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STATISTICAL PROBABILITY EVALUATION OF OPERATING ITS

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Abstract. Intelligent Transport Systems (ITS) work with information and control technologies providing the core of ITS functions. Some of these technologies like loop detectors are well known to transportation professionals. However, there are a number of less familiar technologies and system concepts that are keys to ITS functions. Although information and control technologies act as a technical core of ITS, human factors also remain vitally important and potentially very complex issues. The process of operating ITS is influenced by a number of random factors. Along with an assessment of dependence upon separate random factors, the classification of those in the whole hierarchical structure of operating ITS is presented. Statistical information on operating ITS is renewed and replenished in the course of time. With the growth of information amounts, the costs of storing them also increase. Therefore, the article presents relevant algorithms for obtaining required statistical assessments with the least statistical information. It is deduced that while modelling the process of operating ITS, an analytical description of random factors applying non-parametric assessment is suitable.

Keywords: Intelligent Transport Systems (ITS), information, control systems, ITS efficiency, statistical information, random factors and their classification, minimal amount of stored statistical information.

1. Introduction

Jarašūnienė (2006, 2007) proposes that Intelligent Transport System ITS services can make transport safer and more secure.

The purpose of ITS is to collect information about traffic flows and conditions on roads and to present the obtained data for control systems (GPS, commercial transport control systems, electronic payment systems, route control and creating public transport control systems, tax collecting systems, etc.). Control systems themselves can be defined as ones controlling scope, adaptation, expedition, controlling algorithm and collecting system efficiencies, variety and utility of the information they deliver (Jarašūnienė 2007; Jarašūnienė and Jakubauskas 2007; Daunoras *et al*. 2008; Fan and Machemehl 2006).

Jarašūnienė (2006, 2007) agrees that an intelligent electronic transportation control system usually contains such information collecting, processing, transmission, controlling subsystems and a subsystem of interfaces between separate hierarchical levels.

Jarašūnienė (2006, 2007) also accepts that electronic ITS contain a set of technical tools connected with a general information processing complex. Since at least two systems working according common algorithms and connected using interfaces may be considered as a minimal integrated system, an electronic ITS also is integrated system-video surveillance, signal processing, controlling system, etc (Xu 2000; Lee and Vuchic 2005; Liao 2009). The main part of the system that collects information consists of digital video cameras, video image detectors, vehicle control and incident detectors, weather monitoring tools, electronic toll collection detectors, electronic fee collection detectors, variable message signs, special crosswalk detectors, special video signals processing and transmitting cards installed in proper road sections, crossroads and crosswalks. Such system helps with getting real time information regarding traffic conditions on the road sections of interest. Therefore, all this information can be efficiently used to control vehicle traffic, public transport and pedestrian crossing. All signals from cameras are transmitted to central systems. Presently, statistical information is collected using real operating objects where different detectors, signs, identification tools and video cameras with software support are connected to recognize cars and to estimate the real situation on roads. When the before-mentioned information is collected, it is possible to evaluate not only the efficiency of information collecting a subsystem in detail

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but also a technical fault of such subsystem. An operator should be interested in the quality of operating ITS. One of the regularities of forming random factors is the fact that errors made at the earlier stage of operating ITS pass to the next one. Precise and effective information at all stages facilitates the elimination of random factors.

2. Random Factors and Their Classification

Nowadays, the number of vehicles in the cities is growing very fast. Loading on street intersections, traffic jams, wasting fuel and air pollution also increase. One of the ways to solve these problems is using ITS and increasing the efficiency of similar systems. Quality parameters for ITS systems include the efficiency of data collecting system, the volume and effectiveness of a control system and adaptive and efficient algorithms. In order to select optimal ITS structure, subsystems and local points are important to evaluate the efficiency of these systems.

Overall ITS system efficiency consists of several parts such as the efficiency of information acquisition and transmission, a degree by which information satisfies the needs, control quality, parameters for controlling extent adaptability and partial maintenance duration reduction in transport means. The efficiency of overall ITS using video image detectors, vehicle control and incident detectors, electronic toll collection detectors, electronic fee collection detectors, variable message signs, special crosswalk detectors and video cameras to gather information depends on such parameters as efficiency, placement efficiency, a system of data acquisition via camera operation efficiency, the efficiency of information transmission networks and the efficiency of control algorithm . Not all parameters of video cameras and other detectors used to recognize vehicle means and the real situation on roads are equally significant.

Random factors have to be classified within the whole hierarchic structure of operating ITS. Such structure should comprise national, regional and municipal governments and public authorities, the owners and operators of a transport network, automotive manufactures, fleet operators, industry and commerce, individual travellers, information exchange and decision coordination involving multiple centres, the interaction between a vehicle and road infrastructure, new private sector organizations for information service providers to distribute traffic information through cellular phones or the Internet, non transport organization in electronic payment systems involving financial institutions and in border crossing involving customs and immigration agencies, public transport services, i. e. everything that directly or indirectly influences the formation of random factors. These are very important issues to consider the process of operating ITS. A successful process of operating ITS cannot focus on technological solutions to traffic problems. ITS have implications that go beyond the traditional concerns of road operation and therefore need to be taken into account. Such issues include social, economic and technical questions.

Social issues

According to Yokota *et al*. (2004), for the major part, social issues are concerned with fairness. Some systems might provide disproportional benefit to wealthy people, which may not be desirable. ITS can offer a big promise of providing greater mobility to people with disability as well as to vision-impaired, elderly and poor citizens. The primary reason for offering these services is to improve their quality of life. Improving the mobility of these people also provides them with new and better ways to participate in the overall economy benefiting everyone. For example, improved public transport can provide access to a wider range of employment opportunities for people who cannot afford their own cars.

Economic issues

Yokota *et al*. (2004) also accept that ITS offer the potential for great economic and social benefits. However, ITS have been often expensive both to deploy and to operate and maintain, which sometimes takes a long time for benefits to be realized. Although the recent situation has significantly improved due to the lower cost of ITS and the availability of surrounding ITS infrastructure such as mobile phone networks, nevertheless, the size of necessary outlays still remains significant. Therefore, introducing ITS often requires significant and ongoing investment.

For ITS deployments to get funded, the public and public officials must be persuaded into spending more money on ITS rather than on other projects. This requires the proponents of ITS to do a careful analysis of costs and benefits and to demonstrate the overall value of ITS to life and national economy.

Technical issues

Miles and Chen (2004) suppose that understanding technical issues as a structured framework including the components of ITS systems are brought together - like construction products and services in a building - so that the whole can work efficiently. ITS technical issues specify performance requirements and functions of individual system components and show where standard interfaces can bring big benefits which helps with defining the interaction between the systems and with clarifying the roles of individual actors in the implementation process. It should be firmly based on the analysis of system functions and performance in response to user needs. Technical issues should be generic to allow freedom for system developers to design the worked out optimum solutions while meeting appropriate standards and accommodating interface need for interoperability and future extensions. If design provides for interchangeability of components, it will encourage competition and allow scope for future improvements in cost, design, functionality and safety.

Let us discuss in more detail the scheme of the impact of random factors on the quality of ITS. A team and operator should be interested in the quality of operating ITS. One of the regularities of forming random factors is the fact that the errors made at the earlier stage of operating ITS pass to the next one. Effective and precise information at all stages facilitates the elimination of random factors.

The present classification of reasons of random factor occurrence does not pretend to be absolutely comprehensive, however, in our opinion, it demonstrates the complexity of the phenomena finally determining the efficiency of operating ITS.

A reiterative random factor may occur due to various reasons, for instance an accident while operating ITS. Other reasons may also vary and embrace regulation, incidents, park and ride options, prevailing transport conditions, public transport schedules (timetable and actual), road works including both planned and emergency, tools, weather conditions, roadside phones (roadside services, including call boxes), information terminals at bus stations, rail stations, car parks, in major public places, at transfer points, in vehicles information displays, portable/personal terminals and travel services information.

In general, the technical-economical indices of operating ITS should be analyzed as random factors, which in turn are also random for every meaning of reason. The reasons are time or other parameters of operating ITS. Thus, optimal criteria should also be considered as random rather than determined.

The majority of indices defining an assessment of operating ITS are closely interconnected; therefore it has to be taken into account when these indices are used as optimal criteria. For a definition of statistical information obtained within the process of operating ITS, an assessment of mathematical probability, correlation and dispersion functions should be calculated.

3. Assessment of the Interdependency of Different Random Factors

Different variable parameters in the beginning of operating ITS will be indicated by $X_1, X_2, ..., X_n$, parameters determining operating ITS in progress will be indicated by $Z_1, Z_2, ..., Z_m$ and variable parameters of the exit from operating ITS will be indicated by $Y_1, Y_2, ..., Y_s$ (Figure 1). Then, initial random quantities marked by $X_1, X_2, ..., X_n$ will be analysed as the components of random vector X, random quantities $Z_1, Z_2, ..., Z_m$ – as the components of random vector Z and the exit random quantities $Y_1, Y_2, ..., Y_s$ – as the components of random vector Y (Baublys 2002).

It is not possible to limit ourselves only to the analysis of separate initial and variable factors characterising the quality of operating ITS in progress because they are insufficient for obtaining comprehensive characteristics of the technical-economical indices of operating ITS as well as it is impossible to define an optimal variant of the management of operating ITS. Operating ITS should be analysed as a multi-measurable process with a large number of initial parameters the general assessment of which allows a complex evaluation of the efficiency of operating ITS.

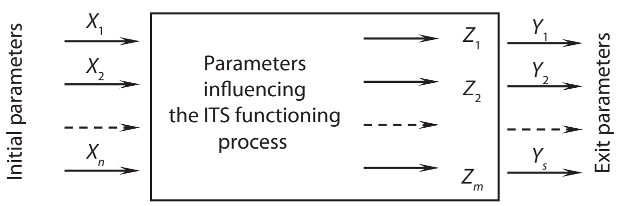

Fig. 1. The scheme of operating ITS

It is obvious that according to the characteristics of the multi-measurable process, the meanings of every exit variable Y should be defined, whereas the exit variables of the multi-measurable process may be independent, correlated or connected by functional interdependence. Practically, the characteristics of operating ITS should be analysed in each of the aforesaid cases. At the beginning, we shall analyse the characteristics of operating ITS when the quantities of the initial variable are interdependent $Y_1, Y_2, ..., Y_s$.

Let us presume that the density of the general probability of the vectored random quantities X, Z and Y is normal (Baublys 2002):

$$\varphi_t\left(U_1, ..., U_t\right) = \frac{1}{\sigma_{u_1}, ..., \sigma_{u_t}\sqrt{\left(2\Pi\right)^t E}} \times$$
$$\exp\left(\frac{-1}{2E}\sum_{\mu,í=1}^{t} E_{\mu,í}\left(\frac{U_\mu - M\left\{U_\mu\right\}}{\sigma_{í\mu}}\right)\left(\frac{U_í - M\left\{U_í\right\}}{\sigma_u}\right)\right), \quad (1)$$

where for the abridgement sake, random vector quantity U made of $X_1, ..., X_n$, $Z_1, ..., Z_m$, $Y_1, ..., Y_s$; $t = n + m + s$ is introduced; E is a determinant of series t.

$$E = \begin{vmatrix} 1 & \rho_{u_1u_2} & \cdots & \rho_{u_1u_t} \\ \rho_{u_2u_1} & 1 & \cdots & \rho_{u_2u_t} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{u+u_1} & \rho_{u+u_2} & \cdots & 1 \end{vmatrix}, \quad (2)$$

or X, Z, Y in marking variables:

$$E = \begin{vmatrix} 1 & \rho_{x_1x_2} & \cdots & \rho_{x_1x_n} & \rho_{x_1z_1} & \cdots & \rho_{x_1z_m} & \rho_{x_1y_1} & \cdots & \rho_{x_1y_s} \\ \rho_{x_nx_1} & \rho_{x_nx_2} & \cdots & 1 & \rho_{x_nz_1} & \cdots & \rho_{x_nz_m} & \rho_{x_ny_1} & \cdots & \rho_{x_ny_s} \\ \rho_{z_1x_1} & \rho_{z_1x_2} & \cdots & \rho_{z_1x_n} & 1 & \cdots & \rho_{z_1z_m} & \rho_{z_1y_1} & \cdots & \rho_{z_1y_s} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{z_nx_1} & \rho_{z_mx_2} & \cdots & \rho_{z_mx_n} & \rho_{z_mz_1} & \cdots & 1 & \rho_{z_my_1} & \cdots & \rho_{z_my_s} \\ \rho_{y_1x_1} & \rho_{y_1x_2} & \cdots & \rho_{y_1x_n} & \rho_{y_1z_1} & \cdots & \rho_{y_1z_m} & 1 & \cdots & \rho_{y_1y_s} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{y_sx_1} & \rho_{y_sx_s} & \cdots & \rho_{y_sx_n} & \rho_{y_sz_1} & \cdots & \rho_{y_sz_m} & \rho_{y_sy_1} & \cdots & 1 \end{vmatrix};$$

$E_{\mu,\nu}$ is algebraic supplement $\rho_{umn\nu}$ in the determinant (2).

For the analysis of the case when the exit variables are independent, it is necessary to determine characteristics of every variable $Y_k\left(k = 1, 2, ..., s\right)$ as well as the influence exercised on them by the initial variables X and variables Z characterising the inner state of the process. Let us indicate the density of general probabilities X, Z and Y $Y_{n+m+1}\left(Y_k, X_1, ..., X_n, Z_1, ..., Z_m\right)$ and the density of the general probabilities of random vectored quantities' X and Z $\varphi_{n+m}\left(X_1, ..., X_n, Z_1, ..., Z_m\right)$. Probability densities $\varphi_{n+m+1}\left(\phi_k, X_1, ..., X_n, Z_1, ..., Z_m\right)$ and $\varphi_{n+m}\left(X_1, ..., X_n, Z_1, ..., Z_m\right)$ are not zero ones and correspond to equation (1) with the meanings of determinants σ and F correspondingly of $(n + m + 1)$ and $(n + m)$ series:

$$\sigma = \begin{vmatrix} 1 & \rho_{x_1x_2} & \dots & \rho_{x_1z_m} & \rho_{x_1y_k} \\ \rho_{x_2x_1} & 1 & \dots & \rho_{x_2z_m} & \rho_{x_2y_k} \\ \dots & \dots & \dots & \dots & \dots \\ \rho_{z_mx_1} & \rho_{z_mx_2} & \dots & 1 & \rho_{z_my_k} \\ \rho_{y_kx_1} & \rho_{y_kx_2} & \dots & \rho_{y_kz_m} & 1 \end{vmatrix}; \quad (3)$$

$$F = \begin{vmatrix} 1 & \rho_{x_1x_2} & \dots & \rho_{x_1x_n} & \rho_{x_1z_1} & \dots & \rho_{x_1z_m} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \rho_{x_nx_1} & \rho_{x_nx_2} & \dots & 1 & \rho_{x_nz_1} & \dots & \rho_{x_nz_m} \\ \rho_{z_1x_1} & \rho_{z_1x_2} & \dots & \rho_{z_1x_n} & 1 & \dots & \rho_{z_1z_m} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \rho_{z_mx_1} & \rho_{z_mx_2} & \dots & \rho_{z_mx_n} & \rho_{z_mz_1} & \dots & 1 \end{vmatrix}. \quad (4)$$

A general characteristic of operating ITS is the density of conditional probability $\varphi\left(Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right)$ according to which the meaning may be defined by the general characteristics of the prior variables X and the inner state variables Z by transforming the distribution laws of these random quantities.

Generally, conditional density $\varphi(Y_k/(X_1, ..., X_n, Z_1, ..., Z_m))$ is determined by general probability densities:

$$\varphi_{n+m+1}\left(Y_k X_1, ..., X_n, Z_1, ..., Z_m\right) \text{ and}$$

$$\varphi_{n+m}\left(X_1, ..., X_n, Z_1, ..., Z_m\right):$$

$$\varphi\left(Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right) = \frac{\varphi_{n+m+1}\left(Y_k, X_1, ..., X_n, Z_1, ..., Z_m\right)}{\varphi_{n+m}\left(X_1, ..., X_n, Z_1, ..., Z_m\right)}. \quad (5)$$

If common probability densities $\varphi_{n+m}\left(X_1, ..., X_n, Z_1, ..., Z_m\right)$ and $\varphi_{n+m+1}\left(Y_k, X_1, ..., X_n, Z_1, ..., Z_m\right)$ are normal, then for the analysed case, conditional probability density is:

$$\varphi\left(Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right) = \frac{1}{\sqrt{2\pi}\sigma_{yk}} \sqrt{\frac{F}{\sigma}} \exp\left(\frac{1}{2F} \sum_{k,\, l=1}^{n+m} F_{kl} \times \left(\frac{\nu_k - M\{\nu_k\}}{\sigma_{\nu_k}}\right)\left(\frac{\nu_t - M\{\nu_l\}}{\sigma_{\nu_l}}\right) - \frac{1}{2\sigma} \sum_{m,\, \nu=1}^{n+m+1} \sigma_{\mu_\nu} \times \left(\frac{U_\mu - M\{U_\mu\}}{\sigma_{\acute{u}\mu}}\right)\left(\frac{U_{\acute{\iota}} - M\{U_{\acute{\iota}}\}}{\sigma_u}\right)\right),$$

where: $F_{k,\,l}$ – element $\rho_{k,\,l}$ algebraic supplement in determinant (4); $\sigma_{\mu,\,\nu}$ – element $\rho_{m,\,\nu}$ algebraic supplement in determinant (3) $\nu(X_1, ..., X_n, Z_1, ..., Z_m)$, $U\left(Y_k, X_1, ..., X_n, Z_1, ..., Z_m\right)$. Given conditional density $\varphi(Y_k/(X_1, ..., X_n, Z_1, ..., Z_m))$ for the definition of characteristics X and Z, the general characteristics of the exit variable Y, i. e. the probability density $\varphi_{yk}\left(Y_k\right)$ is defined with the help of the following integral equation:

$$\varphi_{yk}\left(Y_k\right) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \varphi\left(Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right) \varphi_{n+m} \times \left(X_1, ..., X_n, Z_1, ..., Z_m\right) dX_1, ..., dX_n, dZ_1, ..., dZ_m.$$

While knowing general characteristics $\varphi(Y_k/(X_1, ..., X_n, Z_1, ..., Z_m))$ we may also define the main digital characteristics of X and Z in the aspect of the exit variable Y_k, i.e. conditional mathematical probability and conditional dispersion:

$$M\left\{Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right\} = \frac{\int_{-\infty}^{\infty} Y_k \varphi_{n+m+1}\left(Y_k, X_1, ..., X_n, Z_1, ..., Z_m\right) dy_k}{\int_{-\infty}^{\infty} \varphi_{n+m+1}\left(Y_k, X_1, ..., X_n, Z_1, ..., Z_m\right) dy_k} = \int_{-\infty}^{\infty} Y_k \frac{\varphi_{n+m+1}\left(Y_k, X_1, ..., X_n, Z_1, ..., Z_m\right)}{\varphi_{n+m}\left(X_1, ..., X_n, Z_1, ..., Z_m\right)}.$$

Thus, we shall assess equation (5) and obtain:

$$M\left\{Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right\} = \int_{-\infty}^{\infty} Y_k \varphi\left(Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right) dy_k.$$

Whereas conditional dispersion Y_k in the aspect of X and Z:

$$D\left\{Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right\} = \int_{-\infty}^{\infty} \left(Y_k - M\left\{Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right\}\right)^2 \times \varphi\left(Y_k / \left(X_1, ..., X_n, Z_1, ..., Z_m\right)\right) dy_k.$$

Thus, while defining the characteristics of operating ITS, when random factors are assessed, it is necessary to determine unconditional and conditional dispersion laws. For this reason, the constant accumulation of statistic information is necessary.

For practical purposes, it is often expedient to use digital characteristics instead of the distribution laws of random factors. Although digital characteristics give insufficient information on random factors, however, for a solution to certain information issues they fully suffice and their determination is by far easier. A complete analysis and synthesis of the characteristics of operating ITS is carried out according to the general characteristics, i. e. according to the unconditional and conditional distribution laws that may be employed in defining different characteristics of operating ITS. However, as the presented formula shows, the characteristics of operating ITS may be determined in such cases when the general laws of the distribution of random quantities are known. This condition must be observed in designing systems for accumulating and processing statistical information.

4. Mathematical Models for Accumulating Statistical Information

Statistical information on operating ITS is renewed and replenished in due course. Along with the increasing amounts of information, the costs of its storage are also increasing. Therefore, our aim is to carry out necessary statistical assessment of the minimal amount of statistical information.

The assessment of distributing random factors using the Bayesian method. Presumably, we have sample *X*, made of *N* meanings $x_1, ..., x_N$. Considering meanings *x* as random quantities, we shall assess a distribution function of random quantity *X*. For an analytical description of distribution *X*, relevant prior information on a distribution type is necessary. Let us presume the density of parametric probabilities known as $f\left(X/\theta\right)$,

where: $\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_r \end{pmatrix}$ – a vector of parameters describing the distribution of random quantity *x*; *r* – is the number of parameters and presumably known the prior distribution of vector parameters $\theta f_{apr.}\left(\theta\right)$. The latter may be defined by experimental assessment. Then *X* distribution may be assessed with the help of the Bayesian formula (Baublys 2002, 2008).

At the beginning, we shall obtain the aposterioric density of distributing vector parameters. Considering that observations in the sample $x_1, ..., x_N$ are independent, according to the Bayesian formula it follows that:

$$f\left(\theta / x_1, ..., x_N\right) = \frac{\prod_{i=1}^{N} f\left(x_i/\theta\right) f_{apr.}\left(\theta\right)}{\int_{\Omega_\theta} \prod_{i=1}^{N} f\left(x_i/\theta\right) f_{apr.}\left(\theta\right) d\theta}, \quad (6)$$

where integration is in *r* – measurable area Ω_θ by changing parameter θ. The aposterioric distribution of *X* is obtained by integrating

$$f\left(X / x_1, ..., x_N\right) = \int_{\Omega_\theta} f\left(X/\theta\right) f\left(\theta / x_1, ..., x_N\right) d\theta. \quad (7)$$

By putting (6) into (7) we obtain:

$$f\left(X / x_1, ..., x_N\right) = \frac{\int_{\Omega_\theta} f\left(X/\theta\right) \prod_{i=1}^{N} f\left(x_i/\theta\right) f_{apr.}\left(\theta\right) d\theta}{\int_{\Omega_\theta} \prod_{i=1}^{N} f\left(x_i/\theta\right) f_{apr.}\left(\theta\right) d\theta}.$$

After a new sample of phenomena $x_{n+1}, ..., x_{n+m}$ is obtained, and thus it is necessary to specify distribution *f*(X) using new data; then, instead of the prior distribution $f_{apr.}\left(\theta\right)$ we shall use the former prior one $f\left(\theta / x_1, ..., x_N\right)$. Let us notice that such method does not require storing all the meanings of a sample; only the meanings of aposterioric distribution $f\left(\theta / x_1, ..., x_N\right)$ are subject to storing, namely only a certain number of coefficients characterising it. At the beginning of the data accumulation process, we may not have $f_{apr.}$ on the whole as then the Bayesian postulate is applied and the prior distribution is considered uniform (Baublys 2002, 2007, 2008, 2009).

The presented formulae would lose their practical importance if they did not have concrete distribution types $f\left(X/\theta\right)$ and $f_{apr.}\left(\theta\right)$. Besides, it is preferable that the analytical type of the prior and aposterioric density would be uniform because in that case, calculation algorithms and programmes become simpler.

Normal distribution. Let us analyse the case when random qualities are distributed according to the normal law the density of which is:

$$f\left(X / \mu, \sigma\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\frac{\left(x-\mu\right)^2}{\sigma^2}\right). \quad (8)$$

In the given case, parameter vector $\theta = \begin{pmatrix} \mu \\ \sigma \end{pmatrix}$ and

$$f\left(x_1, ..., x_N / \mu, \sigma\right) = C_0 \sigma^{-N} \exp\left(-\frac{1}{2}\frac{S\left(N-1\right)}{\sigma^2}\right) \exp\left(-\frac{1}{2}\frac{N\left(\bar{x}-\mu^2\right)}{\sigma^2}\right), \quad (9)$$

where: $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$; $S = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2$ – sample medium and dispersion accordingly $C_0 = \left(2\pi\right)^{-N/2}$.

The prior distribution of parameters μ and σ is selected so that its density would be analogous to the density of conditional one

$$f\left(\mu, \sigma\right) = C_1 \sigma^{-n} \exp\left(-\frac{1}{2}\frac{a\left(n-1\right)+n\left(\mu+\mu_0\right)^2}{\sigma^2}\right). \quad (10)$$

Coefficient C_1 depends on *n* and may be obtained from the rationed condition. Coefficient C_1 is not calculated here because it does not depend on μ and σ.

After putting (10) into (6) and following corresponding rearrangement, we shall obtain:

$$f\left(\sigma, \mu / x_1, ..., x_N\right) = \frac{1}{C_3}\sigma^{-N*} \times \exp\left(-\frac{1}{2}\frac{\left(N^*-1\right) a^* + N^*\left(\mu-\mu^*\right)}{\sigma^2}\right), \quad (11)$$

where: C_3 is obtained from the rationed condition;

$N^* = N + n$;

$$a^* = S\frac{N-1}{N+n-1} + a\frac{n-1}{N+n-1} + \frac{Nn\left(\mu_0 - \bar{x}\right)^2}{\left(N+n\right)\left[\left(N+n\right)-1\right]};$$

$$\mu^* = \bar{x}\frac{N}{N+n} + \mu_0\frac{n}{N+n}.$$

Now, let us calculate coefficient C_3.

Having noticed that:

$$\frac{1}{\sqrt{2\pi}\sigma/\sqrt{N^*}}\int_{-\infty}^{\infty}\exp\left(-\frac{1}{2}\frac{\left(\mu-\mu^*\right)^2}{\sigma^2/N^*}\right)d\mu = 1,$$

after relevant rearrangements,

$$C_3 = \frac{1}{2}\sqrt{\frac{2\pi}{N^*}}\left(\frac{a^*\left(N^*-1\right)}{2}\right)^{\left(N^*-2\right)/2}\Gamma\left(\frac{N^*-2}{2}\right).$$

Thus, the aposterioric distribution of parameters σ and μ:

$$f\left(\sigma, \mu / x_1, ..., x_N\right) = \frac{\sigma^{-N^*}\exp\left(-\frac{1}{2}\frac{\left(N^*-1\right)a^* + N^*\left(\mu-\mu^*\right)^2}{\sigma^2}\right)}{\sqrt{\frac{\pi}{2N^*}}\left(\frac{a^*}{2}\left(N^*-1\right)\right)^{\left(N^*-2\right)/2}\Gamma\left(\frac{N^*-2}{2}\right)}. \quad (12)$$

The aposterioric density of x distribution:

$$f\left(X / x_1, ..., x_N\right) = \frac{\Gamma\left(\frac{N^*-1}{2}\right)}{\sqrt{2\pi}\Gamma\left(\frac{N^*-2}{2}\right)\sqrt{\frac{a^*\left(N^*-1\right)}{2}}} \times \left(1+\frac{N^*\left(X-\mu^*\right)^2}{\left(N^{*2}-1\right)a^*}\right)^{-\left(N^*-1\right)/2}. \quad (13)$$

Aposterioric density may be approximated by the normal one (8) putting instead of σ and μ more probable meanings obtained from (12). By differentiation, we obtain the following more expected meanings of parameters $\mu = \mu^*$, $\hat{\sigma}^2 = a^*$.

Then, aposterioric density is approximated by the phenomenon:

$$f\left(X / \hat{\mu}, \hat{\sigma}\right) = \frac{1}{\sqrt{2\pi}\hat{\sigma}}\exp\left(-\frac{1}{2}\frac{\left(X-\hat{\mu}\right)^2}{\hat{\sigma}^2}\right). \quad (14)$$

In the presence of large N (14), (13) approximates density with great precision. As (13) and (14) include only parameters $N^* = N + n$, a^* (or $\hat{\sigma}$), μ^* (or $\hat{\mu}$), then every aposterioric distribution may be very simply used as an aprioric one. For this purpose, only parameters are recalculated:

$$N_i = N_{i-1} + N;$$

$$\hat{\mu}_i = \mu_{i-1}\frac{N_{i-1}}{N_{i-1}+N} + \frac{N}{N_{i-1}+N}\bar{x};$$

$$\hat{\sigma}^2 = \hat{\sigma}^2_{i-1}\frac{N_{i-1}-1}{N+N_{i-1}-1} + S\frac{N}{N+N_{i-1}-1} + \frac{NN_i\left(\mu_i-\bar{x}\right)^2}{N+N_i\left(N+N_{i-1}\right)}.$$

Consequently, suffice it to remember only three coefficients, including N_{i-1}, μ_{i-1} and σ_{i-1}. To demonstrate under which given N it is possible to use simple (14) instead of the complicated formula (13), let us analyse the following example.

Accumulation of data. As it was mentioned before, storing all data is not convenient and not possible, however, there does not exist a precise method of defining parameters m and x_0. Therefore, we shall make a compromise and assess parameters applying the method of moments (Baublys 2002).

Let us presume that the first two moments of the sample are known:

$$m_1 = \frac{1}{r}\sum_{i=1}^{r} t_i, \quad m_2 = \frac{1}{r}\sum_{i=1}^{r} t_i^2. \quad (15)$$

The precise meanings of the prior moments of the Weibull distribution:

$$m_1 = x_0^{1/m}\Gamma\left(1+\frac{1}{m}\right); \; m_2 = x_0^{2/m}\left(1+\frac{2}{m}\right). \quad (16)$$

Having equalised the meanings of sample parameters to their analytical expressions, we shall obtain the system of equations enabling to find x_0 and m:

$$x_0^{1/m}\Gamma\left(1+\frac{1}{m}\right) = m_1; \; x_0^{m/2}\Gamma\left(1+\frac{2}{m}\right) = m_2.$$

After the rearrangement of the system of non-linear equations, i.e. after dividing the second one from the first square:

$$\frac{x_0^{2/m}\Gamma\left(1+\frac{2}{m}\right)}{x_0^{2/m}\Gamma\left(1+\frac{1}{m}\right)\Gamma\left(1+\frac{1}{m}\right)} = \frac{\hat{m}_2}{\hat{m}_1}.$$

Since $\Gamma\left(a+1\right) = \Gamma\left(a\right)a$, then:

$$\frac{\Gamma\left(\frac{2}{m}\right)\frac{2}{m}}{\Gamma\left(\frac{1}{m}\right)\Gamma\left(\frac{1}{m}\right)\frac{1}{m^2}} = \frac{2m\Gamma\left(\frac{2}{m}\right)}{\Gamma\left(\frac{1}{m}\right)\Gamma\left(\frac{1}{m}\right)} = \frac{\hat{m}_2}{\hat{m}_1^2}. \quad (17)$$

By applying the Lagrange's formula for the gamma function, an argument of equation:

$$\Gamma(2z) = 2^{2z-1}\pi^{-1/2}\Gamma(z)\Gamma\left(z+\frac{1}{2}\right),$$

from (17) we obtain a much more convenient phenomenon of obtaining *m*.

$$\hat{m}_1^2 2^{2/m} m\Gamma\left(\frac{1}{m}+\frac{1}{2}\right) - \hat{m}_2^1\sqrt{\pi}\Gamma\left(\frac{1}{m}\right) = 0 . \qquad (18)$$

Quantity *m* is obtained solving equation (18) and x_0 – by the obtained meaning and by one of (16) equations, for example

$$x_0 = \left(\frac{m_1}{\Gamma\left(1+\frac{1}{m}\right)}\right)^m . \qquad (19)$$

Searching for parameters x_0 and *m* by (18), (19) and (15), only statistics is stored:

$$\hat{m}_t = \left(\frac{1}{w}\right)\sum t_i \text{ ; } \hat{m}_2 = \left(\frac{1}{w}\right)\sum t_i^2 .$$

The method of moments gives a non-optimal assessment of meanings *m* and x_0. The quantities of small samples *m* and x_0 are calculated by the method of moments and may differ from the calculated ones using the method of maximum simplicity. Therefore, given small *n*, it is purposeful to calculate *m* and x_0 applying the latter method and for this purpose, it is necessary to remember all meanings $t_1, ..., t_r$. For a large *n*, it is better to use the method of moments.

Finally, it should be mentioned that stochastic models have to reflect the main regularities of the investigated object. The degree of adequacy in the given case depends on how precisely the interdependency between incoming and outgoing parameters, interactive system and environment, ability to correct decisions using a model and finally, the application of stochastic methods in obtaining the optimal behaviour scheme of the investigated system are assessed in the models.

5. Conclusions

1. Generally, the technical-economic indices of operating Intelligent Transport Systems (ITS) should be analysed as random factors which are also random in view of each meaning of the argument. Arguments mean the time and other parameters of operating ITS. Thus, the optimum criteria should be considered as random rather than determined ones.
2. The most indices according to which operating ITS are assessed, are interconnected, and therefore it has to be admitted that these indices are used as optimality criteria. For determining the digital characteristics of technical-economical indices according to statistical information received during the process of operating ITS, the functions of assessing mathematical probability as well as correlation and dispersion should be calculated.
3. For modelling ITS networks, it is analytically purposeful to describe random factors conducting non-parametric assessment. Differently from the parametric one, a non-parametric assessment of distribution density has a number of merits. First, when in the course of time, a type of the flows of distribution density changes, it would be possible to approximate for practical use. Second, such assessment is significantly less sensitive to data errors.
4. Statistical information on operating ITS is renewed and replenished in the course of time. Along with the increasing amounts of information, the costs of its storage are also increasing. Therefore, it is necessary to obtain the required statistical assessment with the least information amounts. For this purpose, mathematical models necessary for accumulating statistics are made and verified. There have been developed models for the following cases when the density of distributing random factors is assessed by:
   a) Bayesian analysis;
   b) normal distribution;
   c) logarithmic-normal;
   d) non-parametric distribution;
   e) Weibull distribution.

   For all these cases, algorithms for accumulating minimum statistical data are created.

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