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VEHICLE CLASS WISE SPEED-VOLUME MODELS FOR HETEROGENEOUS TRAFFIC

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Abstract. Link performance functions commonly used for traffic assignment are often based on Volume Delay Functions (VDF) developed for homogeneous traffic. However, VDFs relating stream speed to the volume of traffic based on homogeneous lane-based traffic are not adequate for traffic assignment in developing countries due to the heterogeneous nature of traffic that is characterized by a mix of a wide range of vehicle classes with significant differences in static and dynamic characteristics and an imperfect lane discipline. Unfortunately, the use of VDFs similar to those for homogeneous traffic flow situations imposes strong restrictions considering two respects: 1) travel times at path and link levels can be obtained for an aggregated stream but not for individual vehicle types; 2) the effect of varying composition and asymmetric interactions is captured only to a limited extent by converting all vehicles into equivalent Passenger Car Unit (PCU). Hence, this paper proposes the development of VDFs specific to different classes of heterogeneous traffic, as it is more realistic in traffic assignment than the use of the same VDF for all classes of vehicles in a link. This study is aimed at developing models to determine the speed of each vehicle class as a function of flow and composition for six lane roads with heterogeneous traffic based on data obtained from Chennai city, India. Heterogeneity in this study mainly refers to differences in vehicle types (two-wheeler, car, bus, etc.) participating in mixed traffic. To develop multiple user class VDFs, the speed and flow of each vehicle class for a wide range of traffic flow conditions need to be recorded. As this is not possible using field measurements, an established micro-simulation model (HETEROSIM) is used for determining speeds for each vehicle type by systematically varying the volume and composition levels over a range of values that represent relevant and practical traffic conditions observed in six lane divided roads in Chennai city. The proposed delay functions are different from standard single user class VDFs in three key respects: first, they enable more realistic behaviour by modelling differences in class wise speeds at a given volume and composition level; second, they allow for capturing asymmetric interactions of different vehicle types on an average speed of a given vehicle class. Finally, speed-flow relationships for each class are also allowed to vary across volume levels which enable the representation of differential interactions at different levels of congestion in mixed traffic. The need for homogenizing the volumes in terms of a single class is obviated. The models significantly outperformed single class VDFs in both calibration and validation datasets. Further, the proposed models are used for analyzing heterogeneous traffic characteristics. Empirical evidence of asymmetric interactions and the impact of composition on class-wise performance are also found and quantified. Finally, two applications of the proposed models are demonstrated for the level of service analysis of different classes and impact analysis of excluding some classes. The proposed models may have applications such as determining class wise road user costs and performance measures (e.g. emissions) that depend on class-specific speeds.

Keywords: traffic flow, vehicle, speed, model, interaction, analysis.

1. Introduction

Speed flow relationships are widely used for predicting link travel times, traffic assignment and evaluating traffic improvement measures. The operating speeds of vehicles affect travel time, road user cost and the Performance/ Level of Service (LOS) measures. Therefore, understanding how traffic speeds vary due to influencing factors is critical. In this context, this paper addresses the problem of developing speed-flow relationship for mixed traffic urban arterials in Indian cities using data obtained from Chennai city.

The volume delay function is a central part of static traffic assignment models and describes how travel time or speed on road link changes along with traffic demand (Janson Olstam *et al.* 2008). Various volume delay functions (VDFs) have been developed mainly for homogeneous lane-based traffic (Spiess 1990; Hoban 1987; Akçelik 1991). These VDFs relate stream speed with stream volume and do not account for speeds of various classes present in the stream. Thus, they may not be directly suitable for many applications in heterogeneous traffic. Mixed traffic in such countries like India does not follow a perfect lane discipline. Thus, the concepts of vehicle density, lane occupancy and queuing from homogeneous lane-based traffic cannot be applied for mixed traffic.

Several other sources of complexity also arise in modelling mixed traffic in developing countries. Heterogeneous traffic is characterized by frequent changes in lanes by vehicles due to the interaction between fast-moving and slow-moving vehicles. Also, the static and dynamic characteristics of vehicles differ vastly (Arasan, Koshy 2005) and all types of vehicles occupy the entire road width with very little lateral separation. Due to these features and asymmetric traffic interactions, the aggregation of different class flows into a single volume measure (passenger car equivalent – PCE) may not be adequate (Rudjito 2006). A better understanding of the nature and extent of such variation across vehicle classes is needed to develop suitable traffic models.

Many researchers have developed simulation-based models for mixed traffic providing valuable insights into stream speed versus flow relationships (Chandra, Sikdar 2000; Arasan, Krishnamurthy 2007). However, vehicle class speeds may deviate considerably from stream speed depending on volume, composition, vehicle type, etc. Unfortunately, literature on theoretical or empirical models of class-specific speeds for different vehicle types and their heterogeneous interactions is relatively sparse.

A standard equilibrium assignment algorithm using volume delay functions involving a single user class assume that all vehicles on a facility have the same speed. Multi-class VDFs are more realistic since asymmetric interactions among vehicle classes can be explicitly captured. Understanding class-wise speed-flow relationships is also critical for applications like the evaluation of class-based traffic management measures such as heavy vehicle curfews at certain times or locations. These models may be used for determining speeds and road user costs, fuel consumption and emissions that depend on vehicle classes and corresponding speeds.

Due to these motivating considerations, this study is aimed at developing vehicle class-specific speed-flow relationships for heterogeneous traffic. The specific objectives include:

- proposing and developing new multiple class multiple regime speed – flow models for mixed traffic;
- analyzing heterogeneous traffic characteristics including asymmetry and interaction across classes, the effect of composition and the role of the volume;
- applying the proposed models to:
  - determine the LOS of different vehicle classes in mixed traffic;
  - analyze the impact of vehicle-class based traffic measures such as the exclusion of certain vehicle types.

The developed estimation method has been exemplified on one road type and road environment. Specifically, this paper focuses on heterogeneous traffic flow at mid-block sections of divided six lane urban arterial roads in Chennai city, India. In this case, heterogeneity refers to variation in static and dynamic characteristics across the classes of vehicles. This paper is distinct from the existing studies on mixed traffic flow in the following respects. A set of new multiple-class multiple-regime speed-flow models for mixed traffic are proposed and estimated. The proposed models are found to significantly outperform single class and single regime models. The application of the model demonstrates that experienced LOS varies across vehicle types even for the same volume level and affects larger vehicles more adversely. Further, the exclusion of certain vehicle types such as heavy vehicles or auto-rickshaws during heavy congestion shows promise for improving system performance. Furthermore, the findings suggest that the average stream speed better represents the effect of dominant vehicle classes (two-wheelers) in mixed traffic than car speed.

The rest of this paper is organized as follows. A review of related work focusing more on mixed traffic is presented in Section 2. The used simulation model and findings of exploratory analysis are discussed in Section 3. Section 4 discusses and validates the proposed multiple class speed-flow models. A comparison of other single regime and single class models is also performed for benchmarking. The salient empirical findings of the study are discussed in Section 5 and are followed by concluding remarks in the final section.

Scope of Work. This study focuses on class-wise volume delay functions for heterogeneous traffic in Indian cities. Specifically, multiple classes considered in the paper refer to different types of vehicles (two-wheeler, car, etc.). The classes of vehicles identified for this study are heavy vehicles, cars, auto rickshaws and motorized two wheelers. In particular, the models are developed using data from many mid-block sections on six lane roads of Chennai city, India. The proposed models are used for developing class-wise volume delays as a function of the volume and composition for particular road width. A primary application of the proposed models is in the context of static traffic assignment in the lieu of a traditional single class of VDFs. Since performance at mid-blocks and intersections vary significantly, this study focuses only on midblock sections. The effect of intersection delays and downstream queues are not included in the scope of this study and are being investigated as a part of another study to be reported in the near future. The effects of grade, side friction, etc. though not considered in this study can be incorporated by applying the same methodology with suitable data. The models may be used as the basis for link performance functions for a static assignment of multiclass traffic in developing countries with similar mixed traffic conditions.

2. Literature Review

Significant research efforts have been made to investigate speed-flow relationships. Volume delay functions may be classified as linear, logarithmic, exponential power and polynomial forms. A review of the existing link travel time models is presented by Mun (2009).

The following review is restricted in scope to four threads relevant to this study, namely, single regime models, multi-regime models, simulation-based mixed traffic models and asymmetry among classes in mixed traffic. Also, a few link performance models used in assignment practice are analyzed.

2.1. Single Regime Model of Mixed Traffic

A series of single regime linear speed-flow models under mixed traffic have been proposed by the Central Road Research Institute, India (CRRI 2001). These relate the speed of a vehicle in kilometres per hour (V, kmph) and the flow in passenger car units (Q, pcu) per hour in both directions in the form of :

$$V = b_0 - b_1 \cdot Q,$$

where: b_0, b_1 – model parameters.

The Indonesian Highway Capacity Manual (1993) also proposes a single regime non-linear model used for estimating the capacity of undivided rural roads. The basic equation proposed to describe traffic operations using data from cities all over Indonesia is:

$$V = 0.5 \cdot V_0 \cdot \left(1 + (1 - Q / C)^{0.5}\right),$$

where: V – stream speed (kmph) at flow Q; V_0 – free-flow speed (kmph); Q – actual flow (pcuph); C – capacity (pcuph).

Speed-flow relation using a single-zone linear model was also estimated in China (Pan, Kerali 1999) using an equivalent passenger car volume. In contrast, Rotwannasin and Choocharukul (2005) developed single regime linear and non-linear speed density relationships for multilane highways in Bangkok, Thailand. Olszewski *et al.* (1995) developed negative exponential models to relate journey speed versus density for six different classes of vehicles in Singapore.

In all above mentioned studies, traffic flow is homogenized by using equivalency factors into a reference vehicle (usually car except for Minh *et al.* 2005). The output of these models is stream speed; however, vehicle class speeds are not determined. Schofield (1986) pointed out there was a wide band of speeds associated with any given level of flow because of variable traffic composition, weather, light conditions and traffic congestion. These observations suggest the need for class-specific models for mixed traffic.

2.2. Multiple Regime Models

Bång and Heshen (2000) developed two regime linear stream speed – flow relationship with a break point at V/C ratio of 0.85 for urban and inter urban sites in Henan and Hebei provinces in China for two lane undivided roads. Hall and Montgomery (1993) proposed two segment linear speed-flow models for U.K. motorways. The percentage of heavy vehicles and the difference between free-flow speeds of light and heavy vehicles influenced the slope of the first segment. Based on German data, Brilon (1994) presented three-segment linear models where slopes were found to increase with traffic volume. Suh *et al.* (1990) suggested a link capacity function for Korea, calibrated based on a BPR type formula. They list some two zone linear travel time models including those with a different slope on either side of practical capacity: Congested travel time T is given by:

$$T = T_a + a \cdot \left(Q' - C'_p\right) \text{ for } Q' < C'_p;$$

$$T = T_a + b \cdot \left(Q' - C'_p\right) \text{ for } Q' > C'_p,$$

where: T_a – free flow travel time; Q' – flow per lane (pcuph); $C_p{}'$ – capacity (pcuph) per lane; a – model parameter; b – model parameter.

All studies indicate that the volume of the flow is measured using PCE values and hence the interaction among the classes of vehicles is not adequately modelled.

2.3. Mixed Traffic Models Based on Simulation

A distinguishing factor of mixed traffic in developing countries is the presence of motorized two wheelers. The predominance of two-wheelers and relatively unique traffic manoeuvres such as filtering, moving abreast of other vehicles in the same lane, oblique following and swerving are difficult to represent adequately with the existing simulation tools (Lee *et al.* 2009). They made a computer simulation system incorporating gap acceptance behaviour and path choice behaviour of motorcycles in a traffic stream.

A review of earlier traffic flow simulation models are presented in Khan and Maini (1999). Mallikarjuna and Rao (2006) produced a simulation model based on the concept of cellular automata to estimate the PCE values of vehicles. More recently, Arasan *et al.* (2009) have developed a set of stream speed-flow curves for one-way traffic movement on the roads of the widths of 7.5 m, 11.0 m and 14.5 m based on simulation. The studies noted above were intended mainly for estimating PCE values, capacity analysis or the development of stream speed functions. Therefore, class-wise speed flow relationships are not adequately investigated.

Fewer studies focus on class-wise speeds. Ramanayya (1988) designed speed-flow models by running a simulation model, named MORTAB, for different volumes and percentage compositions of vehicle types. These models yield class-wise speeds of a car, bus, auto rickshaw, truck, motorcycle as a function of average flow (vph) and the percentage of slow moving vehicles (bicycles, cycle rickshaws, and bullock carts).

2.4. Asymmetry in Multi-Class Traffic

As noted earlier, it is common practice to convert traffic flow in terms of passenger car equivalents. Unfortunately, however, a PCE is not a constant but is expected to vary with flow and composition. Rudjito (2006) noted that the use of a constant PCE is not suitable; the aggregation of faster and slower vehicles into a single class may lead to a loss of information about inter-class interactions.

Toint and Wynter (1996) argued that certain forms of class-wise speed flow relationships are behaviourally incoherent as they result in constant speed ratios across different classes at varying volumes. They also caution against mathematical incoherence due to the use of the PCU based assignment in mixed traffic. Such an assignment results in a single output value of link flow in the PCU but is unable to identify underlying class wise vehicular volumes uniquely.

Further, a car is not the predominant component of traffic in urban areas in many developing countries. Matsuhashi *et al.* (2005) found that the proportion of motorcycles in Hanoi and Ho Chi Minh City was around 90%. In Malaysia, approximately 41% of registered vehicles annually are motorcycles (Vien *et al.* 2008). Stream speed is the volume weighted average of class speeds, and is therefore more likely to be influenced by dominant vehicle types in the stream. Thus, some researchers have argued that making the dominant class (two-wheeler) as a reference unit is more suitable than the use of car equivalents. Along this line, Minh *et al.* (2005) used the MCU (Motor Cycle Unit) instead of the PCU as a unit to represent mixed traffic while developing speed flow relationships in Hanoi.

The literature presented above highlights the need to capture asymmetric interactions present in mixed traffic. Many studies have investigated stream speed versus flow relationship; fewer studies examine class-wise speed flow models. As a result, the nature of class-wise speed-flow relationships (linear or non-linear) and differences across classes are not well understood. Further, the limited ability of PCU-based conversion to capture asymmetric interactions has also been noted in some investigations. To overcome these limitations, it is desirable to directly use class-wise speed as dependent variables and class-wise volumes as independent variables.

2.5. Multi-Class Link Performance Models

Link performance functions are fundamental constructs in traffic assignment. Most link performance functions used in practice and reported in various literature, including the widely popular Bureau of Public Roads (BPR) function, are polynomials in the aggregate volume, the degree and coefficients of which are specified based on a statistical analysis of real data (Nie, Zhang 2005). Class volumes are transformed into the PCE, which on assignment results in unique link volumes in the PCE but are not unique in class volumes.

Si *et al.* (2008) calibrated link performance functions for the roads of different breadths and traffic conditions. Volume v_a of each class was converted into equivalent units of bikes b. Link travel time t_a as a function of free flow time t_0 for a divided two-breadth road is given as:

$$t_a = t_0 \cdot \left(1+\alpha_1\left(\frac{b_{car} v_a^{car}}{C_a}\right)^{\beta_1}\right) \times \left(1+\alpha_2\left(\frac{b_{bus} v_a^{bus}}{C_a}\right)^{\beta_2}\right) \cdot \left(1+\alpha_3\left(\frac{v_a^{bike}}{C_a}\right)^{\beta_3}\right),$$

where: C_a is practical capacity on link a; α and β are parameters.

Wynter (2001) used linear cost functions of the following forms to illustrate the presence of multiple equilibriums in a simple network:

$$t_{1i}(x) = 1.5 \cdot x_{1i} + 5 \cdot x_{2i} + 30;$$
$$t_{2i}(x) = 1.3 \cdot x_{1i} + 2.6 \cdot x_{2i} + 28,$$

where: x_{1i} and x_{2i} are the volumes of class 1 and class 2 respectively; t_{1i} and t_{2i} are the costs in link i.

Simple regression models were developed by Kov and Yai (2010) for mean stream speed while studying the effects of motorcycles and light vehicles separately in different ranges of volume. A sample model, considering the impact of motorcycle percentage (P_{MC}), has the form:

$$Stream\ Speed = 29.638 - 6.173 \cdot P_{MC}.$$

Wu *et al.* (2006) used volume/delay function aggregating volumes using PCE values expressed as a non-linear function mix of trucks and cars and other factors like the slope of the links; However, link costs $C_a(v_a)$ are computed based on the total volume using the function of the BPR type:

$$C_a(v_a) = \beta_a \cdot \left(1 + 0.15 \cdot \left(\frac{v_a}{k_a}\right)^4\right),$$

where: k_a – the capacity of link a; β_a – a parameter; v_a – flow in the PCE obtained from class volumes where the PCE value of a class is a function of traffic and road characteristics of a link.

Transport modelling packages like Cube voyager, Emme/2 are able to model separate speed/flow relationships for light and heavy vehicles of a multi-regime linear form. SATURN represents a two-regime linear form of speed/flow relationships by means of a power function. In all cases, the flow used to calculate speed is the total flow on the link summed up over all user classes.

Difference in class speeds is modelled by:

- adding fixed extra time per kilometre for HV relative to light vehicles over all flow conditions;
- specifying separate speed/flow curves for heavy and light vehicles at lower flow with speeds converging at capacity.

In practice, speed differential is not constant over the volume and representation of speed in terms of the total volume overlooking the effect of composition that is significant in mixed traffic. Multi regime MUC models are needed to address these issues.

3. Simulation Model and Exploratory Analysis

Due to the inadequacy of available speed-flow models, this study aims to develop class-wise speed-flow relationships for heterogeneous traffic conditions. The data required for such models include speed and volume measures for each vehicle class observed over a wide range of the levels of the total volume and composition. Measuring speeds and volumes under a large number of compositions and volumes is highly tedious and time consuming. For analysis over a wide range of practical roadway and traffic conditions, the use of simulation models is necessary for adequate experimental control over the factors of interest. Further, a lack of an adequate and automated infrastructure of traffic data collection in many Indian cities makes it practically infeasible to obtain such extensive and high resolution traffic data from the field. To circumvent this problem, micro simulation models (e.g. MIXSIM, CARSIM, INSWERTS) calibrated with field data have been extensively used to systematically vary the factors of interest and obtain the desired performance measures. A similar approach is adopted in this study. A well established micro-simulation model (HETEROSIM) for modelling non-lane based heterogeneous traffic flow is used for this purpose (Arasan, Koshy 2005). The micro-simulation model was calibrated to ensure that the simulation model replicated the observed field traffic conditions for the mid-block sections of six lane divided urban roads in Chennai.

3.1. Simulation Model

Simulation model HETEROSIM was developed based on extensive field traffic data including free flow speeds of each class, headways, lateral and longitudinal clearances, etc., collected on the roads of Chennai city. The inputs required for the traffic simulation model include road geometrics (road width, shoulder), traffic characteristics (free flow speeds, headways, lateral and longitudinal clearance) and static (dimension) and dynamic characteristics (acceleration and deceleration) of vehicles.

The component of the simulation model includes vehicle generation, vehicle movement and vehicle placement modules. Vehicles can be generated under varying volumes and composition levels from different headway distributions, including negative exponential, shifted exponential and Erlang distribution. Vehicle movement and placement logic reflect an imperfect lane discipline prevailing in mixed traffic. Consequently, the entire road width is treated as a single unit. The generated vehicle is placed at the beginning of the test stretch with the speed that is a function of a free flow speed of the class, the acceleration/deceleration characteristics of the vehicles and the availability of front clearance. Vehicle positions are then updated at the intervals of 0.5 second. The model outputs the average speed of each vehicle over the length of one kilometre after a warm up zone of the length of 200 m. More details about the modelling framework and validation of the HETEROSIM model are presented in Arasan and Koshy (2005).

This model was calibrated using the observed values of traffic flow characteristics and roadway conditions on a six-lane road in Chennai city, India. The model was validated by simulating field observed traffic flow and composition and comparing the characteristics of simulated traffic with those of the observed traffic flow. For the purpose of simulation, a 12.0 m wide and 1400 m long road stretch was considered. The middle portion of 1000 m was the observation stretch. The initial 200 m length at the entry point was used as a warm up zone and the 200 m length at the exit point was also excluded from analysis to ensure a steady flow of traffic in the simulation stretch. Simulation was run using three random number seeds for generating headways and average speeds and the volume of each class of vehicles were the final outputs of the model. Percentage errors in the average seeds simulated and observed in the field make 5.4, 6.4, –3.03 and –6.76% for a bus, car, motorized three wheeler and motorized two wheeler respectively. Thus, for macro-level practical applications of average speeds, these values are within acceptable limits.

The traffic simulation model was run for varying compositions of traffic at different volume levels and corresponding speed for each class of the vehicle was obtained. Realistic composition levels were identified based on field data collected from various roads in Chennai (Arasan, Koshy 2005; Arasan, Vedagiri 2008; Arasan, Krishnamurthy 2007). Due to inherent similarities of their speed characteristics, trucks, buses and commercial vehicles were classified as 'heavy vehicles'. The prominent types of vehicles and their composition ranges from field data were: heavy vehicles (1÷5%), light commercial vehicles (1÷4%), motorized three-wheelers (5÷15%), cars (20÷30%), motorized two-wheelers (40÷60%) and bicycles (0÷10%). The percentage of non-motorized three wheelers was very small in the study corridors and hence is not considered further in this paper. Thus, the vehicles fall into four classes: heavy vehicle (HV), car (C), motorized three wheeler (Auto) and motorized two wheeler (2W).

3.2. Development of Speed-Volume Relationship

The validated simulation model was used for simulating traffic flow considering twenty five sets of compositions drawn uniformly within the representative range mentioned in Section 3.1. The compositions were chosen such that class-wise compositions were summed up to 100%. For each combination, the total volume of traffic was increased in the steps of 500 vph from low volume (500 vph) until capacity. For each volume, the simulation model was run for each of the twenty five sets of compositions separately. Thus, for a particular volume, there are 25 observations of speed corresponding to 25 different compositions. At every volume level, three speed values were used for different simulation runs and the average value of the speed of each class of vehicles over the three runs is used as a dependent variable. Independent variables correspond to the volume classified by the vehicle type.

3.3. Exploratory Analysis of Class Wise Speeds and Volume

An illustrative plot of variation in speed and the volume for cars are given in Fig. 1. As expected, the speeds of different classes decrease with an increase in the volume. When the volume is low, the scatter of class-wise speed over different compositions is small. Thus, composition does not have a strong influence on a low volume. However, as the volume increases, the scatter of speeds increases indicating a greater role of composition and interactions.

Speed volume curves corresponding to a representative base composition (HV – 6%, Car – 25%, Auto – 11%, 2W – 58%) for all classes of vehicles are shown in Fig. 2. It is clear that the average speeds vary across classes. Note that cars are the fastest component at low and moderate volumes followed by two-wheelers. Autos are the slowest due to their smaller free flow speed and heavy vehicles are slightly better at low volumes. These trends reverse above medium volume levels. Two-wheelers are faster than cars, and autos are faster than heavy-vehicles. In relation to stream speed, at low volumes, cars are the only class which is faster than stream speed (two-wheelers are nearly equal to stream speed). At high volumes, only two-wheelers are faster than stream speed. Thus, congestion affects cars and heavy vehicles more adversely than two-wheelers

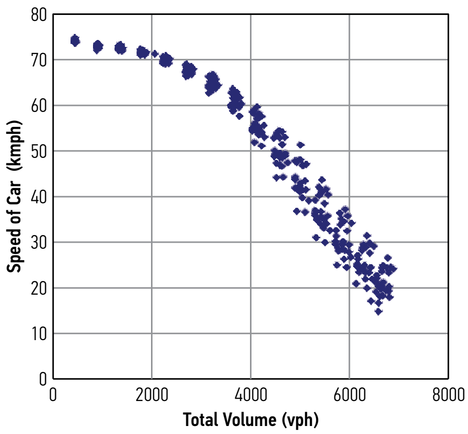

Fig. 1. Car speed (kmph) vs total volume (vph)

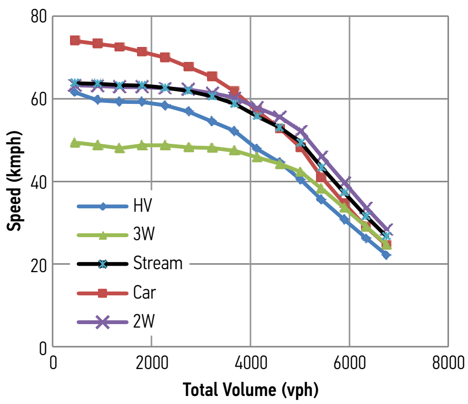

Fig. 2. Plots of class speeds vs volume

and autos respectively. This trend is also evident from the varying slopes across different vehicle classes. At low and moderate volumes, the slope of cars is the largest followed by heavy vehicles, whereas, two-wheelers and autos slopes are nearly flat. At high volumes, the slopes of all classes increase; cars and heavy-vehicles more so than two-wheelers and autos. Therefore, separate speed-flow relationships for different vehicle classes are warranted.

Further, the slopes of speed flow curves vary depending on volume levels. In particular, the slopes at a low volume are small and progressively increase as the volume increases. These findings suggest the need to delineate different regimes of speed-flow in mixed traffic. Thus, there is a need for multi-class, multi-regime volume delay functions for mixed traffic.

The plots also show that the difference between vehicle class speed and stream speed varies across classes and volume levels. At low volumes, car speed is the largest relative to stream speed. However, as the volume increases, this speed differential decreases and becomes negative at high congestion. On the other hand, the autos that are much slower at low volumes than stream speed tend towards stream speed at high congestion. Two-wheelers, in contrast, nearly match stream speed at a low volume, but tend to slightly exceed it under very congested conditions. Thus, no particular class speed represents other classes or stream speed over the entire range of volumes. Car speed declines more sharply than other classes with an increase in the volume. Consequently, the deviations of vehicle class speeds from car speeds are relatively larger. Furthermore, car speed does not adequately represent stream speed in mixed traffic due to a smaller composition of cars. For these reasons, stream speed forms a more suitable basis for measuring mixed traffic performance than average car speeds.

4. Multi-Class Multi-Regime Speed Flow Models

The proposed models are aimed at estimating the speeds of different classes of vehicles in mixed traffic, given the volume and composition of traffic. Based on the results of exploratory analysis, different equations are considered for low, medium and high volume levels. It is possible that cut-offs may vary in view of different vehicle types. To allow for variable cut-offs, various trial values of threshold for low, medium and high volume were considered for each vehicle class and estimated models. The set of thresholds that provided the best fit with data in terms of the RMSE of class-wise speeds was selected for delineating low, moderate and high volumes. Accordingly, medium volume range was found to be 4000÷5600 vph for all classes except heavy vehicles. The range of heavy vehicles was 4000÷6400 vph. This implies that heavy vehicles experience a significant slope change (increase) at a higher volume than other classes.

At low volumes, composition did not affect class speeds significantly (noted in Section 3.3, Fig. 1). Hence, traffic volume (at an aggregate level) in vehicle per hour is used as an independent variable in this regime. In contrast, class-wise flows are used as independent variables at moderate and high volume levels. Therefore, separate regression models are used for estimating class wise speed and stream speed as a function of class-wise vehicular volumes at moderate and high volume levels.

Different functional forms were tested for regression models and linear regression provided reasonable goodness-of-fit. Improvements in other non-linear forms were practically small. Hence, due to adequate fit for data, simplicity and more intuitive interpretation, linear regression analysis was adopted for the development of models in all cases (all volumes, classes and compositions).

For low volumes, speed flow relationships are given as:

$$V_i = a_i + b_i \cdot Volume + \varepsilon_i,$$

where: V_i – the average speed of the vehicles of class i (hv, car, auto, two-wheeler respectively); Volume – the total volume in vph; ε_i – the error term.

For medium and high volumes, speed flow relationships are given by the equations of the following form:

$$V_i = c_i + \beta_{1i} \cdot HV + \beta_{2i} \cdot Car + \beta_{3i} \cdot Auto + \beta_{4i} \cdot TW + \varepsilon_i,$$

where: V_i – the average speed of the vehicles of class i; HV – the volume of heavy vehicles in vph; Car – the volume of cars in vph; $Auto$ – the volume of auto-rickshaws in vph; TW – the volume of two-wheelers in vph; β_{1i}, β_{2i}, β_{3i}, β_{4i} – slope coefficients for a heavy vehicle, car, auto and two-wheeler volumes on the class speed of vehicle class i.

The results of multiple linear regression models relating the speed of different vehicle classes to volume and composition and corresponding to the goodness of fit are shown in Table 1 indicating that the value of determination coefficient (R^2) is high and the models are statistically significant based on the F-test. The model parameters were also found to be statistically significant using the t-test at a usual significance level of 5%. With increasing the volumes of any class, the speed of all classes reduce as expected. However, the magnitude of reduction depends on composition (specifically a class contributing to an increase in volume) and the current volume level (low, medium or high). A detailed discussion of class-wise speed flow relationships and associated insights are given in Section 5.

4.1. Development of Stream Speed Models

Conventionally, the speed of traffic is estimated by stream speed or car speed. For the reasons noted earlier (Section 3.3), stream speed is used as a benchmark to evaluate the performance of various models under mixed traffic (Table 2).

Furthermore, other existing models do not capture class wise speeds, and hence stream speeds form the common basis for comparison. The models compared include:

- a single regime, single class linear model of speed versus total volume;
- a single regime, single class non-linear (polynomial) speed-flow model;
- a three regime, single class linear model;
- a three regime, single class polynomial model;
- the proposed multi-regime, multi-class model.

The results of stream speed show that the proposed multi-regime, multi-class models offer a significant improvement in the MAPE of stream speed compared to other models. In addition, the multi-regime multi-class model is also able to capture differences in speed flow relationship across different component vehicle types. Among single regime models, the polynomial model outperforms the linear one. In contrast, the three zone linear model performs just as well as the three zone polynomial (and better than the single zone polynomial) one. Thus, speed-flow relationship varies non-linearly with the volume that can be adequately captured using the three zone linear model. Further, the error in prediction is the largest at higher congestion for all models, particularly, the single regime ones.

4.2. Validation

Mathematical speed-flow models were validated against the empirically observed video of traffic data on speed, volume and composition collected on three other mid-block sections (1 km long) of Chennai city roads not used for calibration. The videos from both cameras were synchronized and played. A vehicle entering the stretch captured on camera 1 is traced in camera 2 and the time taken is noted to calculate speed. This was done for all vehicles over observation time, yielding class volumes and class speeds. A comparison of the field observed and the model-predicted values of the speeds of different vehicle types as well as stream speed are shown in Table 2 taking into account different volumes. The observed and predicted speeds for each vehicle class and stream speed appear to be reasonably close (within 10% mostly). Thus, the models are able to replicate the observed field data in a satisfactory manner (Table 3).

5. Analysis of Results

5.1. Differences in Speed Flow Relationships across Vehicle Classes

Free Flow Speeds:

Free flow speeds and the rate of acceleration vary widely across the classes and number of lanes. The mean free speeds of heavy vehicles, cars, autos and two-wheelers were recorded as 67, 72, 48 and 61 kmph respectively. Acceleration rates at low and high speeds are 0.89÷0.67 m/s^2 for HV, 1.5÷0.95 m/s^2 for a car, 1.01÷0.3 m/s^2 for autos and 1.35÷0.60 m/s^2 for two wheelers. Note that autos (three-wheelers) have significantly lower free speed than other categories. These vehicles with lesser free speed are observed to impede the movement of vehicles with higher free speeds resulting in frequent overtaking. Also, interference is caused by the frequent the lane-changing behaviour of the vehicles of a smaller size due to an imperfect lane discipline. As the volume increases, the effect of free-speed differ-

Table 1. Multi-regime multi-class speed models

Average speed of the class (kmph)	Coefficient							
	Volume level	Intercept	HV volume (β_{1i})	Car volume (β_{2i})	Auto volume (β_{3i})	Two-wheeler volume (β_{4i})	Total volume (b)	Goodness-of-fit (R^2)
Two-wheeler	low	64.24	–	–	–	–	–0.001	0.77
	medium	105.7	–0.035	–0.015	–0.016	–0.007		0.95
	high	109.3	–0.037	–0.016	–0.017	–0.007		0.89
Car	low	77.3	–	–	–	–	–0.004	0.88
	medium	116.2	–0.04	–0.0174	–0.022	–0.009		0.97
	high	96.61	–0.033	–0.014	–0.016	–0.006		0.81
Auto	low	49.7	–	–	–	–	–0.0007	0.75
	medium	77.74	–0.023	–0.0103	–0.01	–0.004		0.94
	high	90.9	–0.03	–0.013	–0.014	–0.006		0.89
Heavy vehicle	low	64.5	–	–	–	–	–0.0033	0.89
	medium	93.63	–0.03	–0.014	–0.016	–0.007		0.97
	high	77.76	–0.026	–0.011	–0.012	–0.005		0.77
Stream	low	65.9	–	–	–	–	–0.0018	0.76
	medium	107.9	–0.037	–0.017	–0.019	–0.007		0.97
	high	96.33	–0.033	–0.014	–0.015	–0.006		0.78

entials on stream speed comes down and the slope coefficients of different class volumes are significantly different (particularly, two-wheelers and heavy vehicles). Thus, the size of a vehicle and interactions play a greater role in the variation of speed with flow.

5.2. Asymmetric Interactions

This section studies the asymmetric interaction between the pairs of vehicle classes. This asymmetry is a key feature of heterogeneous traffic. Specifically, speed reduction in class s due to the addition of the given number of vehicles of class r and vice-versa are examined. In particular, the asymmetry ratio between vehicle classes i and j is determined as follows:

$$AR_{ij} = \frac{SR_{ij}}{SR_{ji}},$$

where: SR_{ij} – speed reduction (in %) in class i due to the addition of the given number of the vehicles of class j; SR_{ji} – speed reduction (in %) in class j due to the addition of the given number of the vehicles of class i.

Thus, ratio AR_{ij} may be viewed as the impedance of vehicle class j on the average speed of class i relative to the impedance of class i on the average speed of class j for the given volume and composition level.

The above speed reductions are obtained using equations from Table 1 by incrementing 50 vehicles of a corresponding class to the reference volume level (low, medium or high) and base composition level. At the medium volume level, the highest ratio is for a 2W–HV pair (4.04). The asymmetry ratios of a 2W–Auto, Car–HV, Car–Auto and TW–Car were found to be 3.07, 2.39, 1.85 and 1.58 respectively. Higher asymmetry for 2W–HV and Car–HV pairs can be attributed to size differential which alone does not contribute to asymmetric interactions. For instance, the lowest ratio is for an auto-heavy vehicle (1.35) despite size differential. Smaller asymmetry is due to the fact that the slope of the speed-flow curve of heavy vehicles decreases at a faster rate than that that of auto-rickshaws (Fig. 2) resulting in a smaller base speed of heavy vehicles. Consequently, speed reduction in heavy vehicles due to autos is also larger. Higher ratios for a 2W–Auto and Car–Auto, in spite of smaller size differences, can be attributed to poorer dynamic characteristics of autos. Asymmetry ratios at higher volumes are quite different from those at moderate volumes. Higher ratio is noted for a 2W–HV pair as before but is higher (5.93). The asymmetry of Auto–HV increases from 1.35 to 2.34 from the moderate to high volume level. Similarly, TW–Car ratio increases from 1.85 to 2.31, and Car–HV ratio increases marginally from 2.39 to 2.67. Thus, the impedance of a larger vehicle on a smaller one grows at a faster rate (than speed reduction of a larger vehicle due to a smaller vehicle). Consequently, size differential plays a greater role at higher volumes.

On the other hand, asymmetry ratio for some classes reduces at a higher volume. For instance, Car–Auto ratio decreases from 1.85 (moderate volume) to 1.1 (high volume). This decrease is due to the fact that an increase in the impedance of an auto to a car is only marginal (speed reduction increases from 2.38% to 3.06%). However, the impedance of a car to an auto is more than double (from 1.27% to 2.79%). Similarly, 2W–auto asymmetry reduces from 3.07 (moderate volume) to 2.42 (at high volume) as the impedance of a two-wheeler to an auto grows at a faster rate than the impedance of an auto to a two-wheeler at high congestion.

Table 2. MAPE % in stream speeds across different models

Speed (kmph)	Intercept (c)	Coefficient		Goodness -of-fit (R^2)	Volume level	MAPE (%)
		Q	Q^2			
Single zone linear	75.9	–0.007	–	0.85	low	7.4
					medium	9.3
					high	30.1
Single zone parabolic in Q	61.1	0.0051	–1.7E–06	0.96	low	1.4
					medium	4.8
					high	9.4
Three-zone linear	65.9	–0.0018	–	0.76	low	1.3
	106.8	–0.013	–	0.97	medium	3.6
	106.1	–0.013	–	0.67	high	7.0
Three-zone parabolic in Q	63.5	0.0016	–8.5E–07	0.86	low	0.9
	55.8	0.0092	–2.4E–06	0.95	medium	3.3
	65.6	–	1E–06	0.67	high	9.9
Stream speeds using multi-class models	Equations given in Table 1				low	1.06
					medium	2.3
					high	4.04

Table 3. Validation of the proposed speed-volume models

Class of vehicle	Heavy vehicles			Car			Motorized three wheeler			Motorized two wheeler		
Location No.	1	2	3	1	2	3	1	2	3	1	2	3
Observed speed (kmph)	47.8	20.5	48.8	59.4	22.0	56.8	42.6	22.2	44.8	51.4	25.5	55.6
Estimated speed (kmph)	51.8	20.0	44.2	62.5	22.2	52.6	47.1	22.5	44.1	60.3	25.9	55.2

A three-wheeler on the other hand, due to its lower speed and frequent lane changing, impedes other faster vehicles more than cars at a moderate volume rather than at a high volume. Thus, the relative impedance of a car, two-wheeler and heavy vehicles to autos grows faster at higher volumes than reverse impedance. These findings show that interactions in mixed traffic not only vary by class-types depending on vehicle dimensions and dynamic characteristics, but are also influenced by volume levels.

5.3. Difference in the Levels of Service (LOS) Across Different Vehicle Classes

The level of service for a vehicle at any V/C ratio depends on operating speed as well as on its free flow speed of that class. Further, free flow speeds vary with the class. Besides, the average speeds vary with the classes at a given V/C ratio. Consequently, the experienced LOS can also vary across the classes at a given volume. Indian code – IRC 106 (1990) recognizes six levels of service based on percentage difference between stream speed and free flow speed. The level of service is designated 'A' if operating speed is not lower than 90% of free flow speed (FFS); similarly B (70 to < 90% of FFS), C (50 to < 70%), D (40 to < 50%), E (33 to < 40%) and F (25 to < 33%) are delineated. The class-wise LOS can be computed analogously in terms of % deviation between class-wise average speed and class-wise free flow speed. Note that V/C ratio cut-offs for the same LOS may also differ across the classes.

LOS computations are illustrated for the base composition C1 (HV – 6%, Car – 25%, Auto – 11%, 2W – 58%) using class-wise speed flow equations in Table 1. From free flow speed (FFS) of each class, the speeds corresponding to the different levels of service are determined using the model provided in Table 1 (for the above percentages of FFS). V/C ratio cut-offs for each class correspond to 90%, 70%, 50%, 40% and 33% of corresponding class FFS and are summarized in Table 4. V/C ratios at all levels of service based on stream speed are quite different from that of passenger cars and closer to LOS cut-offs for two wheelers which is the dominant class. In this case, the suitability of using PCU (passenger car) as a basis for determining LOS in mixed traffic may be questionable.

The LOS of larger vehicles (cars and heavy vehicles) is affected more (than autos and two-wheelers) by congestion. For instance, a V/C ratio of 0.7 (for the above composition) corresponds to LOS A for a two-wheeler and auto, B for cars and C for heavy vehicles. In contrast, even at high congestion, V/C ratio = 0.95, two-wheelers and autos face only the LOS of C, whereas, the LOS of a car is D and a heavy vehicle – E.

For comparison purposes, the LOS of another composition C2 (HV – 10%, Car – 29%, Auto – 12%, 2W – 49%) is also illustrated in Table 4. Note that C2 has a higher % of heavy vehicles and cars and a lower composition of two-wheelers compared to C1. Table 4 clearly shows that the thresholds of the LOS for V/C ratio differ considerably for two compositions. In particular, for the same LOS, threshold is lower for C2 than C1 implying an earlier onset of congestion for composition C2. Also, when the component of large vehicles is high, the level of service drops at a faster rate with V/C, especially in the low volume range. The performance of cars and heavy vehicles (larger vehicles) deteriorates faster than other vehicles, more so at C2 than C1, highlighting the asymmetry noted earlier. These findings emphasize the significance of the composition for vehicle classes in the performance of mixed traffic.

5.4. Effect of Vehicle-Class Specific Traffic Management Measures

The above multi-regime multi-class model can be applied to study the impact of vehicle class-based traffic management measures in mixed traffic (e.g. the imposition of restrictions on the movement of certain categories of vehicles in a link). Another key advantage of the proposed model is the ability to quantify differential impacts to different classes.

Three scenarios are investigated to analyze the effect of such class-based traffic control measures. The base scenario corresponds to the case where no vehicle classes are excluded and corresponds to a base composition (HV – 5%, C – 24%, Auto – 11%, 2W – 51%, Bicycles – 9%). The other two scenarios correspond to the exclusion of heavy vehicles (Scenario 1) and autos (Scenario 2) respectively. Each of the three scenarios is applied at three different volume levels: a low volume (3000 vph), a moderate volume (5000 vph) and a high volume (6500 vph). The average class wise speeds and stream speed are estimated using the proposed multi-class multi-regime model as well as the simulation of each scenario and volume level. The results are summarized in Table 5.

In terms of stream speed, the speeds of the proposed model were close to simulation results (deviation is less than 5% at all volume levels). As expected, when the total volume is low, percentage increase in speeds due to vehicle class exclusions is not substantial. At higher volumes, an increase in speeds due to such exclusions turns out to be significant. As noted earlier, the extent of benefits/impacts also vary across vehicle types. In all cases, cars benefit most from such exclusion policies with speed increase in 19.6% and 34.3% due to the exclusion of HV at medium and high volumes. The speed of cars has increased by 23.5% and 35.6% due to the exclusion of auto at medium and high volumes respectively. The results also highlight relatively large differences and errors in estimated stream speeds from the use of compared single class stream speed models to the proposed multi-class multi-regime model. Thus, the proposed models show the promise of evaluating scenarios involving the class-based management of mixed traffic systems.

Table 4. V/C ratios at different levels of service

Level of service	V/C ratio for different classes under compositions							
	C1				C2			
	HV	Car	Auto	2W	HV	Car	Auto	2W
A	0.5	0.49	0.71	0.7	0.18	0.46	0.59	0.58
B	0.63	0.71	0.87	0.83	0.57	0.6	0.72	0.7
C	0.82	0.86	1.0	0.96	0.7	0.72	0.86	0.82
D	0.9	0.94	NA	NA	0.78	0.79	0.92	0.87
E	1.0	1.0	NA	NA	0.83	0.85	0.94	0.93
F	NA	NA	NA	NA	0.93	0.91	0.98	0.97

NA – not applicable when volume exceeds capacity

Table 5. A comparison of speeds in different traffic scenarios

Class of a vehicle/model	Speed of vehicles (kmph) under the various scenarios								
	Base Scenario			Scenario 1 (excluding HV)			Scenario 2 (excluding Auto)		
	Volume (vph)			Volume (vph)			Volume (vph)		
	3000	5000	6500	2850	4750	6175	2670	4450	5785
Heavy vehicle	54.7	42.9	27.7	–	–	–	55.7	51.9	36.2
Car	65.9	51.0	30.9	66.5	61.0	41.5	67.1	63.0	42.0
Auto rickshaw	47.7	43.1	30.5	47.8	48.8	40.3	–	–	–
Two wheeler	61.2	53.6	35.6	61.4	62.2	47.7	61.6	62.6	47.9
Stream speed (single class linear)	56.7	44	34.4	57.8	45.7	36.7	59	47.8	39.4
Stream speed (single class parabolic)	62.2	48.9	31.5	62.8	51.4	36.2	63.4	54.1	41.4
Stream speed (multi-class, multi-regime)	60.5	51.0	33.3	61.1	60.1	45.0	63.0	62.0	45.4
Stream speed (Simulated)	61.7	51.1	34.2	62.7	57.3	47.5	65	58.8	46

6. Conclusions

This study has developed multi-regime and class-wise speed-volume equations for mixed traffic for six lane urban road mid-block sections in Chennai city. The models estimate vehicle class speeds and stream speeds as a function of class-wise volumes. The models are then used for investigating and analyzing differences in speed-flow relationships, interaction and asymmetry across vehicle classes. Further, the impact of composition and volume on the mixed traffic stream and class-wise performance is investigated. The proposed models generalize the current speed-flow models for mixed traffic by explicitly including the role of composition and differences in class speeds. The proposed models performed reasonably well in relation to the micro-simulation model for various scenarios and validation cases. Distinct advantages of the proposed models over the existing models are as follows:

1. The proposed multi-class models explicitly capture differential impacts of different class volumes on the average speed of a given vehicle type.
2. The models capture asymmetric and unequal interactions between different pairs of vehicle classes; thus, the need for homogenizing analysis in terms of a single class (e.g. PCU) is obviated.
3. The multi-class model significantly outperforms its single regime and single class (PCU-based) counterparts in terms of the accuracy of speed predictions.
4. The proposed models are computationally less intensive and easier to interpret.

The analysis of flow characteristics using the models yielded the following insights about mixed traffic flow:

1. At the substantive level, the results suggest that the speed flow relationships of major vehicle types in mixed traffic can be well represented by piece-wise linear models.
2. The findings reveal that speed flow relationships vary considerably across different vehicle classes and depend on volume levels.
3. The average car speeds do not represent stream characteristics in mixed traffic due to their smaller composition.
4. Composition does not influence speeds much at low volumes, but plays a prominent role at higher volumes.
5. Considerable evidence of asymmetric interactions is observed. These interactions vary across classes and are stronger for pairs involving heavy vehicles due to size differential and differences in dynamic characteristics.

The proposed model can also support a number of practical applications two of which are presented in this paper. First, it is used to quantify LOS for different classes in mixed traffic. The second application involves the analysis of class-specific traffic management measures. The effect of excluding some vehicle classes on the performance of mixed traffic is studied. Specifically, the effect of excluding autos or heavy vehicles at different volume levels was analyzed. The results show the promise of improved performance by excluding autos or heavy vehicles at higher volumes, which may be applied on some facilities or time-of-day.

The following findings from these applications are noteworthy:

1. The level of service is different for different vehicle classes at a given volume due to differences in their free-speed and asymmetric interactions.
2. It is found that LOS cut-offs vary across the classes. The results also show a wide range of operating characteristics across vehicle types even at high volumes. In addition, the LOS experienced by different classes is also affected by the composition of mixed traffic.
3. LOS cut-offs for stream speed is closest to thresholds for two-wheelers which form the predominant component in many urban roads in India.

Thus, the proposed model can be used for assessing the level of service enjoyed by different classes of vehicles in mixed traffic and for analyzing equity implications of traffic management measures. Several features of mixed traffic merit further investigation including the role of an imperfect lane discipline, the effect of free-flow speeds, the presence of non-motorized vehicles and traffic behaviour on other facilities (four and two lane urban roads).

This study focused mainly on class speed and stream speed as the major performance measure. Developing models for the effect of heterogeneous traffic flow and interactions on vehicle characteristics and performance measures such as fuel consumption, pollution and user costs is an important practical direction for future research.

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