}. \quad (13)$$

With the help of formulae (12) and (13), it is possible to calculate $D_i \tau_n = d_i - m_i^2$ and later on, having used Chabyschev's inequality, it is possible to calculate the probability of preserving freight transport work ability (without reloading) for set time.

At a later stage of the analysed task (the general characteristics of developing private freight transport), the characteristics of resource use per one time unit, for instance, per year (increase of loading) should be known.

For the analysis of the above mentioned characteristics (Reiman *et al.* 1999), the so called proliferation and fall process 'with migration' characterised by the shifting probabilities are introduced:

$$\lambda_i = i\lambda + \beta, \; M_i = iM.$$

Here, constant β is introduced with the aim of avoiding the interruption of the process, in case it reaches zero status at some time. In a certain case, when $\beta = 0$, this process converges into the Candel's process (the process when the intensities of both shiftings are proportional to the number order of the status) (Crainic 2000; Chakroborty and Wivedi 2002).

Let us denote $b(t) = M_{bn}$ as the medium proliferation number over time $(0, t)$ and $d(t) = M_{dn}$ as the medium number of fall per time $(0, t)$. Then, the medium amount of increase will be $m(t) = i + b(t) - d(t)$, where $d(t)$ – the status of the process at time moment 0.

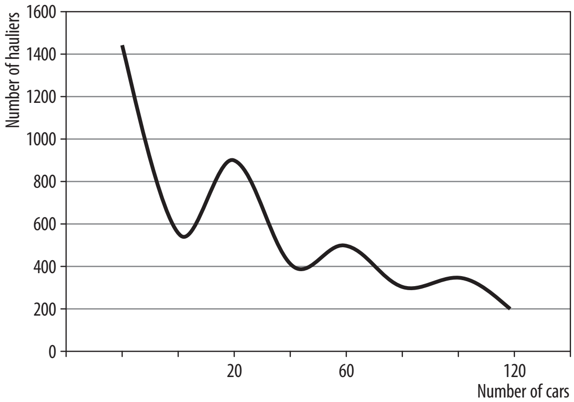
Having expressed by *t* year, it is possible to identify an annual increase:

$$q(t) = \left(m(t+1) - m(t)\right) / m(t).$$

Having used (Crainic 2000), it is possible to find an annual increase:

$$q(t) = \begin{cases} \dfrac{\left[i + \beta/\Delta\right] e^{\Delta t}\left(e^{\Delta} - 1\right)}{\left[i + \beta/\Delta\right] e^{\Delta t} - \beta/\Delta}, & \text{when } \lambda \neq \mu; \\ \beta / \left(i + \beta t\right), & \text{when } \lambda = \mu, \end{cases}$$

where the admitted noting is $\Delta = \lambda - \mu$. It is evident that for selecting the coefficient, it is possible to define increase ratio. Over the course of time, medium increase may be defined:



Calculation results

$$\bar{q} = \begin{cases} 0, \text{ when } \lambda < \mu, \beta \neq 0, \\ 0, \text{ when } \lambda = \mu, \\ e^{\Delta} - 1 \quad \text{in other cases.} \end{cases}$$

Having obtained the statistical data and algorithm of task solution, it is possible to model a demand for suitable transport means.

The figure demonstrates calculation results according to the model presented, using the statistical information of ten years.

Similar results are also obtained in the analysis of passenger road transport.

In transition from the planned to economy market, transport undergoes very big structural changes particularly visible in road transport. It is defined that non-stationary processes are characteristic of the development of freight road transport. Therefore, a conclusion was made that the process of freight road transport development under the conditions of competition might be analysed as the process of proliferation and fall. It is with regard to these processes that changes in Lithuanian freight road transport were assessed. This model may be also applied to other transport modes; however, the obtained results of calculation may be less precise because, for example, the quantities of transport means and statistical information sets are far less.

4. Conclusions

1. To assess transport development efficiency in terms of costs and results, the following principle has to be observed: costs spent on transport development have to be no less than efficiency received using funds in some other branches of national economy.
2. The interaction of different transport modes, their own interests and aims as well as the criteria for assessing the mechanism of functioning determine the occurrence of systemic effects, i.e. parameters that cannot be separated and assessed according to the separate characteristics of transport modes and may be defined only in their interaction.

   The assessment of systemic effects is very important for the analysis of complex transport because of the fact that orientation towards the local effect of different transport modes leads to solutions which are not always useful to the entire system. Therefore, a relevant mechanism for the co-ordination of interests is necessary. The co-ordination of this mechanism has to guarantee the expediency of solutions to users of the entire transport system. In such situation, there may be certain cases when the efficiency of the entire system does not coincide with the sum of efficiencies of its separate elements.

3. Under conditions of market economy, non-stationary processes are characteristic of the development of separate transport modes. A suggestion was made to analyse (under competition conditions) the process of developing freight transport modes as a process of reproduction and break-up. Having obtained relevant statistical data on separate transport modes, with the help of the suggested algorithm, we can model the development trends of certain transport modes.

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