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PRINCIPLES FOR MODELLING TECHNOLOGICAL PROCESSES IN TRANSPORT TERMINAL

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Abstract. A technological process is evaluated as a random process and assessed in respective models. The methodology of formalizing technological processes in the terminal and criteria for optimal control and the quality of the technological process are suggested. In addition, models and algorithms for the optimal control of freight clearing technological process are proposed.

Keywords: transport terminal, technological process, optimal control, models and algorithms.

1. Introduction

The following technological operations are carried out with freight in transport terminal: unloading from vehicles, storage, grouping, distribution according to transportation routes, packing in larger transport units, direct reloading from one vehicle to another, loading goods to vehicles. In case containers are cleared in the terminal, technological freight clearing processes have a higher nomenclature of the executed operations. According to the world experience of transportation in containers, 10–20 per cent of freight is delivered to container terminals in small consignments. Afterwards these consignments are grouped according to dispatch routes and loaded to containers. There are also cases, when forwarders have not their own containers, deliver all goods to the terminal in light packages (after having agreed with terminal authorities) and later, the staff of the terminal loads the goods to containers (upon the agreement) (Baublys 2008; 2003, 2007a, 2007b; Baublys and Petrauskas 2002; Bostel and Dejax 1998; Chakroborty and Wivedi 2002; Crainic 2000; He et al. 2003).

Relevant technological equipment is needed for the execution of technological processes in the terminal, that is loading-unloading and sorting equipment, supplies for transporting goods in the territory of the terminal etc.

Besides, it should not be forgotten that several transport modes interact in the terminal. For example, road transport, railway transport and maritime transport are combined in the sea terminal.

Therefore, a key task of the terminal staff is to optimally coordinate various technological freight clearing processes and to distribute available technological equipment and other relevant resources according to separate transport modes.

The article analyses how technological processes in a terminal could be optimally controlled via mathematical methods and computer hardware.

2. Formalization of Technological Processes in the Terminal

Let $J = \{j,\ j = \overline{1,\ m}\}$ is a set of the stages of the technological process in the freight clearing terminal. For each stage $j \in J$ the amount and time of equipment K_{jp} is needed and their efficiency is m_{jp}, where p indicates the index of a type of equipment realising the operations of the given stage; m_j – the amount of equipment during the stage j.

The technological stage of the process can also be characterised by the amount of the available resources $R_j^t,\ t \in \left[0,\ T\right]$.

The functioning of the terminal is analysed by time interval $[0, T]$ and described by the set of quantum time $T = \{\Delta t,\ \forall \Delta t \in N\}$ characterising the fund of working time of the terminal.

There is a set of goods which have to be cleared,

$$I = D \cup S = \{i \in I,\ i = di,\ di \in D_{vi} = S_i,\ S_i \in S\},$$

where D – a set of the planned goods; S – a set of unplanned goods.

It is assumed that planned goods have to be fully cleared, whereas a certain part of unplanned goods might be uncleared in case of lack of necessary resources.

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Each consignment $j \in J$ is expressed (described) in suite:

$$i \le \alpha_i, \beta_i, \gamma_i, \delta_i, \pi, \tag{1}$$

where $\alpha_i = \begin{cases} t_i^1,\ i = di, & \text{Time when consignment is delivered for clearing;} \\ t_i^1,\ i = S_i, & \text{Time from which a part of resource } W_i \text{ is allocated for consignment } S_i; \end{cases}$

$$\beta_i = \begin{cases} t_i^2,\ i = di, & \text{Planned end time of clearing;} \\ t_i^2,\ i = S_i, & \text{Time from which a part of resource } W_i \text{ is allocated for consignment } i; \end{cases}$$

$$\gamma_i = \begin{cases} V_{ij},\ i = di, & \text{Work volumes according to consignment } i \text{ in stage } j; \\ W_{ij},\ i = S_i, & \text{A part of resource to which consignment } i \text{ is subject for clearing in stage } j; \end{cases}$$

$$\delta_i = \begin{cases} \delta_i - & \text{Fine for incompliance with clearing term;} \\ 0, & \text{If } t_i^a \le t_i^2, i = di; \\ \delta_i - & \text{Fine for refucal to clear;} \\ 0, & \text{If consignment is cleared.} \end{cases}$$

Thus, value δ_i is function t_i^a on $i \in D$ and a function of the allocated resource on $i \in S$.

Clearing time t_i^a of consignment i will be analysed as a linear function of value reverse to the amount of conditional unit resource stage j allocated for consignment i.

In the expression (1), π_i is a set of technological routes possible for i consignment $\pi_i = \{p_{ik}\}$ – here p_{ik} – technological route k, for consignment i $k = 1\theta_i$. More exactly, π_{ik} can be defined as a set $\bar{I} \subset I$, a set of technological stages arranged for consignment i according to binary preference connection distinguished by reflexivity, anti-symmetry and transitivity following the rule $\forall a, \beta \in I, a\left\{B \leftrightarrow |a| < |B|\right\}$.

In general terms, a technological route will be a set of coordinated operations distributed between the technological equipment of the terminal in the process of clearing freight. The summation of technological routes for consignment $j \in J$ is expressed as a set $\pi = \pi_a \cup \pi_s$ anticipating all possible options for clearing goods delivered to the terminal.

Subset $\pi_a = \left\{\bigcup_i \pi_i,\ i = d_i,\ d_i \in D\right\}$ demonstrates technological routes in clearing planned freight in which $i = a_i$, $di \in D$, having stable characteristics, fixed technological routes can be established as they guarantee a planned and efficient clearing of goods.

Subset $\pi_s = \left\{\bigcup_i \pi_i,\ i = S_i,\ S_i \in S\right\}$ indicates technological routes in clearing unplanned freight for which freight $i = S_i$, $S_i \in S$, a set π_i is formed depending on the nature of consignment, its dependency on various consumers and the state of technological processes in the terminal in clearing other consignments. The formation of a set π_i very much depends on the system of priorities and the selection of clearing procedure.

The intersection of a set π_i indicates a possible loading of the equipment (resources) of the technological stage, $V_j = \left\{\bigcap_i \pi_i,\ \ i = d_i,\ \ d_i \in D\right\}$, $W_{i(j)} = \left\{\bigcap_i \pi_i,\ i = S_i,\ S_i \in S\right\}$. In this case, the following situations are possible:

1) Resources of the available equipment meet the emerging demand:

$$V_j \le \sum_p K_{jp} M_{jp};\ W_{i(j)} \le 1; \tag{2}$$

2) Resources of the existing equipment for insufficiently emerging demand:

$$V_j > \sum_p K_{jp} M_{jp};\ W_{i(j)} \le 1; \tag{3}$$

3) Loading of equipment is much lower than its capacity:

$$V_j \ll \sum_p K_{jp} M_{jp};\ W_{i(j)} \ll 1. \tag{4}$$

It is clear that the second situation corresponds with the peak loadings and cannot be considered as satisfactory. This can be solved by a better organisation of the technological process or the introduction of additional equipment.

With regard to the third situation, we can speak about the insufficient organisation of the process since expression (4), as a rule, comply with expression (3) of the second situation.

The technological route sometimes has to be adjusted due to the breakdowns of equipment and other disturbances at separate stages of the freight clearing technological process as well as due to the occurrence of single unplanned consignments.

The technological process as a controlled system P that can be formalized as follows. For technological process P, the incoming flow of planned consignments is attributed $D = \{d_i\}$. The flow of unplanned freight $S = \{S_i\}$ is analysed as a flow of disturbing impacts and is also attributed to technological process P. Each consignment $i \in I = D \cup S$ is described by a set of parameters $i = \langle \alpha_i,\ \beta_i,\ \gamma_i,\ \pi_i \rangle$. Goods are cleared by the existing resources of the terminal, meanwhile one technological route $\pi_{ik} \in \pi_i$ is realised for each consignment.

A passage of consignment i by route π_{ik} is regulated by introducing impacts $U = U(t)$. Function $U = U(t)$ is discrete.

Controlling the technological process of freight clearing is an operative impact on technological routes in changing sequence and connection between technological operations or stages.

A passage of consignment along technological route is characterised by parameters:

$$\delta_i = \delta_i\left(\alpha_i, \beta_i, \gamma_i, \Delta t, \pi_i, u\right), \Delta t \in \left[0, T\right]. \tag{5}$$

Meanwhile, generally, the movement of freight along technological route is restricted by parameters:

$$\delta_i = \left(\Delta t, \pi_i, u\right) \in \left[\alpha_i, \beta_i\right], \Delta t \in \left[0, T\right], \tag{6}$$

and system resources, described in expressions (2)–(4) or general restrictions:

$$\delta_i = \left(\Delta t, \pi_i, u\right) \in \left[\alpha_i, \beta_i\right], \Delta t \in \left[0, T\right], \tag{7}$$

where r_{ij}^t – the amount of resource *j*, allocated for consignment *i* by time quantum *t*.

At the end of technological process *P*, the flow of uncleared consignment $\tilde{\bar{I}} = D \bigcup \tilde{S}$ is released. Subset $\bar{S} = S / \tilde{S}$ consists of uncleared unplanned consignments.

The formalisation of the technological process as a controlled system *P* allows creating a task for controlling the technological process on the basis of the mathematical theory of optimal control according to the various quality criteria of implementing them.

3. Optimal Control of the Technological Process and the Criteria of its Quality

The technological process of freight clearing is a complex control system. The following control aspects could be distinguished: 1) control of clearing planned freight; 2) control of clearing unplanned freight; 3) simultaneous control of clearing planned and unplanned freight.

Control of clearing planned freight is the distribution of the total resources placed in the terminal according to the types of the operations and stages of the technological process and operations. Each consignment $i = d_i$, $d_i \in D$ is characterised by a set of technological routes $\pi_i = \{\pi_{ik}\}$. A respective set of criteria $P = \{P_{k1}, ..., P_{kq}\}$ can be envisaged for a technological route characterising technological route π_{ik}. The above mentioned criteria might indicate duration, price, reliability, stability etc.

The duration of technological route t_{ik} is defined as follows:

$$t_{ik} = \sum_{j \in \pi_{ik}} \sum_P V_{ij} / \mu_{jp} K_{jp},$$

where $\sum_P V_{ij} / \mu_{jp} K_{jp}$ – the duration of stage *j* for *i* consignment.

Reliability in executing *j* stage can be expressed by the preparedness coefficient $0 < \eta_j < 1$. Three states of the technological process are usually possible: 1 – adequate, non-operational; 2 – adequate, operational; 3 – inadequate. Let's say that the probabilities of the indicated states during moment *t* are known, i.e. $Q_j^1(t)$, $Q_j^2(t)$, $Q_j^3(t)$, whereas $\sum_{n=1}^{3} Q_j^n(t) = 1$ – a coefficient of preparedness of *j* stage, defined as the summation of probabilities which will be in states 1 and 2, i.e. $n_j = Q_j^1(t) + Q_j^2(t)$.

The intensity of shifts from one state to another α, β, γ, ξ are values, reverse to certain time parameters and are specified via expressions:

$\alpha = 1/t_n$,

where t_n – the duration of the interval of allocation for operations;

$\beta = 1/t_j$,

where t_j – the duration of operation during stage *j*;

$\gamma = 1/t_{\theta T}$,

where $t_{\theta T}$ – average run-in time for breakdown;

$\xi = 1/t_b$,

where t_b – average restoration time.

These values can be defined by analysing the operations performed in the terminal according to statistical data for a respective time interval.

Having assessed the coefficient of preparedness η_j, the duration of technological process t_{ik} is specified as:

$$t_{ik} = \sum_{j \in \pi_{ik}} 1 / \eta_j \sum_P V_{ij} / \mu_{jp} K_{jp}. \tag{8}$$

Price criteria are used in the tasks of the technological process taken place in the terminal. The clearing price of consignment *i* according to technological route C_{ik} can be estimated depending on clearing time:

$$C_{ik} = t_{ik} C_t, \tag{9}$$

where C_t – consignment clearing price per time unit.

Clearing price C_{jk} can be estimated as an additive function of the prices of technological stages:

$$C_{jk} = F\left(C_j\right) = \sum_{j \in \pi_{ik}} V_{ij} C_j / \eta_j, \tag{10}$$

where C_j – consignment clearing price during *j* stage.

The task of the optimal control of clearing planned freight on the basis of the listed criteria can be formed as follows. We have to define a set $\pi_d = \left\{\bigcup_i \pi_i, i = d_i, d_i \in D\right\}$ which anticipates the possible options of clearing planned freight. In the set π_d, we can define a subset of permitted technological routes $\pi_d = \left\{\pi_{ik}, \forall \pi_{ik} \in \pi_d \middle| t_{ik} \in \left[t_j^1, t_j^2\right]; t_j^1, t_j^2 \in \left[0, T\right]\right\}$. In the subset *pd* we can find a subset of optimal technological routes.

Thus, the above task assesses the restrictions of the duration of technological routes and minimises the price of clearing planned freight.

Clearing unplanned freight is executed simultaneously delivering it from a set of customers $G = \{q\}$; each of them is characterised by a set of technological equipment $K_g = \{K_{qp}\}$. The control of clearing unplanned freight is related to the determination of clearing technology, priorities and queues.

Meanwhile, unplanned freight can be cleared with relative, absolute and dynamic priorities. Relative and absolute priorities are fixed, whereas stable priorities are allocated for a respective period. When $t_i \to t_j^2$, we deal with dynamic priority given by priority function $V_i(t) =$

$\omega_i(t - t_i)$, where ω_i – a coefficient defining the change of the priority of consignment during the time of storing it in the terminal, where t_i – the time of storing consignment in the terminal, $i \subset S$.

An optimal determination of priorities can be done by assessing economic criteria. Let's say that flows P of the goods of various consumers are delivered to the terminal with parameter λ_p, a general loading of the terminal equals to $\sum_{K_p \in K} \lambda_p t_p > 1$; waiting in the queue is detrimental to a consumer α_{ip} and terminal C; we consider that the goods of consumers are distributed by declining order α_p/t_p:

$$\max_P\left(\alpha_{ip}/t_p\right),\ \min_P\left(\alpha_{ip}/t_p\right)\Big] \tag{11}$$

in the entire range of options. We have to select the number of priority groups $\eta = \overline{1, N}$, $N \le r$, $\bigcup_\eta K_\eta = K$ and the distribution of consignment according to these groups, so as to minimise the function:

$$F(\eta) = F_1(y_\eta) + F_2(y_\eta) \to \min, \tag{12}$$

where $F(\eta)$ – total loss due to waiting in the queue during the time unit of terminal operations.

The function consists of two constituents. The first constituent $F_1(\eta)$ can be expressed as a function:

$$F_1(y_\eta) = \sum_{K_p \in K} \sum_{i \in S} \alpha_{ip} \lambda_p \overline{t}_p^I - \sum_{\eta \in N} \sum_{K_p \in K_\eta} \sum_{i \in S} \alpha_{ip} n_\eta t_p^I,$$

where $F_1(y_\eta)$ – difference between the loss due to waiting for goods in the queue before and after their linkage into y_η priority group; $\overline{t}^I$ – average waiting time in the queue; n_η – the amount of goods combined to y_η priority group. The second constituent $F_2(y_\eta)$ indicates the expenditure of the terminal in grouping and clearing consignments and we will refer to these as to technological expenditure (costs):

$$F_2(y_\eta) = \sum_{\eta \in N} \sum_{K_p \in K_\eta} C_p \lambda_p \overline{t}_p^I.$$

Function $F_1(y_\eta)$ is a declining function and reaches the optimal value when $\eta = 1$ and zero, when $\eta = N$. Function $F_2(y_\eta)$ is evenly increasing with the increase $\eta \to N$, and when $\eta = 1$ has minimal value, then F_η is an unimodal cost function.

The general control of freight clearing set $I = D \cup S$ in principle is the control of the technological process of freight clearing and is displayed by the distribution of terminal resources between the goods cleared during the period $[0, T]$. Control should be executed so as to clear as many consignments as possible and gradually use terminal resources. Then, the following optimal control criteria could be specified.

Criteria $F(\delta_i)$ for minimising fines for uncleared (untreated) $i = S_i$ consignment i or the inobservance of finalisation directive terms:

$$F(\delta_i) = \sum_{i \in I} \delta_i (R_j^t, \pi_i) \to \min. \tag{13}$$

In applying criteria $F(\delta_i)$, restrictions should be observed according to freight clearing terminals (6) and restrictions according to terminal resources (7).

Terminal loading minimisation criteria $F(\Delta R)$,

$$F(\Delta R) = \sum_{t \in [0,\, T]} \sum_{j \in J} \left(R_j^t - \sum_{i \in J} r_{ij}^t \right) \to \min \tag{14}$$

i.e. criteria $F(\Delta R)$ for eliminating peak loading situations when terms (3) and (4) are simultaneously executed in various time quanta $t \in [0, T]$.

In realising criteria $F(\Delta R)$, freight clearing terms (6) and terminal resources (7) should be observed (7).

Thus, we analyse various aspects and criteria of controlling the technological process of freight clearing as well as a general control task. Further, we will specify separate tasks and algorithms of solving them.

4. A Model for Optimal Control of Clearing Planned Freight

The model is created by applying the graph theory:

1. The structure of the freight clearing system is modelled in the form of graph $G = \{I, V\}$, where I – graph peak indicating a set of stages, meanwhile each stage $j \in I$ can be presented either via one element or a set of the units of technological devices $K_j = \{K_{jp}\}$; V – graph link indicating possible relations between graph extremes.

   In order to facilitate analysis and formalisation, graph G will be presented in the form of floors. We'll arrange graph G assuming that the peak of j floor – technological equipment K_{jp}, j stage, p type. The peak of the upper floor has no incoming link and the peak of the lower floor has no outgoing link.
2. Subgraph $G_i \subset G$ is found on graph $G = \{I, V\}$; here each subgraph G_i corresponds to subset π_i, $i \in D$ and has peaks and links corresponding to a set of technological routes to i consignment $G_i = \pi_i = \{j, v \in \pi_{ik}, i = di, di \in D\}$, $\bigcap_i G_i \ne \varnothing$; $\bigcup_i G_i \ne G$.

A set $G_0 = \bigcup_{i \in D} G_i$ indicates a general structure of the options of possible technological routes.

According to task formulation presented in the first part, the control of planned freight is expressed as follows:

1. To define rules for the minimisation of a set G_0 down to subset $\tilde{G}$ corresponding to subset π_d. Here, subset $\tilde{G} \subset G_0$ connects technological routes for which:

$$\tilde{G} = \left\{ \pi_{ik} \in \tilde{\pi}_d \,\middle|\, t_{ik} \in \left[t_i^1, t_i^2\right], t_i^1, t_i^2 \in [0, T], i \in D \right\}. \tag{15}$$

2. In subset $\tilde{G}$ to define subset $G^* \in \tilde{G}$, for which conditions are fulfilled:

$$F\left(G^*\right) = \min_{i \in D,\, G_{ik} \in \tilde{G}} F\left(c_{ik}, G_{ik}\right); \tag{16}$$

$$\sum_{i \in D} V_{ij} / \eta_j \mu_{jp} \le K_{jp}, j \in I; \tag{17}$$

$$1/\eta_j \sum_{K_{jp} \in K_j} V_{ij} / \mu_{jp} K_{jp} \in \left[t_i^1, t_i^2\right],\ i \in D,\ j \in I. \tag{18}$$

In other words, subset G^* creates subgraphs $G_i \subset \tilde{G}$ formed by technological routes $\pi_{\delta k} \in \pi_d^*$ meeting in j stage resources restrictions $j \in I$ (17) and the duration of clearing separate consignments (18) and minimises the total price of technological process $F(C_{ik}, G_{ik})$ (16).

Subset $\tilde{G} \subset G$ can be found by different means. Subset $\tilde{G} \subset G$ can be formed by choosing between the permitted $\pi_{ik} \in \tilde{G}_i$ critical technological routes $\pi_{i\varphi}$ defined as:

$$\pi_{i\varphi} = \max_{e.r.\mu} \left\{ \sum_{j=1}^{K} t_l a_l \right\}, \tag{19}$$

where t_l – time loading extremes of graph G_i; j – the number of graph floor; a_l^j – the matrix of the ratios of subgraph G_i extremes j and j – 1 floors; l – the index of the edge of subgraph G_i; $j = \overline{1, m}$, $K \le m$; r, μ – a subset of indexes highlighting the links and peaks of road π_{ik}.

Matrix is equivalent to the subgraph of floors j and j – 1 of subgraph G_i. In the case, when a link in a subgraph crosses j – 1 floor, the zero extreme is introduced in floor j – 1. In forming a critical technological route $\pi_{i\varphi}$, values t_l are taken according to the expressed binary connections of matrix $\left\{a_l^j\right\}$.

The formation of subset $\tilde{G}$ of critical technological routes $\pi_{i\varphi}$ comprises a respective structure of the technological process increasing probability that its option optimal according to (16) does exist.

A critical technological route is defined via the method of dynamic programming the key principle of which is recurrent ratio expressing the principal of optimality R. Belman, i.e. with each step a decision is made and guarantees an optimal continuation of the process with respect to the achieved state at the given moment. Thus, by step K, G_i of the subgraph corresponds to the ratio $\pi_{i\varphi}^k(l^{k-1}) = \max\left\{t_l a_l^k\right\}$, here $1 \le l \le m$, l – graph G_i number of edge in the critical road in floor K. If maximum exists in this road, then, in search for it, function $l^k = l^k(l^{k-1})$ will be found.

G_i of graph for floor K – 1 will be

$$\pi_{i\varphi}^{k-1}(l^{k-2}) = \max_{l^k,\, l^{k-1}} \left\{ t_l,\ a_l^{k-1} + \max_{l^l}\left(l^{k-1}\right) \right\},$$

where $1 \le l \le m$, $l^l \ll l^{k-1} \ll m$.

Having distributed the maximum by floors we get:

$$\pi_{i\varphi}^{k-1}(l^{k-2}) = \max_{l^k,\, l^{k-1}} \left\{ t_l,\ a_l^{k-1} + \max_{l_k}(l^{k-1}) \right\}.$$

Having assessed the obtained expressions:

$$\pi_{i\varphi}^{k-1}(l^{k-2}) = \max_{l^{k-1}} \left\{ t_l a_l^{k-1} + \pi_{i\varphi}^k(l^{k-1}) \right\}.$$

The recurrent ratio can be obtained to any floor of subgraph G_i.

$$\left.\begin{array}{l} \pi_{i\varphi}^{k-r}\left(l^{(k-r-1)}\right) = \max\limits_{l^{k-2}} \left\{ t_l a_l^{k-r} + \pi_{i\varphi}^{k-r}(l^{k-r}) \right\} \\ l^{k-r} = l^{k-r}(l^{k-r-1});\ 1 \le e^{k-r} \le m,\ r \in j,\ j = \overline{1, m} \end{array}\right\}. \tag{20}$$

Ratio (20) is the main functional equation of R. Belman for this task.

In subgraph G_i of critical technologies, the process of finding is repeated until the subset of technological routes is found $\breve{G} = \bigcup_i \breve{G}_i$ meeting restrictions (17), (18).

However, in the above case, we have to define a full set G_i for each consignment or flow by applying a special procedure in order to reduce it to subset $\breve{G}_i$. Calculations can be shortened if to form subset $\tilde{G} \subset G_0$, we'll use a formal procedure omitting the stage of forming a full set $G_0 = \bigcup_{i \in D} G_i$. Meanwhile, criteria (10) of the applied price $F(c_j)$.

Subset $\breve{G}$ is formed under the following assumptions.

When analysing the operating actual terminals and referring to theoretic assumptions, it becomes clear that if we compare technological routes with various indicators c_{ik} and t_{ik}, we can formulate the following propositions providing for the establishment of subset $\breve{G}_i$.

Proposition 1. If $\pi_{ik} \in G_i$, and $F(c_j)$ price increases when c_j = const, then:

$$\breve{G}_i = \left\{ \pi_{in} \in G_i,\ n \in \theta_i \,/\, c_n \le c_{t_{\min}};\ t_n \le t_{c_{\min}} \right\}.$$

Proposition 2. If $\pi_{ik} \in G_i$, and $F(c_j)$ increases with increasing t_j and c_j = const, and exist π_{in} and π_{in-1} such as $t_n \ge t_{n-1}$, $c_n \le c_{n-1}$, it means that $\frac{c_{nj}}{t_n} < \frac{c_{(n-1)j}}{t_{n-1}}$, and therefore $\pi_{in} \in \breve{G}_i$.

On the basis of these propositions, subset $\tilde{G}$ is formed as follows.

Let Ω_k – space of states in step k; $\omega_{kj} \in \Omega_k$ – an element of set Ω_k; D_k – space of solutions in step k; $b_{kj} \in D_k$ – an element of set D; m – the total number of steps; $c_{kj} t_{kj}$ – the projection of function in k step.

Consequently, we assume that this task belongs to the type of recursive tasks and the subset is formed as follows:

$$c_j = \sum_{k=1}^{m} c_{kj}\left(\omega_{k-1}, j;\ b_{kj}\right) \le c_{t\min};$$

$$t_j = \sum_{k=1}^{m} t_{kj}\left(\omega_{k-1}, j;\ b_{kj}\right) \le t_{c\min}.$$

Subset $\tilde{G}$ is established by the following algorithm:

1. In the subset D_m for step m, a set of arranged pairs has to be formed $A_{mj} = (t_{mj}, C_{mj})$, here $t_{mj} = t_{mj}\ (\omega_{m-1}, b_{mj})$, $C_{mj} = C_{mj}(\omega_{m-1}, b_{mj})$.

2. To define a set $A_m^1 \subset A$ where:

$$A_m^1 = \left\{ A_{mj} \middle| 0 \le t_{mj} \le t_{\text{cmin}};\ 0 \le C_{mj} \le C_{\text{tmin}} \right\}.$$

3. A_m^1 finding corresponds to the definition of p solutions in step m $b_m^* = \left\{ b_{mj}^* \right\}$, i.e. $a_{mj}^* \sim b_{mj}^*$, $a_{mj}^* \in A_m^1$, $b_{mj}^* \in \left\{ b_{mj}^* \right\}$. $t_{(m-1,j)m}^t C_{(m-1,j)m}^t$ – a full winning in the last two steps at any solutions in step $m-1$ and in a step of perspective solutions $m\{b_{mj}\}$.
4. In the set D_{m-1} for step $m-1$, a set of arranged pairs has to be defined $A_{m-1,j} = \left\{ t_{(m-1,j)m}^+;\ C_{(m-1,j)m}^+ \right\}$ where

$$t_{(m-1,j)m}^+ \left(\omega_{m-2}, b_{m-1} \right) = t_{m-1,j} \left(\omega_{m-2}, b_{m-1} \right) +$$
$$t^* \left[\omega_{m-1} \left(\omega_{m-2}, b_{m-1} \right] ;$$
$$C_{(m-1,j)m}^+ \left(\omega_{m-2}, b_{m-1} \right) = C_{m-1,j} \left(\omega_{m-2}, b_{m-1} \right) +$$
$$C^* \left[\omega_{m-1} \left(\omega_{m-2}, b_{m-1} \right] .$$

5. To specify subset $A_{m-1}^1 \in A_{m-1}$, where

$$A_{m-1}^1 = \left\{ A_{m-1} \middle| 0 \le t_{(m-1,j)m}^+ < t_{\text{cmin}};\right.$$
$$\left. 0 \le C_{(m-1,j)m}^+ < C_{\text{tmin}} \right\}.$$

6. Finding A_{m-1}^1 corresponds to the specification of a set in the step of perspective solutions $(m-1) b_{m-1}^* = \left\{ b_{m-1,j}^* \right\}$, i.e.

$$a_{m-1,j} \sim b_{m-1,j}^*;$$
$$a_{m-1,j} \in A_{m-1}^1;$$
$$b_{m-1,j}^* \left\{ b_{m-1,j}^* \right\}.$$

Accordingly, for the first stage, we receive the set $b_1^* = \left\{ b_{1j}^* \right\}$, each element and continuation (extension) of which specify a technological route depending on set $\tilde{G}_i$.

In order to extend set $\tilde{G}_i$, including more than one technological route for i consignment, the selective procedure changes and is of the following sequence:

1. In the set D_m, the set $A_m = \{C_{mj}\}$, here $C_{mj}(\omega_{m-1}, b_m)$ is formed – this is a criterion according to which we execute optimization.
2. The subset

$$A_m^1 \subset A_m,\ A_m^1 = \left\{ C_m^*(\omega_{m-1}),\ C_m^{**} \left(\omega_{m-1} \right) \right\} \subset A_m$$

is formed,

where $C_m^* \left(\omega_{m-1} \right) = \max \left\{ C_{mj} \left(\omega_{m-1}, b_m \right) \right\}$;

$$C_m^{**} \left(\omega_{m-1} \right) \sim b_m^{**} \left(\omega_{m-1} \right);$$
$$C_m^{**} \left(\omega_{m-1} \right) = \max \left\{ m_j \left(\omega_{m-1}, b_m \right) \right\};$$
$$C_m^{**} \left(\omega_{m-1} \right) \sim b_m^{**} \left(\omega_{m-1} \right).$$

Consequently, in step m we get a set of perspective answers $\left\{ b_m^*, b_m^{**} \right\}$. Here

$$C_m^{**} \left(\omega_{m-1} \right) = C_m^* \left[\omega_{m-1} \left(\omega_{m-2}, b_{m-1} \right) \right],$$

whereas $C_{(m-1)m}^+$ is a full winning in two last steps at any step of the answer $(m-1)$ and rational answers $b_m^* \left(\omega_{m-1} \right)$, $b_m^{**} \left(\omega_{m-1} \right)$, in step m. In the set D_{m-1} (step $m-1$), the set $A_{m-1}^* = \left\{ C_{m-1,j}^+ \right\}$ is formed, here

$$C_{m-1,j}^+ \mu_j \left(\omega_{m-2}, b_{m-1} \right) = C_{m-1,j} \left(\omega_{m-2}, b_{m-1} \right) +$$
$$C_m^+ \left[\omega_{m-1} \left(\omega_{m-2}, b_{m-1} \right) \right.. $$

3. The set

$$A_{m-1}^1 = \left\{ C_{m-1}^* \left(\omega_{m-2} \right),\ \left\{ C_{m-1}^{**} \left(\omega_{m-2} \right) \subset A_{m-1}^* \right. \right.$$

is formed,

where

$$C_{m-1}^* \left(\omega_{m-2} \right) = \max \left\{ C_{m-1,j} \left(\omega_{m-2}, b_{m-1} \right) + \right.$$
$$\left. C_{mj}^+ \left[\omega_{m-1} \left(\omega_{m-2}, b_{m-1} \right) \right] \right\};$$
$$C_{m-1}^{**} \left(\omega_{m-2} \right) = \max \left\{ C_{m-1,j} \left(\omega_{m-2}, b_{m-2} \right) + \right.$$
$$\left. C_{m-1,j} \left(\omega_{m-2}, b_{m-2} \right) + C_{mj}^+ \left[\omega_{m-1} \left(\omega_{m-2}, b_{m-1} \right) \right] \right\}.$$

Following this procedure until the end, we get

$$C_{1,2,\ldots,m}^* = \max_{\omega_0 \in '_0} \left\{ C \left(\omega_0, b_1 \right) + C_2^* \left[\omega_1 \left(\omega_0 b_1 \right) \right] \right\},$$

$C_{1,2,\ldots,m}^* \sim b_1^*$ – a rational technological route included into $\tilde{G}_i$.

The above pressure is executed by applying the general methods of successive analysis and its peculiarity is expressed only by the rules of selecting the options and ways for using the obtained results.

The procedures of the compression of options allow to indirectly establish a subset $\breve{G}$ corresponding to respective restrictions and criteria, by omitting the creation of a full set G_0 and reviewing all its elements. Since the set $\tilde{G}$ usually has few options, the subset G^* can be established through a direct review.

5. Models and Algorithms for Optimal Control of the Technological Process of Freight Clearing

As mentioned above, clearing goods at the various stages of the technological route is affected by various (random) obstacles; as a result, the technological process in the terminal is also random. Therefore, the moment of the finalisation of freight clearing can be defined only with a certain probability. Besides, when the needs of consumers are not in compliance with terminal capacities, peak loadings and situations also occur. In the above case, a distribution task is established and criteria (10) have to be optimised.

The capacity of the terminal (throughput) during period $[0, T]$ is N and can clear a set of consignments $I = D \bigcup S = \{i = d_i,\ d_i \in D \forall_i = S_i,\ S_i \in S\},\ i = \overline{1, m}$, each consignment is described by $i = \langle \alpha_i, \beta_i, \gamma_i \rangle$, here α_i – the beginning of freight clearing; β_i – the planned end of freight clearing; $\gamma_i = t_i$ – the time of clearing consignment i.

Let's say that period $[0, T]$ is divided into a set by equal interval $T = \{\Delta t_z\},\ z = 1, m,\ \bigcup_z \Delta t_z = T$, meanwhile $\Delta t_z > \max \gamma_i$.

We'll define the set $\{N_z^z\},\ z = \overline{1, m}$, here terminal capacity (budget of the main operational time in the interval Δt_z). Each consignment i has defined a set of intervals $\{\Delta t_\varphi, t_{\varphi+1}, ..., \Delta t\psi\} \in T,\ \varphi_1 \psi \in z,\ \varphi < \psi$ which can be given as a section $[\Delta t_j, \Delta t\psi]$, here $\Delta t_{i\varphi} = \alpha_i$ a term of starting clearing goods $\Delta t_{i\psi} = \beta_i$ – obligatory term for the finalisation of clearing goods. If $\Delta t_{i\varphi} = \Delta t_{i\psi}$, then consignment can be cleared only during interval $\Delta t_{i\varphi} = z$. If $\Delta t_{i\varphi} \neq \Delta t_{i\psi}$, then consignment is cleared in any interval $\{\Delta t_\varphi, \Delta t_{\varphi+1}, ..., \Delta t\psi\} \in [0, T]$.

It is necessary to distribute freight clearing so as to gradually load the terminal during the entire T. Thus, in the set of consignments $I = D \cup S$, we have to find subsets $I_z \in I,\ z \in Z$ which guarantee the optimal loading of the technological equipment of the terminal and clearing according to the terms envisaged for all consignments.

$$x_{iz} = \begin{cases} 1, & \text{If consignment } i \text{ is cleared in interval } z; \\ 0 & \text{– otherwise.} \end{cases}$$

In general, the task for optimizing the technological loading of the terminal can be formulated as follows. To find vector:

$$\bar{x} = \{x_{iz}\},\ i = \overline{1, m};\ i \in I;\ Z = \overline{1, m}. \tag{21}$$

Minimising target function:

$$F = \sum_{r=1}^{m} \left(N_z - \sum_{i=1}^{n} t_i x_{ir} \right) \to \min \tag{22}$$

under restrictions:

$$x_{iz} \in \left[\Delta t_{i\varphi}, \Delta t_{i\psi}\right],\ \forall z,\ \varphi,\ \psi \in [0, T],\ i \in I; \tag{23}$$

$$\sum_{i=1}^{n} t_i x_{iz} \leq N_z,\ Z = \overline{1, m}; \tag{24}$$

$$\sum_{z=1}^{m} x_{iz} = 1,\ i \in I; \tag{25}$$

$$x_{iz} \in \{0, 1\},\ Z = \overline{1, m},\ i \in I. \tag{26}$$

A peculiarity of the given task is that it is moderate and condition (24) restricts the margins of $\bar{x}$ existing. For task solving, the algorithm created on the basis of margins and branches is used.

Vector $\bar{x}$, meeting (24)–(26) restrictions, will be referred to as an answer, vector $\bar{x}$, meeting (23)–(26) restrictions – as a permitted answer, whereas a permitted answer optimising (22) function – as an optimal answer. The main idea of the suggested algorithm is to find the base vector $\bar{x}^0$ which is the answer of task (23)–(26) and at a later stage, to execute its gradual optimisation.

The base of vector $\bar{x}^0 = \{x_{iz}^0\}$ is found as follows. For each i consignment according to a given term, $t_{i\psi}$ is defined as Δt_z, for which $x_{iz}^0 = 1$, if $z = \psi$, z, $\psi \in [0, T]$, and $x_{iz}^0 = 0$ for all $z \neq \psi$. A received vector $\bar{x}^0 = \{x_{iz}^0\}$ is a permitted answer, since term (23)–(26) is fulfilled. However, vector $\bar{x}^0$ is not within the margins of the optimal formulated task as in case of the other criteria of the schedule theory.

We will analyse the possibilities of optimising vector $\bar{x}^0$. Having distributed consignments $\{x_{iz}^0\}$, it is considered that loading terminal equipment is uneven, and therefore in the set of intervals $\{\Delta t_z\} \in [0, T]$, $z = 1, m$ the set of several intervals $\{\Delta t_z\}$ can be defined, $r \in Z$, for which the condition (23) is a strict inequality. All other intervals $\Delta t_z / \Delta t_r\} \in [0, T]$, $r, z = 1, m$ will be referred to as full. In order to get the permitted answer, it is necessary to fill in the pursued intervals Δt_r, and in order to get an optimal answer, it is necessary to highlight an answer in the set of the permissible answers $\{\bar{x}^S\} = \bar{x},\ S = \overline{1, s}$ minimising the target function (22).

Vector $\bar{x}^0$ is optimised by the iterative procedure in the freely chosen full interval $\Delta t_x \in \Delta t_z / \Delta t_r \in [0, T]$, α, z, $r \in m$ in the set of consignments $I_\alpha^1 = \{i \in I \mid x_{i\alpha} = 1, \alpha \in z\}$ the subset $\tilde{\tilde{I}}_\alpha^1 = \{i \in I_\alpha^1, t_{i\varphi} \neq t_{i\alpha}, \varphi < \alpha;\ \varphi, \alpha \in z_j$ is found. The subset $\tilde{\tilde{I}}_\alpha^1$ is redistributed according to intervals $\{\Delta t_r\}$, here $r = \alpha - 1, \alpha - 2, ..., \varphi$, by optimising the target function (22).

The permissible answers $\{\bar{x}^S\},\ S = \overline{1, s}$ are found via oriented movement according to the extremes of the tree of the options of freight clearing distribution. Ramification strategy is as follows. At tree level p, option k of the distribution of consignment p is formed, meanwhile $p_i \in I_\alpha^1,\ t_k \in t_r \subset [1, T],\ \varphi_{pi} \leq 2 \leq \alpha.$ At each tree level p, the received distribution options are assessed according to condition (23). The set of the received options $\{t_k\}$ is defined according to assessment $t_{pi} \leq N_k - \sum_{i \in n} t_i x_{ik}^0,\ \forall k \in m$. Later, in the set of the permissible options $\{\Delta t_k\}$, the lower evaluations of distribution are introduced which, based on the optimum (22), can be estimated according to formula:

$$F_{pk} = \min \left\{ \sum_{i \check{z} 1}^{n} t_i x_{ik}^0 + t_p \right\}. \tag{27}$$

The extreme, complying with the option of the lowest assessment (evaluation) (27), is chosen as active for further split. The remaining extremes of the given level are final.

If (27) complies with several indices k, then we select the smallest index $\bar{k} = \min\{k\}$. Further, we read $x_{pk} = 1$ and $x_{iz} = 0$ for all $z \in m$, $z \neq m$. The process continues until further split becomes impossible. The answer is optimal if the tree of options has no final extremes with evaluations.

$$F(\bar{x}^S) < F^*, \quad \forall i \notin I_k^1, \tag{28}$$

where $F^* = \min_s F\left(\bar{x}^S\right)$ – the value of the target function of the received answer. Otherwise, the answer is verified and split from extremes, corresponding to (28) is specified. Verification should start from lower levels, since then it is possible to find the answer quite quickly; besides the number of the options of upper levels to be verified would decrease. The split from the verified extreme is terminated if an assessment of a lower margin in some of the levels reaches or exceeds F^*. When a new answer is received, a respective value of a target function is used for verification. An optimisation procedure of vector $\bar{x}^0$ iteratively is repeated for intervals $\dfrac{\left(\Delta t_{\alpha+1}, \Delta t_{\alpha+2}, ..., \Delta t_z\right)}{\Delta t_r} \in \left[0, T\right]$, where $\alpha + 1, \alpha + 2, ..., z$, $k \in m$ until $\left\{\Delta t_r\right\} \neq \varnothing$. If the above condition is not fulfilled, it is considered that further optimisation is not possible and calculation is finished.

For further vector $\bar{x}^0$ optimisation iterations $\alpha + 1$, $\alpha + 2, ..., \gamma, ..., z$. In estimating assessments (27), we assume that $\bar{x}_{1r}^0 = x_{ir}^{\gamma-1}$, where $x^{\gamma-1} = \left\{x_{iz}^{\gamma-1}\right\}$ – distribution vector, formed $\gamma - 1$ iteration.

Algorithm of calculations:

1. Base vector $\bar{x}^0$ meeting (23)–(26) is formed.
2. Condition (24) is verified for the received distribution and in the set $\{\Delta t_z\}$, the subset $\{\Delta t_r\}$ is found.
3. In the subset $\{\Delta t_r\}$, interval Δt_α is selected. The subset $I_\alpha^1 = \varnothing$ is defined, the procedure is repeated three times due to $\Delta t_{\alpha+1} \in \left\{\Delta t_r\right\}$.
4. Extremes of level p are formed according to (23) and (24), the margins of the options of answers are assessed. The options of level p are gradually re-selected until Δt_k is defined.
5. Extreme $\Delta t_{\bar{k}} = \min\left\{\Delta t_k\right\}$ is defined and read $x_{p\bar{k}} = 1, x_{iz} = 0, z \in m, z \neq k$.
6. Answer $\left\{x_s^*\right\}$ is fixed, for which (28) is met, for verification procedure No 3 is repeated.
7. In case if during verification it turns out that $\left\{\bar{x}_s\right\}$ exists, $F\left\{\bar{x}_s\right\} < F^*$ then the value of an answer is renewed and we return to 3, otherwise a shift to 8.
8. End of calculations.

The optimal control of the technological process of freight clearing can be executed by applying criteria (13).

Let‘s introduce additional markings. Let $T_i \in [0, T]$ time interval during which consignments can be cleared $i \in I$, $T_j = \left[t_i^1, t_i^2\right]$, x_{ij}^t – a pursued variable, resource j stage, given for clearing consignment during t, x_{ij}^t quantum has discrete values and equals to:

$$x_{ij}^t = \begin{cases} 0 \leq r_{ij}^t \leq R, \ t \in T_i; \\ 0, \ t \notin T_i. \end{cases} \tag{29}$$

From the set I we will specify the subset of cleared consignments $\theta_1 = \left\{i \in I \middle| t - t_i^2 \geq 0, \ x_{ij}^t \neq 0, t \in [1, T]\right.$ and the subset of uncleared consignments $\theta_2 = \left\{i \in I \middle| t - t_i^2 < 0,\right.$ $x_{ij}^t = 0$, $t \in [1, T]$, meanwhile $I = \theta_1 \cup \theta_2$, $\theta_1 \cap \theta_2 = \varnothing$, $\theta_2 = I / \theta_1$.

Formally the task looks as follows:

$$F(\delta_i) = \sum_{i \in I} \delta_i - \sum_{i \in \theta_1} \delta_i \sum_{j \in \pi} \sum_{t \in T_i} x_{ij}^t \Big/ \sum_{j \in \pi_i} V_{ij} \to \min \tag{30}$$

$$\sum_{i \in \theta_1} X_{ij}^t \leq R_j^t, j = \overline{1, J}; \ t \in \left[0, T\right], 0 \leq X_{ij}^t \leq R_j^t, \tag{31}$$

where x_{ij}^t – discrete values;

$$\forall j \in \pi_i, \forall i \in I, t \in \left[0, T\right], \tag{32}$$

where the upper t indicates time $t \in [0, T]$ quantum; π_i – a technological route, run during clearing i by consignment $t \in \theta_i$. The target function (30) minimises the sum of fines for uncleared consignments.

Task (30)–(32) is the task of a dynamic distribution of the vector resource in the set; its meaning is to re-distribute the transformation of the arranged phases of resources between competing processes according to the fine minimum for the unfulfilled planned terms of freight clearing.

6. Conclusions

1. The suggested methodology for the formalisation of technological processes in transport terminal provides for the management of these processes by means of a dialogue between an employee and a computer and solves tasks for optimal control of freight clearing.
2. The technological process of freight clearing is a complex control system, and therefore the following aspects of controlling it could be specified:
   - control of clearing planned freight;
   - control of clearing unplanned freight;
   - simultaneous control of clearing planned and unplanned freight.
3. The above specified compression of technological routes is executed through the general methods of the successive analysis, and its peculiarity is expressed only by the option selection rules and the ways of using the obtained results.
4. Clearing goods at the various stages of the technological route is influenced by various (random) obstacles; therefore, the technological process itself is also random. Thus, the moment of finishing the process of clearing goods can be defined only with a certain probability. Besides, peak loadings as well as situations when the needs of consumers do not comply with terminal capacities are also possible. In this case, a distribution task is formed and has to be optimised.
5. The task for a dynamic distribution of the vector resource in the set is formed; its meaning is to re-distribute the transformation of the arranged phases of resources between competing processes

(consignments) according to the fine minimum for the unfulfilled planned terms of freight clearing.
6. Generally, the technical-economic indicators of terminal operation should be analyzed in terms of random factors which are also random with respect to any argument value. The arguments include time or other parameters of terminal operation (technological process). Thus, the criteria of optimality should also be considered as being random rather than determined.
7. Most criteria used for assessing terminal operation and individual technological processes are interlinked and this should be taken into account when using them as optimality criteria. In determining the numerical characteristics of technical-economic indicators according to statistical data obtained during the process of terminal operation, mathematical expectation as well as correlation and variance functions should be calculated.
8. Random emergency situations cause failures in the transport terminal. To consider them, the theory of probability value functions should be used.
9. A great number of various factors influence the operation of the terminal and may cause its malfunction; however, their influence may differ to a great extent. Therefore, simulation data should be optimized for usage during a further decision-making process.

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