TRANSPORT
2009
24(2): 170–179

INVESTIGATION INTO THE STABILITY OF DRIVING AN AUTOMOBILE ON THE ROAD PAVEMENT WITH RUTS

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Received 13 October 2008; accepted 10 April 2009

Abstract. The article reviews the stability of an automobile on the road pavement with ruts. The problems of road safety depend on many factors. One of those is the quality of the road surface. Big heavy lorries deform the road surface causing ruts especially in the first line of the road. The article analyzes the influence of the geometry of ruts on the stability of the automobile. A mathematical model of a 3D automobile and the interaction between wheels and the road surface and the geometry of the road surface are presented. The mathematical criterion of the stability of the moving automobile on the road surface with ruts is laid down. The results of mathematical simulation and the criterion of the stability of the moving automobile (60 km/h) on the road surface depending on the depth of ruts are introduced.

Keywords: automobile dynamics, 3D dynamics model, roughness, ruts, interaction, stability criterion, numerical method.

1. Introduction

Due to improvements in technology, the necessity of having a complex evaluation of the interaction among the automobile, its wheel and the road appears. The evaluation is based on determining general touch points and searching for optimal decisions. Such a complex attitude allows increasing the efficiency of the power of the automobile, improvements to driving comfort and the reduction of road impact on the exploitation features of the automobile.

The continuity of motion is one of the exploiting features of the automobile as it determines the driving comfort of the passenger, the driver him/herself and the safety of cargo. When the automobile is moving on a rough (rutted) road in the wide speed stretch, many various elements make an influence on its stability and the safety of people. Although we are living in modern times, traffic accidents are still one of the sorest problems. The elements influencing traffic accidents fall into several groups embracing the road, the automobile, the environment and human. All these elements can be subdivided into smaller groups in order to evaluate the elements making an impact on traffic accidents.

One of the pressing problems is ruts on the road. When driving an automobile on the road with ruts and in case asphalt pavement is wet or covered with snow or ice, the stability of the automobile can be quickly reduced, and thus a traffic accident with severe results may happen.

In order to evaluate forces having an impact on the automobile and the way it moves in regard with ruts and asphalt pavement, it is necessary to create a mathematical model for the automobile and evaluate its stability.

The majority of scientists have a goal to create a precise model of the automobile able to be close to the real one and to obtain correct parameters allowing the determination of the following aspects: how does the automobile react to driving on the uneven road, how does its frame deform in case of a traffic accident and what kind of deformations it would incur at the moment of the strike. For this reason, in order to recreate the automobile, the model using finite elements or mobile hinges is produced (Sousa *et al.* 2008). The model of the created automobile is tested by using a large quantity of various crash scenarios determined by the international standards such as EuroNCAP. The automobile model also can be used for developing some kind of safety systems or for improving the already used safety systems of the automobile body.

In order to avoid crashing in case of an obstruction, the automobile has to be stable. Stability is greatly

ISSN 1648-4142 print / ISSN 1648-3480 online
www.transport.vgtu.lt
DOI: 10.3846/1648-4142.2009.24.170-179

influenced by a running gear. One of the main elements of the running gear is buffer (Gonçalves and Ambrósio 2003 and 2005) the main features of which are processing road roughness, the quality of controlling the automobile and the comfort of passengers. These functions are evaluated and analyzed in an article of one of the Italian scientists (Arcidiacono *et al.* 2001). When modeling passive and active running gears, the tightness of springs has to be evaluated (Schiehlen *et al.* 2007; Liberzon *et al.* 2001).

In order to analyze the fluctuation of the automobile due to changing the acceleration of deceleration, the fluctuation processes of amortizing and non amortizing masses have to be determined when there is a tangential effect on the wheel at its contact point with a supporting surface. After determining the processes of mass fluctuation, the calculations of automobile braking parameters could be specified. For this reason, (Pečeliūnas *et al.* 2005) analysis methodology suggested by the author covers making the mathematical algorithms of analyzing processes and theoretical calculations and giving various elements having an influence on the fluctuation of the automobile when breaking. Numerical and analytical methods are applied to conduct research. Also experimental researches are performed using decelometer VZM 100 for recording acceleration and deceleration forces. Using the made mathematical model, the expert possibilities of modeling the motion of the automobile when investigating the circumstances of the traffic accident related to breaking the automobile are increasing (Pečeliūnas *et al.* 2005; Peceliunas and Prentkovskis 2006).

To ensure the stability of the automobile, the interaction between the road and the tire is very important. For this reason, various road pavement – automobile models are created with the help of which the impact of the road roughness on the automobile is sought to be described (Rouillard 2008; Gonçalves and Ambrósio 2003 and 2005; Pacejka 2006).

Moreover, close attention is paid to modeling a situation on vehicle movement on a particular road surface. The suggested way to solve this problem is given in the model of the mathematical system 'vehicle – road'. This article presents that the automobile is modelled by concentrated masses interacting between each other by tough and dissipative connections. This model evaluates the movement of the body in space, the movement of the front and back suspension in respect of the automobile's body, the interaction of wheels in respect of the surface of the road pavement, locking wheels and the alternation of cohesion with the surface of road pavement forces having an influence on the vehicle. The road pavement surface is modeled with the help of finite elements. The specific roughness height of the road pavement surface and the cohesion coefficients of the pavement and automobile wheel in the longitudinal or transversal directions are chosen in every knot of the finite element. The supplied results of modeling include, i.e. graphs illustrating vehicle movement on the road, its braking in case of various conditions and vehicle movement on the road with a speed reduction knoll ('lying policeman') (Prentkovskis and Bogdevičius 2002; Bogdevičius *et al.* 2009).

The article (Prentkovskis and Sokolovskij 2008) describes a traffic accident where the automobile crashes into a sidewalk losing its balance and overturns or careers off the road to a ditch. When modeling this traffic accident, the automobile was imitated like a single concentrated mass joined by elastic 'Kelvin-Foight' elements. Using this model, the forces inducing inertia have been determined.

The dynamics of the automobile, its stability assuring the additional measures of controlling steadiness and their influence on the safety of the automobile and passengers are described in the book released in the United States of America (Rajamani 2006).

The article discusses a dynamic model of the automobile having eight degrees of freedom and an active running gear. The model moves along the road without definite logic (Fuzzy Logic) and is used to obtain information about the influence made on a driver in case of such fluctuation. Modelling is performed in several cases. First, fluctuations in a driver's seat are evaluated. Second, fluctuations in the position of the automobile are separately evaluated. In the third case, fluctuations in the automobile and its driver are simultaneously evaluated. Thus, the level of comfort provided in the automobile in respect of its driver and passengers is going to be evaluated (Guclu 2005).

Calculation methods used for the accuracy evaluation of the results of road accident examination are analyzed by Nagurnas *et al.* (2008).

Prentkovskis *et al.* (2007) investigated potential deformations developed by the elements of transport and pedestrian traffic restricting gates during motor vehicle–gate interaction.

Kinderytė-Poškienė and Sokolovskij (2008) investigated the impact of traffic control elements on accidents, mobility and the environment.

Research by Sokolovskij (2007) depicts the results of investigation into automobile braking parameters and traction characteristics on different road surfaces.

The evaluation of the veracity of car braking parameters used for analyzing road accidents is reviewed by Nagurnas *et al.* (2007).

Sokolovskij *et al.* (2007) investigated the interaction between the automobile and road border.

Viba *et al.* (2009) reviewed the theory of vehicle collision with pit corner.

2. The Mathematical Model of Moving the Automobile on the Uneven Road Surface

The geometry of the uneven road surface is described by isoparametric finite elements having nine nodes (Fig. 1).

Contact condition between wheel point $\left\{R_p\right\}$ and the element of the road pavement is:

$$\left\{n_9\right\}^T \left\{R_{9p}\right\} < 0, \tag{1}$$

where

$$\{n_9\} = \{e_{91}\} \times \{e_{92}\} = \left[\tilde{e}_{91}\right]\{e_{92}\}. \quad (2)$$

Is the unit vector normal to the element surface in the ninth node,

$$\{e_{91}\} = \left(\frac{\partial\{R_9(\eta=0,\xi=0)\}}{\partial\xi}\right),$$
$$\{e_{92}\} = \left(\frac{\partial\{R_9(\eta=0,\xi=0)\}}{\partial\eta}\right), \quad (3)$$

where $\left[\tilde{e}_{91}\right]$ is the skew-symmetric matrix associated with $\{e_{91}\}$ vectors,

$$\left[\tilde{e}_{91}\right] = \begin{bmatrix} 0 & -e_{91z} & e_{91y} \\ e_{91z} & 0 & -e_{91x} \\ -e_{91y} & e_{91x} & 0 \end{bmatrix},$$

$$\{R_{9p}\} = \{R_p\} - \{R_9\}, \quad (4)$$

where $\{R_9\}$ is the vector of the ninth node of the finite element in the global coordinate system *XY*; η, ξ are the local coordinates of the element.

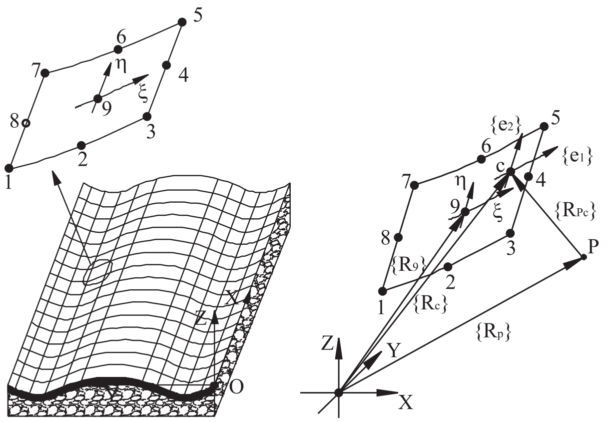


Fig. 1. The geometry of the uneven road surface

Contact point $\{R_c(\xi_c,\eta_c)\}$ on the surface of the road pavement in the global coordinate system *XYZ* is equal:

$$\{R_c(\xi_c,\eta_c)\} = \begin{Bmatrix} X_c \\ Y_c \\ Z_c \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{9} N_i(\xi_c,\eta_c) X_i \\ \sum_{i=1}^{9} N_i(\xi_c,\eta_c) Y_i \\ \sum_{i=1}^{9} N_i(\xi_c,\eta_c) Z_i \end{Bmatrix}, \quad (5)$$

where $N_i(\xi_c,\eta_c)$ is the shape function of the ninth node of the finite element, i=1,.., 9; X_i, Y_i Z_i are the global coordinates of the finite element nodes; ξ_c, η_c are the local coordinates of points C.

Local coordinates ξ_c, η_c in the finite element are obtained from the next system of nonlinear algebraic equations:

$$\{\Phi\} = \begin{Bmatrix} \left(\frac{\partial\{R_c\}}{\partial\xi_c}\right)^T \left(\{R_c(\xi_c,\eta_c)\} - \{R_P\}\right) = 0 \\ \left(\frac{\partial\{R_c\}}{\partial\eta_c}\right)^T \left(\{R_c(\xi_c,\eta_c)\} - \{R_P\}\right) = 0 \end{Bmatrix} = 0. \quad (6)$$

The system of algebraic equations is solved by the Newton-Raphson method:

$$[J_i]\{\Delta X_i\} = -\{\Phi(X_i)\}, \quad (7)$$

where $[J]_i$ is Jacobs matrix, $[J]_i = \left[\frac{\partial\{\Phi\}}{\partial\{X\}^T}\right]$; $\{X_i\}^T = [\xi_{ci}, \eta_{ci}]$; $\{\Delta X_i\}^T = [\Delta\xi_c, \Delta\eta_c]$ is the vector of increments; $\{X_{i+1}\} = \{X_i\} + \{\Delta X_i\}$.

Penetration and its derivative by the time of point $\{R_p\}$ on the road surface are equal:

$$\Delta = \sqrt{\{R_{pc}\}^T \{R_{pc}\}}, \quad (8)$$

$$\dot{\Delta} = \frac{1}{\Delta}\{R_{pc}\}^T \{\dot{R}_{pc}\}, \quad (9)$$

$$\{R_{pc}\} = \{R_c(\xi_c,\eta_c)\} - \{R_p\},$$

and

$$\{\dot{R}_{pc}\} = \{\dot{R}_c(\xi_c,\eta_c)\} - \{\dot{R}_p\}.$$

The mechanical forces of virtual work acting on the element *e* are equal:

$$\delta W = \delta\{q\}^T \{Q_{12}\}, \quad (10)$$

where $\delta\{q\}$ is the variation of the displacement vector; $\{Q_{12}\}$ is the vector of generalized forces.

The distance between points 1 and 2 (see Fig. 2) is equal:

$$L_{12} = \sqrt{\{X_{21}\}^T \{X_{21}\}}, \quad (11)$$

where

$$\{X_{21}\} = \{R_2\} - \{R_1\},$$

$$\{R_1\} = \{R_{i0}\} + \{q_i\} + [A_i(\theta_i)]\{r_{i1}\},$$

$$\{R_2\} = \{R_{j0}\} + \{q_j\} + [A_j(\theta_i)]\{r_{j2}\},$$

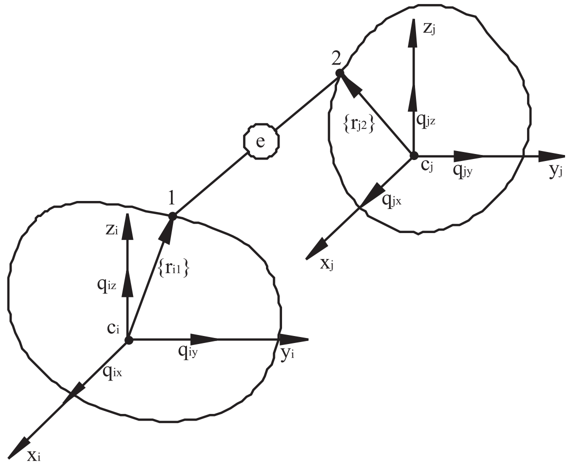


Fig. 2. The distance between points 1 and 2

where: $\{R_{i0}\}$, $\{R_{j0}\}$ are the initial vectors of the mass centers of bodies *i* and *j*; $\{q_i\}$ and $\{q_j\}$ are the generalized coordinates of bodies *i* and *j* in the coordinate systems of the bodies, respectively; $[A_i(\theta_i)]$, $[A_j(\theta_j)]$ are the rotation matrices of bodies *i* and *j*, respectively; $\{\theta_i\}$ and $\{\theta_j\}$ are the vectors of Cardan angles of bodies *i* and *j*, respectively.

Rotation matrix $[A(\theta)]$ with Cardan angles $\{\theta\}^T=[\theta_1 \quad \theta_2 \quad \theta_3]$ is equal:

$$[A(\theta)]=\begin{bmatrix} c_2c_3 & -c_2s_3 & s_2 \\ c_1s_3+s_1s_2c_3 & c_1c_3-s_1s_2s_3 & -s_1c_2 \\ s_1s_3-c_1s_2c_3 & s_1c_3+c_1s_2s_3 & c_1c_2 \end{bmatrix}, \quad (12)$$

where $s_i=\sin(\theta_i)$, $c_i=\cos(\theta_i)$, $i=1,2,3$; $\{r_{i1}\}$ is a vector between the mass center of body *i* and point 1; $\{r_{j2}\}$ is a vector between the mass center of body *j* and point 2.

The relation between a derivative of the vector of Cardan angles by time and an angular velocity of the body in the body coordinate system:

$$\{\dot{\theta}\}=[G_1(\theta)]\{\omega_{body}\}, \quad (13)$$

$$[G_1(\theta)]=\begin{bmatrix} \frac{c_3}{c_2} & -\frac{s_3}{c_2} & 0 \\ s_3 & c_3 & 0 \\ -\frac{s_2}{c_2}c_3 & \frac{s_2}{c_2}s_3 & 1 \end{bmatrix}; \; \{\dot{\theta}\}\equiv\frac{d}{dt}\{\theta\};$$

$$\{\omega_{body}\}=[\dot{\phi}_x \quad \dot{\phi}_y \quad \dot{\phi}_z]=\frac{d}{dt}\{\phi\},$$

where $\{\phi\}^T$ is the vector of the angles of the axes of body coordinates.

Relation between an angular velocity of the body in the body coordinate system and the derivative of the vector of Cardan angles by time is equal:

$$\{\omega_{body}\}=[G_2(\theta)]\{\dot{\theta}\}, \quad (14)$$

where $[G_2(\theta)]=\begin{bmatrix} \cos(\theta_2)\cos(\theta_3) & \sin(\theta_3) & 0 \\ -\cos(\theta_2)\sin(\theta_3) & \cos(\theta_3) & 0 \\ \sin(\theta_2) & 0 & 1 \end{bmatrix}$.

The variation of length L_{21} is equal:

$$\delta L_{21}=\frac{1}{L_{21}}\delta\{X_{21}\}^T\{X_{21}\}, \quad (15)$$

where $\delta\{X_{21}\}$ is the variation of vector $\{X_{21}\}$,

$$\delta\{X_{21}\}=[[B_i], \;[B_j]]\delta\begin{Bmatrix}\{q_i\}\\ \{q_j\}\end{Bmatrix}=[B]\delta\{q_{12}\},$$

$$[B]=[[B_i], \;[B_j]],$$

$$[B_i]=[-[E] \quad [A_i(\theta_i)][\tilde{r}_{i1}][G_1(\theta_i)]],$$

$$[B_j]=[[E] \quad -[A_j(\theta_j)][\tilde{r}_{j2}][G_1(\theta_j)]],$$

$$\{q_{12}\}^T=[\{q_i\}^T \quad \{q_j\}^T],$$

where $[\tilde{r}_{i1}]$, $[r_{j2}]$ are skew-symmetric matrices associated with vectors $\{r_{i1}\}$, $\{r_{j2}\}$, respectively; $\{q_{12}\}$ is the vector of generalized displacements.

The generalized force vector is obtained using expression:

$$\{Q_{12}\}=\begin{Bmatrix}\{Q_{i12}\}\\ \{Q_{j12}\}\end{Bmatrix}=\begin{Bmatrix}\frac{F_{12}}{L_{12}}[B_i]^T\{X_{21}\}\\ \frac{F_{12}}{L_{12}}[B_j]^T\{X_{21}\}\end{Bmatrix}, \quad (16)$$

where $\{Q_{i12}\}$, $\{Q_{j12}\}$ are the generalized force vectors of bodies *i* and *j*, respectively; F_{12} is force in the element *e*,

$$F_{12}=-k_{12}(L_{12}-L_{120})-c_{12}\dot{L}_{12}-F_{f12}sign(\dot{L}_{12}), \quad (17)$$

where $k_{12}=k_{12L}+k_{12NL}(\Delta L_{12})$ and $c_{12}=c_{12L}+c_{12NL}(\Delta L_{12},\dot{L}_{12})$ are linear and nonlinear stiffness and the damping coefficient, respectively; L_{120} is the initial length of element *e*; $\dot{L}_{12}$ is a derivative by the time of L_{12},

$$\dot{L}_{12}=\frac{1}{L}\{X_{21}\}^T\{\dot{X}_{21}\}=\frac{1}{L_{12}}\{X_{21}\}^T[B]\{\dot{q}_{12}\}, \quad (18)$$

where F_{f12} is dry friction force.

Aerodynamic forces acting on driving the automobile depend on the wind, the characteristics of the automobile and time. In the global coordinate system *OXYZ*, the aerodynamic force is described by the wind velocity V_{wind} and its direction angle α_{wind} (angle between X axes and velocity direction, Fig. 3). Fig. 3 shows normal and unit vectors $\{e_i\}$, i=1,..4 described the in body coordinate system $cx_cy_cz_c$:

$$\{e_1\}^T = [1 \quad 0 \quad 0]; \{e_2\}^T = [-1 \quad 0 \quad 0];$$
$$\{e_3\}^T = [0 \quad 1 \quad 0]; \{e_4\}^T = [0 \quad -1 \quad 0]. \qquad (19)$$

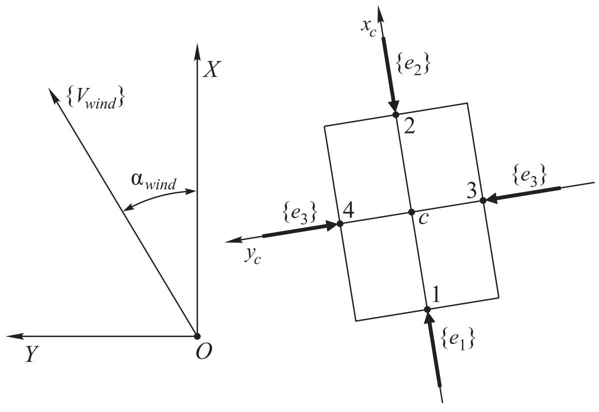


Fig. 3. The angle between X axes and velocity direction

The vector of wind direction in the body coordinate system $cx_cy_cz_c$ is equal:

$$\{v_{wind}\} = [A(\theta)]^T \{V_{wind}\}, \qquad (20)$$

where $\{V_{wind}\}$ is the vector of wind direction in the global coordinate system $OXYZ$; α_{wind} is the angle of wind direction (see Fig. 3)

$$\{V_{wind}\}^T = \left[V_{wind}\cos(\alpha_{wind}) \quad V_{wind}\sin(\alpha_{wind}) \quad 0\right]. \qquad (21)$$

The total vector of aerodynamic forces in the body coordinate system $cx_cy_cz_c$ is equal:

$$\{f_{wind}\} = \sum_{i=1}^{4} f_{wi}\{e_i\}, \qquad (22)$$

where $f_{wi} = \frac{1}{2}\rho_{air}C_iS_iv_i^2$; ρ_{air} is air density,

$$\rho_{air} = 1.225\left(\frac{p_{air}}{0.101325} + \frac{288.16}{T_{air}}\right), \qquad (23)$$

where p_{air} , T_{air} are the pressure and temperature of the air; C_i is the aerodynamic drag coefficient; S_i is the area of the i-th side of the automobile; v_i velocity,

$$v_i = \begin{cases} \{e_i\}^T(\{v_{wind}\} - \{\dot{q}_c\}), \text{ if } \{e_i\}^T(\{v_{wind}\} - \{\dot{q}_c\}) > 0, \\ 0, \text{ if } \{e_i\}^T(\{v_{wind}\} - \{\dot{q}_c\}) \le 0, \end{cases} \qquad (24)$$

where $\{\dot{q}_c\}$ is the vector of displacing the automobile's body in the body coordinate system $cx_cy_cz_c$.

The vector of aerodynamic moments acting on the automobile's body is equal:

$$\{M_{wind}\} = \sum_{i=1}^{4}[\tilde{r}_{wi}]\{f_{wind}\}, \qquad (25)$$

where $[\tilde{r}_{wi}]$ is the skew-symmetric matrix associated with $\{r_{wi}\}$ vector. Vector $\{r_{wi}\}$ describes the geometrical centre in the i-th side of the automobile in the body coordinate system $cx_cy_cz_c$.

The system of the equations of the automobile's body in the body coordinate system is equal:

$$[M_c]\{\ddot{q}_{body}\} = \{F_{body}\}, \qquad (26)$$

where $[M_c]$ is the mass matrix of the automobile's body,

$$[M_c] = \begin{bmatrix} [M_{q_cq_c}] & [0] \\ [0] & [I_{\phi_c\phi_c}] \end{bmatrix};$$

$[M_{q_cq_c}]$, $[I_{\phi_c\phi_c}]$ are the mass matrices of rotational and rotational movement, respectively; $\{\ddot{q}_{body}\}$ is the acceleration vector, $\{\ddot{q}_{body}\}^T = [\{\ddot{q}_c\} \quad \ddot{\phi}_c]$; $\{F_{body}\}$ is the vector of generalized forces,

$$\{F_{body}\} = \begin{Bmatrix} \{F_{q_c}\} \\ \{F_{\phi_c}\} \end{Bmatrix} = \begin{Bmatrix} \{F_{q_c}(t, q_{body}, \dot{q}_{body})\} \\ \{F_{\phi_c}(t, q_{body}, \dot{q}_{body})\} - [\tilde{\omega}_c][I_c]\{\omega_c\} \end{Bmatrix}; \qquad (27)$$

$[\tilde{\omega}_c]$ is the skew-symmetric matrix associated with $\{\omega_c\} = \begin{Bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{Bmatrix}$ vector; $[I_c]$ is the inertia tensor of the automobile's body, $[I_c] = \begin{bmatrix} I_{x_cx_c} & I_{x_cy_c} & I_{x_cz_c} \\ I_{y_cx_c} & I_{y_cy_c} & I_{y_cz_c} \\ I_{z_cx_c} & I_{z_cy_c} & I_{z_cz_c} \end{bmatrix}$.

Longitudinal and lateral forces in the k-th wheel are given by:

$$F_{xk} = F_{zk}\mu_0\left\{\frac{\lambda_x}{\lambda_0}\cos(\alpha_k) + \frac{\lambda_y}{\lambda_0}\sin(\alpha_k)\right\};$$
$$F_{yk} = F_{zk}\mu_0\left\{\frac{\lambda_y}{\lambda_0}\cos(\alpha_k) - \frac{\lambda_x}{\lambda_0}\sin(\alpha_k)\right\}, \qquad (28)$$

where μ_0 is the friction coefficient; α_0 is slip angle; F_{zk} is normal force to surface,

$$F_{zk} = \begin{cases} -k\left(z_p - z_k\right) - c\dot{z}_p, \text{ when } z_p < z_k \\ 0, \text{ when } z_p \geq z_k, \end{cases} \quad (29)$$

where: k, c are the coefficients of stiffness and damping; z_k is z coordinate of the road surface; z_p is z coordinate of the contact point of k-th wheel; λ_{xk} and λ_{yk} are longitudinal and lateral slip ratios, respectively,

$$\lambda_{xk} = \frac{R_{dk}\omega_k - v_{xk}}{\max\left(R_{dk}\omega_k, v_{xk}\right)};$$

$$\lambda_{yk} = \begin{cases} \mathrm{tg}\left(\alpha_k\right), \text{ when } R_{dk}\omega_k \sin\left(\alpha_k\right) > v_{xk}\cos\alpha_k, \\ \dfrac{v_{xk}\mathrm{tg}\left(\alpha_k\right)}{R_{dk}\omega_k}, \text{ when } v_{xk}\cos\alpha_k \neq 0, \end{cases} \quad (30)$$

where: ω_k is wheel angular velocity; R_{dk} is dynamic wheel radius; λ_{0k} is the total slip

$$\lambda_{0k} = \sqrt{\lambda_{xk}^2 + \lambda_{yk}^2}; \quad (31)$$

v_{xk} is the longitudinal velocity of the automobile.

The vector of the velocity of the contact point of the k-th wheel in the local wheel coordinate system is equal:

$$\{v_k\} = \left[A\left(\alpha_k\right)\right]^T \left[A(\theta)\right]^T \{V_k\}, \quad (32)$$

where $\{V_k\} = \left[V_{xk} \quad V_{yk} \quad V_{zk}\right]$ is the vector of velocity in the global coordinate system ($OXYZ$):

$$\begin{aligned} &\{V_k\} = \{V_c\} + \\ &\left[A(\theta)\right]\left[\tilde{\dot{\theta}}\right]\left(\{r_{cok}\} + \left[A\left(\alpha_k\right)\right]\{q_k\}\right) + \\ &\left[A(\theta)\right]\{\dot{r}_{cok}\} + \left[A(\theta)\right]\left[A\left(\alpha_k\right)\right]\{\dot{q}_k\} + \\ &\dot{\alpha}_k\left[A(\theta)\right]\frac{d\left[A\left(\alpha_k\right)\right]}{d\alpha_k}\{q_k\}, \end{aligned} \quad (33)$$

where: $\{V_c\}$ is the vector of the velocity of the automobile's body; $\left[\tilde{\dot{\theta}}\right]$ is the skew-symmetric matrix associated with vector $\{\theta\}$, $\{r_{cok}\}$, $\{\dot{r}_{cok}\}$ are the vectors between the center of the automobile's body and contact point k in the body coordinate system and a derivative by time, respectively; α_k, $\dot{\alpha}_k$ is rotation angle and the angular velocity of the wheel.

The system of the equations of the automobile in motion is written as (Prentkovskis and Bogdevičius 2002; Prentkovskis and Sokolovskij 2008; Bogdevičius *et al.* 2009):

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F(q, \dot{q}, t)\}, \quad (34)$$

where $[M]$, $[C]$, $[K]$ are mass, damping and stiffness matrixes, respectively; $\{F(q, \dot{q}, t)\}$ are load vectors.

In order to evaluate road and the stability of the automobile in motion on the uneven surface with ruts, the following criterion is suggested:

$$MBV = \frac{1}{gI_{z_c z_c}}\sqrt{\frac{1}{T}\int_0^T M_{z_c}^2(t)\,dt}, \quad (35)$$

where M_z is the summary moment of cohesion forces around body's axis z; T is general movement time; $I_{z_c z_c}$ is the overall automobile mass moment of inertia around z axis; g is gravity acceleration.

3. Simulation Results

The automobile's movement on the uneven road surface is analyzed in case of the ruts of specific width and depth. The wheels of the automobile are rotated following this law:

$$\alpha = a \sin(\omega_\alpha t),$$

where a is wheel turn amplitude; ω_α is the angular speed of the wheel turn, $\omega_\alpha = 2\pi \cdot 0.7$ rad/s.

The amplitude of the wheel turn has to be chosen placing a wheel in a rut. The dependency of the amplitude of the wheel turn on the speed and width of a rut when the automobile is moving can be seen in Fig. 4.

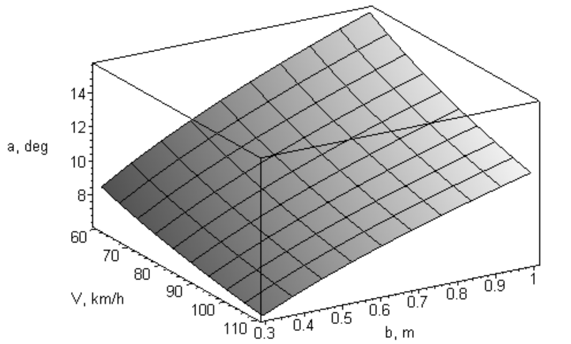


Fig. 4. The dependencies of wheel turn amplitude a on the width of rut b and automobile movement speed v

The body mass of the analyzed automobile is 1455 kg and mass inertia moments are: $I_{x_c x_c} = 603\,\mathrm{kg\,m^2}$; $I_{y_c y_c} = I_{z_c z_c} = 2012\,\mathrm{kg\,m^2}$. Research on the movement stability of the automobile on the uneven road surface with ruts of $\Delta = 0$ ...0.05 m depth when the speed of the automobile is 60 km/h has been carried out. The analyzed road surface includes dry, wet, snowed and icy asphalt road. Depending on a selected surface of the asphalt road, different friction coefficients μ were chosen while modeling. These coefficients depend on the coefficient of longitudinal skidding (Fig. 5). Fig. 6. shows the alternation of every wheel friction coefficient in case of different types of road pavement. Fig. 7 shows the alternation of MBV – the stability criterion of the automobile in case of a different type of the road pavement and the depth of ruts. Fig. 8 shows the alternation of MBV – the stability criterion of the automobile in case of different road pavement types, the depth of ruts and side wind impact.

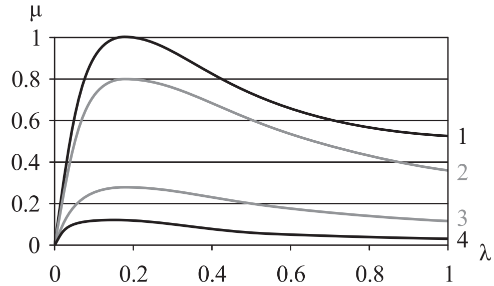
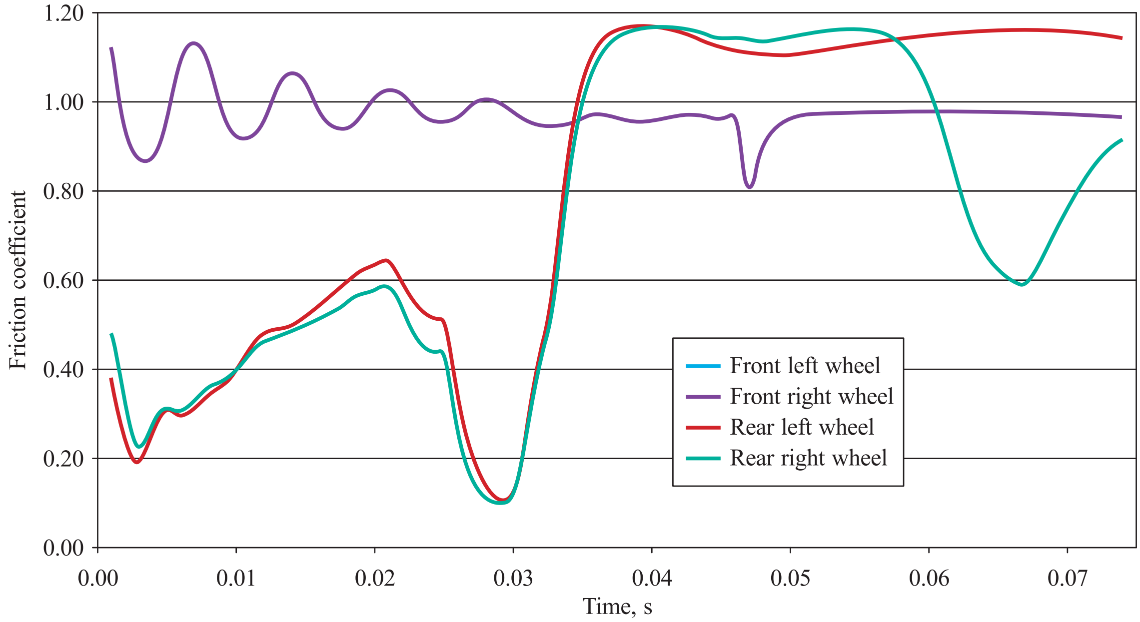
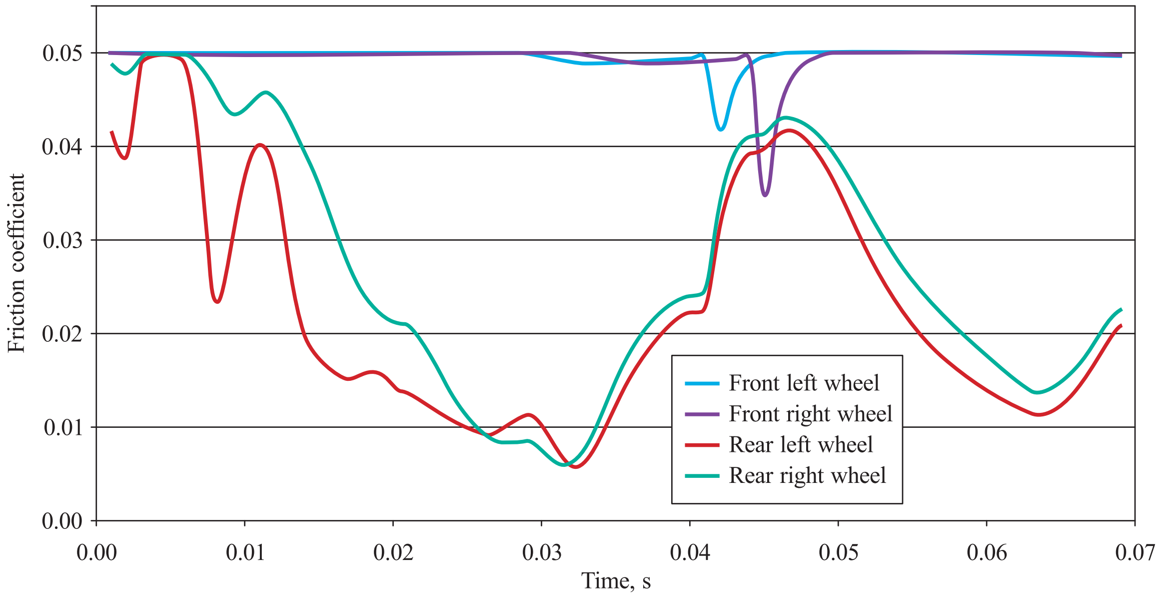


Fig. 5. The dependence of the friction coefficient μ on the longitudinal relative skidding coefficient λ: 1 – dry asphalt, 2 – wet asphalt, 3 – snowed asphalt; 4 – icy asphalt

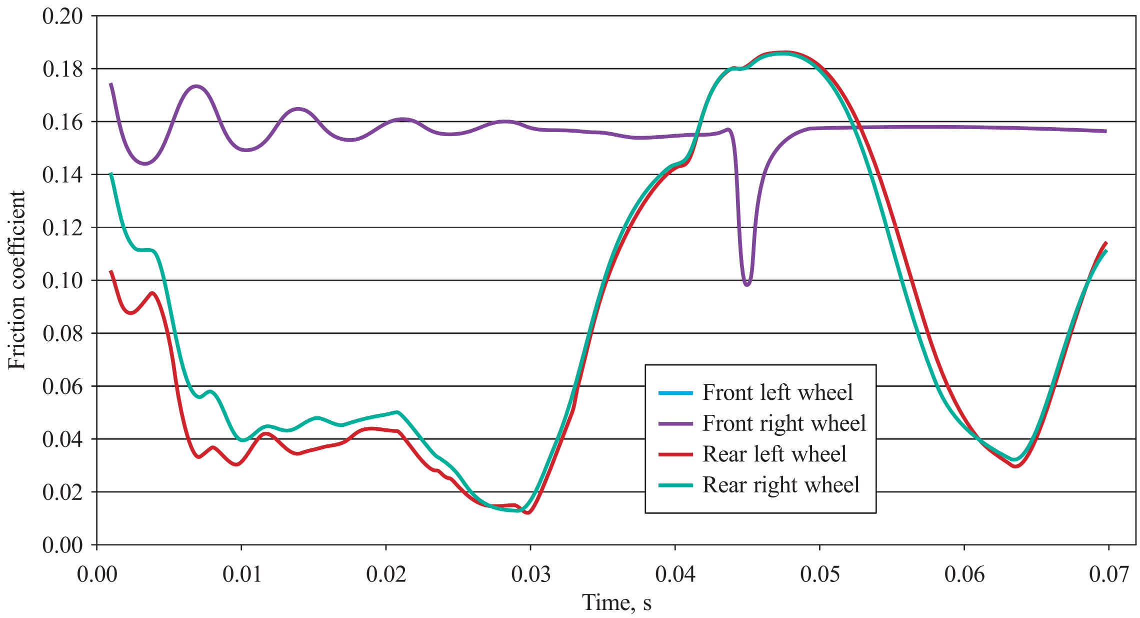


a

b



c



d

Fig. 6. The alternation of the wheel friction coefficient in time: rut depth – 0.05 m; driving speed – 60 km/h; wheel rotation amplitude – 7 degrees: a – dry pavement; b – wet pavement; c – snowed pavement; d – icy pavement

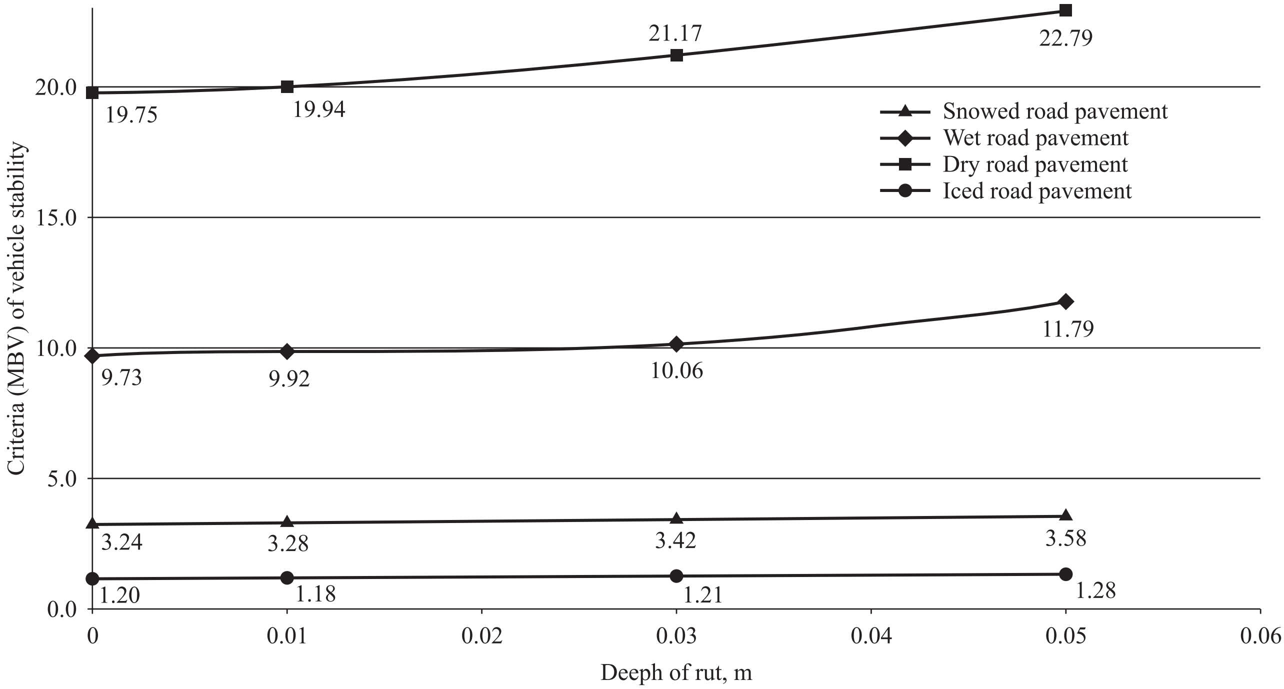


Fig. 7. The dependency of stability criterion MBV of the automobile on the depth of ruts and the type of the road pavement: the depth of ruts – 0.05 m, driving speed – 60 km/h; wheel rotation amplitude – 7 degrees

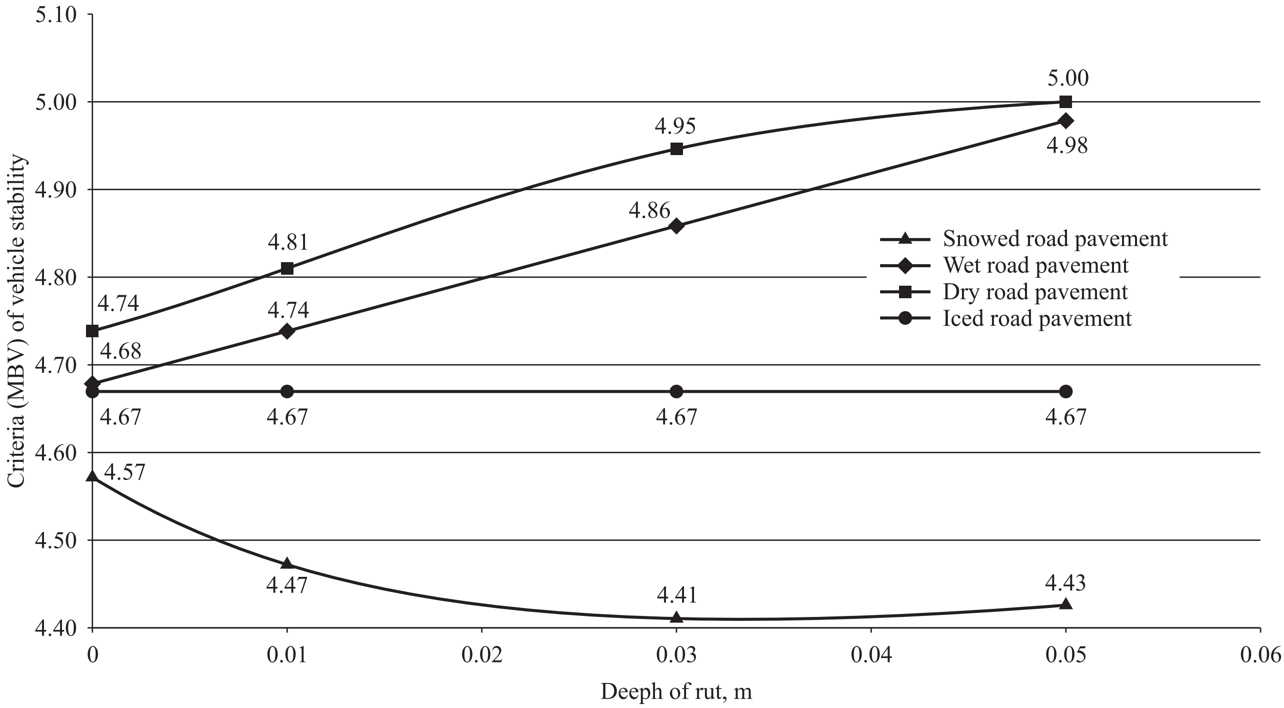


Fig. 8. The dependency of stability criterion MBV of the automobile on the depth of the ruts and type of road pavement: the depth of ruts – 0.05 m; driving speed – 60 km/h; side wind – 30 m/s

4. Conclusions

1. It is determined that when the driving speed of the automobile is 60 km/h on the uneven road surface with ruts, its movement is not stable and safe if the depth of ruts is up to 0.05 m and the width makes approx. 0.50 m.
2. It is determined that when the driving speed of the automobile is 60 km/h on the snowed or icy road with ruts, it becomes dangerous.
3. It is determined that when the driving speed of the automobile is 60 km/h on the road with ruts in case of a different type of the road pavement, its stability criterion MBV alternates from 0 to 21. The suggested automobile stability criterion can be used in order to describe the automobile's movement on the uneven road with ruts; however more detailed analysis should be made.
4. The created mathematical 3D model of the automobile can be used for further research on the automobile's stability when driving on uneven road surfaces.

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