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MODELLING THE DYNAMIC BEHAVIOUR OF THE TRUCK-CRANE

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Abstract. The paper deals with the problem of a dynamic analysis of truck-cranes. Therefore, the article has developed a mechanical-mathematical model having 18 generalized coordinates. Depending on the type of truck crane operation as well as on the fact whether the aim of the article is to conduct the dynamic analysis of the whole truck crane or only that of one of its components, simpler mechanical models are also offered. The presented model is more realistic than those describing the dynamic behaviour of the truck-crane performing all necessary functions, i.e. a mobile means of lifting, transportation and reloading.

Keywords: truck-crane, dynamic, mechanical-mathematical model, Lagrange equations.

1. Introduction

A truck-crane represents a mobile means of lifting, transportation and reloading intended for lifting the load of different mass to various heights at various radiuses. Similarly to other cranes, the truck-crane has a possibility of lifting, transporting and manipulating the load. Thus, space for manipulating the load is considerably enlarged and a possibility of changing the location of the truck crane rapidly appears. Due to the possibilities of performing different types of motion under different ways of operating power units, the truck-crane must be considered a dynamic object. A great number of possible operations, a dynamic character of truck-crane operation as well as plenty of assistant workers are likely to cause a stronger possibility of casualties that can result in the material loss and destruction of workers' lives.

There are many reasons for analyzing this problem. Along with an assessment of statistical indicators, we have established that the truck-crane participates in 73% of the accidents related to all types of cranes (Yow *et al.* 2000). Also, nearly 3% of the accidents are fatal and in 8% of those, permanent disability occurs (MacCollum 2011).

The presented models are mostly defined with the purpose of obtaining results taking into account the dynamic behaviour of one or several assembles of the truck-crane (Mijajlović *et al.* 2000; Šelmić 1979; Jerman *et al.* 2004; Maczynski, Wojciech 2003; Šelmić, Mijailović 1998; Mijailović, Šelmić 2002a; Posiadał 1997; Towarek 1998; Volkov *et al.* 2000; Dubowsky *et al.* 1991). In this way, a certain number of parameters having a significant influence on the dynamic behaviour of the truck-crane have been ignored. Literature does not provide a paper that would characterize the conditions under which the negligence of certain parameters is allowed.

2. Mechanical-Mathematical Model

Analyzing the truck-crane as a complex dynamic object, the mechanical-mathematical model is described (Mijailović 2005) (Fig. 1) with reference to the following generalized coordinates:

- ξ_0, ξ_1, ξ_2, ξ_3 – the distance between outriggers (pneumatics) and non-deformed ground;
- ξ_4, ξ_5, ξ_6 – deflection in the middle of beams (a part of chassis);
- ξ_7 – a top boom slope in the horizontal plane;
- ξ_8 – a top boom slope in the vertical plane;
- ξ_9 – sway angle projection in the vertical plane;
- ξ_{10} – sway angle projection in the horizontal plane;
- ξ_{11} – a slewing angle of the slewing platform;
- ξ_{12} – a slewing angle of the boom;
- ξ_{13} – rope length;
- ξ_{14} – bearing slope;
- ξ_{15} – telescopic boom length;
- ξ_{16}, ξ_{17} – the components of the vector of truck-crane position during driving.

A detailed explanation of a mechanical-mathematical model (figures, equations) of the truck-crane can be found in papers by (Mijailović 2005; Mijailović, Šelmić 2004).

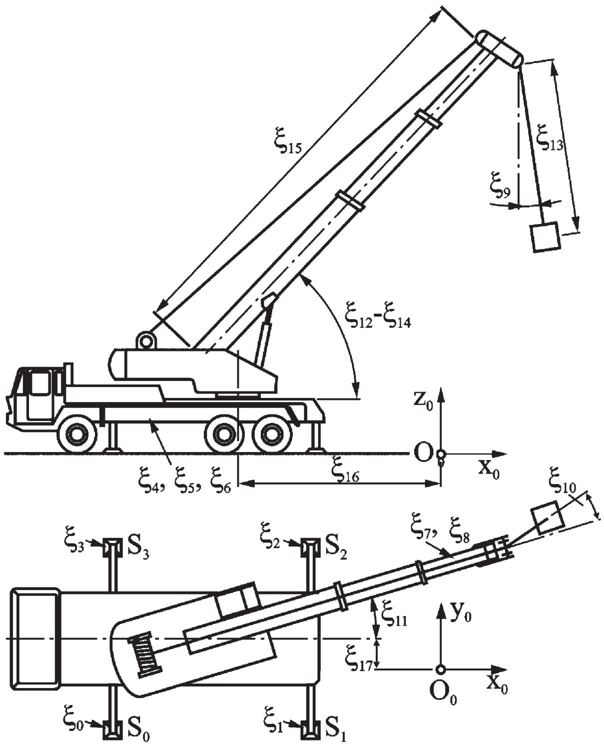

Fig. 1. A mechanical model of the truck-crane

To establish differential equations for motion, Lagrange's equations of the second kind are used:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\xi}_i}\right)-\frac{\partial E_k}{\partial \xi_i}+\frac{\partial \Phi}{\partial \dot{\xi}_i}+\frac{\partial E_p}{\partial \xi_i}=Q_i^n,\ i=0,\ ...,17\ , \quad (1)$$

where: E_k, E_p, Φ and Q_i^n represent kinetic energy, potential energy, a dissipation function and appropriate generalized non-conservative forces respectively.

More complex spatial systems encounter the major problem of how to define the velocities and coordinates of the characteristic points of the analyzed system. The derivation of their expressions in the developed form is an exceptionally complicated task. Also, the following differentiation gives us the final forms of differential equations for motion that are exceptionally complex considering their scope. As such, they also produce difficulties in their numerical solutions. The faced difficulties are evident either in the impossibility of solving them or in increased computer time consumption necessary for calculation. The examined problem is solved by specifying the coordinates of any characteristic point of the truck-crane at any instant of time in relation to stationary coordinate system $O_0x_0y_0z_0$ (Fig. 2). Within the frame of the model, a number of mobile coordinate systems are denoted $O_kx_ky_kz_k$ ($k \geq 1$). The motion of the systems is also determined in relation to the static coordinate system. In this way, we have obtained expressions by which necessary coordinates, velocities and accelerations are explained in the form of a matrix (Šelmić 1979). The application of this method enables a simple way of defining the velocities of very complex mechanical systems, considerably facilitates the process of deriving the final forms of differential equations for motion and shortens time needed for their solution.

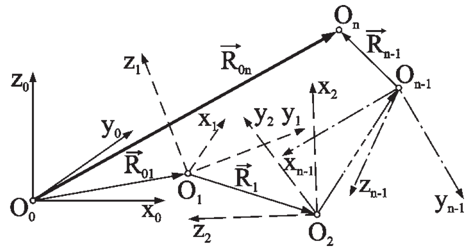

Fig. 2. Coordinate systems (mobile and stationary)

A position vector of origin O_{k+1} in relation to coordinate system $O_kx_ky_kz_k$ can be defined by the following expression:

$$\mathbf{R}_k=\mathbf{O}_k\mathbf{O}_{k+1}=R_{k,x}\cdot\mathbf{i}_k+R_{k,y}\cdot\mathbf{j}_k+R_{k,z}\cdot\mathbf{l}_k\ , \quad (2)$$

where: $\mathbf{i}_k, \mathbf{j}_k, \mathbf{l}_k$ are unit vectors of coordinate system $O_kx_ky_kz_k$.

A position vector of origin O_n in relation to the stationary coordinate system can be defined in the form of the matrix:

$$\mathbf{R}_{0n}=\mathbf{R}_{01}+\sum_{k=1}^{n-1}A_{0k}\cdot\mathbf{R}_k\ ;$$
$$\mathbf{R}_k=\begin{bmatrix}R_{k,x} & R_{k,y} & R_{k,z}\end{bmatrix}^T ; \quad (3)$$
$$A_{0k}=A_{01}\cdot A_{12}\cdot ...\cdot A_{k-1,k}\ ,$$

where: A_{0k} – the transformation of matrices (a matrix of the cosine of the angles) from stationary coordinate system $O_0x_0y_0z_0$ to coordinate system $O_kx_ky_kz_k$; $A_{k-1,k}$ – the transformation of matrices (a matrix of the cosine of the angles) from coordinate system $O_{k-1}x_{k-1}y_{k-1}z_{k-1}$ to coordinate system $O_kx_ky_kz_k$.

By differentiating expression (3), the velocity of origin O_n can be also defined as follows:

$$\mathbf{v}_{0n}=\dot{\mathbf{R}}_{0n}=\mathbf{R}_{01}+\sum_{k=1}^{n-1}\dot{A}_{0k}\cdot\mathbf{R}_k+\sum_{k=1}^{n-1}A_{0k}\cdot\dot{\mathbf{R}}_k\ , \quad (4)$$

where: $\dot{\mathbf{R}}_k$ is local time for deriving vector $\mathbf{R}_k$.

2.1. Position Vectors and Transformation Matrices

Origin O_0 is placed on the ground and can be defined as a point placed on the axis of rotating a slewing platform. Coordinate axis z_0 is perpendicular to the horizontal ground space. Coordinate axis x_0 overlaps with the longitudinal axis and axis y_0 – with the transversal axis of the track-crane. The axes are defined in the case when the truck-crane is based on the horizontal ground space.

Position vectors and transformation matrices for the values of ground slope (α_1, α_2) (Fig. 3) are the forms of:

$$\mathbf{R}_{01}=\mathbf{R}_1=\mathbf{0}\ ; \quad (5)$$

$$A_{01}=\begin{bmatrix}\cos\alpha_1 & 0 & \sin\alpha_1\\ 0 & 1 & 0\\ -\sin\alpha_1 & 0 & \cos\alpha_1\end{bmatrix}; \quad (6)$$

$$A_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_2 & \sin\alpha_2 \\ 0 & -\sin\alpha_2 & \cos\alpha_2 \end{bmatrix}. \tag{7}$$

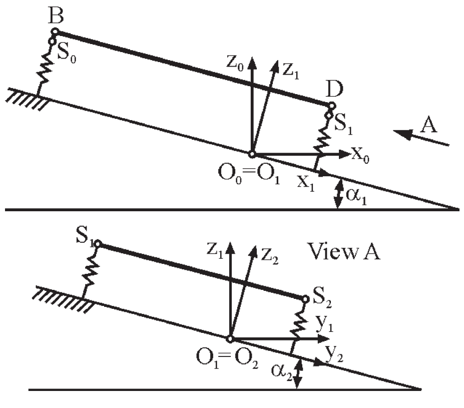

Fig. 3. Coordinate systems $O_0x_0y_0z_0$, $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$

The method that was used for obtaining parameters of moving point S_0 (the point of outrigger and chassis connection) is employed for receiving parameters of points S_1, S_2 and S_3. In case of contact between the outrigger and the ground, the distance between outriggers and non-deformed ground under point S_0 (generalized coordinate ξ_0) corresponds to the deflection of the ground when measured under the observed point. The following points and planes required for further analysis are presented in Fig. 4: 1 – outrigger, 2 – point S_0 with non-deformed outrigger and non-deformed ground, 3 – point S_0 with non-deformed outrigger and deformed ground, 4 – point S_0 with deformed outrigger and deformed ground, 5 – non-deformed ground, 6 – deformed ground. In a mechanical sense, it can be considered that chassis at point S_0 is rested across two in a series connected springs of different rigidity. Fig. 4 also includes the following signs: h – the height of outriggers, $\Delta_{t,0}$ – the deformation of the ground, $\Delta_{s,0}$ – the deformation of outriggers.

Using Figs 4 and 5, the position vectors of points S_0, S_1, S_2 S_3 and R_2 can be written in the following form:

$$\mathbf{R}_{2,S_0} = \begin{bmatrix} -L_4 - \xi_{16} \\ -0{,}5\cdot L_1 + \xi_{17} \\ h - k_0\cdot\xi_0 \end{bmatrix}, \quad k_0 = 1 + \frac{c_{t,0}}{c_{h,0}}; \tag{8}$$

$$\mathbf{R}_{2,S_1} = \begin{bmatrix} L_5 - \xi_{16} \\ -0{,}5\cdot L_2 + \xi_{17} \\ h - k_1\cdot\xi_1 \end{bmatrix}, \quad k_1 = 1 + \frac{c_{t,1}}{c_{h,1}}; \tag{9}$$

$$\mathbf{R}_{2,S_2} = \begin{bmatrix} L_5 - \xi_{16} \\ 0{,}5\cdot L_2 + \xi_{17} \\ h - k_2\cdot\xi_2 \end{bmatrix}, \quad k_2 = 1 + \frac{c_{t,2}}{c_{h,2}}; \tag{10}$$

$$\mathbf{R}_{2,S_3} = \begin{bmatrix} -L_4 - \xi_{16} \\ 0{,}5\cdot L_1 + \xi_{17} \\ h - k_3\cdot\xi_3 \end{bmatrix}, \quad k_3 = 1 + \frac{c_{t,3}}{c_{h,3}}; \tag{11}$$

$$\mathbf{R}_2 = \begin{bmatrix} -\xi_{16} \\ \xi_{17} \\ R_{2z} \end{bmatrix}; \tag{12}$$

$$R_{2z} = k_4\cdot\xi_0 + k_5\cdot\xi_1 + k_6\cdot\xi_2 + k_7\cdot\xi_3 + {}$$
$$+ k_8\cdot\xi_4 + k_9\cdot\xi_5 + k_{10}\cdot\xi_6 + k_{11},$$

$$k_4 = \frac{k_0}{2}\cdot\left(\frac{L_4}{L_6} - 1\right); \quad k_5 = -\frac{k_1}{2}\cdot\frac{L_4}{L_6}; \quad k_6 = -\frac{k_2}{2}\cdot\frac{L_4}{L_6};$$

$$k_7 = \frac{k_3}{2}\cdot\left(\frac{L_4}{L_6} - 1\right); \quad k_8 = \frac{L_4}{L_6} - 1; \quad k_9 = -\frac{L_4}{L_6};$$

$$k_{10} = \frac{4\cdot L_4}{L_6}\cdot\left(\frac{L_4}{L_6} - 1\right); \quad k_{11} = h + L_7,$$

where: $c_{t,i}$ – ground rigidity below outrigger 'i'; $c_{h,i}$ – outrigger rigidity ($i = 0\div 3$).

Transformation matrix A_{23} is determined by the following expression:

$$A_{23} = \begin{bmatrix} 1 & 0 & -\theta_0 \\ 0 & 1 & -\theta_1 \\ \theta_0 & \theta_1 & 1 \end{bmatrix}. \tag{13}$$

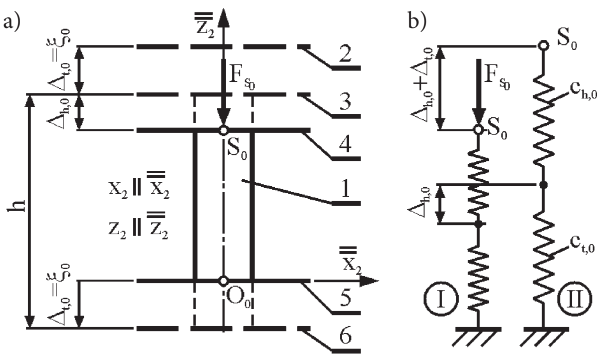

Fig. 4. Modelling outriggers and ground connection

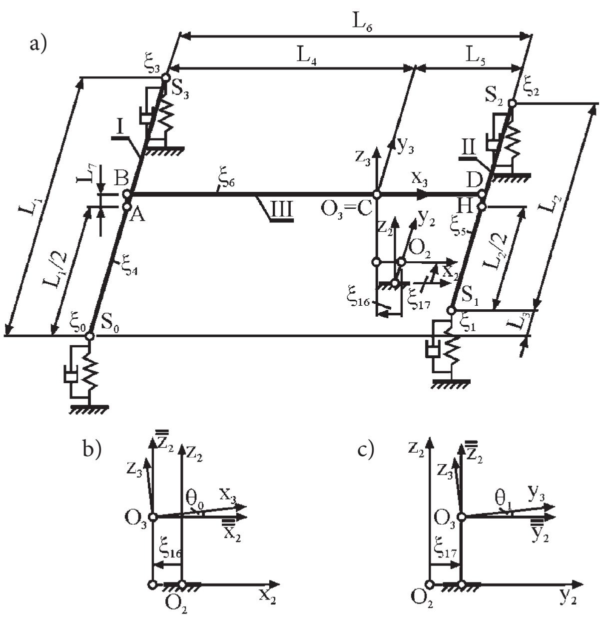

Fig. 5. Modelling the chassis of the truck-crane

Rotation angles θ_0 and θ_1 are caused by outriggers and chassis deformation. A paper by Mijailović (2005) presents a detailed explanation (equations) of angles θ_0 and θ_1 below given by expressions:

$$\theta_0 = \frac{k_0}{2L_6}\xi_0 - \frac{k_1}{2L_6}\xi_1 - \frac{k_2}{2L_6}\xi_2 + \frac{k_3}{2L_6}\xi_3 + \frac{1}{L_6}(\xi_4 - \xi_5) + (\frac{8L_4}{L_6^2} - \frac{4}{L_6})\xi_6; \tag{14}$$

$$\theta_1 = \frac{k_0 L_5}{L_1 L_6}\xi_0 + \frac{k_1 L_4}{L_2 L_6}\xi_1 - \frac{k_2 L_4}{L_2 L_6}\xi_2 - \frac{k_3 L_5}{L_1 L_6}\xi_3. \tag{15}$$

Generalized coordinate ξ_{11} is determined using mobile coordinate systems $O_3x_3y_3z_3$ and $O_4x_4y_4z_4$ (Fig. 6).

Boom rotation and bearing deformation are specified using mobile coordinate system $O_5x_5y_5z_5$ (Figs 6 and 7).

The analysis of Figs 6 and 7 displays the following equations:

$$\mathbf{R}_3 = \begin{bmatrix} -L_8 \cdot \cos\xi_{11} \\ -L_8 \cdot \sin\xi_{11} \\ L_9 \end{bmatrix}; \tag{16}$$

$$A_{34} = \begin{bmatrix} \cos\xi_{11} & -\sin\xi_{11} & 0 \\ \sin\xi_{11} & \cos\xi_{11} & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{17}$$

$$A_{45} = \begin{bmatrix} \cos(\xi_{12} - \xi_{14}) & 0 & -\sin(\xi_{12} - \xi_{14}) \\ 0 & 1 & 0 \\ \sin(\xi_{12} - \xi_{14}) & 0 & \cos(\xi_{12} - \xi_{14}) \end{bmatrix}; \tag{18}$$

$$\mathbf{R}_K = \begin{bmatrix} L_{12} \cdot \sin\xi_{14} - L_{13} \cdot \cos\xi_{14} \\ 0 \\ L_{13} \cdot \sin\xi_{14} + L_{12} \cdot \cos\xi_{14} \end{bmatrix}; \tag{19}$$

$$\mathbf{R}_4 = \mathbf{R}_5 = \mathbf{0}. \tag{20}$$

An equation for the boom of the neutral axis remains unknown. The papers by Komarov (1969), Mijailović and Šelmić (2000, 2002a, 2002b) analyze the problem of boom stability. The authors have used different equations for the boom of the neutral axis. This paper puts forward the equation for the boom (Fig. 8) as a polynomial expression:

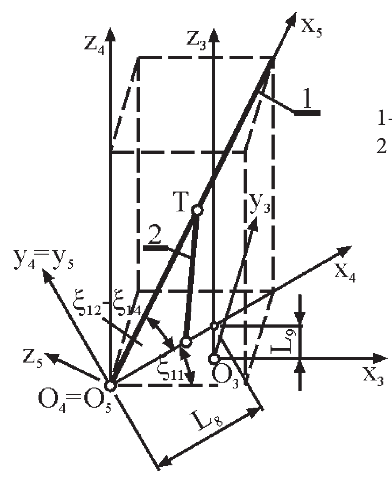

Fig. 6. Coordinate systems $O_3x_3y_3z_3$, $O_4x_4y_4z_4$ and $O_5x_5y_5z_5$

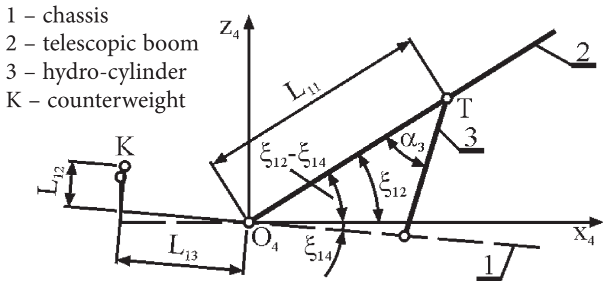

Fig. 7. The coordinate system $O_4x_4y_4z_4$

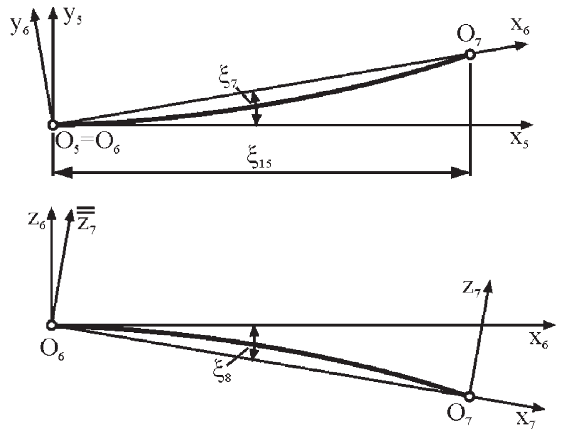

Fig. 8. The boom of the neutral axis in two perpendicular planes

$$y_5 = \frac{\xi_7}{\xi_{15}} x_5^2, \tag{21}$$

$$z_6 = -\frac{\xi_8}{\xi_{15}} x_6^2. \tag{22}$$

The analysis of Fig. 8 and equations (21) and (22) points to obtaining the following equations:

$$A_{56} = \begin{bmatrix} 1 & -\xi_7 & 0 \\ \xi_7 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{23}$$

$$A_{67} = \begin{bmatrix} 1 & 0 & \xi_8 \\ 0 & 1 & 0 \\ -\xi_8 & 0 & 1 \end{bmatrix}; \tag{24}$$

$$\mathbf{R}_6 = \begin{bmatrix} \xi_{15} \\ 0 \\ -\xi_8 \cdot \xi_{15} \end{bmatrix}. \tag{25}$$

Pulleys are placed on the top of the boom. The rope is placed over the pulleys and connected to the crane drum on one end and to the crane hook on the other. We presume that the rope is not deformable. The length of the rope (distance from the boom to the hook) is changing during the process of lifting the load. Such change is defined by generalized coordinate ξ_{13} describing the distance from the boom top to the centre of the load (Fig. 9). Position vector R_7 is given by expression:

$$\mathbf{R}_7 = \begin{bmatrix} \xi_{13} \cdot \sin\xi_9 \cdot \cos\xi_{10} \\ \xi_{13} \cdot \sin\xi_9 \cdot \sin\xi_{10} \\ -\xi_{13} \cdot \cos\xi_9 \end{bmatrix}. \quad (26)$$

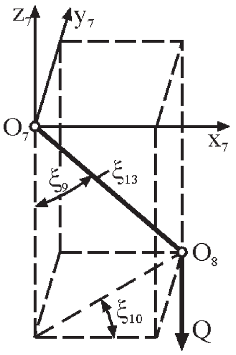

Fig. 9. Swaying the load

2.2. Kinetic Energy, Potential Energy, Dissipation Function and Generalized Non-Conservative Forces

The potential energy of the observed mechanical system is denoted as follows:

$$E_p = \sum_{i=1}^{8} E_{p,i} \,, \quad (27)$$

where: E_{p1} – the potential energy of outriggers and the ground; E_{p2} – the potential energy of chassis; E_{p3} – the potential energy of the driver's cab and a drive unit when driving the track-crane; E_{p4} – the potential energy of bearing (Mijailović, Šelmić 2004); E_{p5} – the potential energy of the operator's cab and a drive unit of crane operations; E_{p6} – the potential energy of counterweight; E_{p7} – the potential energy of the telescopic boom; E_{p8} – the potential energy of the load.

The kinetic energy of the mechanical system is determined by the following expression:

$$E_k = \sum_{i=1}^{6} E_{k,i} \,, \quad (28)$$

where: E_{k1} – the kinetic energy of chassis; E_{k2} – the kinetic energy of the driver's cab and a drive unit when driving the track-crane; E_{k3} – the kinetic energy of the operator's cab and a drive unit of crane operations; E_{k4} – the kinetic energy of counterweight; E_{k5} – the kinetic energy of the telescopic boom; E_{k6} – the kinetic energy of the load.

A dissipation function has the form:

$$\Phi = \sum_{i=1}^{4} \Phi_i \,, \quad (29)$$

where: Φ_1 – the dissipation function of outriggers and the ground; Φ_2 – the dissipation function of the hydraulic cylinder of boom elevation; Φ_3 – the dissipation function of chassis; Φ_4 – the dissipation function of the telescopic boom (Šelmić, Mijailović 2006).

Generalized non-conservative force is determined by the following expression:

$$\begin{aligned} & Q_i^n = \sum_{k=1}^{p} \left(\mathbf{F}_k^n, \frac{\partial \mathbf{R}_k}{\partial q_i} \right); \\ & \mathbf{F}_k^n = F_{k,x}^n \cdot \mathbf{i}_0 + F_{k,y}^n \cdot \mathbf{j}_0 + F_{k,z}^n \cdot \mathbf{l}_0 \,; \\ & \mathbf{R}_k = R_{k,x} \cdot \mathbf{i}_0 + R_{k,y} \cdot \mathbf{j}_0 + R_{k,z} \cdot \mathbf{l}_0 \,, \end{aligned} \quad (30)$$

where: $\mathbf{F}_k^n$ – non-conservative force ($F_{k,x}^n, F_{k,y}^n, F_{k,z}^n$ – non-conservative forces directed along axes x, y and z respectively); $\mathbf{R}_k$ – position vector ($R_{k,x}, R_{k,y}, R_{k,z}$ – position vectors directed along axes x, y and z respectively); i – the number of generalized coordinates ($i = 0$, ..., 17); k – an ordinal number of the points at which non-conservative forces act.

Non-conservative forces include forces and torques that come from engines and induce the respective motion of the truck-crane. Non-conservative forces also include the forces of friction.

On the basis of expressions (30), it follows that:

$$\begin{aligned} & Q_0^n = \ldots = Q_{10}^n = Q_{14}^n = 0,\, Q_{11}^n = M_{p11} - M_{\mu 11}, \\ & Q_{12}^n = M_{p12} - M_{\mu 12},\, Q_{13}^n = F_{p13} - F_{\mu 13}, \\ & Q_{15}^n = F_{p15} - F_{\mu 15},\, Q_{16}^n = F_{p16},\, Q_{17}^n = F_{p17}, \end{aligned} \quad (31)$$

where: M_{p11} – the driving torque of rotating the slewing platform; $M_{\mu 11}$ – the friction torque of rotating the slewing platform; M_{p12} – the driving torque of rotating the boom; $M_{\mu 12}$ – the friction torque of rotating the boom; F_{p13} – driving force (acting in the rope) for lifting the load; $F_{\mu 13}$ – friction force for lifting the load (between the rope and pulleys, between the rope and drum, in bearings of pulleys and the drum); F_{p15} – driving force for telescoping the boom; $F_{\mu 15}$ – friction force for telescoping the boom (a result of friction that occurs in slides during the relative motion of two adjacent boom segments); F_{p16}, F_{p17} – driving forces during truck-crane driving.

Driving forces and torques are included in generalized non-conservative force. Their values are defined by engine exploitation characteristics. Generalized non-conservative force also includes friction forces. As an example for defining generalized non-conservative forces, we will define Q_{11}^n that corresponds to generalized coordinate ξ_{11} (in case of rotating the slewing platform). The rotation of the slewing platform can be managed by the engine, i.e. driving torque M_{p11}. Friction torque $M_{\mu 11}$ acts opposite to this rotation. We come to the conclusion that generalized non-conservative force Q_{11}^n is equal to the difference between the drive and friction torques. In the same way, other generalized non-conservative forces are given.

2.3. Solving Differential Equations for Motion

At the initial instant of time, the generalized coordinates defining deformations will have the values equal to their static deformations ($\xi_{i,st}$). The distance from the boom top to the load centre at the initial instant of time is designated by l_0 and determined by rope length. Telescopic boom length at the initial instant of time is designated by $L_{st,0}$. The first derivatives of the generalized coordinates through time are equal to zero. In this case, the initial conditions can be expressed as:

$$\xi_i(0) = \xi_{i,st} \text{ for } i = 0, ..., 9, 14;$$
$$\xi_i(0) = 0 \text{ for } i = 10, 11, 12, 16, 17; \quad (32)$$
$$\xi_{13}(0) = l_{u,0},\ \xi_{15}(0) = L_{st,0},\ \dot{\xi}_i(0) = 0 \text{ for } i = 0 \div 17.$$

Static deformations are defined on the basis of real load acting on the truck-crane. The analyzed load also includes load weight, except in case when the procedure of lifting the load from the ground is analyzed. Namely, when the load lies on the ground, it has no influence on static deformation.

By applying the expression of potential energy (27), kinetic energy (28), dissipation function (29) and generalized non-conservative forces (31), the Lagrange equations of the second kind (1) obtain their final form:

$$\ddot{\mathbf{q}} = \mathbf{B}, \quad (33)$$

where: $\ddot{\mathbf{q}} = \left[\ddot{\xi}_0 \ \ddot{\xi}_1 \ ... \ \ddot{\xi}_{16} \ \ddot{\xi}_{17}\right]^T$;
$\mathbf{B} = f\left(\xi_i, \dot{\xi}_i, L_i, c_i, \delta_i ...\right)$ – a column vector of dimension 18, which is the function of generalized coordinate ξ_i ($i = 0, 1, ..., 17$), distances (L_i), the rigidity of the ground, outriggers and bearing (c_i), the coefficient of ground damping, outriggers and a hydraulic cylinder for boom elevation.

Differential equations (33) can be numerically solved introducing changes:

$$X_0 = \xi_0;\ X_1 = \dot{\xi}_0;\ X_2 = \xi_1;\ X_3 = \dot{\xi}_1, ...\ X_{34} = \xi_{17}; \quad (34)$$
$$X_{35} = \dot{\xi}_{17}.$$

In this way, 36 differential first kind equations convenient for a numerical solution to such software as *Matlab* and *Mathcad* are obtained.

The above mechanical model includes the following parameters:

- a deformable character of the boom, chassis, bearing, outriggers, pneumatics, a hydraulic cylinder and the ground;
- a damping coefficient of the boom, chassis, outriggers, pneumatics, a hydraulic cylinder and the ground;
- ground slope.

The application of the model considers that the dynamic behaviour of the truck-crane can be analyzed during the following operations (simultaneously or separately): the rotation of the slewing platform, the rotation of the boom, lifting the load, telescoping the boom and truck-crane driving.

3. The Analysis of the Dynamic Behaviour of the Truck-Crane during Different Operations

This section presents the analysis of the dynamic behaviour of truck-crane AD-16 (maximum lifting capacities m_q= 16 t) – Ivo Lola Ribar (Belgrade).

3.1. The Analysis of the Dynamic Behaviour of the Truck-Crane in Case of Rotating the Slewing Platform

The dynamic behaviour of the truck-crane is analyzed in case of rotating the slewing platform. The conducted analysis contains data such as the mass of the load 3 t; $\xi_{12} = 45°$; $\xi_{11}(t = 0) = 0°$; $\dot{\xi}_{11,\max} = 0.0675 s^{-1}$.

One of the most significant parameters influencing the dynamic behaviour of the truck-crane is ground compressibility in the place of contact under points S_0, S_1, S_2 and S_3 respectively (outrigger or pneumatic) (Fig. 10). The type of the ground has the greatest influence on the amplitude and extreme values of generalized coordinates. The analysis of Fig. 10 concludes that generalized coordinate ξ_0 for c_p = 30 MN/m^3 is 1.6 times larger than c_p = 50 MN/m^3 and 3.5 times larger than c_p = 100 MN/m^3 (c_p – ground compressibility).

The minimal values of ξ_0, ..., ξ_3 decrease with an increasing coefficient of ground compressibility. The minimal value of ξ_0 for c_p = 30 MN/m^3 is 1.7 times higher than c_p = 50 MN/m^3 and 3.3 times higher than c_p = 100 MN/m^3. The analysis of numerical data indicates that along with an increase in ground compressibility, the period of ξ_0, ..., ξ_3 decreases. During exploitation, in order to reduce additional dynamic load, it is necessary to reduce the value of the oscillation amplitude, and, at the same time, to provide a contact between supports and the ground. The amplitudes can be decreased by increasing ground rigidity.

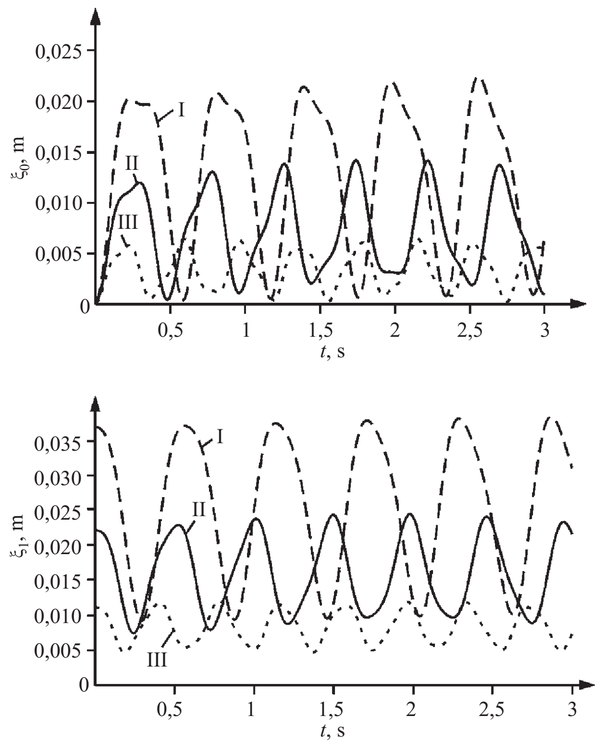

Fig. 10. The dependences of $\xi_0 = \xi_0(t)$ and $\xi_1 = \xi_1(t)$ on the different values of the coefficient of ground compressibility (c_p): I – c_p = 30 MN/m^3; II – c_p = 50 MN/m^3; III – c_p = 100 MN/m^3

During its exploitation, the truck-crane can rest by means of the supports that are in contact with various types of the ground, i.e. the ground under each of the supports can have different values of the compressibility coefficient. The minimum of generalized coordinate ξ_0 for the case of ground characteristics III is less than zero and makes 0.00022 m, which means that in such a case, the separation of support S_0 from the ground occurs. Manufacturers define the load diagram that does not include dependence on the ground type and other factors. To reduce the possibility of accidents, the manufacturers introduce the factor of safety. Therefore, the truck-crane is not used at its full capacity. To achieve this goal, it is necessary to make a computer code that will enable the operator to introduce defined exploitation characteristics in the offered computer code and to get maximum load value permitted for the load of the observed truck-crane. In that way, the task of a designer becomes more responsible and more demanding in respect of time. On the other hand, the owner of the truck-crane gets important information referring to the limited characteristics. Similar problems also appear discussing other types of cranes.

The analysis of literature shows that mechanical-mathematical models approximate bearing as a rigid body. In this article, bearing was approximated as an elastic body. The results related to this approximation can be found in the paper by (Mijailović, Šelmić 2004).

A comparison of the maximum values of the amplitudes of the generalized coordinates defining deformations of chassis carriers (ξ_4, ξ_5 and ξ_6) suggests that they are approximately two times higher for the analyzed chassis which has two times lower values of geometrical characteristics. It results in the appearance of higher values of additional dynamic load. The amplitude of generalized coordinate ξ_0 for chassis construction with the support cross-sections of weaker characteristics is 22% greater than in case of real construction.

A great number of professional papers have investigated the models of the boom where its deformations in the horizontal plane are ignored. The analyzed results conclude that if we exclude boom deformation in the horizontal plane when calculating the total deformation of the boom top we make error in about 8%.

The examination of changes in the distance between the outrigger and non-deformed ground shows that a negative effect of a simple increase in the boom cross-section is noticed in greater amplitudes of generalized coordinates (ξ_0, ..., ξ_3) which can cause the loss of truck-crane stability.

The amplitude of ξ_0 for the model with swaying the load is 34% higher than the model where swaying the load is ignored. It should not be forgotten that swaying the load is the fact that also has a negative effect on the stability of the truck-crane as an object.

Swaying the load has a greater influence on boom deformation in the horizontal rather than in vertical plane. An interesting point is that in case where swaying the load is not ignored and the initial value of real sway angle projection on the vertical plane is 20°, by ignoring boom deformation in the horizontal plane, an error is made in defining the total boom deformation, which now has the maximum value of 19%. The previous results confirm the need for spatial modelling of the boom.

During the period of time when power units operate under non-stationary conditions, additional dynamic load appears and can cause the loss of truck-crane stability as well as permanent plastic deformations of its constituent elements and assemblies. The given phenomenon is most frequently introduced into calculations by means of the dynamic factor, which, in case of rotating the slewing platform, makes 1.19. For the case of the boom, the dynamic factor is 1.28.

In our case, we present errors caused by neglecting different quantities and assemblies where swaying the load is ignored.

The analysis of graphs $\xi_0 = \xi_0(t)$ (Fig. 11) concludes that in light of the aspect of truck-crane stability, it is necessary to analyze the model having deformable beam carriers and component assemblies. In case that, as a result of analysis, data important for the dynamic behaviour of chassis should be obtained, it is necessary that the model should contain the deformable character of chassis as well as the carriers and assemblies placed above the chassis. Such conclusion can be drawn taking into account significant differences among the curves defining deflections in the middle of chassis beam carriers for the cases when the boom is considered as absolutely rigid, i.e. deformable. The effect of the deformable characteristics of chassis on the dynamic behaviour of the boom is considerably weaker, which demonstrates that the dynamics of the object such as the truck-crane should be analyzed in a complex way, i.e. the influence of the deformable characteristics of a number of elements and their complex structure must be taken into account.

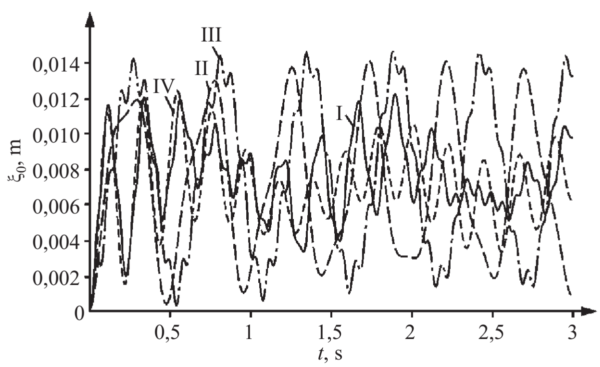

Fig. 11. The dependences of $\xi_0 = \xi_0(t)$ on the models having a different level of complexity: curve I – a model having generalized coordinates ξ_0 ... ξ_8; curve II – a model having generalized coordinates ξ_0 ... ξ_3 (absolutely rigid truck-crane construction with the exception of the deformable characters of the ground and outriggers); curve III – a model having generalized coordinates ξ_0 ... ξ_6 (absolutely rigid truck-crane construction with the exception of the deformable characters of the ground, outriggers and chassis); curve IV – a model having generalized coordinates ξ_0, ..., ξ_3, ξ_7 and ξ_8 (absolutely rigid truck-crane construction with the exception of the deformable characters of the ground, outriggers and boom)

3.2. The Analysis of the Dynamic Behaviour of the Truck-Crane in Case of Lifting (Putting Down) the Load

This chapter comprises a comparison of results for the case when the rope is tight and force within the rope at the start of load lifting is equal to load weight ($m_Q(t = 0) = 3$ t), and for the case when the rope is not tight and force within the rope before the start of the load lifting procedure is equal to zero ($m_Q(t = 0) = 0$ t). The minimum discrepancies between two cases occur for generalized coordinates ξ_0, ..., ξ_3, ξ_9 and ξ_{10} (distances between outriggers and non-deformed ground, sway angles). The amplitudes of chassis differ from 6 to 20%. The value of force within the rope before the start of lifting has the greatest effect on the deformation of the boom and bearing. Therefore, the ratio of bearing amplitudes is 1.96 and the ratio of the amplitudes of the top boom slopes in the vertical plane make 2.62. The ratio between maximum boom deflections in the vertical plane for the analyzed systems is 1.46.

4. Conclusions

1. The process of designing and employing the truck-crane always presents the problem of providing its reliable and safe operation. Thus, there is a need for introducing the new and improving previous mechanical models to describe the problems of exploitation more precisely. Such contribution provides a possibility of introducing appropriate devices, measures and procedures that will result in improving the safety of construction and personnel in charge.
2. This paper offers a contribution to the solution to the dynamic behaviour of the truck-crane as a whole and to some of its main components. The article defines the mechanical-mathematical model having 18 generalized coordinates.
3. In case of analyzing the stability of the truck-crane, in respect to overturn, it is necessary to deal with the model made of real deformable carriers and assemblies.
4. In case when the primary aim is to analyze the dynamic behaviour of chassis, the process of including a deformable character of that in the model as well as carriers and assemblies above the chassis is required.
5. The influence of the deformable character of chassis on the dynamic behaviour of the boom is insignificant.
6. The unfavourable dynamic behaviour of bearing mostly appears during the operation of lifting (lowering) the loaded boom.
7. Boom deformations in the horizontal plane can be ignored when analyzing lifting (lowering) the boom, boom telescoping and lifting (lowering) the load.
8. It should be noted that lifting (lowering) the load most unfavourably affects the dynamic behaviour of the boom and bearing.

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