



# STUDY ON HETEROGENEOUS TRAFFIC FLOW CHARACTERISTICS OF A TWO-LANE ROAD

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**Abstract.** The paper presents traffic studies conducted by using a video capturing technique on the uninterrupted heterogeneous mix of vehicles plying on an undivided two-lane road facility. On the basis of the collected data, traffic characteristics pertaining to arrival, headway and speed distributions have been plotted considering suitable mathematical distributions to fit field observed values. The curves representing fundamental traffic flow relationships among three basic variables, namely speed, density and flow have also been established. Thus, a systematic attempt to enable the understanding of heterogeneous traffic flow parameters has been made through this exploratory study.

Keywords: heterogeneous mix, flow parameters, traffic characteristics, fundamental traffic relationships, chisquare test, regression-fit analysis.

### 1. Introduction

Road traffic in India displays a heterogeneous mix condition wherein vehicles possessing distinguished physical and operational attributes constitute the vehicular flow. The behaviour of homogeneous flows, commonly observed in the developed Western nations, is characterized by a strict lane discipline and single-file motion of vehicles with restricted movement across the lanes. A heterogeneous flow, on the contrary, is differentiated by the presence of a loose lane discipline and use of the entire road space without any confinements for manoeuvring. The lateral movement of vehicles, apart from usual longitudinal motion, results in mass queue formations that operate two-dimensionally. Furthermore, wide ranging vehicle types moving in these traffic flows add to the dynamic quality of the flow. The behaviour of such traffic is ensued by the existence of flow variables that vary over space and time. Knowledge about these parameters is, therefore, essential for understanding the nature of a heterogeneous traffic mix moving on the road. Information derived from parametric studies can provide a crucial base for accomplishing tasks such as road design, planning and operation.

A large proportion of the transport network in India consists of two-lane roads. The majority of them are undivided road facilities that are uniquely identified by the subjection of vehicles to traffic moving on the oncoming lane. Thus, the behaviour of flows travelling in a particular direction is predominantly influenced by the prevailing traffic conditions on the opposing lane, as it constrains the passing/overtaking activity of vehicles. In India, these facilities carry a high volume of traffic comprising a variety of public, private and commercial vehicles. They serve a wide range of traffic requirements concerned with infrastructural potential and indirectly affect the economy and commerce of the country.

The paper hereby aims to examine the traffic flow parameters of the uninterrupted heterogeneous traffic flow on a two-lane undivided road facility. The primary objectives desired to be accomplished through the study are listed as follows:

- to obtain and extract traffic data from the field employing a suitable approach;
- to assess data with respect to arrival patterns, time headway characteristics and vehicle speed distributions;
- to derive fundamental relationships among key variables speed, density and flow.

#### 2. Literature Review

Several empirical studies were performed in the past to evaluate traffic flow characteristics of vehicles and to determine appropriate representations of traffic flow parameters. Katti *et al.* (1985) found through their study on arterials that for volumes ranging from 500 to 1000 vehicles/hour, negative exponential distribution was

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suitable for representing headways between vehicles. Mukherjee et al. (1988) evaluated the suitability of negative and shifted negative exponential distributions to generate vehicles approaching roads at the intersections in Calcutta, India. The suitability of theoretical distributions was judged by using the chi-square test. It was found that the shifted negative exponential distribution gave a close fit for the observed headways. It was suggested that a comparison of theoretical headway distributions along with the observed distributions could be made based on cumulative frequencies. Sahoo et al. (1996) used a 3-second class interval for grouping headway data and compared cumulative frequencies of the observed and theoretical distributions. The maximum traffic volume was about 850 vehicles per hour, and headways were found to fit into negative exponential distribution. The authors also found that vehicular speeds fitted well to normal distribution with a mean of 42÷45 km/h and a standard deviation of 9÷13 km/h on different intercity roads in India. Hossain and Iqbal (1999) attempted to study headway patterns on twolane, two-way highways in Bangladesh. The carried out analysis determined that for the volume of the observed field ranging between 200 and 708 vehicles/hour, exponential and log-normal distributions were adequate for fitting time headways between vehicles. Al-Ghamdi (2001) analyzed the time headways of vehicle arrivals on urban roads in Riyadh based on lane-wise traffic data collected under different volume levels. It was found that negative exponential, shifted exponential and gamma distributions reasonably fitted time headways at low and medium flow rates on freeways, whereas the Erlang distribution was found to be appropriate in high traffic flows. An appropriate methodology to extract headway data was suggested by Arasan and Koshy (2003) in their study on a heterogeneous traffic flow on an urban arterial in Chennai, India. They also recommended Sturges' rule for computing the range of headway classes and established that headway data representing varying flows could be fitted as negative exponential distribution. Arkatkar and Arasan (2010) determined that vehicle arrivals could be fitted well into the Poisson distribution, whereas inter-arrival times could be fitted well into negative exponential distribution. Kadiyali et al. (1981) discovered that free speeds of different types of vehicles followed normal distribution. May (1990) suggested choosing the Poisson distribution for modelling vehicle arrivals under low-flow random headway state conditions. He purported that the speeds of the observed speeds could be fitted as normal or log-normal distribution based on the traits of their frequency distribution plots. Tseng et al. (2005), as a part of effort to revise Taiwan Area Highway Capacity Manual, analyzed the collected free-flow speed data at the midpoints of seventy-six multilane rural and suburban highway segments in Taiwan. The analysis regarding the distribution of free flow speeds for different vehicle categories showed that free speeds of different vehicle categories followed normal distribution. Minh et al. (2005) performed a study to analyze motorcycle behaviour and operation on four

selected locations in Hanoi, Vietnam. Speed - flow relationships were developed using the adjustment factor for the presence of vehicles, other than motorcycles, and was based on the motorcycle equivalent unit. Speed data on motorcycles were also plotted as normal distribution. Hall and Montgomery (1993) pointed out the important features of a speed-flow fundamental diagram. Speedvolume relationships employed in the U.K. comprises two-segment linear functions. Speed may remain constant with increasing flow for some considerable range of flows. The break point, at which speed starts decreasing, is somewhere around the two-thirds or three-quarters of maximum flows. Speed at maximum flows, in the absence of congestion (queues), may be 10 to 25 km/h lower than free-flow speed. It is unlikely, that the whole data pertaining to the speed-flow curve can be obtained at any location. Within the bottleneck, data may cover an uncongested segment along with some portion of the queue discharge segment. On the other hand, in the location that experiences queue formation, some portion of the uncongested segment will be observed along with some part of the within queue. Sahoo et al. (1996) studied traffic flow characteristics on National Highway No. 5. The conducted analysis concluded that with an increase in traffic volume, speed decreased. The relationships among speed, flow and density were studied by Kumar and Rao (1998) for road stretches on NH 5 and NH 6 in India. They established fundamental diagrams based on the collected data and observed it was inadequate to estimate capacity values since the flow of vehicles did not encompass an appropriate regime. However, Haefner et al. (1998) were able to determine capacity, free-flow speeds and critical density from the relationships plotted for traffic data collected from an urban freeway in St. Louis, as a wide range of flow regimes could be identified. Akcelik et al. (1999) developed a time-dependent speed-flow model popularly known as Akcelik's function based on queuing theory concepts, providing a smooth transition between a steady-state queuing-delay function for unsaturated conditions and a deterministic-delay function for over saturated conditions. Arasan et al. (2009) modelled a set of speed-flow curves using HETEROSIM simulation software for heterogeneous traffic on road spaces having widths of 7.5 m, 11.0 m and 14.5 m. Relationships thus developed were observed to follow the standard pattern indicating the goodness of the simulation model. Van As and Van Niekerk (2004) evaluated the available traffic models for an operational analysis of two-lane highways and proposed an alternative queuing model for platoons in a traffic flow under conditions of South African roadways. It was found that the available models such as those from the Highway Capacity Manual had various limitations and were inappropriate or inadequate for modelling South African road traffic. A new model based on microscopic simulation was proposed and an alternative measure of effectiveness, named follower density, was introduced which serves as a better indicator of warranting capacity upgrading. Roshandeh et al. (2009) performed spot-speed studies on a Malaysian highway and

plotted frequency distribution curves useful for understanding traffic flow characteristics and making speedrelated decisions on a particular roadway system. Speed distribution curves were plotted for field observed data and basic parameters related to speed were derived for the selected highway with a heterogeneous traffic mix. The importance of speed studies, with respect to traffic engineering, was also discussed. Polus and Cohen (2009) developed and estimated new, theory-based queuing relationships considering the quality of the flow on 15 two-lane rural highways in northern Israel and proposed a new Level of Service (LOS) variable measuring the quality of the flow both inside and between platoons in a traffic flow. The study presented five flow measures, namely - the platoon flow rate, the average platoon length, traffic intensity, percent-time-spent following and the freedom of the flow. These measures are highly relevant to estimate flow characteristics on two-lane rural highways. The relationship between the freedom of the flow and two-way flow was also calibrated from traffic observations. LOS thresholds based on the flow and the freedom of the flow were presented and discussed.

The reviewed literature presents that the selection of field location as well as the application of a suitable approach for collecting and extracting data form a crucial part of traffic studies. Also, various distributions and relationships pertaining to flow parameters take into account the quality of the flow that prevails on road spaces, which subsequently determines the choice and appropriateness of an adequate representation of field observed variables.

## 3. Data Collection

For the purpose of collecting traffic data, a 500 m long road stretch on National Highway (NH) No 11 corridor between Sikar and Jaipur cities situated in the western part of India was selected. It is an undivided two-lane two-way road with an overall uniform carriageway width of 7.5 m and paved shoulder width of 1.25 meters in each direction. The width of the unpaved shoulder, made of gravels and stones, ranges from 2 to 2.5 m in each direction; however, it had no effect on the movement of traffic as vehicles travelled on the road pavement. The chosen traffic plying on the study section is not influenced by any nearby intersections or road geometry such as a gradient. The conditions prevailing on the study section thus ensured an uninterrupted flow of traffic.

Data was collected within the period of two hours – from 4 p.m. to 6 p.m. Four observation points were chosen for installing video cameras: two in each direction, for recording the entry and exit of vehicles. The recordings were synchronized so that extraction errors due to time lag could be avoided. Discernible markings were made on the unpaved shoulder that functioned as reference points. A verbal recording of vehicle categories and license plate numbers was also done to assist the data extraction process. In this manner, continuous data was collected on the two-lane roadway ramp without any loss of information pertaining to the vehicles crossing the road section.

## 4. Data Extraction

Traffic data collected from the field section was extracted using the FMV6 (Full Motion Video) player. Mpeg format video files were played on a computer screen after copying recordings from each camera on compact discs. The FMV6 player is capable of fragmenting video into frames, which can then be noted down by stopping playback and navigating forward and backward to obtain frame reading when the vehicle just crosses the reference point. The frame numbers can be changed to time values by applying a simple conversion of the unitary method. Time thus computed is denoted in seconds and has the accuracy of up to two decimal places. Careful consideration was given to obtain the exact frame number as, in many instances, more than one vehicle crossed the reference point almost together. This was observed in cases where an overtaking manoeuvre took place near the reference points.

Four observation recordings were processed one at a time. Certain key features such as the number of passengers in case of open vehicles, vehicle-model descriptions etc., were noted down so that the identification of a vehicle could be facilitated when the other recording for the same direction was processed. The videos were also repeatedly played at normal speeds around the instances when the vehicle crossed the reference point, so that information from verbal recording could also be obtained for verification. The data incurred from video recordings was tabulated as in-time and out-time values ranging from 0 to 7200 and denoted in seconds. Time headways and speeds for vehicles in each direction were calculated by operating on time values. The hourly volume of vehicles travelling towards Jaipur and Sikar was found to be about 733 vehicles/hour and 654 vehicles/ hour respectively.

Within the period of two hours considered for the study, there were a few vehicles for which the in-time and out-time values could not be accounted either because they had been already present on the road section when the recording began, or the recording ended before they could exit the study section. However, single time values obtained for these vehicles were retained as they were needed for examining parameters measured at a point such as arrival and headway. Vehicles for which both intimes and out-times were within the period of two-hours were accounted during speed calculations. A number of vehicle types were observed on the study section and then broadly grouped into nine categories, namely:

1) motorized 2-wheeler;

2) motorized 3-wheeler;

- 3) bus;
- 4) car;
- 5) light commercial vehicle (L.C.V.);
- 6) truck;
- 7) tractor;
- 8) bicycle;
- 9) other (tricycle, animal-pulled cart).

Traffic composition for each direction at the study section is illustrated in Table 1.

Table 1.	Vehicle	composition	(denoted as %)
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Vehicle Category	Towards Jaipur	Towards Sikar
Car	33.9	35.3
L.C.V.*	9.9	11.6
M-2w*	36.7	36.7
M-3w*	1.4	1.1
Bus	5.1	6.0
Truck	11.0	7.6
Tractor	1.1	1.4
Bicycle	0.8	0.3
Other	0.1 (rickshaw)	0.1 (camel cart)

\*M-2w – motorized 2-wheeler; M-3w – motorized 3-wheeler; L.C.V. – light commercial vehicle

It can be noticed that motorized 2-wheelers dominated in the traffic flow. It can also be seen that non-motorized vehicles such as bicycles, tricycles and camel carts were found to comprise the heterogeneous traffic flow.

## 5. Evaluation of Traffic Flow Characteristics

Three main evaluated traffic characteristics taken from the extracted data include arrival patterns, headway characteristics and speed distributions. The corresponding results obtained from the data analysis procedure are discussed in the following sub-sections.

#### 5.1. Arrival Pattern

The uninterrupted heterogeneous traffic flow observed on the selected study section had vehicles mostly travelling independently of each other and resulting in a random arrival pattern. Therefore, for the conditions observed on the road section, the Poisson distribution was determined to be a suitable choice for modelling arrival data. The probability mass function for the Poisson distribution is given by:

$$P(r) = \frac{m^r e^{-m}}{r!}$$

In the above equation, P(r) is the probability of vehicles r arriving in time t and m is the average vehicle arrivals per time interval.

Vehicle arrival is point-based measurement, and hence, the arrival pattern was analyzed separately for four observation points in the study section. Vehicles entering the study section were marked as in-vehicles and vehicles exiting the study section were marked as out-vehicles. The vehicles examined for evaluating arrival characteristics were, therefore, grouped as  $J_{In}$  (invehicles travelling towards Jaipur), J<sub>Out</sub> (out-vehicles travelling towards Jaipur),  $S_{In}$  (in-vehicles travelling towards Sikar) and S<sub>Out</sub> (out-vehicles travelling towards Sikar). A combined arrival pattern was also examined by taking into account vehicles arriving at a point on the road section from both directions. The point where in-vehicles travelling towards Jaipur and out-vehicles travelling towards Sikar arrive was labelled as  $J_{In}S_{Out}$ , whereas the point where out-vehicles travelling towards Jaipur and in-vehicles travelling towards Sikar arrive was labelled as  $J_{Out}S_{In}$ .

For each case, an appropriate time interval was chosen and the number of vehicles arriving in each of these time intervals was counted in order to obtain grouped data on vehicle arrivals. The chi-square test was performed to assess the goodness-of-fit of field observed data as the Poisson distribution. The results of analyses have been summarized in Tables 2, 3 and 4.

Degrees of freedom (*df*) have been computed by the formula given below:

df = (I - 1) - p.

In the above formula, *df* refers to the number of degrees of freedom, *I* is the number of class intervals being compared during analysis and p is the number of parameters estimated for defining the distribution.

Table 2. Chi-square tests on arrival data (towards Jaipur)

Description	J <sub>In</sub>	J <sub>Out</sub>
Sample size	1465	1463
Range of time intervals, sec	60	80
Number of time intervals	120	90
Average arrivals per interval, m	12.21	16.26
Number of classes, I	13	12
Degrees of freedom, I-2	11	10
$\chi^2$ [calc.]	10.13	11.82
$\chi^2 [c]$	19.68	18.31

 Table 3. Chi-square tests on arrival data (towards Sikar)

Description	S <sub>In</sub>	S <sub>Out</sub>
Sample size	1307	1303
Range of time intervals, sec	60	60
Number of time intervals	120	120
Average arrivals per interval, m	10.89	10.86
Number of classes, I	12	12
Degrees of freedom, I-2	10	10
$\chi^2$ [calc.]	7.20	9.46
χ <sup>2</sup> [c]	18.31	18.31

Table 4. Chi-square tests on arrival data (combined)

Description	$J_{In}S_{Out}$	$J_{Out}S_{In}$
Sample size	2768	2770
Range of time intervals, sec	60	60
Number of time intervals	120	120
Average arrivals per interval, m	23.07	23.08
Number of classes, I	9	11
Degrees of freedom, I-2	7	9
$\chi^2$ [calc.]	2.09	9.35
$\chi^2$ [c]	14.07	16.92

For the Poisson distribution, the value of p is 1, as only '*m*' referring to the average arrival of vehicles per class interval is estimated for defining the distribution. Thus, the generic equation evaluating degrees of freedom (*df*) becomes:

df = I - 2.

Depending on the number of arrival time intervals (I) being compared during analysis in each case, the value of df is obtained and the corresponding critical chi-square value is checked from the chi-square table for a user-specified level of significance.

In all cases, the calculated chi-square value  $(\chi^2[\text{calc.}])$  is found to be less than critical chi-square value  $(\chi^2[\text{c}])$  obtained from the chi-square table corresponding to the computed degrees of freedom and having the significance level of 0.05. This indicates a good fit of the observed arrival values as the proposed distribution. Thus, for all cases, arrival data was successfully modelled as the Poisson distribution and the hypothesis was ascertained by evaluating chi-square tests.

## 5.2. Headway Characteristics

Vehicles constituting the uninterrupted traffic flow moving on the study section were travelling independently of each other and resulting in random headway distribution. Therefore, for the conditions observed in the field study section and based on literature review, negative exponential distribution was selected as the mathematical model to fit the observed headway data. Unlike the Poisson distribution which is discrete distribution considering the counts of vehicles, a negative exponential model is a continuous curve taking into account frequency class interval. It describes times between events in the Poisson process and can be derived from the equation of probability mass function for Poisson. The function is then given as:

$$P(h \le t) = 1 - e^{\lambda t}.$$

In the equation,  $P(h \le t)$  is the probability of time headways equal to or less than time t.  $\lambda$  is a parameter involved in estimating theoretical distribution and is denoted as the inverse of the mean time headway.

Similar to vehicle arrivals, time headway is pointbased measurement, and therefore a headway pattern was analyzed separately in light of four observation points in the study section. As mentioned in the previous section, vehicles entering the study section were marked as in-vehicles and vehicles exiting the study section were marked as out-vehicles. The vehicles examined for evaluating headway characteristics were, therefore, grouped as J<sub>In</sub> (in-vehicles travelling towards Jaipur), J<sub>Out</sub> (out-vehicles travelling towards Jaipur), S<sub>In</sub> (invehicles travelling towards Sikar) and S<sub>Out</sub> (out-vehicles travelling towards Sikar). A combined headway pattern was also examined by combining the headways of in-vehicles and out-vehicles for each direction. The combined headway data for vehicles travelling towards Jaipur was labelled as  $J_{In}J_{Out}$ , whereas for vehicles moving towards Sikar, the combined data was labelled as  $S_{In}S_{Out}$ .

A sample data set representing different flow-levels was selected for each case and an appropriate range of time interval was chosen using the Sturges' rule expressed as:

$$I = \frac{R}{1 + (3.322) \log_{10}(N)} \, .$$

In the equation, I is the range of class interval, Ris the total range of the observed values and N is the number of observations. As exclaimed by Arasan et al. (2003), the Sturges' rule only gives an estimate of the range of class intervals. Several different range values around the computed range should be tried out and the smallest one that gives a smooth histogram must be selected. A desirable way is to interpret the histogram plot of headway data by statistically mapping the field measured values, and select bin range for which a smooth graph of a negative exponential curve is obtained. For this study, MINITAB was used to plot the data set of the headway sample. The selection of an appropriate class interval is a vital step for beginning the analysis process as the selected range will decide the capability of the observed values fitting into the proposed mathematical distribution.

After selecting the range, the chi-square test was performed to assess the goodness-of-fit of field measured data as negative exponential distribution. The results of analysis for each case are summarized in Tables 5 and 6.

Table 5. Chi-square tests on Headway data (towards Jaipur)

Description	J <sub>In</sub>	$J_{Out}$	J <sub>In</sub> J <sub>Out</sub>
Sample size	784	804	1588
Minimum headway, sec	0.01	0.00	0.00
Maximum headway, sec	42.78	59.16	59.16
Range of class intervals, sec	4.0	5.5	5.0
Parameter, λ	0.1375	0.1371	0.1369
Number of classes, I	9	7	8
Degrees of freedom, I-2	7	5	6
$\chi^2$ [calc.]	4.07	8.49	9.57
$\chi^2[c]$	14.07	11.07	12.59

Table 6. Chi-squ	are tests on	Headway da	ata (toward	s Sikar)
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Description	S <sub>In</sub>	S <sub>Out</sub>	$S_{In} S_{Out}$
Sample size	771	783	1554
Minimum headway, sec	0.02	0.00	0.00
Maximum headway, sec	51.31	51.01	51.31
Range of class intervals, sec	4.8	4.8	4.4
Parameter, λ	0.1338	0.1338	0.1326
Number of classes, I	7	8	9
Degrees of freedom, I-2	5	6	7
$\chi^2$ [calc.]	9.74	8.79	13.38
χ <sup>2</sup> [c]	11.07	12.59	14.07

The tables show that minimum headways were recorded to be as low as 0.01 seconds, and in some cases, even zero time headways were observed. This can be attributed to the fact that on many occasions, vehicles engaged in overtaking manoeuvres near the observation points only if traffic on the opposing lane was very light or negligible in order to allow the pass. In some cases, two smaller vehicles such as motorized 2-wheelers were involved in overtaking activity; however, owing to their small sizes, it could be executed on the same one-way lane without being moderated by traffic moving over the oncoming lane.

Degrees of freedom have been computed by the formula given in the previous sub-section. Even for headway analysis, the value of p is 1 as only  $\lambda$  referring to the inverse of the mean time headway is estimated for defining the distribution. Thus, the generic equation for evaluating degrees of freedom becomes:

df = I - 2.

In the equation, T pertains to the number of class intervals being compared.

In all cases, the calculated chi-square value  $(\chi^2[\text{calc.}])$  is found to be less than critical chi-square value  $(\chi^2[\text{c}])$  obtained from the chi-square table, corresponding to the computed degrees of freedom and having the significance level of 0.05. This indicates a good fit of the observed headway values as the proposed distribution. Thus, for the cases, headway data was successfully modelled as negative exponential distribution and the hypothesis was ascertained by evaluating chi-square tests.

## 5.3. Speed Distributions

Speed is a qualitative measure of traffic flow. In a traffic flow, vehicles move with different individual speeds. Instead of having single characteristic speed, a traffic mix has the distribution of speeds that can then describe the behaviour of vehicle movements on the road facility. The basic speed measures, as observed for the vehicles plying on the study section, are illustrated in Tables 7 and 8 for each vehicle category and for both directions. Max/Min refer to maximum and minimum speeds recorded for the vehicle category.  $\mu$  is the mean and  $\sigma$  is a standard deviation of the observed speed values.

The Tables indicate that high speed values were recorded for cars, whereas vehicle categories such as a motorized 3-wheeler, tractor, bicycle etc., were found to traverse the road section with considerably lower speeds. As a result, these categories had lower mean speeds while high mean values were detected for cars. Also, a maximum deviation of speed values from the mean was observed for cars since they were found to travel with wide ranging speeds, for example, in the range of 85.82 km/hr for cars moving towards Jaipur. Motorized 2-wheelers, buses and L.C.V's were also noticed to have substantially high standard deviation values. The reason for this occurrence can be attributed to the various models or types of these vehicle categories having different power capacities and operational capabilities plying on the road. For example, certain models of motorcycles can be accelerated to higher speeds, whereas scooters that fall under the same motorized 2-wheeler category have comparatively low acceleration capabilities. The same observation can also be ascribed to the age of the vehicle, since old vehicles will have lower performance. Comparatively smaller deviation values have been identified for motorized 3-wheelers, trucks and tractors as operational variation in models is comparatively less.

Sample data sets from vehicle categories with the size above 30 observations were analyzed for each direction. A combined data set was also created by uniting vehicle samples from each vehicle category moving in each direction, and analyzed for evaluating speed characteristics. For starting the analysis process, an appropriate speed class interval was computed from the Sturges' rule described in the previous sub-sections. By supplying various bin range values around the computed range, the observed speeds were plotted as frequency histograms in MINITAB.

Table 7. Speed measurement (towards Jaipur)(denoted as km/hr)

Vehicle Category	Min.	Max.	Mean, µ	Standard Deviation, $\sigma$
Car	30.53	116.35	67.12	15.54
L.C.V.*	22.96	89.69	56.99	12.77
M-2w*	30.24	87.76	49.58	10.27
M-3w*	24.03	49.50	39.78	5.68
Bus	36.32	96.67	60.27	11.84
Truck	30.28	84.75	49.82	9.86
Tractor	12.15	28.77	23.06	4.46
Bicycle	12.36	24.43	17.64	3.81
Other**	22.78	22.78	_	_

\*M-2w – motorized 2-wheeler; M-3w – motorized 3-wheeler; L.C.V. – light commercial vehicle; \*\*camel cart; \*\*\*tricycle

 
 Table 8. Speed measurement (towards Sikar) (denoted as km/hr)

Vehicle Category	Min.	Max.	Mean, µ	Standard Deviation, $\sigma$
Car	25.66	108.30	69.60	13.83
L.C.V.*	24.11	94.79	55.73	10.83
M-2w*	20.38	91.70	46.88	10.01
M-3w*	25.59	47.83	38.72	6.99
Bus	36.52	88.85	64.19	11.74
Truck	25.51	75.66	51.46	9.19
Tractor	9.72	46.11	27.18	9.48
Bicycle	11.75	16.82	14.36	2.17
Other***	5.92	5.92	_	_

\*M-2w – motorized 2-wheeler; M-3w – motorized 3-wheeler; L.C.V. – light commercial vehicle; \*\*camel cart; \*\*\*tricycle The shape of the spread was then examined and interpreted for facilitating the choice of mathematical distribution. For all vehicle types, except motorized 2-wheelers, the histogram assumed a normal curve. Therefore, for these categories, normal distribution was selected to represent the field values. In order to ease calculations, standard normal distribution was opted, which has the sample mean set at zero with dispersion on either side of *y*-axis. The probability density function for a standard normal curve is given by:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}.$$

In the equation, *z* is a parameter computed as  $z = (x - \mu)/\sigma$ , where *x* is an observation of the investigated speed,  $\mu$  is the mean of the sample and  $\sigma$  is a standard deviation from the sample.

In the case of motorized 2-wheelers, the histogram plots revealed that the distribution was unimodal and asymmetric around the mean with a considerably long tail extending to the right. The plot indicated that the mean was located at a lower side with respect to the wide spread of the observed speeds. This behaviour is typical of a log-normal curve. Therefore, log-normal distribution was selected as a suitable model to represent data sets for motorized 2-wheelers. The probability density function for a log-normal curve is expressed by the following equation:

$$f(z) = \frac{1}{x\sigma_{In}\sqrt{2\pi}}e^{\frac{-(\ln(x)-\mu_{In})^2}{2(\sigma_{In})^2}}$$

In the above equation, *x* refers to the investigated speed value;  $\mu_{ln}$  is the mean of the natural logarithm of speed values and  $\sigma_{ln}$  is a standard deviation the natural logarithm of speed values.

After selecting a suitable range to group the field values, the chi-square test was performed to assess the goodness-of-fit of field measured data as the proposed distribution. The results of the analysis of each vehicle category that were travelling towards Jaipur and Sikar are summarized in Tables 9 and 10 respectively. The results incurred from the combined data analysis are summarized in Table 11. In the Tables, L.C.V. refers to Light Commercial Vehicles.

Table 9. Chi-square tests on data about speed (towards Jaipur)

Vehicle Category	Sample	Distribution	Range of class intervals, km/hr	Number of classes, <i>I</i>	Degrees of freedom, <i>I</i> –3	χ <sup>2</sup> [calc.]	$\chi^2 \left[ c  ight]$
Bus	74	Normal	7.1	6	3	1.36	7.81
Car	487	Normal	7.2	10	7	12.98	14.07
L.C.V.	143	Normal	8.1	7	4	5.98	9.49
Motorized 2-wheeler	535	Log-normal	5.2	11	8	13.16	15.51
Truck	160	Normal	6.4	7	4	6.25	9.49

Table 10. Chi-square tests on data about speed (towards Sikar)

Vehicle Category	Sample	Distribution	Range of class intervals, km/hr	Number of classes, <i>I</i>	Degrees of freedom, <i>I</i> –3	χ <sup>2</sup> [calc.]	$\chi^2 \left[ c  ight]$
Bus	78	Normal	7.0	7	4	2.20	9.49
Car	458	Normal	8.3	9	6	2.35	12.59
L.C.V.	151	Normal	8.5	6	3	2.01	7.81
Motorized 2-wheeler	477	Log-normal	7.1	9	6	11.59	12.59
Truck	98	Normal	6.5	6	3	6.13	7.81

Table 11. Chi-square tests on data about speed (combined)

Vehicle Category	Sample	Distribution	Range of class intervals, km/hr	Number of classes, <i>I</i>	Degrees of freedom, <i>I</i> –3	χ <sup>2</sup> [calc.]	$\chi^2 [c]$
Bus	152	Normal	7.3	7	4	2.80	9.49
Car	952	Normal	8.1	10	7	12.65	14.07
L.C.V.	294	Normal	7.2	8	5	7.83	11.07
Motorized 2-wheeler	1012	Log-normal	6.2	10	7	13.32	14.07
Motorized 3-wheeler	34	Normal	4.1	5	2	0.16	5.99
Truck	258	Normal	6.1	7	4	3.38	9.49
Tractor	32	Normal	6.0	4	1	0.52	3.84

Degrees of freedom have been accounted for by the formula given in the 'arrival pattern' sub-section. For a standard normal curve, the value of p is 2 as z value, estimated for each speed observation, considers mean ( $\mu$ ) and standard deviation ( $\sigma$ ) for defining the distribution. In case of a log-normal curve, the value of p is also 2, since a mean and standard deviation from natural logarithms of speed ( $\mu_{ln}$  and  $\sigma_{ln}$ ) are two parameters determined for defining the distribution. Thus, a generic equation for evaluating degrees of freedom for both distributions is given by: df = I - 3.

In the equation, '*I*' refers to the number of compared class intervals.

In all cases, the calculated chi-square value  $(\chi^2[\text{calc.}])$  is found to be less than critical chi-square value  $(\chi^2[\text{c}])$  obtained from the chi-square table corresponding to the computed degrees of freedom and having the significance level of 0.05. This indicates a good fit of the observed speed values as the proposed distribution. Thus, for all cases, data on speed was successfully modelled as log-normal for motorized 2-wheelers and normal distribution for the rest of vehicle categories. The hypothesis was ascertained by evaluating chi-square tests.

## 6. Fundamental Traffic Flow Relationships

Empirical relationships among three basic variables – speed, density and flow – can be explained on the basis of fundamental diagrams of traffic flow. It is a macroscopic theoretical model which is an eminent indicator of the operational capability of road facility. On the basis of the data collected from the field for this study, three primary relationships have been established and discussed in the following sub-sections.

## 6.1. Speed-Density

According to the theoretical diagrams of traffic flow, linear relationship exists between the density and speeds of vehicles travelling on the road (May 1990). Density is space measurement accounted for a particular instant of time. Therefore, in order to explain the same phenomenon for the observed traffic data, a total of 36 random control points were evaluated separately for each direction. Data on a two-hour period was extracted identifying time (in seconds) at which vehicles crossed observation points. Hence, the times of entry and exit ramps ranging between 0 and 7200 seconds were obtained. These time values were plotted in MINITAB by supplying specific bin values and the regions of varying flows were identified. Control points were selected within the identified classes and density was computed by counting the number of vehicles present on the road section at those instants of time. The average speed of vehicles constituting density was then calculated as space mean speed. On the basis of the obtained values, graphical plots between speed, as kilometres per hour and density, as vehicles per kilometre, were mapped and regressionfit analysis was performed to examine how well data

points approximated to a linear model. For vehicles travelling towards Jaipur and Sikar, speed–density linear plots were found to incur  $R^2$  values of 0.810 and 0.745 respectively. This indicates a decent fit of data points as a linear model. Both fitted line plots are illustrated in Figs 1 and 2 along with the equation for the regression line.

The obtained fitted linear plots were extrapolated in order to determine axis intercepts on x-axis and y-axis representing jam-density and free-flow speeds of vehicles respectively. For vehicles travelling towards Jaipur, jam density and free-flow speeds were estimated to be 109.75 vehicles/km and 68.88 km/hr respectively. In case of vehicles moving towards Sikar, jam density and free flow speeds were determined to be 106.88 vehicles/km and 69.02 km/hr respectively.

It may be noted that these parameters have been estimated utilizing an empirical approach, and this exercise was conducted to gain some basic understanding of such relationship.

Control points selected for both directions were united to obtain a combined plot of 72 data points. Regression-fit analysis was performed to examine the goodness-of-fit of the linear model. The  $R^2$  value of the combined plot was found to be 0.796 indicating a decent fit. The fitted line was extrapolated to obtain jam density and free-flow speed determined to be 109.07 vehicles/ km and 68.89 km/hr respectively. The regression-fit plot



Fig. 1. Speed vs. density plot (towards Jaipur)

![](_page_7_Figure_14.jpeg)

Fig. 2. Speed vs. density plot (towards Sikar)

![](_page_8_Figure_1.jpeg)

Fig. 3. Speed vs. density plot (combined)

for a combined data set is illustrated in Fig. 3 along with the equation for the regression line.

In all the plots, free-flow speeds are the space mean speed of vehicles. The established relationships reveal that the average speed of vehicles decreases with an increase in density since the movement of vehicles on the road gets impeded. According to the linear model, this trend continues until density reaches jam condition wherein vehicles come to a complete stop. The same phenomenon was explained for the traffic flow plying on the selected two-lane road stretch.

Data on traffic collected from the study site represents an uninterrupted vehicular flow. The flow of vehicles on the road did not reach the capacity level at any point of time during the study. Thus, only the uncongested regime was obtained for the relationship and region at and beyond capacity flow could not be mapped. However, some basic idea about traffic behaviour as well as linear relationship that exists between the speed and density of vehicles was acquired through the plots.

#### 6.2. Speed-Flow

According to the theoretical diagrams of traffic flow, a parabolic relationship exists between the flow and speeds of vehicles in a traffic flow (May 1990; Highway Capacity Manual 2000). The flow is point-based measurement accounted for a time period usually shorter than an hour. However, it is expressed as a measure of vehicles per hour. For data collected from the study on the two-lane section, traffic regimes at and beyond capacity flow could not be mapped as behaviour observed for an uninterrupted flow was insufficient for evaluating relationships during congested condition. In order to explicate the uncongested regime, in total, 36 control intervals of 1 minute length were evaluated separately for each direction. These intervals of varying flow-levels were identified after plotting in MINITAB. The flow of vehicles and the average speed of vehicles constituting the flow were computed for each time interval. The average speed represents the space mean speed of vehicles. Graphical plots between speed, as kilometres per hour, and the flow, as vehicles per hour, were mapped for each case as shown in Figs 4 and 5. Regression-fit

analysis was performed to analyze the approximation of data points as a quadratic model. For vehicles travelling towards Jaipur and Sikar, speed–flow quadratic model plots were found to incur  $R^2$  values of 0.630 and 0.631 respectively. These values indicate a decent fit of field observed data as a quadratic relationship model. Regression curve equations are also displayed in the figures.

A combined data set of 72 values, after uniting control points from each direction, was also created. The speed vs. flow graph for combined values is presented in Fig. 6 along with the equation for the regression curve.

![](_page_8_Figure_10.jpeg)

Fig. 4. Speed vs. flow plot (towards Jaipur)

![](_page_8_Figure_12.jpeg)

Fig. 5. Speed vs. flow plot (towards Sikar)

![](_page_8_Figure_14.jpeg)

Fig. 6. Speed vs. flow plot (combined)

Flow and speed values were plotted in MINITAB and regression-fit analysis was conducted to assess the goodness of the quadratic model. For a combined data plot,  $R^2$  value was observed to be 0.592 which indicated a decent fit of data points as the quadratic model.

The established relationships disclose that the average speed of vehicles decreases gradually with an increase in flow. This occurs due to the fact that along with a rise in the flow, the number of vehicles moving on road facility increases thus impeding the free movement of vehicles. The same phenomenon was explained for the traffic flow plying on the selected section of the twolane road. According to the theoretical model, this trend continues until capacity flow is reached wherein vehicles travel with critical speed. Beyond this point, the flow decreases gradually along with speed under congested conditions. Data on traffic collected from the study site represents an uninterrupted vehicular flow. The flow of vehicles on the road did not reach the capacity level at any point of time during the study. Thus, as mentioned earlier in this section, only the uncongested regime was obtained for the relationship and region at and beyond capacity flow could not be mapped. However, some basic idea about traffic behaviour as well as quadratic relationship that exists between the speed and flow of vehicles was acquired through the plots.

#### 6.3. Flow-Density

The relationship between flow and density follows a parabolic curve according to the theoretical models of traffic flow (May 1990; Highway Capacity Manual 2000). Associating these two variables for traffic data collected from the study section was a bit tricky, since the flow is measured over a time interval while density is accounted for an instant of time. The obtained data discloses that minimum time taken by a vehicle to cross the road section of 500 meters, considered for the study, was a bit more than 15 seconds. Hence, a time interval of 15 seconds was chosen for computing the flow of vehicles entering the road section. The end point of these 15 second intervals was selected as the instant of time for calculating density. By selecting a period of the remarked length of time, it was ensured that whatever vehicles constituted the flow could also be accounted for density measurement. The vehicles that may have been present on the road section before the start of the interval and remained on stretch for the entire 15 seconds were also included in density measurements. This can be justified by the fact that vehicles already moving in a traffic flow also have an influence on the inflow of vehicles.

For data collected from the two-lane study, traffic regimes at and beyond capacity flow could not be mapped as the behaviour observed for an uninterrupted flow was insufficient for evaluating relationships during the congested condition. In order to explicate the uncongested regime, a total of 36 control intervals of 15 seconds were considered and evaluated separately for each direction. These intervals of varying flow-levels were identified from the plots obtained through MINIT-AB. The flow and density of vehicles were computed for each time interval. The graphical plots between the flow, as vehicles per hour, and density, as vehicles per kilometre, were mapped in MINITAB and regression-fit analysis was performed to examine the approximation of data points as a quadratic model. For vehicles travelling towards Jaipur and Sikar, the plots of the flow-density quadratic model were found to incur  $R^2$  values of 0.788 and 0.845 respectively. Two graphs, along with the equation for the regression curve, are displayed in Figs 7 and 8.

A combined relationship was also established with respect to flow-levels related to both directions by considering a separate set of 50 class intervals of 15 seconds. For a particular time period of 15 seconds, the flow from both directions was aggregated and noted down. Similarly, taking the end point of these intervals as an instant of time, the total density of vehicles on the entire road section was computed. The relationship between flow and density for the combined data was plotted and regression fit analysis was conducted to assess the goodness-of-fit of data points as a quadratic model. The  $R^2$  value of the fitted line plot was determined to be 0.704, which indicates a decent fit of the observed data points for a quadratic plot. The graph along with the equation for the regression curve is shown in Fig. 9.

The established relationships indicate that the density of vehicles on the road increases gradually with an increase in the flow. This is attributed to the fact that along with a rise in the flow, the number of vehicles us-

![](_page_9_Figure_10.jpeg)

Fig. 7. Flow vs. density plot (towards Jaipur)

![](_page_9_Figure_12.jpeg)

Fig. 8. Flow vs. density plot (towards Sikar)

![](_page_10_Figure_1.jpeg)

Fig. 9. Flow vs. density plot (combined)

ing the road facility increases, thus subsequently raising density measure. The same phenomenon has been explained while discussing the traffic flow plying on the selected section of the two-lane road. According to the theoretical model, this trend continues until capacity flow is reached wherein vehicles reach critical density. Beyond this point, the flow decreases gradually while density keeps increasing as more vehicles enter the road section resulting in the congested flow. Traffic data collected from the study site represents an uninterrupted vehicular flow. The flow of vehicles on the road did not reach the capacity level at any point of time during the study. Thus, only the uncongested regime was obtained and relationship with the region at and beyond capacity flow could not be mapped. However, some basic idea about traffic behaviour as well as a quadratic relationship existing between the flow and density of vehicles was acquired through the plots.

#### 7. Summary and Conclusions

The study discussed in this paper was performed in order to understand primary traffic parameters and their features pertaining to vehicle behaviour on the two-lane road. Traffic data on a long road section of 500 meters was collected and extracted for a period of two hours. A suitable methodology was incorporated in the process that ensured the collection of a wide range of traffic plying on the study site without any loss of information and the development of accurate readings for evaluating traffic variables. The obtained traffic data was utilized to compute arrival, headway, speed, flow and density parameters analyzed using statistical methods for studying traffic characteristics and fundamental relationships of the vehicle flow moving in both directions on the road section. The outcomes of the procedures performed in this study are listed below:

- Arrival patterns of vehicles were fitted as the Poisson distribution and chi-square test analysis validated the likelihood of field values conforming to the proposed distribution.
- Headway characteristics were modelled as negative exponential distribution and the choice was confirmed by conducting the analysis of the chisquare test.

- 3) Speed distributions of different vehicle types observed on the road section were fitted for a normal curve, except the speed distributions for motorized 2-wheeler that were modelled as lognormal distribution. The performed chi-square test analysis validated the likelihood of the observed speed values approximating to the proposed distributions.
- 4) Speed vs. density plots were established for traffic data that revealed a linear relationship between the two flow parameters. Regressionfit analysis was carried out to verify the linear model. Jam density and free-flow speeds were estimated through the plots.
- 5) Speed vs. flow plots were developed for the observed values and a quadratic relationship between the two parameters was verified through regression-fit analysis.
- 6) Flow vs. density plots were established for the observed data which revealed a quadratic relationship between the two traffic flow parameters. The model was verified conducting regression-fit analysis.

Thus, for an uninterrupted heterogeneous traffic flow moving on the two-lane road section selected for the survey, parameter distributions and fundamental relationships among the key variables were examined and ascertained through this exploratory study.

In the case of density measurements, it should be noted that the executed methodology is sufficient only for gaining a basic understanding of parameters. In this study, the density of vehicles formulated for the road section was adequate for accomplishing the basic task of presenting the fundamental relationships of the traffic flow. Therefore, jam densities and free-flow speeds evaluated from speed-density fitted line plots are only estimated empirical values derived through a methodology that enables understanding of primary flow attributes.

One principal limitation of the study pertains to the vehicle composition quality of the traffic flow. The conducted two-hour survey presented only a snapshot of traffic flow properties of the composition observed during the stipulated period. However, vehicular traffic flow is a dynamic attribute and keeps changing over time. In case of a heterogeneous mix, where a variety of vehicle-types with distinguished physical and operational characteristics comprise the traffic flow, the composition becomes a major determinant as it influences the parametric properties of traffic flow. Hence, observations made between two different periods of the study may not develop exactly the same results. Also, data points evaluated from the study could only present flow regimes for an uninterrupted traffic flow. The region pertaining to the over-saturated flow could not be obtained as the conditions persisting on the study section were not sufficient to plot the related relationship. However, traffic studies, such as the one explained in this paper, can provide a basic understanding of the nature and behaviour of vehicle flows moving on the road facility.

Real-time headways and speed measurements computed for a traffic flow can also be regenerated on computer via simulation software that requires three main inputs for recreating the observed traffic flow, including vehicle generation, vehicle placement and vehicle movement. Headways and arrivals account for the first two input variables, whereas speed values can determine the movement of vehicles. Simulated traffic can then be studied by dealing with various parameters and observing effects on the nature of the flow. In this way, consequences of changing the behaviour of the traffic flow can be evaluated and certain key factors with respect to design considerations can be determined. Using data information taken from this study, traffic simulation exercises can be performed as further research on widening the scope of understanding the behaviour of the vehicular flow for a heterogeneous mix.

Traffic is a stochastic quantity and it varies from place to place depending on local factors and driver's behaviour. Therefore, traffic studies are important for acquiring knowledge about the nature of vehicular flows. The information acquired assists in building better, safer and efficient road facilities. Roads are a part of the infrastructure and traffic that flows through them determines the aesthetic as well as economic value of the country. Hence, traffic engineering plays a vital part in ensuring that the vehicles travelling on the road facilities can reach their intended destinations with comfort and ease and without compromising safety issues.

#### References

- Akcelik, R.; Roper, R.; Besley, M. 1999. Fundamental Relationships for Freeway Traffic Flows. Vermont South, Vic.: Australian Road Research Board. Research Report ARR 341. 104 p.
- Al-Ghamdi, A. S. 2001. Analysis of time headways on urban roads: case study from Riyadh, *Journal of Transportation Engineering–ASCE* 127(4): 289–294. doi:10.1061/(ASCE)0733-947X(2001)127:4(289)
- Arasan, V. T.; Koshy, R. Z. 2003. Headway distribution of heterogeneous traffic on urban arterials, *Journal of the Institution of Engineers. India. Civil Engineering Division* 84: 210–215.
- Arasan, V. T.; Vedagiri, P.; Manu, S. 2009. Development of speed-flow relationship for heterogeneous traffic using computer simulation, *Journal of the Institution of Engineers*. *India. Civil Engineering Division* 89: 3–7.
- Arkatkar, S. S.; Arasan, V. T. 2010. Effect of gradient and its length on performance of vehicles under heterogeneous traffic conditions, *Journal of Transportation Engineering*– *ASCE* 136(12): 1120–1136.
  - doi:10.1061/(ASCE)TE.1943-5436.0000177
- Haefner, L. E.; Li, M.-S.; Porrello, L. A.; Blanco, M. V.; Van Pelt, D. B. 1998. Examination of fundamental traffic characteristics and implications to ITS, in *Transportation Conference Proceedings*, 12–17.
- Hall, F. L.; Montgomery, F. O. 1993. The investigation of an alternative interpretation of the speed-flow relationship for U. K. motorways, *Traffic Engineering Control* 34(9): 420–425.

- Hossain, D. M.; Iqbal, G. A. 1999. Vehicular headway distribution and free speed characteristics on two-lane two-way highways of Bangladesh, *Journal of the Institution of Engineers. India. Civil Engineering Division* 80: 77–80.
- Kadiyali, L. R.; Vishwanathan, E.; Gupta, R. K. 1981. Free speeds of vehicles on Indian roads, *Journal of the Indian Roads Congress* 42(3): 387–458.
- Katti, B. K.; Pathak, R. H. 1985. A study on headway distribution models for the urban road sections under mixed traffic conditions, *Highway Research Bulletin – Indian Roads Congress* 26: 1–30.
- Kumar, V. M.; Rao, S. K. 1998. Headway and speed studies on two lane highways, *Indian Highways* 26(5): 23–36.
- May, A. D. 1990. Traffic Flow Fundamentals. Prentice Hall. 464 p.
- Minh, C. C.; Sano, K.; Matsumoto, S. 2005. The speed, flow and headway analyses of motorcycle traffic, *Journal of the Eastern Asia Society for Transportation Studies* 6: 1496–1508.
- Mukherjee, S. K.; Rao, S. K.; Raichowdhury, M. L. 1988. Fitting a statistical distribution for headways of approach roads at two street intersections in Calcutta, *Journal of the Institution of Engineers. India. Civil Engineering Division* 69: 43–48.
- Polus, A.; Cohen, M. 2009. Theoretical and empirical relationships for the quality of flow and for a new level of service on two-lane highways, *Journal of Transportation Engineering–ASCE* 135(6): 380–385. doi:10.1061/(ASCE)0733-947X(2009)135:6(380)
- Roshandeh, A. M.; Nesheli, M. M.; Puan, O. C. 2009. Evaluation of traffic characteristics: A case study, *International Journal of Recent Trends in Engineering* 1(6): 62–68.
- Sahoo, P. K.; Rao, S. K.; Kumar, V. M. 1996. A study of traffic flow characteristics on two stretches of national highway No. 5, *Indian Highways* 24(4): 11–17.
- *Highway Capacity Manual.* 2000. TRB Special Report 209. 3rd Edition. Transportation Research Board. 634 p.
- Tseng, P.-Y.; Lin, F.-B.; Shieh, S.-L. 2005. Estimation of freeflow speeds for multilane rural and suburban highways, *Journal of the Eastern Asia Society for Transportation Studies* 6: 1484–1495.
- Van As, S. C.; Van Niekerk, A. 2004. The operational analysis of two-lane rural highways, in *Proceedings of the 23rd Annual Southern African Transport Conference*. 12–15 July, 2004. Pretoria, South Africa, 622–633. Available from Internet: <a href="https://repository.up.ac.za/upspace/bitstream/2263/5708/1/Vanas\_Operational%282004%29">https://repository.up.ac.za/upspace/bitstream/2263/5708/1/Vanas\_Operational%282004%29</a>. pdf>.