DESIGNING CONTAINER SHIPPING NETWORK UNDER CHANGING DEMAND AND FREIGHT RATES

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Abstract. This paper focuses on the optimization of container shipping network and its operations under changing cargo demand and freight rates. The problem is formulated as a mixed integer non-linear programming problem (MINP) with an objective of maximizing the average unit ship-slot profit at three stages using analytical methodology. The issues such as empty container repositioning, ship-slot allocating, ship sizing, and container configuration are simultaneously considered based on a series of the matrices of demand for a year. To solve the model, a bi-level genetic algorithm based method is proposed. Finally, numerical experiments are provided to illustrate the validity of the proposed model and algorithms. The obtained results show that the suggested model can provide a more realistic solution to the issues on the basis of changing demand and freight rates and arrange a more effective approach to the optimization of container shipping network structures and operations than does the model based on the average demand.

Keywords: container shipping, ship-slot allocating, empty container repositioning, container configuration, average unit ship-slot profit, optimization.

1. Introduction

With the growth of the global economy, the container shipping industry is playing a more and more important role in international cargo transportation (Jaržemskienė and Jaržemskis 2009; Liu *et al.* 2009; Su and Wang 2009; Paulauskas and Bentzen 2008; Vasilis Vasiliauskas and Barysienė 2008; Jaržemskis and Vasilis Vasiliauskas 2007; Rohács and Simongáti 2007). To adapt to greater container cargo shipment demand, shipping companies are increasing capacity via new super-size container ships. Companies have also begun to pay special attention to optimizing container shipping network designs and operations in order to promote higher quality service. This paper looks at the issues of container shipping network structures and operations considering changing demand and freight rates in dealing with empty container repositioning, ship-slot allocating, ship sizing and container configuration.

The container shipping network design problem (CSNDP) involves selecting a group of calling ports from a set of candidate ports and determining the calling sequence. The objective is to make optimal decisions regarding the following issues: voyage itinerary, the scale of ship assets and containers to be deployed, allocating ship-slots at each calling port in a specified sequence, container quantities loaded at each route and maximizing ship-slot profits in a round-trip operation.

In most existing studies, the CSNDP is solved based on the assumption that cargo demand is given only as a set of constants by a demand matrix that represents either a set of stable values or a set of the average values of annual demand. This assumption arises from the belief that ship sizes can be determined by the given demand and that the costs of route shipping are fixed. It further assumes that freight rates are not directly affected by fluctuations in the real-world demand for a fixed ship capacity. However, this assumption does not reflect the reality of container shipping network design. In fact, cargo traffic demand and freight rates fluctuate periodically. In this case, the shipping network operations may result in large capacity over plus when demand is low and make a great loss on revenue when demand is high. For example, within a year, in Sino-Japan shipping line, the highest cargo demand and freight rates are often three times higher than the lowest ones. Therefore, how to move or lease empty containers in a timely and efficient manner, what size ships maximize revenues during peak seasons and minimize loss during off-seasons and how to determine container configurations to reduce the risk of excessive containers in off-seasons and guarantee enough available containers in peak-seasons are the problems to be solved. These questions have already become critical and fundamental issues of the CSNDP that should be influenced primarily by container cargo distri-

bution among all ports in the trade area. Since it is essential to consider the impact of changing demand and changing freight rates, the CSNDP can be broken down into a series of sub problems, including the ship routing problem (SRP), calling sequencing problem (CSP), ship-slot allocating problem (SAP), ship-sizing problem (SSP) and container constituting problem (RCCP).

Based on the characteristics and attributes of the CSNDP, this study will propose an integrated model. The issues such as empty container repositioning, ship-slot allocating, ship sizing and container configuration are simultaneously considered based on a series of the matrices of demand for a year. The problem is formulated using the analytical method and the average revenue expected value technique by the Knapsack problem (KP), Salesman Travelling problem (STP) and Mixed Integer Nonlinear problem (MIP) basis. To solve the model, the bi-level genetic algorithm based method is proposed. Finally, numerical experiments are provided to illustrate the validity of the introduced model and algorithms.

The rest of this paper is organized as follows: In Section 2, a brief review of previous works is given. The descriptions of the problem are presented in Section 3. In section 4, the model for the CSNDP is developed. A bi-level genetic algorithm is designed in section 5. Numerical examples are used to test the performance of the worked out method in Section 6. Conclusions are given in Section 7.

2. Literature Review

A number of the existing research papers have focused on container shipping transportation. The larger part of them can be divided into two major categories covering ship routing and related operations.

On the issue of containership routing, the existing literature is rather limited. A comprehensive survey of vehicle routing problems can be found in Bodin *et al.* (1983), Laporte (1992) and Christiansen *et al.* (2004). Boffey *et al.* (1979) developed a heuristic optimization model and an interactive decision support system for scheduling container ships on the North Atlantic route. Rana and Vickson (1988 and 1991) tried to find the optimal sequence of calling ports for a fleet of ships operating on a trade route in order to maximize liner operation profit while also determining an optimal calling port sequence. They assumed that non-profitable ports should be rejected as calling ports on the route. They formulated the problem as a mixed integer nonlinear programming model and solved it by using Lagrangean relaxation techniques and the decomposing method. Perakis and Jaramillo (1991) and Jaramillo and Perakis (1991) designed a Linear Programming model for a routing strategy to minimize total operating and lay-out cost over a planning time horizon. They also studied the assignment of the existing fleet of container ships to a predetermined set of routes (sequence of calling ports) based on the realistic models of shipping operation costs. Cho and Perakis (1996) proposed optimal models for the fleet size and design of liner routes by taking into account future cargo demands both in

real-life situations and future forecasts. The problem is formulated as Mixed Integer Linear Programming and solved by devising a flow-route incidence matrix in the models to examine a number of candidate ports for different ships. Fagerholt (1999) studied the problem of determining the optimal fleet and liner routes based on a weekly frequency formulated as a multi-trip vehicle routing problem solved by a partitioning approach. Bendall and Stent (2001) proposed a determination model for optimal fleet configuration while taking into account the fleet deployment plan applied in a hub-spoke container shipping network. Lam *et al.* (2007) used a simple shipping route with two ports and operation with two voyages (TPTV) and its extension with multiple ports and multiple voyages (MPMV) to demonstrate the effectiveness of an approximate dynamic programming approach in finding operational strategies for empty container allocation. Since temporal difference learning for average cost minimization is utilized in the above suggested approach, only two voyages may not be sufficient to represent complete shipping route system operation. Hsu and Hsieh (2007) formulated a two-objective model to determine optimal liner routing, ship size and sailing frequency for carriers and shippers by minimizing shipping costs and inventory costs simultaneously, based on a trade-off between the two costs. From the viewpoints of carriers and shippers, the proposed approach may be of practical value.

On the issue of shipping route operations, considerable research has been done focusing primarily on empty container repositioning. Gavish (1981) developed a system for making decisions regarding container fleet management. In his study, if empty containers were not relocated at the requested time, the system would assign the owned and leased containers to satisfy demand based on the marginal cost criterion. It should be further noted that the extra leased containers affected liner operation total cost without consideration for the inventory of the idle owned containers. Crainic *et al.* (1993) introduced dynamic and stochastic models for empty container relocation in a land distribution and transportation system. Similarly, to deal with the problem of leased and empty container relocation, the authors ignored difference between short-term leasing cost and long-term cost. This seems impractical and not in keeping with the practice of dealing with long-term leased containers as the owned ones. Cheung and Chen (1998) also considered the sea-borne empty container allocation problem. In their paper, the dynamic container allocation problem was formulated as a two-stage stochastic network model. The model assists liner operators in allocating empty containers and consequently in reducing leasing cost and inventory level at calling ports. However, their work failed to consider the duration of leasing time. Imai *et al.* (2009) studied the optimization problem of container shipping network design proposing an approach to solve empty container repositioning problems. In their paper, port calling sequence and empty container repositioning are considered simultaneously by designing the objective function with a penalty cost

factor. Thus, the issue is integrated and formulated as a two-stage problem. The idea of adding penalty cost in the proposed model and using virtual points in designing network structure should be certainly valuable. Nevertheless, due to a lack of cargo traffic demand fluctuations and cargo flow distributions among ports in their experiments, there are evident flaws in ship-slot allocations among calling ports. More recently, Chang *et al.* (2008) studied a heuristic method to provide an optimal solution to reduce the cost of empty container interchange. Using the available data, they tested the effectiveness of computational time and solution quality. Di Francesco *et al.* (2009) developed a multi-scenario, multi-commodity, time-extended optimization model to deal with the empty container repositioning problem. Some uncertain model parameters that cannot be estimated through historical data are treated as the sets of a limited number of values according to the shipping company opinions. Bandeira *et al.* (2009) created a decision support system (DSS) to deal with full and empty container trans-shipment operations. The arrangement of repositioning empty containers can be determined by adjusting several parameters in the DSS model.

None of the above introduced studies has looked into the encountered problem and approach in this paper: namely container shipping network design and operation should be incorporated into a single, coordinated problem to be addressed by considering the revenue-loss risk control of ship sizing and container configuration based on periodic fluctuations in cargo demand and freight rates.

3 Problem Descriptions

In general, the optimization of the CSNDP should be completed by a series of decision-making processes that involve selecting appropriate calling ports from candidate ports in a trade area determining the reasonable order of calling sequence with the fixed regular frequency service and settling rational ship-slot allocation at each calling port with the suitable scale of the deployed assets that include ship size, container quantity and container configurations in the network. These decision-making processes depend upon the following influencing factors and are also called controllable factors mainly covering distances and cargo traffic demand together with freight rates among candidate ports in a trade area, the investment costs of ships and containers and company's policies regarding the shipping market and investment etc. Based on these controllable factors, the decision-making process ought to determine factors, including the optimal set of ports to be called, the optimal order of calling sequence, the optimal size of ships and the optimal series of ship-slot allocations on shipboard at each calling port. Since the ship size is unchangeable during a planning period and fluctuating demand produces a significant effect on ship size, it is more feasible to use a series of the matrices of demand in order of time to represent fluctuating demand rather than to use only a matrix of the average demand.

The container shipping network structures generally can be divided into two types of forms according to their operation characteristics. One is circular and another is pendulum, as shown in Fig. 1. From the viewpoint of topology, they can be essentially reduced to the circular route as a basic form because any pendulum type can be converted to a circular one by adding virtual nodes representing the ports in the backward direction and by constructing an adequate matrix for demand distributions. Shipping network operation is generally performed by a fleet of ships with a series of ship-slot allocations for calling ports. The fleet of ships travelling on the route ought to be split into two groups where one group travels in a clockwise direction while another travels at the same time in a counterclockwise direction. In this way, cargo traffic at any calling port is conveniently transported to its adjacent ports in different directions. For example, cargo traffic from Port 1 to Port 2 must be carried by one group of ships in a clockwise direction and cargo traffic from Port 1 to Port 9 can be carried by another group of ships in a counterclockwise direction, as shown in Fig. 2. The ships are only required to pick up the containers to be transported to other calling ports located in a half voyage travelling in the same direction as the ships.
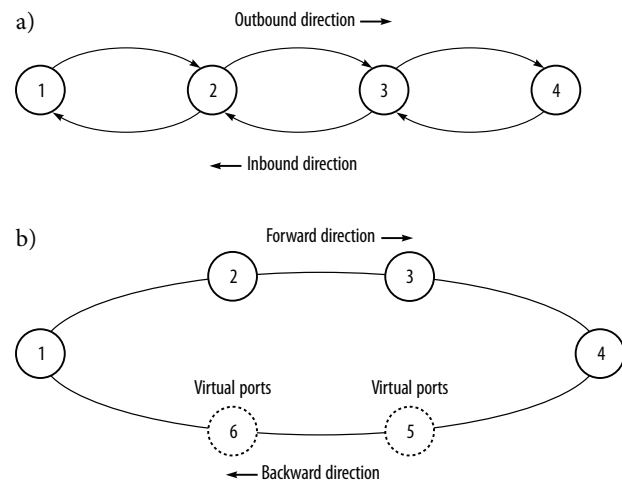

Fig. 1. The example: a – of pendulum route;
b – of circular route

In addition, due to the imbalanced directional cargo flows between some calling ports, there must be difference between the total cargo traffic originating from the port and the one arriving to it. Since load rejection is very unlikely in practice assuming that ship capacities have spare slots, liner shipping companies must decide whether to reposit empty containers or lease extra containers and store the idle owned containers at the specific ports. Since comparisons of the costs in a single voyage are not reasonable, comparing these average costs in a sufficient number of voyages under consideration is necessary. These elements must be represented in the formulation as the opportunity costs with mutual-substitution relation between them.
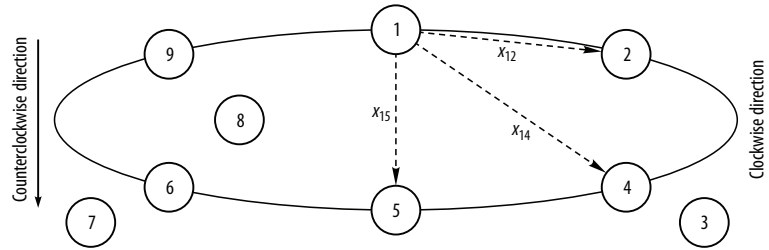
Fig. 2. The example of circular route operation

Thus, the model we will construct should include the above influencing factors and elements. The model with an objective of the average unit ship-slot profit maximization can be formulated by designing an average closed voyage trip in a circular route with the appropriate scales of ships and containers deployed. In ship routing, it is not necessary for ships to call at all ports in the trade area, for example, Ports 1, 2, 4, 5, 7 and 9 are selected but Ports 3, 7, and 8 are given up, as shown in Fig. 2. Other assumptions are as follows:

(a) As a key influencing factor, fluctuating cargo traffic demand among all ports is presented by a series of demand matrices in order of time and with relevant homogenous freight rates rather than by a matrix of the average demand in a planning horizon. The reason is mainly because the practical number of containers transported in the planning horizon should be limited by the accepted ship size once it is determined, as shown in Fig. 3.

(b) There must be an appropriate quantity of containers equipped at every calling port corresponding to the quantity handled at them, according to container shipping network design. Additional containers can be leased at any port but must ultimately be returned to the original port.

(c) The ships deployed in the network or route must be the same with capacities and cruising speeds.

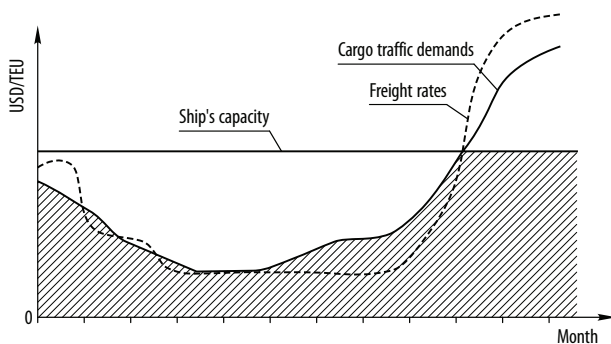(d) The ship's capacity must not be exceeded by a total number of containers loaded on shipboard at any route leg.

Fig. 3. Periodic fluctuations in cargo traffic demand and freight rates

4. Model Formulations

As described above, the CSNDP consists of four sub-problems. The first one is choosing the best group of calling ports for the optimal network or route. The second is identifying the calling sequence of the chosen group of calling ports for an optimal arrangement of voyage itinerary. The third is optimizing ship-slot allocations at each calling port with a series of container quantities handled on each voyage at each calling port for the average unit ship-slot profit maximization. The last problem is determining rational container configurations deployed in the networks depending upon the above container quantities handled at each calling port. Since there are interrelations and interactions among these sub-problems, the CSNDP can be formulated as a mixed integer non-linear programming problem (MINP) at three stages, based respectively on the Knapsack Problem (KP), Salesman Travelling Problem (STP) along with the Operation Problem (OP) and Container Configuration Problem (CCP). The optimal model can be developed as follows:

Stage 1:

[KP] Maximize $\sum_{k \in V} \omega_k \times P^k$,   (1)

Subject to $\sum_{k \in V} \omega_k = 1$,   (2)

$\omega_k \in \{0,1\}, \ \forall k \in V$,   (3)

where: V is a set of combinations of calling ports taken from a set of candidate ports N in the trade area; $\omega_k = 1$ if the route constructed by a candidate combination of calling ports k is selected and 0 otherwise; P^k is the values of the objective function under the candidate combination of calling ports k.

Stage 2:

Given a set of calling ports, an optimal calling sequence can be formulated by constructing the MINP with the STP and OP. In order to find decision variables as described, let $w_{ij} \ (i, j \in N, i \neq j)$ be binary flow variables, $x_{ij}, y_{ij} \ (i, j \in N, i \neq j)$ – respectively full and empty ship-slot allocation variables at each calling port, u – the ship-size variable and $X_{ijg}, Y_{ijg} (i, j \in N, \ g \in G)$ express respectively the real quantities of full and empty containers as auxiliary variables loaded in the scenario $g(g \in G)$ of the series of cargo traffic demand $d_{ijg} (i, j \in N, g \in G)$. In consideration of the period fluctuations of cargo traffic among calling ports, the unit ship-slot profit an average voyage in a planning horizon is introduced which

may be more reasonable and effective and can be represented by the expected revenue in a year with total G voyages. Thus, if route operation by a single ship with capacity (u) is considered, [MINP] may be formulated by the unit ship-slot profit an average voyage in a year with total G voyages as follows: Maximize:

$$\overline{P} = \frac{1}{G \cdot u}\left(\sum_{i \in N}\sum_{j \in N}\sum_{g \in G}[(R_{ij}^f - C_{ij}^f)\cdot X_{ijg} + w_{ij}\cdot(R_{ij}^e - C_{ij}^e)\cdot Y_{ij}] \right) -$$

$$\sum_{i \in N}\sum_{j \in N} t_{ij}\cdot w_{ij}\cdot\left(\sigma\cdot\frac{(G+1)}{2}\cdot[\lambda\cdot LS_i + \mu\cdot ST_i] + C_{cn} + C_{sh}(u)\right).$$

$$(4)$$

Subject to:

$$\sum_{j \in N} w_{ij} - \sum_{j \in N} w_{ji} = 0, \ \forall i \in N, \tag{5}$$

$$\sum_{i \in \phi}\sum_{j \notin \phi} w_{ij} \geq 1, \ \forall \phi \subset N, \tag{6}$$

$$w_{ij} \in \{0,1\}, \ \forall i, j \in N, \tag{7}$$

$$z_{ij} \leq d_{ijg}, \ \forall i, j \in N, \ \forall g \in G, \tag{8}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij} + y_{ij})a_{ijm} \leq u,$$

$$\forall i, j \in N, \ \forall m \in M, \ \forall g \in G, \tag{9}$$

$$X_{ijg} = d_{ijg}, \ \forall x_{ij} \geq d_{ijg}, \ i, j \in N, g \in G, \tag{10}$$

$$X_{ijg} = x_{ij}, \ Y_{ijg} = y_{ij}, \ \forall x_{ij} < d_{ijg}, \ i, j \in N, g \in G, \tag{11}$$

$$x_{ij}, y_{ij}, X_{ijg}, Y_{ijg} \in N \cup \{0\}, \ \forall i, j \in N, \ \forall g \in G, \tag{12}$$

$$L_i = \max\{[\sum_{j \in N}(X_{ji} + Y_{ji}) - \sum_{j \in N}(X_{ij} + Y_{ij})], 0\}$$

$$\forall i, j \in N \tag{13}$$

$$S_i = \max\{[\sum_{j \in N}(X_{ij} + Y_{ij}) - \sum_{j \in N}(X_{ji} + Y_{ji})], 0\}$$

$$\forall i, j \in N, \tag{14}$$

where:

$$C_{cn} = \eta\cdot\sum_{i \in N}\{\max\{\sum_{j \in N}(x_{ij} + y_{ij}), \sum_{j \in N}(x_{ji} + y_{ji})\}\} + v \times u; \tag{15}$$

$$C_{sh}(u) = (\alpha\cdot u^2 + \beta\cdot u + \gamma) \times (v + \varepsilon\cdot n); \tag{16}$$

t_{ij} the time of ship's travel from port (i) to port (j); v the number of ships deployed in the route with the fixed cruising speed; N a set of calling ports for $k \in V$; ϕ a non-empty subset of N; $C_{sp}(\cdot)$ a shipping cost function of the selected arcs (i, j represented in a linear part of the scope; $C_{cn}(\cdot)$ the requested quantity of containers (including at ports and on the shipboard); LS_i the number of leasing containers (TEU) at port (i); ST_i the number of storing containers (TEU) at port (i); M the number of the route equal to the number of calling ports in the circular route form; C_{ij}^f, C_{ij}^e the unit cost of handling full and empty containers (TEU) at a calling port; R_{ij}^f, R_{ij}^e the unit revenue of transporting full and empty containers (TEU) from port (i)to port (j); $a_{ijm} = 1$ if route-leg covering cargo traffic flows between port pairs (i, j) = 0 otherwise.

Functions (1)–(3) provide the method by which an optimal set of ports to be called in the route can be selected from the candidate ports in the trade area.

Constraint (5) ensures that each ship that arrives at a calling port must leave from it.

Constraint (6) guarantees that all the ports to be called must be connected via the constructed route in which there is no such sub-voyage that it does not visit all the ports selected in N.

Constraints (8)–(9) are constraints for full containers loaded at each calling port and ship's capacity on any route-leg, respectively.

Constraints (10)–(12) indicate the real quantity of containers loaded at each calling port and equal to real cargo traffic demand when the quantity of ship-slots allocated on shipboards is greater than real cargo traffic demand at it, whereas otherwise, the real quantity of containers loaded at the port only equals to the quantity of ship-slots allocated on shipboards.

Constraints (13) and (14) are leasing and storing container constraints.

Functions (15)–(16) represent the relationships between the assets of ships and containers deployed in the route and shipping operation cost with ship size, where η, α, β, γ respectively denote the weighted factors for the relative terms.

Objective function (4) is to maximize the unit ship-slot profit of an average voyage which is an algebraic sum of the total revenue, repositioning cost, leasing and storage costs and assets operation costs divided by ship's capacity, where σ, λ, μ, ε express the cost coefficients of the relative terms, respectively.

Stage 3:

The problem of container configuration is to determine and arrange the optimal configurations of containers with the owned container quantity, long-term leasing container quantity and short-term leasing container quantity deployed in the networks in order to minimize the total using container cost. If short-term leasing time be set to less than three months and long-term leasing time be more than three months, and let Q_i^O, Q_i^L and Q_i^S respectively signify the quantity variables of the owned containers, long-term and short-term leasing containers deployed at calling port i, the total container cost, including using costs and the idling costs of containers can be formulated as follows:

$$\text{Minimize } C = \sum_{i \in N}(C^O \times Q_i^O + C^L \times Q_i^L + C^S \times Q_i^S), +$$

$$\sum_{i \in N}(\Delta^O\cdot C_{I\,i}^O + \Delta^L\cdot C_{I\,i}^L + \Delta^S\cdot C_{I\,i}^L). \tag{17}$$

Subject to:

$$Q_i^O + Q_i^S + Q_i^L \leq \sum_{\substack{j \in N \\ j \neq i}}(x_{ij} + y_{ij}), \ \forall i, j \in N, \tag{18}$$

$$C^O \leq C^L \leq C^S, \tag{19}$$

$$0 \leq Q_i^O \leq \min_{g \in G}\{\sum_{\substack{j \in N \\ j \neq i}}(X_{ijg} + Y_{ijg})\}, \ \forall i, j \in N, \forall g \in G, \tag{20}$$

$$0 \leq Q_i^L \leq (\frac{1}{G} \sum_{\substack{j \in N \\ j \neq i}} \sum_{g \in G} (X_{ijg} + Y_{ijg}) - Q_i^O),$$

$$\forall i, j \in N, \forall g \in G, \tag{21}$$

$$0 \leq Q_i^S \leq (\max_{g \in G}\{\sum_{\substack{j \in N \\ j \neq i}} (X_{ijg} + Y_{ijg})\} - \frac{1}{G} \sum_{\substack{j \in N \\ j \neq i}} \sum_{g \in G} (X_{ijg} + Y_{ijg})),$$

$$\forall i, j \in N, \forall g \in G, \tag{22}$$

$$\Delta^O = Q_{ig}^O - \sum_{\substack{j \in N \\ j \neq i}} (X_{ijg} + Y_{ijg}), \forall i, j \in N, \forall g \in G, \Delta^O \geq 0 \tag{23}$$

$$\Delta^L = (Q_{ig}^O + Q_{ig}^L) - \sum_{\substack{j \in N \\ j \neq i}} (X_{ijg} + Y_{ijg}),$$

$$\forall i, j \in N, \forall g \in G, \Delta^L \geq 0, \tag{24}$$

$$\Delta^L = (Q_{ig}^O + Q_{ig}^L + Q_{ig}^S) - \sum_{\substack{j \in N \\ j \neq i}} (X_{ijg} + Y_{ijg}),$$

$$\forall i, j \in N, \forall g \in G, \Delta^S \geq 0. \tag{25}$$

Objective function (17) represents the total using cost of all containers during an average voyage where coefficients C^O, C^L and C^S respectively imply the unit using costs of the owned containers, long-term leasing containers and short-term leasing containers and where variables Δ^O, Δ^L and Δ^S signify the idle quantities

of the owned containers, long-term leasing containers and short-term leasing containers deployed in the route, respectively. Homogenously, there are coefficients C_{Ii}^O, C_{Ii}^L and C_{Ii}^S respectively denoting the unit idle costs of the owned containers, long-term leasing containers and short-term leasing containers. Constraint (18) gives the limit requirement between the quantities handled and the quantities of all container configurations deployed at each calling port. Constraints (19) show requirements for general relations among C^O, C^L and C^S. Constraints (20)–(22) represent constraint requirements between the variables deployed and the quantities handled at each calling port. Equalities (23)–(25) represent the idle quantity of each part of container configurations at every calling port, respectively.

5. Design of Algorithms

To reflect the interrelation between the three stage models, a bi-level genetic algorithm (GA) is designed. The upper level genetic algorithm is used to searching for the ports to be called by ships and the lower level genetic algorithm is applied for searching an optimal port calling a sequence of ships. Based on the ports selected by the upper level, the lower level algorithm optimizes calling sequence and container configuration is obtained. Then, the outcome of the lower level is feedback to the upper level to calculate the objective function of the lower level algorithm. The process is shown in Fig.4.
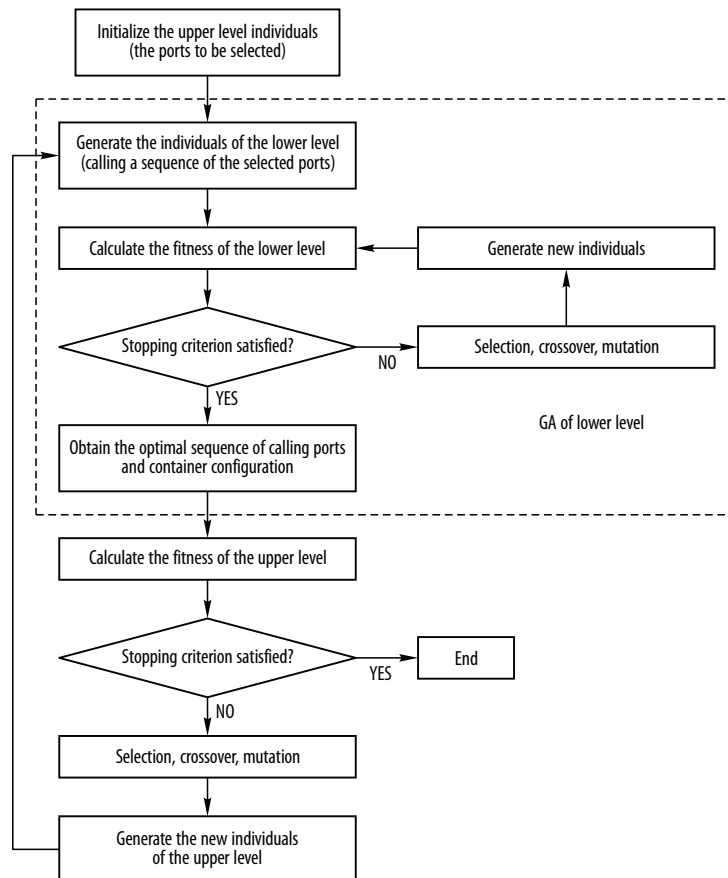

Fig. 4. Bi-level genetic algorithms

(1) Representation of chromosomes

The chromosomes of the upper level algorithm are represented as binary bit strings. The length of a chromosome equals to the number of ports. In the figure depicting chromosomes, '0' denotes the port that is not selected and '1' indicates the selected port, e.g. chromosome '1–0–1–1–0–1–1' denotes that there are 7 ports from which ports 1, 3, 4, 6, 7 are selected.

The chromosomes of the lower level are all represented as character strings. Each chromosome denotes a stowage plan of outbound containers and each integer in the chromosome denotes an outbound container. For example, chromosome 4–1–7–5–2–6–3 denotes that containers 4, 1, 7, 5, 2, 6, 3 are loaded in position 1, 2, 3, 4, 5, 6, 7 of ship.

(2) Initialization

The initialization method of the upper level algorithm is based on selecting the number of ports to be called. The initialization method of the lower level algorithm is randomly selected calling sequence for the selected ports. M_1 and M_2 individuals are generated for the upper and lower levels.

(3) Calculation of the fitness value

Maximization is the problem of the paper, thus the larger is the objective function value the higher the fitness value must be. Therefore, the fitness function of the upper and lower levels can be defined as equations (26) and (27):

$$F_1(x) = \omega_k \times P^k \; ; \tag{26}$$

$$F_2(x) = \overline{p} \; , \tag{27}$$

where: C_M is a constant to ensure $F(x) \geq 0$ and is determined by the problem scale.

(4) Reproduction

Reproduction is a process in which individual chromosomes are copied according to their scaled fitness function values. Chromosomes with a higher fitness value would be selected with higher probabilities. Selection probability can be expressed in the following way:

$$p(x_i) = \frac{F(x_i)}{\sum\limits_{i=1}^{M} F(x_i)}, \quad i = 1, 2, \dots, M. \tag{28}$$

(5) Crossover operator

The process of crossover for upper and lower GA is as follows: selecting two parents, generating the point randomly and swapping the genes for two parents.

(6) Mutation

Mutation introduces random changes in the chromosomes by altering the value to a gene with user-spec-

ified probability p_m called the mutation rate. The mutation method of the upper and lower levels generates two random numbers between 1 and the length of chromosomes first and exchanges the values of the gene at these two positions second.

(7) Stopping criterion

Having reached the pre-determined stopping generations, the algorithm stops.

6. Numerical Experiments

This section presents sample cases to demonstrate the application of the proposed formulation. The case experiments focus on container shipping network design in the trade area of Far East Asia. Since there are a number of relevant factors to be considered that have an impact on shipping network design, we implemented the case experiments according to some empirical knowledge about shipping operation and management in this trade area.

6.1. Parameter Settings

(1) Candidate ports in the trade area (10 ports): Dalian (DL), Tianjin (TJ), Qingdao (QD), Shanghai (SH), Busan (BSN), Kaohsiung (KSH), KeeLung (KL), Kitakyushu (KTK), Osaka (OSK) and Tokyo (TKY).
(2) Planning horizon: one year.
(3) Weekly service frequency: once.
(4) The turnaround time of containers at each port: less than or equal to service interval.
(5) Storage cost at each port (i): $USD2/TEU·day.
(6) Short-term leasing cost at each port (i): $USD2/TEU·day.
(7) Given ship cruising speed: 21 knots.
(8) Total handling and standby time at each port: 0.5 day/per port.
(9) Given cargo traffic demand in the matrix: from January to December.
(10) Fuel oil and diesel oil cost: $USD 320 /metric ton and $USD 560 /metric ton respectively.

The above parameters (5)–(8) are set to be average value. Due to a lack of detailed data on ship expense criteria at each port, we assume that they are the same for all ports under consideration. However, according to such assumptions, the reliability of the solution cannot be affected in the decision making process.

6.2. Demand Matrices

With the characteristics of periodic fluctuations, historic cargo traffic demand may be obtained through market surveys or provided by liner companies. The distributions of cargo traffic demand with relevant freight rates can be represented by a series of matrices consisting of weekly data based on bi-months in a year. In this case, the series of the matrices of fluctuating demands are displayed in the Tables 1–6.

Table 1. Weekly distributions of the average demand and freight rates in January and February (TEU/USD)

$d_{ij6}R_{ij6}^{f}$	DL	TJ	QD	SH	BSN	OSK	KTK	TKY	KL	KHS
DL	0/0	0/0	0/0	0/0	230/250	220/250	200/250	220/230	250/430	200/440
TJ	0/0	0/0	0/0	0/0	240/270	230/260	220/260	250/240	270/450	250/430
QD	0/0	0/0	0/0	0/0	250/240	230/240	240/230	220/240	260/420	250/420
SH	0/0	0/0	0/0	0/0	280/260	220/250	280/230	250/250	260/380	250/400
BSN	105/230	125/220	200/220	200/220	0/0	250/270	300/250	300/270	350/330	300/350
OSK	50/230	40/250	40/240	50/240	60/260	0/0	0/0	0/0	170/350	190/380
KTK	20/220	30/240	40/220	40/220	150/250	0/0	0/0	0/0	160/300	150/300
TKY	25/230	35/250	30/230	55/240	40/260	0/0	0/0	0/0	145/350	125/380
KL	233/370	350/380	320/360	420/340	350/450	325/380	300/360	330/380	0/0	0/0
KHS	260/390	240/380	300/380	400/340	340/460	300/360	300/360	290/360	0/0	0/0

Table 2. Weekly distributions of the average demand and freight rates in March and April (TEU/USD)

$d_{ij6}R_{ij6}^{f}$	DL	TJ	QD	SH	BSN	OSK	KTK	TKY	KL	KHS
DL	0/0	0/0	0/0	0/0	100/220	200/230	180/220	200/210	200/300	200/300
TJ	0/0	0/0	0/0	0/0	160/220	200/230	200/220	220/240	220/320	220/330
QD	0/0	0/0	0/0	0/0	160/210	150/220	160/210	160/220	220/360	250/380
SH	0/0	0/0	0/0	0/0	210/160	200/160	260/160	250/160	260/340	240/350
BSN	55/220	75/230	100/210	160/210	0/0	230/240	210/240	220/240	250/380	250/400
OSK	30/220	30/240	30/220	40/200	50/280	0/0	0/0	0/0	100/320	120/320
KTK	20/220	30/240	30/220	28/200	45/280	0/0	0/0	0/0	110/350	100/350
TKY	25/220	30/240	30/220	45/200	40/280	0/0	0/0	0/0	115/350	100/350
KL	260/280	250/280	240/270	360/240	300/400	220/30	220/280	220/300	0/0	0/0
KHS	220/280	210/280	200/270	300/240	300/400	240/300	220/280	240/300	0/0	0/0

Table 3. Weekly distributions of the average demand and freight rates in May and June (TEU/USD)

$d_{ij6}R_{ij6}^{f}$	DL	TJ	QD	SH	BSN	OSK	KTK	TKY	KL	KHS
DL	0/0	0/0	0/0	0/0	200/200	200/210	240/200	240/220	220/280	240/280
TJ	0/0	0/0	0/0	0/0	230/210	240/210	210/210	220/220	230/300	230/310
QD	0/0	0/0	0/0	0/0	180/180	250/200	250/180	280/200	240/360	280/360
SH	0/0	0/0	0/0	0/0	250/180	280/200	300/180	280/200	280/340	260/340
BSN	50/200	75/210	150/210	220/210	0/0	220/210	200/180	200/210	200/350	200/360
OSK	30/200	30/210	30/200	50/180	50/200	0/0	0/0	0/0	100/300	100/300
KTK	20/200	30/220	35/200	50/180	45/190	0/0	0/0	0/0	110/320	100/320
TKY	25/200	30/210	30/200	45/180	40/200	0/0	0/0	0/0	100/320	90/320
KL	200/280	200/280	250/270	380/240	340/380	255/300	250/300	240/300	0/0	0/0
KHS	200/280	160/280	210/270	340/240	320/380	200/300	230/320	240/300	0/0	0/0

Table 4. Weekly distributions of the average demand and freight rates in July and August (TEU/USD)

$d_{ij6}R_{ij6}^{f}$	DL	TJ	QD	SH	BSN	OSK	KTK	TKY	KL	KHS
DL	0/0	0/0	0/0	0/0	230/200	230/370	390/300	360/320	280/500	270/480
TJ	0/0	0/0	0/0	0/0	200/220	260/390	300/310	360/320	270/480	300/490
QD	0/0	0/0	0/0	0/0	210/210	300/320	350/300	360/320	300/450	350/470
SH	0/0	0/0	0/0	0/0	360/220	390/320	400/300	400/320	350/430	340/450
BSN	105/250	125/250	200/250	220/300	0/0	280/380	330/280	350/380	350/400	300/400
OSK	50/230	40/240	40/210	50/220	60/280	0/0	0/0	0/0	210/350	190/380
KTK	20/210	30/220	40/200	40/200	100/280	0/0	0/0	0/0	200/300	180/300
TKY	25/220	35/240	30/210	55/220	80/260	0/0	0/0	0/0	190/350	185/380
KL	280/320	250/330	300/300	410/280	400/520	325/490	350/480	330/490	0/0	0/0
KHS	260/310	240/320	300/300	400/280	380/500	320/490	320/480	300/490	0/0	0/0

Table 5. Weekly distributions of the average demand and freight rates in September and October (TEU/USD)

$d_{ij6}R_{ij6}^{f}$	DL	TJ	QD	SH	BSN	OSK	KTK	TKY	KL	KHS
DL	0/0	0/0	0/0	0/0	280/300	280/500	400/490	400/540	360/520	320/540
TJ	0/0	0/0	0/0	0/0	240/320	260/500	440/500	450/550	390/540	400/540
QD	0/0	0/0	0/0	0/0	280/320	300/500	500/460	480/550	400/500	400/500
SH	0/0	0/0	0/0	0/0	400/350	560/480	550/460	560/540	450/460	400/480
BSN	120/370	150/380	210/380	300/400	0/0	550/540	500/350	480/560	450/460	400/480
OSK	60/240	50/250	50/240	60/240	80/310	0/0	0/0	0/0	200/400	210/400
KTK	30/220	40/240	50/220	50/220	100/300	0/0	0/0	0/0	200/360	200/370
TKY	35/240	45/250	40/240	65/240	50/310	0/0	0/0	0/0	165/400	145/400
KL	250/430	300/450	350/420	510/420	500/680	425/600	450/580	430/600	0/0	0/0
KHS	280/430	300/450	320/420	500/420	500/680	420/600	400/580	410/610	0/0	0/0

Table 6. Weekly distributions of the average demand and freight rates in November and December (TEU/USD)

$d_{ij6}R_{ij6}^{f}$	DL	TJ	QD	SH	BSN	OSK	KTK	TKY	KL	KHS
DL	0/0	0/0	0/0	0/0	360/330	530/600	560/600	560/650	450/620	370/610
TJ	0/0	0/0	0/0	0/0	400/350	560/650	580/650	560/650	500/650	500/620
QD	0/0	0/0	0/0	0/0	380/380	600/620	600/600	620/620	500/600	460/600
SH	0/0	0/0	0/0	0/0	450/400	660/620	670/600	650/620	540/550	500/550
BSN	110/400	140/450	300/400	300/450	0/0	480/470	600/450	500/470	500/500	420/550
OSK	60/240	50/270	50/260	50/250	120/340	0/0	0/0	0/0	190/420	210/440
KTK	40/220	40/250	70/240	60/240	150/320	0/0	0/0	0/0	210/400	250/420
TKY	40/240	45/270	40/260	65/250	120/340	0/0	0/0	0/0	190/420	225/440
KL	480/470	450/480	440/500	600/500	600/700	525/680	600/660	530/680	0/0	0/0
KHS	360/470	440/480	400/500	550/500	580/700	520/620	550/660	540/650	0/0	0/0

6.3 Results of Experiments

Ship-size, as the main variable, should be represented by a relevant function, including shipping cost. Actually, it is very difficult to construct the precise relationship between ship-size and shipping cost with an exact function. Generally, the function should be presented by a quadratic approximation. Taking into consideration cargo traffic in the trade area of the case, the ship-size to be deployed may be located in the categories of 2,000 TEU to 7,000 TEU. In this section, the relationship between ship-size and shipping cost can be represented approximately by a linear function. The function consists of three sub models: one is associated with fuel oil and diesel oil consumption, the second is associated with ship leasing and the last one is associated with the cost of handling the ship at calling ports. By quoting relevant models and performing regression analysis based on the above cost data, we set up the following linear shipping cost function using TEU (twenty-foot equivalent unit) capacity as an independent variable:

$$C_{sp} = 9.54 \cdot u + 21,973.38.$$

The model is an algebraic sum of three sub-models: the fuel oil cost model is $1.64 \cdot u + 5,440$ per day, the diesel oil cost model is $0.2066 \cdot u + 12,208$ per day,

the ship rental model is $6.54 \cdot u + 1,422.52$ (2005) per day and the cost of ship handling at calling ports is $1.95 \cdot u + 3,453.36$ (2005) per entry. Then, the proposed formulation is solved by Mat Lab based on the GA and results are shown as the following tables and figures.

The optimal set of calling ports with the optimal order of calling sequence based on weekly service frequency is as follows:

Qingdao → Shanghai → KeeLung → Kaohsiung → Busan → Kitakyushu → Qingdao.

Based on fluctuating demand, the optimal ship-size has an approximate capacity of 1,715 TEU with the maximal total profit of USD 102,578.5 and the maximal unit ship-slot profit of USD 60 per average voyage. Based on the average demand, the optimal ship-size has an approximate capacity of 2508 TEU with the maximal total profit of USD 42,509 and the maximal unit ship-slot profit of USD 17 per average voyage. Their ship-slot allocations are shown in Tables 7 and 8.

When the range of fluctuating demand expands 10% and 30% respectively based on original fluctuating demand, the optimal ship sizes with other relevant values based on fluctuation and the average demand vary as shown in Tables 9–11.

Table 7. Optimal ship-slot allocations at each calling port based on fluctuating demand

X_{ij},Y_{ij}	QD F	QD E	SH F	SH E	KL F	KL E	KHS F	KHS E	BS F	BS E	KTK F	KTK E	QD F	QD E	X_{ij},Y_{ij}
QD →			5	500	230								–	–	← QD
	–	–					140		180		490				
SH →					435		450						–	–	← SH
	–	–			105	350	200		150	410					
KL →							210		600		225		–	–	← KL
	–	–			500		420								
KHS →	150								580		160		–	–	← KHS
	–	–	500					80				180			
BS →	300	25	150	175							530		–	–	← BS
	–	–	455								175	405			
KTK →	70	190	60	495	100								–	–	← KTK
	–	–		225					180		550				

Table 8. Optimal ship-slot allocations at each calling port based on the average demand

X_{ij},Y_{ij}	QD F	QD E	SH F	SH E	KL F	KL E	KHS F	KHS E	BS F	BS E	KTK F	KTK E	QD F	QD E	X_{ij},Y_{ij}
QD →			320	320	162	162							–	–	← QD
	–	–					166	166			335	335			
SH →					363	363	327	327					–	–	← SH
	–	–			83	83	163	163	98	98					
KL →							338		415	415	181	181	–	–	← KL
	–	–			350	350	312	312							
KHS →	144	144							403	403	111	111	–	–	← KHS
	–	–	415	415									144	144	
BS →	185	185	439	113							357	357	–	–	← BS
	–	–	197	197							181	181	317	317	
KTK →	43	43	41	787	82	82							–	–	← KTK
	–	–							162	162	410	410			

Table 9. Comparison between two different demand forms based on original fluctuation data

Original data	Based on fluctuating demand	Based on the average demand
Optimal ship-size	2·1,715 TEU	2·2,508TEU
Maximal total profit per average voyage	$USD 205,157	$USD 85,018
Maximal unit ship-slot profit per average voyage	$USD 60 /TEU	$USD 17 /TEU

Table 10. Comparison between two different demand forms based on 10% expansion of original fluctuation data

+10% expansion of original data	Based on fluctuating demand	Based on the average demand
Optimal ship-size	2·1,826 TEU	2·2,508TEU
Maximal total profit per average voyage	$USD 282,079	$USD – 340,881
Maximal unit ship-slot profit per average voyage	$USD 77 /TEU	$USD – 68 /TEU

Table 11. Comparison between two different demand forms based on 30% of original fluctuation data

+30% expansion of original data	Based on fluctuating demand	Based on the average demand
Optimal ship-size	2·1,978 TEU	2·2,508TEU
Maximal total profit per average voyage	$USD 494,358	$USD – 498,917
Maximal unit ship-slot profit per average voyage	$USD 125 /TEU	$USD – 99 /TEU

Table 12. Container configurations deployed at each calling port based on fluctuating demand

Constitutions ╲ Calling ports	QD	SH	KL	KHS	BS	KTK
Owned quantity	340	365	480	390	565	425
Long-term quantity	235	300	253	239	236	384
Short-term quantity	235	290	302	261	379	406
Total quantity at port	810	955	1035	890	1180	1215

The above tables reveal that the larger the range fluctuating demand expands, the better the optimal ship size with relevant values would be based on fluctuating demand; however, the ones based on the average demand would vary in reverse. Consequently, the proposed formulation based on fluctuating demand has distinct superiority in container shipping network design to the one based on classic average demand.

As the results of optimal shipping network design, the maximal container quantities handled at all calling ports are also solved to be 810 TEU at Qingdao, 955 TEU at Shanghai, 1,035 TEU at Keelung, 890 TEU at Kaohsiung, 1,180 TEU at Busan and 1,215 TEU at Kitakyushu, respectively. Based on the maximal quantities handled and ship-slot allocations at all calling ports, optimal container configurations at each calling port are obtained as shown in Table 12.

All determinant factors concerning container shipping network design have been obtained solving the proposed formulations. Results are shown in accordance with the real-world cases of container shipping route operations. They are therefore more practical and applicable than the ones based on classic average demand and generally utilized in the existing studies.

7. Conclusions

1. This study addressed the optimization problem of container shipping network design based on fluctuating demand along with freight rates. Through numerical experiments, we have reached the following conclusions. In considering the influence of fluctuating demand, the unit ship-slot profit of optimal service network operation in binary directions is the best in comparison with the ones based on the fixed average demand. As a result, the problem based on fluctuating demand along with freight rates results in optimizing the smallest ship-size and corresponding container configurations that do not only gain the best voyage profit but also largely reduce the costs of asset deployment.

2. The proposed approach is very useful for assessing shipping network operations from both strategic and tactical viewpoints. Furthermore, it is also extremely effective at employing unit ship-slot profit per average voyage to deal with the issue of comparison between repositioning and leasing empty containers and at optimizing ship-size to deal with revenue-loss control problems.

3. In fact, container shipping network structure and operation should be designed not only for fluctuating demand combined with freight rates determined by historical usage but also for the projections of future fluctuating demand. A combination of these two approaches may provide an interesting topic for future research.

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