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A FUZZY MODEL FOR AN INCREASE IN LOCOMOTIVE TRACTION FORCE

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Abstract. This paper deals with the process of traction force realization described by a suitable mechanical model and is pointed to the adhesion phenomenon as a physical one, i.e. is a suitable factor that the value of traction force depends on. The model for the process of optimizing locomotive traction force based on using the fuzzy set theory is explained. The projecting process of a fuzzy controller regulating the value of skidding and the value of traction torque by increasing the value of traction force that can be realized according to adhesion conditions is described. Finally, testing the optimization model in several numerical examples under specific conditions of wheel-rail adhesion is done.

Keywords: railway, locomotive, traction force, adhesion, skidding, fuzzy control, linguistic variables, optimization.

1. Introduction

Adhesion between wheel and rail is a crucial factor in operating railway industry. A minimum level of adhesion is required for a good traction and braking performance of rail vehicles (Arias-Cuevas and Li 2008; Lata 2008). Increasing traction force represents an extremely complicated problem because of the complexity of wheel-rail adhesion nature, i.e. its stochastic character as a consequence of a great number of exploitation factors with nonlinear and time changeable dependences. To determine the most suitable adhesion coefficient value for particular operating conditions, factors influencing this value are analysed by Bureika (2008), Bureika and Mikaliūnas (2008), Dailydka *et al.* (2008).

The paper shows the fuzzy model for optimizing locomotive traction force based on determining the skidding value by which the maximum traction force value is reached. The fuzzy controller regulates the value of skidding and the value of traction force by increasing its value that can be realized according to conditions for adhesion.

Fuzzy controllers provide many advantages having in mind that they are designed using fuzzy linguistic rules based on expert knowledge and specific numeric data without the existence of a suitable mathematical model. Solving the optimization problem considering fuzzy logic favours, adhesion force dependence on the skidding value is related to track conditions, and therefore this kind of dependence cannot be described by one meaning function (Mei *et al.* 2008).

2. Train Movement Equation

The practical problems of the traction and braking process mostly relate to defining train weight, velocity, maximal acceleration, time, braking distance etc. and are solved by the train movement equation which can be expressed as:

$$\int F_t dl = \int m_t v dv + \int W dl \tag{1}$$

and shows the equality of the mechanical work of traction force F_t, with the sum of the kinetic energy of translating train movement and the work force of resistance W where m_t is the mass of a train.

Since the total kinetic energy of train is:

$$E_k = \frac{m_t v^2}{2} + \sum_{i=1}^{n} \frac{I_i \omega^2}{2}, \tag{2}$$

where: I and ω are the polar moment of inertia and the angular velocity of rotation body, thus:

$$F_{ta} = F_t - W = m_t (1+\rho) \frac{dv}{dt}, \tag{3}$$

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where: ρ is the coefficient of rotation mass; F_{ta} is active traction force the type of train movement depends on and going along with:

- an increase in velocity ($F_{ta} > 0 \Rightarrow F_t > W$, $dv/dt > 0$);
- constant velocity ($F_{ta} = 0 \Rightarrow F_t = W$, $dv/dt = 0$);
- a decrease in velocity ($F_{ta} < 0 \Rightarrow F_t < W$, $dv/dt < 0$).

The realization of traction force high values appears as one of the basic demands for railway exploitation and is based on the fact that the mass of a train that locomotives of a certain type and series can pull is determined. Besides, the maximum acceleration value depends on the traction force value that is very important for city and suburban traffic vehicles from the aspect of travelling time because short distances between stops do not allow much participation in speed limit travelling.

3. Realization of Traction Force

In most railroad vehicles, the inner force made in a power group is realized through the outer force made in the contact zone between a wheel drive axle and a rail. With the effect of adhesion weight and traction torque along with the horizontal reaction of the rail, we get traction force and its effect is transmitted over the traction hook to the pulled composition.

The torque of the traction motor is transmitted to the drive axle through axle gear. Traction torque on axle T_o (Fig. 1) can be represented as the couple of forces $\vec{F_t} = -\vec{F_t'}$ working at points *O* and *K* (with leg r_w) and its intensity is determined by:

$$T_o = r_w F_t = r_w F_t' , \tag{4}$$

where: r_w is the radius of the wheel and $\vec{N} = -\vec{G_a}$ while $\vec{N}$ is a normal reaction to the wheel-rail contact (point *K*). Force $\vec{F_t}$ represents traction force working in the centre of axle *O*, and $\vec{F_t'}$ is tangential force working at point *K*.

While:

$$F_t' \le F_a^l \tag{5}$$

and the limit value of adhesion force is:

$$F_a^l = G_a \Psi , \tag{6}$$

where G_a is the adhesion weight of the locomotive and Ψ is the coefficient of adhesion; force $\vec{F_t'}$ is balanced

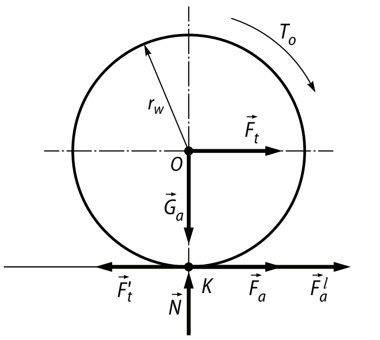

Fig. 1. The realization of traction force

with the force of adhesion $\vec{F_a}$ representing the horizontal reaction of the rail equal to force $\vec{F_t'}$ in intensity but is of the opposite direction. As a consequence, forces $\vec{F_t'}$ and $\vec{F_a}$ are mutually annulled tightening the rail in the wheel–rail contact zone and as a result, force $\vec{F_t}$ remains in the centre of axle O that moves the train. In that case, the immediate velocity pole is in the wheel–rail contact (point $P_v = K$) and consequently, the wheels roll without skidding.

Then, the linear velocity of the wheel centre is:

$$v_o = r_w \omega , \tag{7}$$

where: ω is the angular velocity of the wheel.

When force F_t' exceeds the limit value of adhesion force F_a^l ($F_t' > F_a^l$) mostly because of an excessive increase in traction torque or the local aggravation of adhesion conditions, the wheels skid into point *K*. The friction of rolling then moves to the friction of skidding while the adhesion coefficient is reduced to size Ψ′ (Ψ′ < Ψ) and constantly falls along with an increase in angular velocity. In that case, the limit value of adhesion force is reduced, and therefore the traction force value can be realized. One part of torque T_o goes to creating traction force equal to the reduced value of adhesion force $G_a\Psi'$ and the other one – to the acceleration of wheels spinning around their axle. The latter part of torque is called skidding moment T_s and represents the surplus of torque T_o that was not realized in the sense of traction force realization because the base did not accept it. Immediate velocity pole P_v is in the direction of *O-K* which is from *K* to *O* and thus:

$$v_o = \overline{OP_v} \omega , \tag{8}$$

that is, $v_o < r_w \omega$, where $\overline{OP_v} < \overline{OK}$. When the immediate velocity pole is at point *O*, the axle will spin only around its axle, i.e. it will have complete skidding and the vehicle will still stand.

The skidding value (Kim *et al.* 1999; Nayal *et al.* 2006) can be expressed through skidding velocity v_s:

$$v_s = r_w \omega - v_o \tag{9}$$

or skidding ratio *s*:

$$s = \frac{v_s}{r_w \omega} = \frac{r_w \omega - v_o}{r_w \omega} = 1 - \frac{v_o}{r_w \omega} \tag{10}$$

that goes from 0 for the theoretical rolling of the wheel on the rail without skidding ($v_o = r_w\omega$) to 1 when it comes to the complete skidding of the wheels ($v_o << r_w\omega$).

To avoid the excessive skidding of wheels, it is necessary to lower the value of force F_t' by regulating the work of the power group or improving conditions for wheel adhesion to the rail (for example, by sanding), i.e. it is necessary to fulfil the condition defined by equation (5) to establish adhesion.

The coefficient of adhesion shows physical condition under which the contact surface between the wheel and the rail is, and therefore a certain adhesion force appears. In a narrower sense, it represents a part of the coefficient of friction between the wheel and the rail that can be used for realizing adhesion force while in a broad sense, it represents a complex value including conditions for the adhesion and distortion of contact surfaces, uneven performance of traction motors and driving gear and the parts of oscillatory mass inertia forces. Adhesion conditions can often be changed along the track as a consequence of a large number of exploitation factors such as non homogeneous wheel materials, difference in the radius of rolling and wheel profile, non homogeneous rail material, change in the head of rail dimensions, changes in rail conditions regarding cleanliness, humidity and temperature, track configuration etc. The most important parameters on which the value of the adhesion coefficient depends are material characteristics of the wheel and the rail (steel) and the cleanliness of wheel–rail contact surface. Thus, its value can be changed in a wide range from 0.3÷0.4 with favourable traction conditions (dry and clean rail) to 0.1÷0.15 with unfavourable traction conditions (moist and dirty rails) (Vasic *et al.* 2003).

4. Adhesion Phenomenon as a Physical Phenomenon

The existence of adhesion is caused by the elastic deformations of wheel and rail material as a consequence of strong pressure in the wheel–rail contact zone. Contact surface, in general sense, is in a shape of a convex ellipse and its parameters depend on the characteristics of wheel and rail material, the size of vertical load and the radius of the wheel and the head of the rail curve. Having in mind that surfaces formed at the wheel–rail contact point do not match to kinematical surfaces, the rolling of a wheel with micro sliding to the part of contact surface can occur.

Wheel–rail contact surface (Fig. 2) consists of a sliding zone and a rolling zone (Harrison *et al.* 2002; Bureika and Mikaliūnas 2002). When the vehicle is not in motion, the ellipse has a symmetrical shape, whereas in case it moves, it is reduced in the direction of moving and expands in the opposite direction when there is a certain move of the tightened zones and when there are differences between the path elements of wheel centre and wheel edge point. With an increase in traction torque, traction force is increases and the size of micro sliding makes s = 0.015÷0.02 (the linear part of function $F_t = f(s)$ shown in Fig. 3) when the rolling zone is reduced while the sliding zone expands.

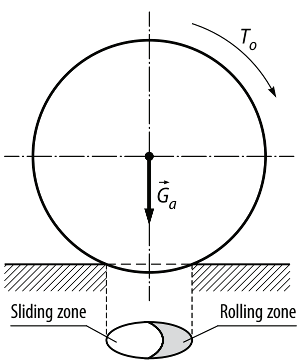

Fig. 2. Wheel–rail contact surface

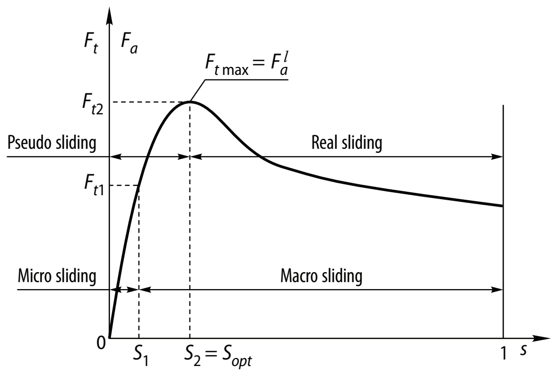

Fig. 3. The analysis of $F_t = f(s)$ curve (adhesion characteristic)

Along with a further increase in traction force, the whole contact surface skids which leads to macro skidding. Traction force shows a further increase in the maximum value (adhesion limit), and consequently starts to fall when adhesion stops, i.e. pseudo sliding becomes real sliding (Lučanin 1996) which represents the most important moment because in this case, the maximum traction force can be realized ($F_{t\max} = F_a^l = G_a \Psi_{\max}$) and the skidding ratio reaches its optimal value s_{opt}.

It is considered that in the zone of pseudo sliding (stable zone), there is complete adhesion called the adhesion zone (Ohishi *et al.* 2000). When the traction force value exceeds adhesion limit, adhesion stops and traction force that can be realized in accordance with conditions for adhesion is suddenly reduced (unstable zone) as a consequence of the fact that sliding friction coefficient Ψ' is smaller than adhesion coefficient Ψ, i.e. Ψ' < Ψ. Therefore, the adhesion coefficient is related to the one with the maximum value of traction force without stopping adhesion (Radojković 1990). Adhesion limit is reached for the skidding ratio value varying from 0.03 to 0.18 depending on the type of a locomotive, velocity and conditions for wheel–rail contact surface (Polach 2005).

The variable friction coefficient can be expressed applying the following equation (Polach 2005):

$$\Psi = \Psi_o\left((1-A)e^{-Bv_s} + A\right), \quad (11)$$

where: Ψ_o is the maximum friction coefficient, v_s is the magnitude of the slip velocity vector [m/s], B is the coefficient of exponential friction decrease [s/m] and A is the ratio of limit friction coefficient Ψ_∞ at infinity slip velocity to maximum friction coefficient Ψ_o:

$$A = \frac{\Psi_\infty}{\Psi_o}. \quad (12)$$

To adjust the theoretical curve to the real one according to adhesion conditions, a reduction of Kalker's coefficient is applied:

$$k = \frac{k_A + k_S}{2}, \quad (13)$$

where: k_A is the reduction factor in the adhesion zone and k_S is the reduction factor in the sliding zone. The typical values of Kalker's reduction coefficient are 0.2÷0.5 for wet rails and 0.6÷0.85 for dry rails (Fig. 4).

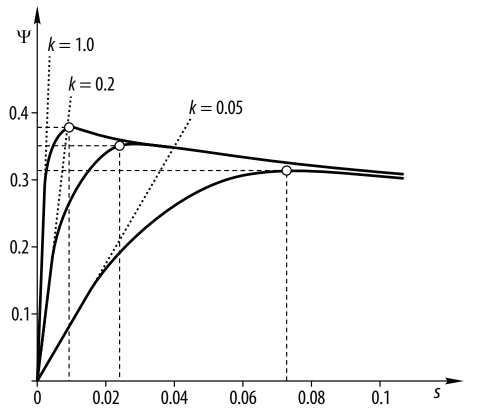

Fig. 4. The shape of $\Psi = f(s)$ curve for different values of Kalker's coefficient

5. Fuzzy Control

Fuzzy logic is often used for complex process modelling in case it is very difficult to determine dependences existing between some variables using other methods. In regard to the fuzzy controller classic approach, fuzzy control represents a natural methodology of using human (heuristic) knowledge since it is possible for experts to apply their knowledge forming a control strategy using linguistic rules (Sugeno and Tanaka 1991). Besides, fuzzy controllers are more flexible in regard to the classic ones because control variables can be easily modified using the principle of 'error and trial'.

Fuzzy logic represents the extension of conventional (Bull's) logic developed to enable work on the accurate values existing between limit values 'true' and 'false'. It is based on the fuzzy set theory the basic principles of which were formulated by the American professor *Lotfi Zadeh* (Zadeh 1965).

If $X = \{x_1, x_2, \ldots, x_n\}$, then fuzzy set A in universal set X is defined as a set of the ordered pairs:

$$A = \{(x, \mu_A(x)) \mid x \in X\}, \qquad (14)$$

where: $\mu_A(x)$ is the membership function of x in A, $\forall x \in X$. The membership function in the fuzzy sets can have any value taken from interval [0,1]. The membership value defines the membership measure of elements in a set. If the value of membership is higher, then it is more likely that element x belongs to set A.

Fuzzy controllers have found their use and proved to be effective in industry, automotive engineering, transportation, power electronics etc. (Sivinandam *et al.* 2007). Moreover, it has been established that they also represent universal function approximators (Wang 1992). The subway train control system developed by Hitachi for the city of Sendai in Japan was among the first commercial applications of fuzzy logic (Yasunobu and Miyamoto 1985).

Basically, a fuzzy control system or a controller should be considered as an artificial decision maker that operates in a feedback system in real time. It 'collects' data from the output of process $y(t)$, compares it to referent $r(t)$ and 'decides' on what should be the input of process $u(t)$ at that moment in order to fulfil the desired performances and given specification goals. Fuzzy controller (Fig. 5) is composed of the following four elements (Passino and Yurkovich 1998):

- *a rule base* (a set of 'if–then' rules) containing the fuzzy logic quantification of expert's linguistic description how to achieve proper control;
- an *inference mechanism* emulating expert's decision on interpreting and applying knowledge of how to most effectively control the plant;
- a *fuzzification* interface converting controller inputs into information that can be easily used by the inference mechanism to activate and apply rules;
- a *defuzzification* interface converting conclusions about the inference mechanism into real variables, i.e. controller output.

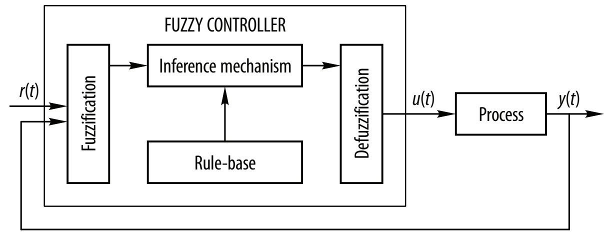

Fig. 5. The fuzzy control system

The relation between the inputs and output of the variable represents a nonlinear function of more variables. With the help of fuzzy logic that enables a great number of operators, many different copies can be produced to fuzzy controllers.

6. Optimization Model

The analysis of traction force change ΔF_t and skidding ratio change Δs in interval Δt indicates that the distance between the immediate and optimal value of skidding ratio can be concluded, i.e. traction force that can be realized under conditions for adhesion, where:

$$\Delta F_t = F_t(t+1) - F_t(t); \qquad (15)$$

$$\Delta s = s(t+1) - s(t), \qquad (16)$$

where: $F_t(t+1)$ and $F_t(t)$ are traction force values, i.e. $s(t+1)$ and $s(t)$ are the values of skidding ratio at the moments $t+1$ and t, respectively. If signs ΔF_t and Δs are identical, then an immediate value of skidding ratio is in the stable zone that corresponds to the increasing part of curve $F_t = f(s)$. In that case, the optimal value of skidding ratio is higher than the immediate one. If signs ΔF_t and Δs are different, then the immediate value of skidding ratio is in the unstable zone that corresponds to the decreasing part of curve $F_t = f(s)$. In that case, the optimal value of skidding ratio is lower than the immediate one. When ΔF_t is small and Δs is big, the immediate value of skidding ratio is close to optimal.

The above rules enable the traction torque regulator, i.e. traction force to be included in the whole process. For example, if the immediate value of skidding ratio is in the unstable zone of curve $F_t = f(s)$, it is necessary to lower the value of traction torque quickly. On the other hand, if the immediate value of skidding ratio is in the stable zone of curve $F_t = f(s)$, then there is a possibility of increasing traction torque values.

Considering the possibility that the traction torque regulator can be included, an algorithm for determining the traction force optimal (maximum) value is defined. The optimization algorithm (Fig. 6) is of an iterative type according to which the optimal value of traction force is determined by traction torque regulation based on the immediate values of skidding ratio, especially for each previously determined time interval in accordance with conditions for adhesion on rail that are of stochastic nature and a subject of certain changes.

The whole procedure begins at moment t where traction force $F_t(t)$ and skidding ratio $s(t)$ values are given. Along with changes in traction torque, traction force changes including skidding ratio and the following moment t +1 and their values are $F_t(t+1)$ and $s(t+1)$ respectively. Based on traction force and skidding ratio values at moments t and t+1, values ΔF_t and Δs are set according to equations (15) and (16).

Depending on values ΔF_t and Δs, skidding ratio values are set at the next moment t+2:

$$s(t+2) = s(t+1) \pm |s'|, \tag{17}$$

according to which the adjustment of traction torque values, i.e. a decision on its increase or decrease, is made. The value of skidding ratio change s' represents the value of the fuzzy controller output variable. The resulting values of traction force and skidding ratio at moment t + 2 become the current values F_{tcurr} and s_{curr}, i.e. the input values for the next iteration ($t = t + 1$). The procedure is repeated again and lasts to the moment when the optimal values of traction force and skidding ratio are reached for the given adhesion conditions.

7. Fuzzy Controller Project

A fuzzy controller is designed as a variable of the system having two inputs and one output. The nonlinear function of the system bonds the values of traction force changes ΔF_t and skidding ratio Δs (input variables) with the value of the next skidding ratio change s' (output variable).

In the fuzzification step, the field of defining the numeric values is set and the selection of the suitable membership functions is done in order to define fuzzy sets describing linguistic variables. The number and schedule of fuzzy sets directly influence the performances of fuzzy controllers through the level of the preciseness and accuracy of controllers, i.e. the number of fuzzy rules that are later generated in the fuzzy rules base (Kosko 1995). The suggested sets of the linguistic values of input and output variables (Fig. 7) are:

- ΔF_t, Δs: negative big – NB, negative medium – NM, negative small – NS, neutral – NE, positive small – PS, positive medium – PM, positive big – NB;

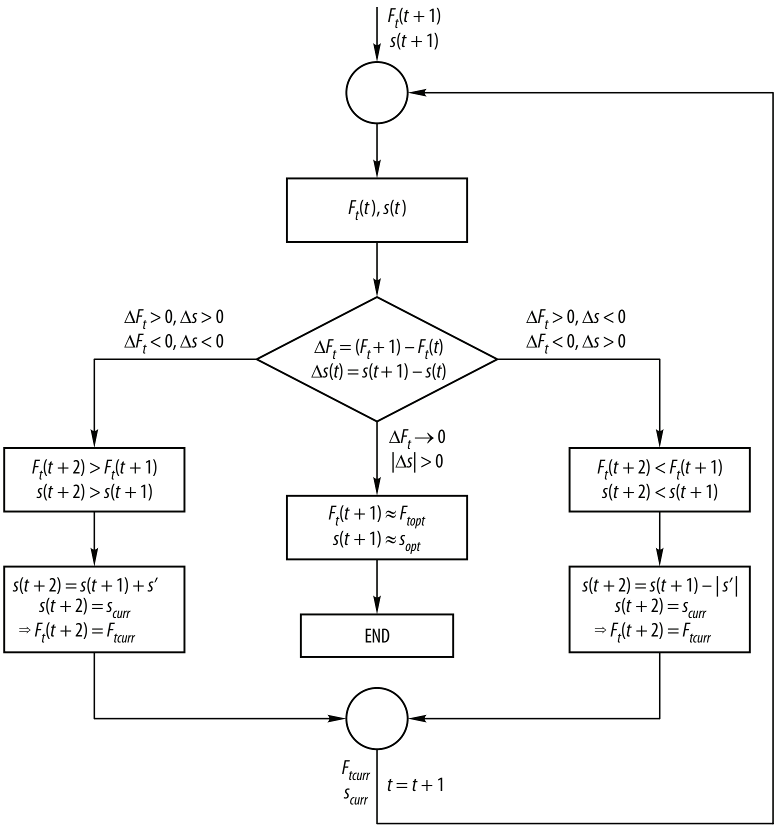

Fig. 6. An algorithm for determining the optimal (maximum) value of traction force

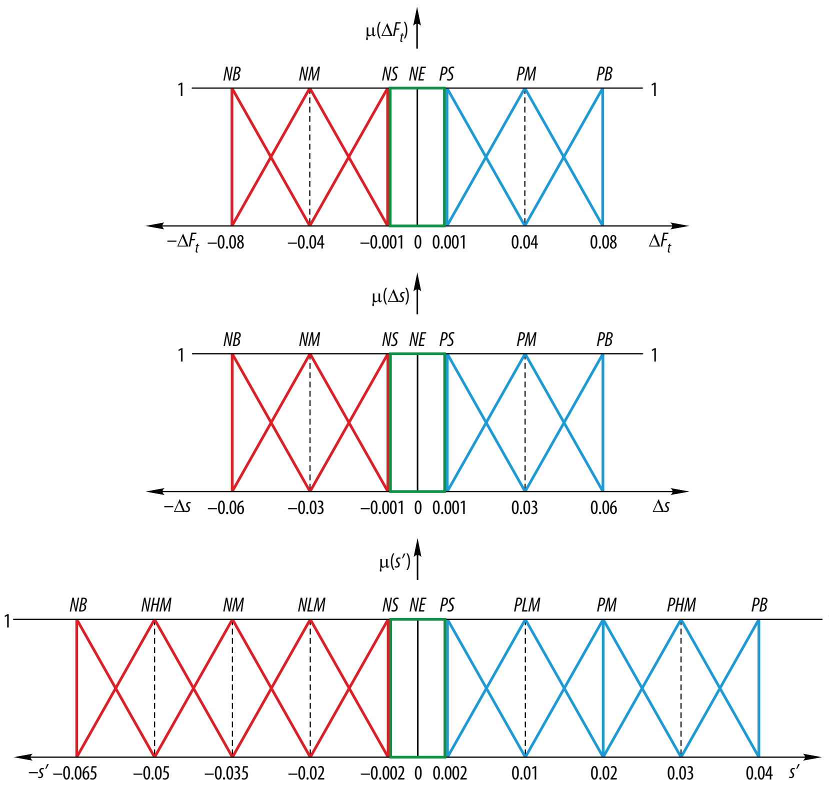

Fig. 7. Linguistic values of the input and output fuzzy variable shown with the suitable fuzzy sets

- *s*': negative big – NB, negative high medium – NHM, negative medium – NM, negative low medium – NLM, negative small – NS, neutral – NE, positive small – PS, positive low medium – PLM, positive medium – PM, positive high medium – PHM, positive big – PB.

The numeric values of fuzzy variable ΔF_t are shown in the form of values F_t / G_a.

After fuzzification, the base of fuzzy rules is formed (Table 1) by which the control strategy between the output and input variable is described and system modelling is formed. These heuristic rules are of the type 'if–then' and are defined by an expert. For example, if ΔF_t is positive big and Δs is positive small, then *s*' is positive big.

At a later stage, training input and output variable pairs is done in order to confirm the resulting numeric values.

After training, the created fuzzy rules will define function *f*:

$$f\!: (\Delta F_t, \Delta s) \rightarrow s'. \quad (18)$$

Based on defined fuzzy rules, their transformation into fuzzy relations between the considered variables is done. The whole process of expert conclusion is described by the algorithm of fuzzy reasoning based on fuzzy relations where each rule represents fuzzy relation between different categories of fuzzy variables (Zadeh 1973).

For example, ΔF_t – positive big and Δs – positive small, is fuzzy phrase *P* in Cartesian space *PB*×*PS* with the membership function:

$$\mu_P(\Delta F_t, \Delta s) = \min\{\mu_{PB}(\Delta F_t), \mu_{PS}(\Delta s)\}. \quad (19)$$

Table 1. Fuzzy rule base

s'		Δ*s*						
		NB	*NM*	*NS*	*NE*	*PS*	*PM*	*PB*
ΔF_t	*NB*	PLM	PM	PHM	NE	NLM	NM	NB
	NM	PS	PLM	PM	NE	NS	NLM	NHM
	NS	PS	PS	PS	NE	NS	NLM	NM
	NE	NE	NE	NE	NE	NE	NE	NE
	PS	NM	NLM	NS	NE	PS	PS	NE
	PM	NHM	NLM	NS	NE	PM	PLM	PS
	PB	NHM	NM	NLM	NE	PB	PM	PLM

The rule 'if P , then s' is positive big' is also fuzzy phrase Q in Cartesian space $\Delta F_t \times \Delta s \times s'$ with the membership function:

$$\mu_Q(\Delta F_t, \Delta s, s') = \min\{\mu_P(\Delta F_t, \Delta s), \mu_{PB}(s')\}. \quad (20)$$

For the conclusion procedure with approximate reasoning, a 'min-max' composition is used.

In the defuzzification step, from the resulting fuzzy set obtained through the fuzzy reasoning algorithm, a numeric (representative) input variable value is chosen. This paper shows the values closest to the centre of the gravity of the resulting fuzzy set.

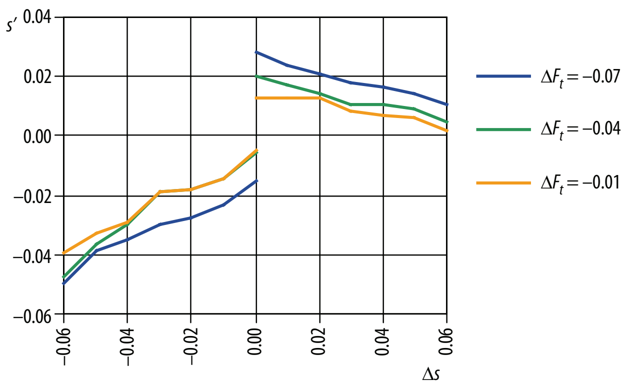

Fig. 8. Diagram $s' = f(\Delta s)$ for positive values ΔF_t

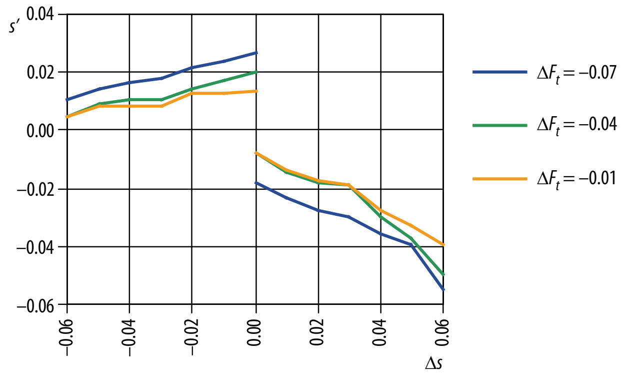

Fig. 9. Diagram $s' = f(\Delta s)$ for negative values ΔF_t

The developed controller is tested for different values ΔF_t and Δs and the resulting values for fuzzy controller output variable s' depending on the values of input variables ΔF_t and Δs are shown in Figs. 8 and 9.

8. Realization of the Optimization Model

Based on the defined optimization algorithm with the principal scheme shown in Fig. 6 and the resulting values of the fuzzy controller output variable, optimization model testing is done taking several numeric examples under conditions for wheel adhesion on rail. To compare the resulting numeric values with the real ones, the adhesion characteristics of electric locomotives series SBB/CFF/FFS 420 (type Re 4/4 II) (Fig. 10) are used.

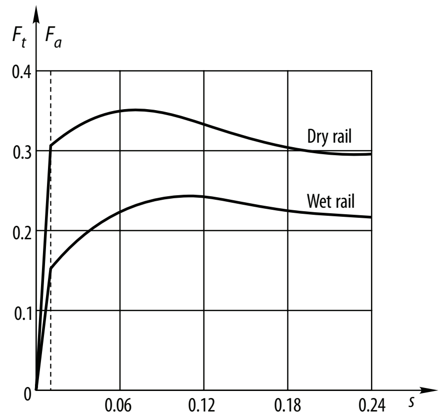

Fig. 10. Adhesion characteristics of locomotives series SBB/CFF/FSS 420 (type Re 4/4 II)

8.1. A Procedure for Determining the Maximum Value of Traction Force on Dry and Wet Rail

The whole procedure starts at moment t = 0 where the values of traction force $F_t(0)$ = 0 and skidding ratio $s(0)$ = 0 are given. The given value of skidding ratio is $s(1)$ = 0.01 according to which the value of traction torque is adjusted which means that at the next moment t = 1, the value of traction force is $F_t(1)$ = 0.3086. Based on the traction force value and skidding ratio at moments t = 0 and t = 1, according to equations (15) and (16), values ΔF_t and Δs are determined. According to the defined fuzzy strategy for the resulting values ΔF_t and Δs, control action represents an increase in skidding ratio of s' = 0.0273 which means that the value of traction torque should be adjusted to the value of skidding ratio that at moment t = 2 is $s(2)$ = 0.0373, equation (17). For the resulting value of skidding ratio the value of traction force is $F_t(2)$ = 0.3371. Having in mind that values ΔF_t are Δs are positive, the immediate value of skidding ratio is in the stable curve zone $F_t = f(s)$. The values of traction force and skidding ratio at moment t = 2 become the current values and represent the input values for the next iteration (Table 2). The procedure is repeated.

Traction force and skidding ratio grow to the moment t = 6 and reach the optimal values $F_t(6)$ = 0.3516 and $s(6)$ = 0.0773. A further increase in skidding ratio would be followed by a decrease in traction force having control action in the form of decreasing the values of traction torque as a consequence. In that case, the current value of skidding ratio would be in the unstable part of curve $F_t = f(s)$.

In case of wet rail, the procedure also begins at moment t = 0 where values $F_t(0)$ = 0 and $s(0)$ = 0 are given. If the value is $s(1)$ = 0.01 according to which the value of traction torque is adjusted, it means that at the next moment t = 1, the value of traction force is $F_t(1)$ = 0.1543. Based on the resulting values ΔF_t and Δs, control action represents an increase in skidding ratio for s' = 0.0273 which means that the value of traction torque should be adjusted to the values of skidding ratio which at the moment t = 2 is $s(2)$ = 0.0373. For the result-

Table 2. A procedure for determining the optimal value of traction force on dry rail

t	$s(t)$	$F_t(t)$	$s(t+1)$	$F_t(t+1)$	Δs	ΔF_t	s'	$s(t+2)$	$F_t(t+2)$
0	0	0	0.01	0.3086	0.01	0.3086	0.0273	0.0373	0.3371
1	0.01	0.3086	0.0373	0.3371	0.0273	0.0285	0.0118	0.0491	0.3456
2	0.0373	0.3371	0.0491	0.3456	0.0118	0.0085	0.0124	0.0615	0.3497
3	0.0491	0.3456	0.0615	0.3497	0.0124	0.0041	0.0097	0.0712	0.3513
4	0.0615	0.3497	0.0712	0.3513	0.0097	0.0016	0.0061	0.0773	0.3516
5	0.0712	0.3513	0.0773	0.3516	0.0061	0.0003	0	0.0773	0.3516

Table 3. A procedure for determining the optimal value of traction force on wet rail

t	$s(t)$	$F_t(t)$	$s(t+1)$	$F_t(t+1)$	Δs	ΔF_t	s'	$s(t+2)$	$F_t(t+2)$
0	0	0	0.01	0.1543	0.01	0.1543	0.0273	0.0373	0.1914
1	0.01	0.1543	0.0373	0.1994	0.0273	0.0451	0.0141	0.0514	0.2173
2	0.0373	0.1994	0.0514	0.2173	0.0141	0.0179	0.0145	0.0659	0.2334
3	0.0514	0.2173	0.0659	0.2334	0.0145	0.0161	0.0144	0.0803	0.2394
4	0.0659	0.2334	0.0803	0.2394	0.0144	0.0060	0.0115	0.0918	0.2433
5	0.0803	0.2394	0.0918	0.2433	0.0115	0.0039	0.0094	0.1012	0.2447
6	0.0918	0.2433	0.1012	0.2447	0.0094	0.0014	0.0057	0.1069	0.2451
7	0.1012	0.2447	0.1069	0.2451	0.0057	0.0004	0	0.1069	0.2451

ing value of skidding ratio, the traction force value is $F_t(2) = 0.1914$. Since values ΔF_t and Δs are positive, the immediate value of skidding ratio is in the stable zone of curve $F_t = f(s)$. Values $F_t(2)$ and $s(2)$ become the current values and represent the input values for the next iteration (Table 3).

Traction force and skidding ratio grow to the moment $t = 8$ and reach the optimal values $F_t(8) = 0.2451$ and $s(8) = 0.1069$.

8.2. A Procedure for Determining the Maximum Value of Traction Force Moving from Dry to Wet Rail

The given values are $F_t(0) = 0.3516$ and $s(0) = 0.0773$. With the deterioration of adhesion conditions there comes to a decrease in traction force values and a sudden increase in skidding ratio. The given traction force values at moment $t = 1$ are $F_t(1) = 0.2161$ and skidding ratio is $s(1) = 0.2194$. Since values ΔF_t and Δs have different signs, the immediate value of skidding ratio is in the unstable zone of curve $F_t = f(s)$. According to the defined fuzzy strategy, control action represents a decrease in skidding ratio for $s' = -0.0603$ which means that the value of traction torque should be adjusted to the value of skidding ratio which at moment $t = 2$ is $s(2) = 0.1591$. For the resulting value of skidding ratio, the traction force value is $F_t(2) = 0.2334$. Values $F_t(2)$ and $s(2)$ become the current values, i.e. the input values for the next iteration (Table 4).

Table 4 shows that a decrease in traction torque lasts until moment $t = 4$, i.e. when the current value of skidding ratio is in the stable part of curve $F_t = f(s)$ and as a consequence there is control action in the form of an increase in traction torque. Traction force and skidding ratio at moment $t = 7$ reach the optimal values $F_t(7) = 0.2450$ and $s(7) = 0.1060$.

Table 4. A procedure for determining the optimal value of traction force in case of deterioration under adhesion conditions

t	$s(t)$	$F_t(t)$	$s(t+1)$	$F_t(t+1)$	Δs	ΔF_t	s'	$s(t+2)$	$F_t(t+2)$
0	0.0773	0.3516	0.2194	0.2161	0.1421	-0.1355	-0.0603	0.1591	0.2334
1	0.2194	0.2161	0.1591	0.2334	-0.0603	0.0173	-0.0415	0.1176	0.2437
2	0.1591	0.2334	0.1176	0.2437	-0.0415	0.0103	-0.0296	0.0880	0.2414
3	0.1176	0.2437	0.0880	0.2414	-0.0296	-0.0023	0.0061	0.0941	0.2435
4	0.0880	0.2414	0.0941	0.2435	0.0061	0.0021	0.0067	0.1008	0.2447
5	0.0941	0.2435	0.1008	0.2447	0.0067	0.0012	0.0052	0.1060	0.2450
6	0.1008	0.2447	0.1060	0.2450	0.0052	0.0003	0	0.1060	0.2450

8.3. A Procedure for Determining the Maximum Value of Traction Force Moving from Wet to Dry Rail

The given values are $F_t(0) = 0.2451$ and $s(0) = 0.1069$. Considering improvement in adhesion conditions, there comes a decrease in the skidding ratio value that makes $s(1) = 0.0069$ at moment $t = 1$. As a consequence of the fact that the immediate value of skidding ratio has moved to the beginning of the stable zone of curve $F_t = f(s)$, there is a possibility of increasing traction torque and traction force. For the value of the starting step from $\Delta s = 0.01$, the size of traction torque should be adjusted to the value of skidding ratio $s(2) = 0.0169$ with the corresponding traction force value $F_t(2) = 0.3149$. Based on resulting values ΔF_t and Δs, control action represents an increase in skidding ratio for $s' = 0.0238$ which means that the value of traction torque should be adjusted to the skidding ratio value making $s(3) = 0.0407$ at moment $t = 3$. For the resulting value of skidding ratio, the value of traction force is $F_t(3) = 0.3402$. Since values ΔF_t and Δs are positive, the immediate value of skidding ratio is still in the stable zone of curve $F_t = f(s)$. Values $F_t(3)$ and $s(3)$ become the current values, i.e. the input values for the next iteration (Table 5).

Traction force and skidding ratio grow at moment $t = 7$ and reach optimal values $F_t(7) = 0.3516$ and $s(7) = 0.0777$.

8.4. Analysis of Results

Having compared the values of traction force and skidding ratio by testing the optimization model with the measured values shown in Fig. 10, an appropriate level of correspondence can be noticed which leads to the high level of projected fuzzy controller application. Differences in the resulting optimal values are a consequence of defined fuzzy sets describing linguistic variables. Based on the defined optimization algorithm, the optimal values of traction force and skidding ratio are reached when $\Delta F_t \rightarrow 0$ where $|\Delta s| > 0$, i.e. value $|\Delta F_t| < 0.001$ when $s' = 0$. The above mentioned discrepancies do not exceed the value of 1%.

Having in mind that the preciseness and accuracy of fuzzy controller depend on the number and position of fuzzy sets describing linguistic variables, i.e. the number of fuzzy rules that are later generated in the rule base membership functions describing linguistic variables and the final fuzzy rule set were described following a number of trials and valuations of results. The possibility of improving the projected fuzzy controller is seen in the possible expansion of models, introducing more input variables, increasing the number of the linguistic values of defined fuzzy variables, change in verbal descriptions, a new set of fuzzy rules, different composition on conclusions, choosing different ways of defuzzification etc. That means that projecting a fuzzy controller is still an open problem of future research.

9. Conclusions

1. The paper shows the systematization of research on the process of traction force realization. A complex mechanical model for traction force realization on locomotive power axle is formed and carefully analyzed. The complexity of adhesion nature as a physical phenomenon and its stochastic character which is the consequence of a great number of exploitation factors with nonlinear and time changeable dependencies are pointed out.
2. The problem of increasing locomotive traction force is solved by the fuzzy set theory, and thus a suitable adhesion fuzzy model is formed. A project of a fuzzy controller for regulating the value of traction force by maximizing its value that can be realized in accordance with the conditions of adhesion is carried out. An advantage to using fuzzy logic is that the controller can be designed using linguistic knowledge which means that the mathematical model is not required. It is accepted to be a great advantage because adhesion characteristics are very difficult to be modelled. The fuzzy controller also provides a nonlinear control action and can be easily modified and tuned. Comparing the values obtained by testing the above described optimization model with measured values, a suitable degree of matching can be noticed which points to the high level of using the projected fuzzy controller.
3. By using the specified fuzzy model, the value of traction force is adjusted according to the real conditions of adhesion along the track. In that way, adhesion is better which is extremely important in case of a sudden deterioration of adhesion conditions. Having in mind the fact that at the moment of reaching the

Table 5. A procedure for determining the optimal value of traction force in case of improving adhesion conditions

t	$s(t)$	$F_t(t)$	$s(t+1)$	$F_t(t+1)$	Δs	ΔF_t	s'	$s(t+2)$	$F_t(t+2)$
0	0.1069	0.2451	0.0069	0.2443					
1	0.0069	0.2443	0.0169	0.3149	0.01	0.0706	0.0238	0.0407	0.3402
2	0.0169	0.3149	0.0407	0.3402	0.0238	0.0253	0.0130	0.0537	0.3486
3	0.0407	0.3402	0.0537	0.3486	0.0130	0.0084	0.0126	0.0663	0.3503
4	0.0537	0.3486	0.0663	0.3503	0.0126	0.0017	0.0064	0.0727	0.3514
5	0.0663	0.3503	0.0727	0.3514	0.0064	0.0011	0.0050	0.0777	0.3516
6	0.0727	0.3514	0.0777	0.3516	0.0050	0.0002	0	0.0777	0.3516

maximum traction force value, an optimal skidding ratio value is achieved for the given adhesion conditions, the introduced model can be effectively used as a base for projecting control systems for preventing the excessive skidding of locomotive wheels.

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