TRANSPORT
ISSN 1648-4142 print / ISSN 1648-3480 online
2013 Volume 28(2): 193–203
doi:10.3846/16484142.2013.805434

A MULTIPLE-VEHICLE TYPE REACTIVE DYNAMIC USER EQUILIBRIUM MODEL AND ALGORITHM WITH PHYSICAL QUEUES

ShuGuang Li

School of Electronic and Control Engineering, Chang'an University, Xi'an 710064, Shaanxi, China

Submitted 27 November 2011; accepted 26 March 2012

Abstract. A multiple-vehicle type reactive dynamic user equilibrium model with physical queues is presented. The single-vehicle type cumulative flow given by Newell is extended into multiple-vehicle types. The flow conditions for multiple-vehicle types on link can be divided into two classes: Class 1 is a congested condition – various types of vehicles cannot overtake; Class 2 is an uncongested condition – each type of vehicles can run according to each type of free-flow velocity. Further, multiple-vehicle type reactive dynamic user equilibrium condition is described. Finally, one iterative algorithm is proposed, and the results show that the model can demonstrate multiple-vehicle type physical queue conditions among the links and multiple-vehicle type reactive dynamic user equilibrium condition.

Keywords: multiple-vehicle type; cumulative flow; reactive dynamic user equilibrium; physical queue.

Reference to this paper should be made as follows: Li, S. 2013. A multiple-vehicle type reactive dynamic user equilibrium model and algorithm with physical queues, *Transport* 28(2): 193–203. http://dx.doi.org/10.3846/16484142.2013.805434

Introduction

Dynamic traffic assignment method as the core theory of intelligent transportation system can predict the network traffic conditions and further ease urban traffic congestion. Dynamic traffic assignment consists of two parts: one is network traffic flow simulation model which can model the changes of network traffic conditions and another is traveler's choice behavior model which simulates the tendency of traveler's travel choices.

In general, these traveler's choice behavior models can be classified into two classes: one class is predictive dynamic user equilibrium model which requires that the travelers take the travel choice decisions according to actual travel time at the same time. Further, some researchers (Ran *et al*. 1993; Ran, Boyce 1996; Huang, Lam 2002; Lo, Szeto 2002; Szeto, Lo 2004; Nie, Zhang 2010) have paid attention to predictive dynamic user equilibrium problems. Another is the reactive dynamic user equilibrium model which assumes that each traveler chooses the shortest path from the decision node to destination according to instantaneous travel time at each time instant. Ran and Boyce (1996) presented a reactive dynamic user equilibrium model based on link and path. Kuwahara and Akamatsu (1997) presented a reactive dynamic traffic assignment with point queue and further presented an iterative algorithm based on the time decomposition. Li *et al*. (2000) presented a discrete time reactive dynamic user equilibrium model with deterministic point queue which was then formulated as a variational inequality problem over a polyhedral set by constructing a new network. But these formulations didn't consider the physical effect of queuing, since a queue physically backs up and may reduce the link capacity of the upstream link, and then the traveler's choice behaviors may be affected. Therefore, Kuwahara and Akamatsu (2001) integrated the cumulative curve method presented by Newell (1993) into a reactive dynamic user optimal assignment in considering physical queue; Gentile *et al*. (2007) presented a dynamic network model which simulated the propagation of congestion among adjacent links due to queue spillovers according to the link cumulative curve. However, Kuwahara and Akamatsu (2001) and Gentile *et al*. (2007) only have paid attention to single vehicle type analytical dynamic traffic assignment problem with physical queue.

Bliemer and Bovy (2003), Bliemer (2007), Li and Su (2005), Li and Ju (2009) and Li and Xu (2009) presented multiple vehicle type predictive dynamic user equilibrium models with point and physical queues, respectively. However, reactive dynamic user equilibrium problem was not considered in these studies. Therefore, multiple vehicle type reactive dynamic user equilibrium models with physical queue is presented in this study.

E-mail: lxlsg@vip.sina.com

The remaining paper is organized as follows: First, according to cumulative link vehicle and simple kinematics wave proposed by Newell (1993), a physical queue discrete-time dynamic traffic network model with multiple vehicle types is presented. Second, multiple vehicle type reactive dynamic user equilibrium condition is described and an iterative algorithm is given according to the method proposed by Kuwahara and Akamatsu (2001). Finally, numerical examples illustrate the simplicity and applicability of the proposed approach.

1. Model formulation

Consider a road network $G(N,L)$, where N and L are the finite sets of nodes and links, respectively. In our setting, nodes are the locations where vehicles enter or leave links, the links with directions are connected to nodes. Let R and S be the sets of origin nodes and destination nodes, respectively, Let r and s represent an origin and a destination, apparently, $r \in R$ and $s \in S$. Here, Let I denote the set of all vehicle types and include passenger cars, trucks, and public bus, etc. Denote pcu_i: as the passenger car equivalents the parameter of vehicle type i.

Notations. The variables used in the formulation include the following:
N – the finite sets of nodes;
L – the finite sets of links;
R – the sets of origin nodes;
r – an index denoting an origin;
S – the sets of destination nodes;
s – an index denoting a destination;
I – the set of all vehicle types;
i – the vehicle type, $i \in I$;
$q_i(x, t)$, $k_i(x, t)$, $v_i(x, t)$ – flow, density, and speed of vehicle type i in the space–time domain (x, t), respectively;
$q_{tot}(x, t)$, $k_{tot}(x, t)$, $v_{tot}(x, t)$ – total flow, total density, and total speed in the space–time domain (x, t), respectively;
pcu_i – passenger car units of vehicle type i;
k_{tot}^{cr} – the critical density of mixed type;
v_i – the free flow speed of vehicle type i;
w_{tot} – the backward wave speed of total flow;
$A_i(x, t)$ – the cumulative flow of type i on x–t plane;
$A_{tot}(x, t)$ – the total cumulative flow on x–t plane;
k_{tot}^{jam} – the maximum congested jam density of total flow;
(m,n) – the link from entry m to exit n;
$A_i(x, t)$, $A_i(n, t)$ – the cumulative flow at link entry m and link exit n by time t, respectively;
$u_i^{m,n}(t)$, $v_i^{m,n}(t)$ – the link actual inflow and outflow at time t on link (m,n), respectively;
δ – the length of discretized unit time interval;
$X^{m,n}(t)$ – the total possible link inflow rate at time t on link (m,n);
$S_{in}^{m,n}$ – the entry capacity (link storage capacity) of link (m,n);
$Y_i^{m,n}(t)$ – the possible link departure flow rate of type i at time interval t on link (m,n);
$q_i^{m,n}(t)$ – the queue vehicle number of type i at time interval t on link (m,n);
$u_i^{m,n}(t)$ – the actual link inflow rate of type i at time interval t on link (m,n);
$v_i^{m,n}(t)$ – the actual link departure flow rate of type i at time interval t on link (m,n);
$Y^{m,n}(t)$ – the total link departure flow at time interval t on link (m,n);
$Y^{m,n,j}(t)$ – the total turning flow at time interval t from link (m,n) to link (n,j);
$C_{out}^{m,n}(t)$ – the actual link exit capacity at time interval t on link (m,n);
$S_{out}^{m,n}$ – the link capacity on link (m,n);
$Y_i^{m,n,s}(t)$ – the possible exit flow of type i on link (m,n) for destination s at time interval t;
$v_i^{m,n,s}(t)$ – the actual exit flow of type i on link (m,n) for destination s at time interval t;
$q_i^{n,s}(t)$ – the trip rate of type i generated from node n to destination s at time interval t (given);
$u_i^{n,j,s}(t)$ – the actual inflow of type i on link (n,j) for destination s at time interval t;
$ql^{m,n}(t)$ – the mixed queue length on link (m,n) at time interval t;
$q^{m,n}(t)$ – the total queue number on link (m,n) at time interval t;
$t_i^{m,n}(t)$ – the travel delay of type i on link (m,n) at time interval t;
$s^{m,n}(v^{m,n}(t))$ – the speed of the mixed queue on link (n,m) at time interval t;
$v^{m,n}(t)$ – the total link exit flow on link (n,m) at time interval t;
$t_i^{p,ms}(t)$ – the instantaneous travel time to traverse path p from node m to destination s for type i at time interval t;
P^{ms} – the path set from node m to destination s;
$f_i^{p,ms*}(t)$ – the inflow rate of the travelers for type i entering into path p between node m and destination s at time interval t;
$q_i^{ms}(t)$ – the demand of type i between node m and destination s at time interval t;
$t_{\min,i}^{ms}(t)$ – the minimum instantaneous travel time of travelers of type i between node m and destination s at time interval t.

With the first-order traffic flow Lighthill–Whitham–Richards (LWR) model, many simple traffic problems can be represented analytically such as a shock formation. However, the original LWR model cannot predict some nonlinear phenomena such as two-capacity, traffic hysteresis, and platoon dispersion, and so on. Therefore, some researchers extended the first-order traffic flow model into multiclass one, each vehicle type is only characterized by their flow–density relationship. The model given by Wong, G. C. K. and Wong, S. C. (2002) consider the distribution of heterogeneous drivers characterized by their choice of speeds in a traffic flow, which reflects the dynamic behavior of heterogeneous users whereby faster vehicles could overtake slower ones under different traffic conditions. Following the similar ideas, some researchers (Lebacque *et al*. 1998; Zhang, Jin 2002; Zhu *et al*. 2003; Ngoduy, Liu 2007; Ngoduy 2008) used different methods to reflect the differences among different type of vehicles. Logghe and Immers (2008) proposed a unified framework for the different existing multitype extensions of the kinematic wave theory and proved that the differences of all models lied in the assumption on how several vehicle types interact.

However, the previous models could only apply to road traffic flow simulation and did not take into account the network traffic. In this study, the single vehicle type cumulative curve method proposed by Newell (1993) is extended into multiple vehicle type conditions, Further the theoretical analysis is given for the multiple vehicle type dynamic traffic network model with cumulative curve presented by Li and Xu (2009), Li (2010) and then the multitype dynamic traffic network model with diverge and merge nodes is integrated (Bliemer 2007; Li, Xu 2009).

Let $q_i\ (x,\ t)$, $k_i\ (x,\ t)$, $v_i\ (x,\ t)$ be, respectively, the flow, density, and speed of vehicle type i in the space–time domain. The total flow, total density, and total speed on a highway section in the space–time domain can be defined as $q_{tot}\ (x,\ t)$, $k_{tot}\ (x,\ t)$, $v_{tot}\ (x,\ t)$.

From the law of conservation of vehicles, the conservation equation for each vehicle type can be written as follows:

$$\frac{\partial k_i(x,t)}{\partial t}+\frac{\partial q_i(x,t)}{\partial x}=0,\ \ \forall t,x,i. \tag{1}$$

The relationship between densities and speeds for each vehicle type is:

$$q_i(x,t)=k_i(x,t)\cdot v_i(x,t),\ \ \forall t,x,i. \tag{2}$$

In previous studies, many methods were used to reflect the differences among various types. In this study, the passenger car units of vehicle type i, pcu_i, and the speeds of different vehicles is used to show the interactions of multiple vehicle types. In order to simplify the following analysis, the mixed density, flow, and speed of mixed flow can be represented as follows:

$$k_{tot}(x,t)=\sum pcu_i\cdot k_i(x,t),\ \ \forall t,x;$$

$$q_{tot}(x,t)=\sum pcu_i\cdot q_i(x,t),\ \ \forall t,x;$$

$$v_{tot}(x,t)=\frac{q_{tot}(x,t)}{k_{tot}(x,t)},\ \ \forall t,x. \tag{3}$$

1.1. Basic speed–density relationship

Different speed–density relationships will induce models with different features. Following Li and Ju (2009) and Li and Xu (2009), multi-type speed–density relationship can be seen in Figure 1. The characteristic curves for multiple vehicle types can be divided into two classes:

- *Class 1* – when the density of mixed type is less than the critical density, k_{tot}^{cr}. Each type of travelers can run according to their own characteristics (v_i is the free flow speed of vehicle type i). For example, the fast vehicles run faster than the slow ones.
- *Class 2* – when the density of mixed type is more than the critical density, various types of vehicles cannot overtake on the congested part due to a smaller distance between the vehicles, the backward wave speed of total flow, w_{tot}, under congested conditions is similar and the speeds of different types approach consistency.

In order to solve the above first-order LWR models for multiple vehicle types, the single vehicle type cumulative curve method proposed by Newell (1993) is extended into multiple vehicle type conditions.

1.2. Cumulative flows

According to the cumulative flow of type i, $A_i(x,\ t)$ on x–t plane, one can obtain:

$$k_i(x,t)=-\frac{\partial A_i(x,t)}{\partial x};$$

$$q_i(x,t)=\frac{\partial A_i(x,t)}{\partial t},\ \ \forall t,x,i. \tag{4}$$

At the uncongested condition along the characteristic flow of each type on x–t plane, the corresponding flow and density of each type keep constant. Therefore, the cumulative flow of each type can be calculated as follows:

$$dA_i(x,t)=\left(\frac{\partial A_i(x,t)}{\partial x}\right)\mathrm{d}x+\left(\frac{\partial A_i(x,t)}{\partial t}\right)$$

$$dt=-k_i(x,t)\mathrm{d}x+q_i(x,t);$$

$$dt=(-k_i(x,t)+q_i(x,t)\cdot w_i)dx\ \ \forall t,x,i, \tag{5}$$

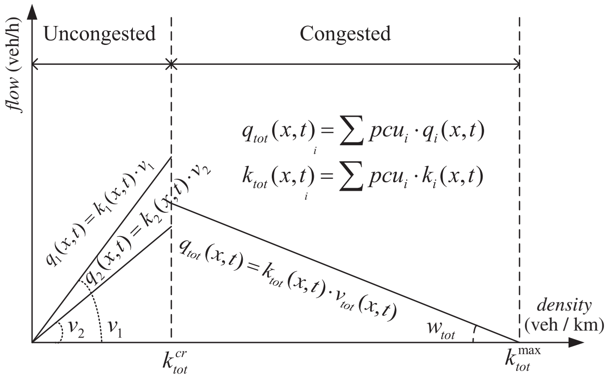

Fig. 1. Basic speed–density relationship

where:

$$w_i = \left(\frac{dt}{dx}\right)_i. \tag{6}$$

According to Eqns (5 and 6), one can see:

$$\frac{dA_i(x,t)}{dx} = (-k_i(x,t) + q_i(x,t) \cdot w_i(x,t)), \quad \forall t, x, i. \tag{7}$$

Because the free flow speed of type i, v_i, is equal to the wave speed of type i, $1/w_i$, at the uncongested condition:

$$v_i = \frac{1}{w_i} = \left(\frac{dx}{dt}\right)_i. \tag{8}$$

Then substituting Eqn (8) into Eqn (7), the below equation can be obtained:

$$\frac{dA_i(x,t)}{dx} = \left(-k_i(x,t) + \frac{q_i(x,t)}{v_i(x,t)}\right) = 0, \quad \forall t, x, i. \tag{9}$$

Similar with Newell (1993), the cumulative flow of each type keeps constant along the trajectory of a vehicle of each type at the uncongested condition.

At the congested condition, the backward wave speed of each type is similar and equal to the backward wave speed of total flow, w_{tot}, according to Eqn (5), the total cumulative flow, $A_{tot}(x, t)$, can be expressed as follows:

$$\frac{dA_{tot}(x,t)}{dx} = \frac{d\left(\sum_i pcu_i \cdot A_i(x,t)\right)}{dx} = \sum_i pcu_i \cdot (-k_i(x,t) + q_i(x,t) \cdot w_i), \quad \forall t, x. \tag{10}$$

Further, $w_{tot} = w_1 = w_2 = \cdots = w_n$.

Therefore, Eqn (10) can be expressed as below:

$$\frac{dA_{tot}(x,t)}{dx} = -\sum_i pcu_i \cdot k_i(x,t) + w_{tot} \cdot \sum_i pcu_i \cdot q_i(x,t) = -k_{tot}(x,t) + w_{tot} \cdot q_{tot}(x,t) = -k_{tot}^{jam}, \quad \forall t, x, \tag{11}$$

where: k_{tot}^{jam} is the maximum congested jam density of total flow.

1.3. Multiple-vehicle type shock wave conditions

At the multiple-vehicle type dynamic traffic conditions, if one should calculate more than one value of the total cumulative flow A_{tot} (x, t) at some (x, t) point from different boundary data, one simply chooses the smallest value. Therefore, the calculation of total cumulative flow on link (m,n) can be expressed as below. First, given the flow–density relationship as well as the cumulative flow at link entry m, A_i (m, t) and the cumulative flow at link exit n, A_i (n, t) by time t. then, the total cumulative flow, $A_{tot}(x, t)$, can be in principle be determined at any location x on link (m,n) at time t. According to Eqn (9), one of the values of $A_{tot}(x, t)$ at location x on link (m,n) at time t is derived from $\sum_i pcu_i \cdot A_i(t - \frac{x}{v_i}, m)$ by the cumulative flow of each type at link entry m; further, according to Eqn (11), another one of the values of $A_{tot}(x, t)$ at location x on link (m,n) at time t is derived from $\sum_i pcu_i \cdot A_i(n, t - \frac{l^{m,n} - x}{w_{tot}}) + k_{tot}^{jam} \cdot (l^{m,n} - x)$ by the cumulative flow of each type at link exit n, where $l^{m,n}$ is the length of link (m,n).

Further, the real value of $A_{tot}(x, t)$ can be calculated as below:

$$A_{tot}(x,t) = \min\left\{\sum_i pcu_i \cdot A_i\left(m, t - \frac{x}{v_i}\right), \sum_i pcu_i \times A_i\left(n, t - \frac{l^{m,n} - x}{w_{tot}}\right) + k_{tot}^{jam} \cdot (l^{m,n} - x)\right\}, \quad \forall t, x \in (m,n). \tag{12}$$

If $\sum_i pcu_i \cdot A_i\left(m, t - \frac{x}{v_i}\right) > \sum_i pcu_i \times A_i\left(n, t - \frac{l^{m,n} - x}{w_{tot}}\right) + k_{tot}^{jam} \cdot (l^{m,n} - x)$, then, the physical queue of the link exit will back up to x, otherwise, total cumulative curve at location x and time t will depend on the flow at link entry n.

1.4. Link outflow rate

A discrete time scheme is presented in this section in such a way that the network equations are reformulated so that they are consistent with the discrete time. Let δ be the length of discretized unit time interval. Here, it is assumed that the study period is long enough to ensure all travelers can exit from the network after the time T. On the other hand, the duration of δ is assumed short enough so that the discrete-time model can approximately represent its continuous time counterpart. For simplicity, the flow rate, either specified by link or by path during a discrete interval is assumed to be constant.

First, the possible link inflow and outflow as presented by Li and Xu (2009) is given; further, actual total capacity is calculated according to node flow assignment method in considering reactive dynamic assignment; finally, each type of actual link outflow is given.

Where the link actual inflow, $u_i^{m,n}(t)$, and outflow, $v_i^{m,n}(t)$, at time t on link (m,n) can be expressed as, respectively:

$$u_i^{m,n}(t) = \frac{A_i(m,t) - A_i(m,t-\delta)}{\delta};$$

$$v_i^{m,n}(t) = \frac{A_i(n,t) - A_i(n,t-\delta)}{\delta}, \quad \forall (m,n), t, i. \tag{13}$$

1.4.1. Link possible inflow

The possible link inflow calculations of multiple types are similar with that of single type. Under the multitype spillback queue assumptions, the link capacity may change over time since only a queue physically backs up and may reduce the link capacity of the upstream link.

Following Kuwahara and Akamatsu (2001), Gentile *et al.* (2007), Li and Ju (2009), Li and Xu (2009), the total possible link inflow rate, $X^{m,n}(t)$, at time t on link (m,n) can be expressed as follows:

$$X^{m,n}(t) = \begin{cases} a \text{ if } b > c; \\ S_{in}^{m,n} \text{ otherwise}, \end{cases}$$
$$\forall t, (m,n);$$
$$a = \sum_i pcu_i \cdot u_i^{m,n}\left(t - \frac{l^{m,n}}{w_{tot}}\right);$$
$$b = \sum_i pcu_i \cdot A_i(m,t);$$
$$c = \sum_i pcu_i \cdot A_i\left(n, t - \frac{l^{m,n}}{w_{tot}}\right) + k_{tot}^{jam} \cdot l^{m,n}, \quad (14)$$

where: $S_{in}^{m,n}$ is the entry capacity (link storage capacity) of link (m,n), if a mixed queue reaches the link entry at time t, the possible link inflow is restricted to the actual exit flow rate at time $t - \frac{l^{m,n}}{w_{tot}}$, otherwise, $X^{m,n}(t)$ is equal to the link entry capacity.

1.4.2. Link possible outflow

Under multitype flow propagation conditions, the possible link departure flow calculations of various types are different due to the difference of the vehicles free-flow velocity. The various types of vehicles entering into link (m,n) at the same time will arrive at the link exit part at the different times.

The possible link departure flow rate of type i at time interval t on link (m,n), $Y_i^{m,n}(t)$, is consisted of two parts; one is the actual link inflow rate of type i at time $t - \frac{l^{mn}}{v_i}$, $u_i^{m,n}\left(t - \frac{l^{m,n}}{v_i}\right)$, another is the queue vehicle number of type i at time $t - \delta$, $\frac{q_i^{m,n}(t-\delta)}{\delta}$. The equation is written as follows:

$$Y_i^{m,n}(t) = u_i^{m,n}\left(t - \frac{l^{m,n}}{v_i}\right) + \frac{q_i^{m,n}(t-\delta)}{\delta}, \quad \forall t, i, (m,n). \quad (15)$$

The queue vehicle number of type i at time t on link (m,n), $q_i^{m,n}(t)$ is equal to the possible link departure flow rate of type i at time t on link (m,n) minus the actual link departure flow rate of type i at time t on link (m,n) $(v_i^{m,n}(t))$:

$$q_i^{m,n}(t) = \max\{0, \delta \cdot (Y_i^{m,n}(t) - u_i^{m,n}(t))\} =$$
$$\max\{0, q_i^{m,n}(t-\delta) + \delta \cdot (u_i^{m,n}(t - l^{m,n}/v_i) - v_i^{m,n}(t))\},$$
$$\forall t, i, (m,n). \quad (16)$$

The calculation method of the spillback queue vehicle number is similar with that of the point queue vehicle number; however, both meanings are distinct in essence. Under the point queue concept, one assumes a vehicle is a point without length; therefore, a point queue vertically forms at the exit point of link, however, a spillback queue forms horizontally.

According to the passenger car equivalents parameter of each vehicle type, the total link departure flow, $Y^{m,n}(t)$, at time t on link (m,n) can be calculated as follows:

$$Y^{m,n}(t) = \sum_i pcu_i Y_i^{m,n}(t), \ \forall t, (m,n). \quad (17)$$

1.5. Node flow assignment

The method as presented by Li and Xu (2009) is used in modeling the cell-based multitype node flow assignment method. Under the multitype flow propagation assumptions, actual exit capacity on link a at time t can be calculated according to the total link departure flow and the possible inflow on the downstream links, further, actual departure flow rate of each type is calculated by actual exit capacity. Therefore, the calculation way of actual link exit capacity of multiple types is similar with that of single type.

The upstream link time-dependent exit capacity of various node types is determined on the basis of the possible link inflow rate of the downstream links and the turning flows. To simplify the exposition, we first consider only two typologies of nodes: "merging" and "diversion".

When considering a diversion node n with entering link (m,n) and exiting links (n,j), that is, an intersection with a singleton backward star. The actual link exit capacity at time t on link (m,n), $C_{out}^{m,n}(t)$, is given by:

$$C_{out}^{m,n}(t) = \min\left\{S_{out}^{m,n}, \sum_j X^{n,j}(t), \min_j\left(\frac{X^{n,j}(t)}{Y^{m,n,j}(t)/Y^{m,n}(t)}\right)\right\},$$
$$\forall t, (m,n), \quad (18)$$

where $Y^{m,n,j}(t)$ is the total turning flow at time t from link (m,n) to link (n,j), which can be obtained by reactive dynamic user equilibrium algorithm; $Y^{m,n,j}(k) = \sum_i pcu_i \cdot Y_i^{m,n,j}(k)$. The first term of the right-hand side means the time-dependent link exit capacity cannot be more than the link capacity on link (m,n), $S_{out}^{m,n}$, the second term means the time-dependent link exit capacity mustn't be smaller than the total possible inflow of all downstream links, the third term shows the time-dependent link exit capacity is restricted to the possible link inflow of downstream links, since $C_{out}^{m,n}(t)$ multiplied by the proportion of the link turning flow should not exceed $X^{n,j}(t)$, the constraint is derived as follows:

$$C_{out}^{m,n}(t) \cdot \frac{Y^{m,n,j}(t)}{Y^{m,n}(t)} \le X^{n,j}(t);$$

$$\therefore C_{out}^{m,n}(t) \le \frac{X^{n,j}(t)}{\frac{Y^{m,n,j}(t)}{Y^{m,n}(t)}}. \quad (19)$$

In order to conform with the link First in First Out (FIFO) characteristic, this condition must be satisfied for each of the downstream links, therefore, the time-dependent link exit capacity is less than $\min_{bj}(X^{n,j}(t)/Y^{m,n,j}(t)/Y^{m,n}(t))$.

When considering a merging node n with entering links (m,n) and exiting links (n,j), that is an intersection with a singleton forward star, the link exit capacity at time interval t on link (m,n) is given by:

$$C_{out}^{m,n}(t) = \min\left\{S_{out}^{m,n}, \lambda_{n,j}(t) \cdot X^{n,j}(t)\right\}, \quad \forall t, (m,n), \quad (20)$$

where: $\lambda_{n,j}(t)$ expresses the supply flow on link (m, n) at time t from the downstream link (n,j). One assumes $\lambda_{n,j}(t) = \frac{S_{out}^{m,n}}{\sum_m S_{out}^{m,n}}$, the $\lambda_{n,j}(t)$ is calculated in using the method presented by Kuwahara and Akamatsu (2001) and Gentile *et al.* (2007) too.

When considering a generic node $n \in N$ with entering links $a_1, \cdots, a_i$ and exiting links $b_1, \cdots, b_j$, the time-dependent link exit capacity can be expressed as follows:

$$C_{out}^{m,n}(t) = \min\left\{S_{out}^{m,n}, \sum_j \lambda_{n,j}(t) X^{n,j}(t), \min_j\left(\frac{\lambda_{n,j}(t)}{Y^{m,n,j}(t)/Y^{n,j}(t)} X^{n,j}(t)\right)\right\}, \quad \forall t, (m,n). \quad (21)$$

The second term of the right-hand side means the total supply flow at time t on link (m,n) is provided by the downstream links. The third term of the right-hand side shows the minimum flow of upstream link (m,n) accepted by the downstream links in order to conform with the link FIFO characteristic. The above equation is consistent with the calculation methods of both merging and diversion nodes.

1.6. Actual link-destination flow

The actual link exit flow rate of type i at time t on link (m, n) for destination s is written as follows:

$$v_i^{m,n,s}(t) = \begin{cases} \frac{Y_i^{m,n,s}(t)}{Y^{m,n}(t)} \cdot C_{out}^{m,n}(t) & \text{if } Y^{m,n}(t) > C_{out}^{m,n}(t); \\ Y_i^{m,n,s}(t) & \text{otherwise,} \end{cases}$$

$$\forall (m,n), i, t, s, \quad (22)$$

where $Y_i^{m,n,s}(t)$ is the possible exit flow of type i on link (m,n) for destination s at time t and $v_i^{m,n,s}(t)$ is the actual exit flow of type i on link (m,n) for destination s at time t.

The node flow conservation equation can be expressed as follows:

$$\sum_j u_i^{n,j,s}(t) = \sum_m v_i^{m,n,s}(t) + q_i^{n,s}(t),$$

$$\forall n \ne s, i, t, \quad (23)$$

where $q_i^{n,s}(t)$ is the trip rate of type i generated from node n to destination s at time t (given) and $u_i^{n,j,s}(t)$ is the actual inflow of type i on link (n,j) for destination s at time t.

1.7. Link instantaneous travel time

When the link has the different exit and entry capacities, the method of the link instantaneous travel time for single-vehicle type given by Kuwahara and Akamatsu (2001) cannot give the link instantaneous travel time for multiple-vehicle types. In order to calculate the link instantaneous travel time of each type, a simple method is given as below.

First, one assumes that mixed queue on links is at jam density. The mixed queue length on link (m,n), $ql^{m,n}(t)$ is equal to the total queue number on link (m,n), and $q^{m,n}(t)$, divided by the jam density at time t, as one can see:

$$ql^{m,n}(t) = \frac{q^{m,n}(t)}{k_{tot}^{jam}}, \quad \forall t, (m,n), \quad (24)$$

where: $q^{m,n}(t) = \sum_i pcu_i \cdot q_i^{m,n}(t)$.

The travel delay of type i, $t_i^{m,n}(t)$, on link (n,m) is a sum of two components: the free flow time of type i on link (n,m) and the queuing delay incurred by a vehicle that enters link (n,m) at time t. This is expressed in the following equation:

$$t_i^{m,n}(t) = \frac{l^{m,n} - ql^{m,n}(t)}{v_i} + \frac{ql^{m,n}(t)}{s^{m,n}(v^{m,n}(t))},$$

$$\forall t, i, (m,n), \quad (25)$$

where: $s^{m,n}(v^{m,n}(t))$ is the speed of the mixed queue on link (n,m) at time t; $v^{m,n}(t)$ is total link exit flow on link (n,m) at time t; $v^{m,n}(t) = \sum_{i,s} pcu_i \cdot v_i^{m,n,s}(t)$, the speed of the mixed queue can be known as a function of the total exit flow rate in congested region as below:

$$s^{m,n}(v^{m,n}(t)) = \frac{v^{m,n}(t)}{k_{tot}^{jam} - \frac{v^{m,n}(t)}{w_{tot}}}. \quad (26)$$

2. Multiple-vehicle type reactive user equilibrium condition

The instantaneous travel time to traverse path p from node m to destination s for travelers of type i at time t, $t_i^{p,ms}(t)$, is calculated in using the following function:

$$t_i^{p,ms}(t) = \sum_{mn \in P^{ms}} t_i^{mn}(t), \quad \forall t, i, p \in P^{ms}. \tag{27}$$

where: P^{ms} is the path set from node m to destination s.

For each vehicle type of travelers and for each node–destination pair, the instantaneous path travel time for each type of travelers is equal and minimum and less than (or equal to) the instantaneous path travel time for each type of travelers on any unused paths. Further, each type of travelers of follow reactive user equilibrium condition, expressed as follows:

$$t_i^{p,ms}(t,f^*)\begin{cases} = t_{\min,i}^{ms}(t) & \text{if } f_i^{p,ms^*}(t) > 0; \\ > t_{\min,i}^{ms}(t) & \text{if } f_i^{p,ms^*}(t) = 0; \end{cases}$$

$$\forall p, s, m \neq s, t; \tag{28}$$

$$\sum_p f_i^{p,ms^*}(t) = q_i^{ms}(t), \quad \forall p, s, m \neq s, t; \tag{29}$$

$$f_i^{p,ms^*}(t) > 0, \ \forall p, s, m \neq s, t, \tag{30}$$

where: $f_i^{p,ms^*}(t)$ is the inflow rate of the travelers for type i entering into path p between node m and destination s at time t; $q_i^{ms}(t)$ is the demand of type i between node m and destination s at time t; $t_{\min,i}^{ms}(t) = \min\{t_i^{p,ms}(t), p \in P^{ms}\}$ is the minimum instantaneous travel time of travelers of type i between node m and destination s at time t. Eqn (29) represents the flow conservation of travelers of type i between node m and destination s at time t, and Eqn (30) represents the nonnegativity of all path inflow rates.

3. Algorithm

Following Kuwahara and Akamatsu (1997, 2001), one iterative algorithm is used to calculate multiple-vehicle type reactive dynamic user equilibrium condition. The overall idea of algorithm can be described as follows: According to the instantaneous link travel time, each vehicle type of travelers at each decision node chooses the downstream turning link for each time interval. The proposed algorithm is explained step by step as follows:

Step 1: Initialize $t_i^{m,n}(t) = \frac{l^{m,n}}{v_i}$, and according to the origin and destination the virtual links is generated, $t = 0$.

Set δ as $\delta < \min_{m,n,i}\left\{\frac{l^{m,n}}{v_i}\right\}$.

Step 2: Calculate the possible inflow, $X^{m,n}(t)$, and outflow, $Y^{m,n}(t)$ by Eqs (14, 15 and 17) at time interval t.

Step 3: Find the shortest path of type i from node m to destination s based on $t_i^{m,n}(t)$, and give $Y^{m,n,j}(t)$ for all link (m,n) and destination s from the shortest path search.

Step 4: Calculate the time-dependent link exit capacity, $C_{out}^{m,n}(t)$ by Eqn (21) and further determine the actual link outflow $v_i^{m,n,s}(t)$ by Eqn (22).

Step 5: Determine the total arrival flow $\sum_m v_i^{m,n,s}(t) + q_i^{r,s}(t)$ by Eqn (23) and further load the flow onto a link starting from node m on the shortest path, $u_i^{n,j,s}(t) = \sum_m v_i^{m,n,s}(t) + q_i^{r,s}(t)$.

Step 6: Calculate the total link queue number and then determine $t_i^{m,n}(t)$ by Eqns (24 and 25).

Step 7: Update $t = t + \delta$, return to Step 2 if t is less than at the end of the study period; otherwise stop.

4. Simulation

The network shown in Figure 2 has 13 nodes, 19 links, and 2 OD pairs: (1,11), (3,13). Table 1 gives six path-link series between origin 3 and destination 13 and four path-link series between node 2 and destination 11. Two different types on the network are passenger cars and trucks, respectively. The passenger car equivalents parameter of passenger cars and trucks is $Pcu = 1$ and $Pcu_t = 2$, respectively. One assumes that every arc $(m,n) \in A$ is characterized by $w_{tot}^{mn} =$ 80 km/h, $k_{jam}^{mn} = 200$ veh/km. The free flow speed of passenger cars and trucks is 80 and 50 km/h, respectively. The time interval length is 0.01 h.

The given demand is as follows:

$$q_{(1,11)}^{m}(t) = \begin{cases} 5000 \cdot \sin\left(\frac{t-1}{0.3}\right), & m = \text{car}, \ 0.1 \leq t \leq 0.4; \\ 2500 \cdot \sin\left(\frac{t-1}{0.3}\right), & m = \text{truck}, \ 0.1 \leq t \leq 0.4; \end{cases}$$

$$q_{(3,13)}^{m}(t) = \begin{cases} 4000, & m = \text{car}, \ 0.05 \leq t \leq 0.5; \\ 2400, & m = \text{truck}, 0.05 \leq t \leq 0.5. \end{cases}$$

Figures 3 and 4 give the inflows and the outflows of car and truck on links 2 and 18 and the instantaneous travel time of six paths (in Table 1) between OD pair (3 and 13), respectively. The X axis represents time, the first and second Y axis express the path travel time and link inflow rate, respectively.

First of all, the inflows of car and truck on link 18 are 2728 veh/h and 1636 veh/h, respectively, during period (0.05 and 0.48) and total inflow is equal to the link entry capacity (6000 veh/h) on link 18 in Figures 3 and 4. After time 0.48, one can find the car and the truck inflows on link 18 suddenly decline because the queue on link 18 which back up the link entry after time 0.48 results in the reduction of the total possible inflow on link 18, further the reduction of the car and truck inflows on link 18 is induced. One can find that dynamic queue model in the paper can reflect physical queue conditions.

According to Figure 3, first, one can find the flows of car entirely enter into the link 18 during period (0.05 and 0.5), the reason is that the instantaneous travel time of car on path OD2–1 between OD pair (3 and 13) is minimal during period (0.05 and

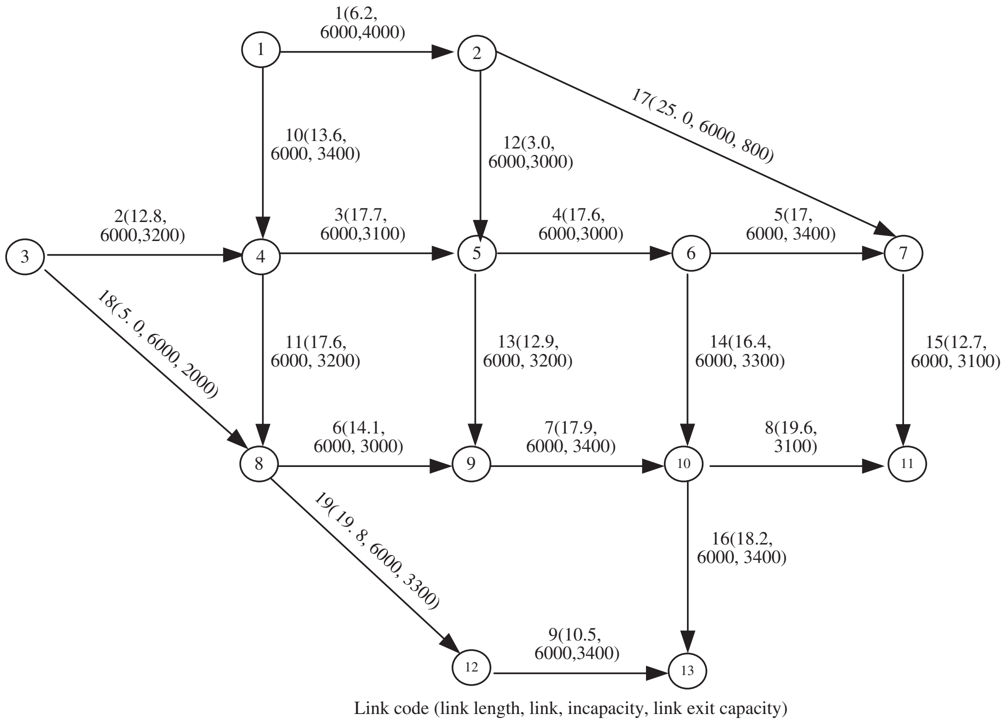

Fig. 2. Simulation network

Table 1. Path-link series

From origin 3 to destination 13		From node 2 to destination 11	
Path code	Path-link series	Path code	Path-link series
OD2–1	18–19–9	OD3–1	17–15
OD2–2	2–11–19–9	OD3–2	12–4–5–15
OD2–3	18–6–7–16	OD3–3	12–4–14–8
OD2–4	2–3–4–14–16	OD3–4	12–13–7–8
OD2–5	2–3–13–7–16		
OD2–6	2–11–6–7–16		

0.5), and the link 18 is the first link of path OD2–1 between OD pair (3 and 13) (in Table 1). Further, after time 0.05, the instantaneous travel time of car on path OD2–2 between OD pair (3 and 13) become minimal, thus the flows of car start to turn into the first link 2 of path OD2–2 between OD pair (3 and 13), and at the same time, the inflow of car on link 18 disappear.

From Figure 4, the instantaneous travel time of the truck on path OD2–1 between OD pair (3 and 13) is minimal during the whole study period, so the truck travelers will choose path OD2–1 between OD pair (3 and 13) to travel, then the flows of truck enter into the first link 18 of path OD2–1 between OD pair (3 and 13), and at the same time, the inflows of truck on link 2 do not occur.

According to the above example, one can find the travelers of the car and the truck always choose minimal instantaneous travel time path to travel at each time interval between one OD pair. In other words, the travelers by car and truck follow the reactive dynamic user equilibrium condition.

Meanwhile, in order to reflect the travel choice behaviors at the nodes (not origin) of network, Figures 5 and 6 give the inflows and the outflows of car and truck on links 12 and 17, and the instantaneous travel time of four paths (in Table 1) from node 2 to destination 11, respectively.

From Figure 5, because the instantaneous travel time of car on path OD3–1 from node 2 to destination 11 is minimal until time 0.67, the car travelers enter entirely into the first link 17 of the path OD3–1 between node 2 and destination 11 (in Table 1). Then the instantaneous travel time of the car on path OD3–2 between node 2 and destination 11 become minimal after time 0.67, and the flows of car turn into the first link 12 of path OD3–2 between node 2 and destination 11. Similarly, according to Figure 6, the instantaneous travel time of the truck on path OD3–1 between node 2 and destination 11 is minimal until time 0.73, thus the truck travelers enter into the first link 17 of the path OD3–1 between node 2 and destination 11. After time 0.73, the instantaneous travel time of the truck on path OD3–2 between node 2 and destination 11 become minimal, thus the truck travelers turn into the first link 12 of path OD3–2.

According to the explanations of Figures 5 and 6, one can find the travelers of car and truck

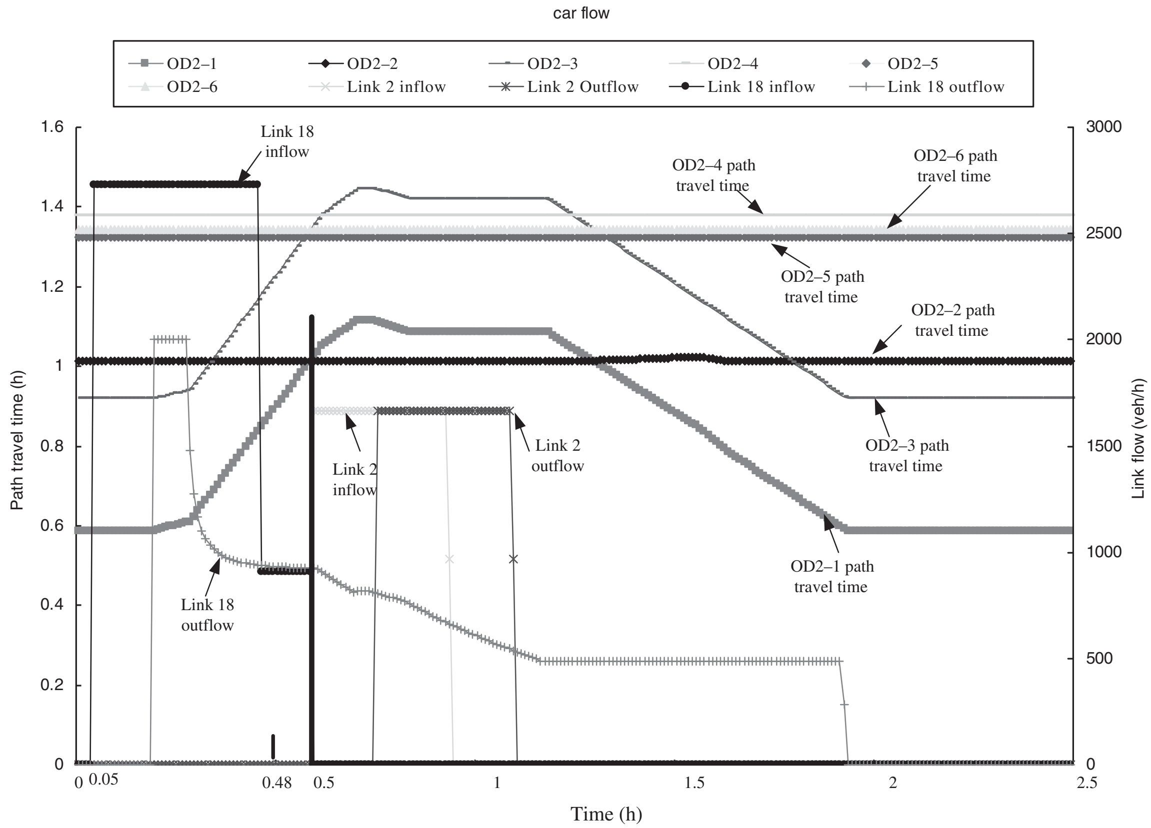

Fig. 3. The car flow and instantaneous travel time on Links 2 and 18

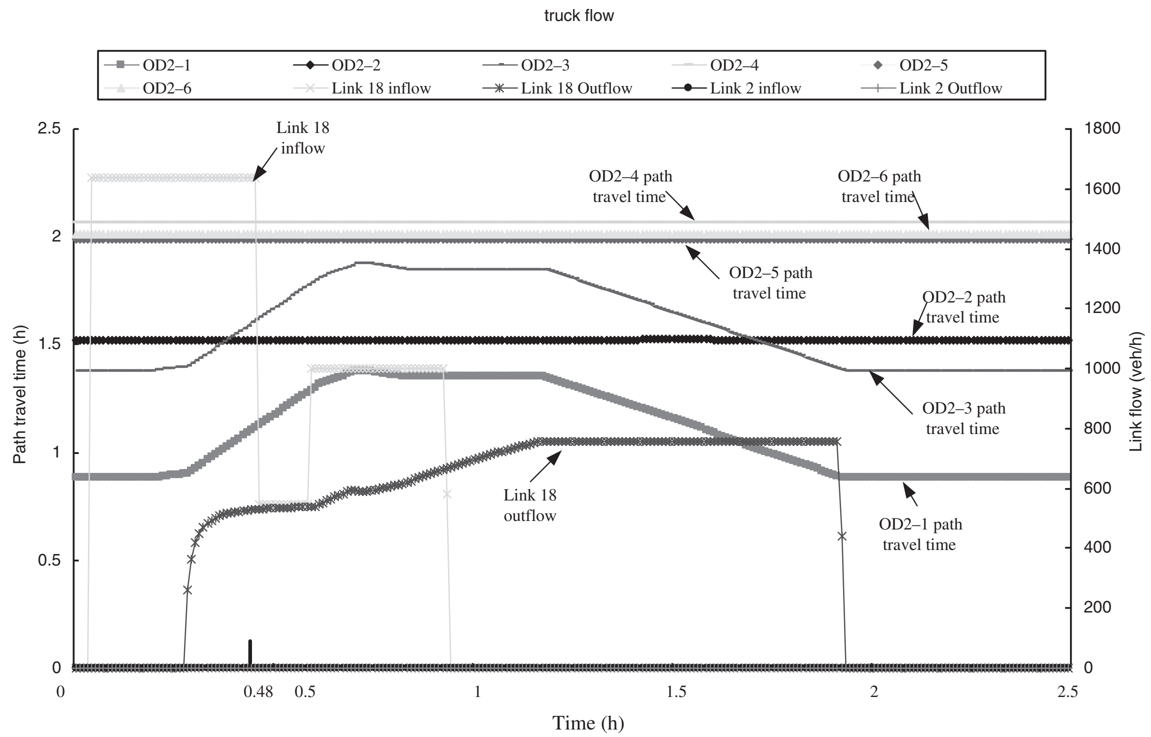

Fig. 4. The truck flow and instantaneous travel time on Links 2 and 18

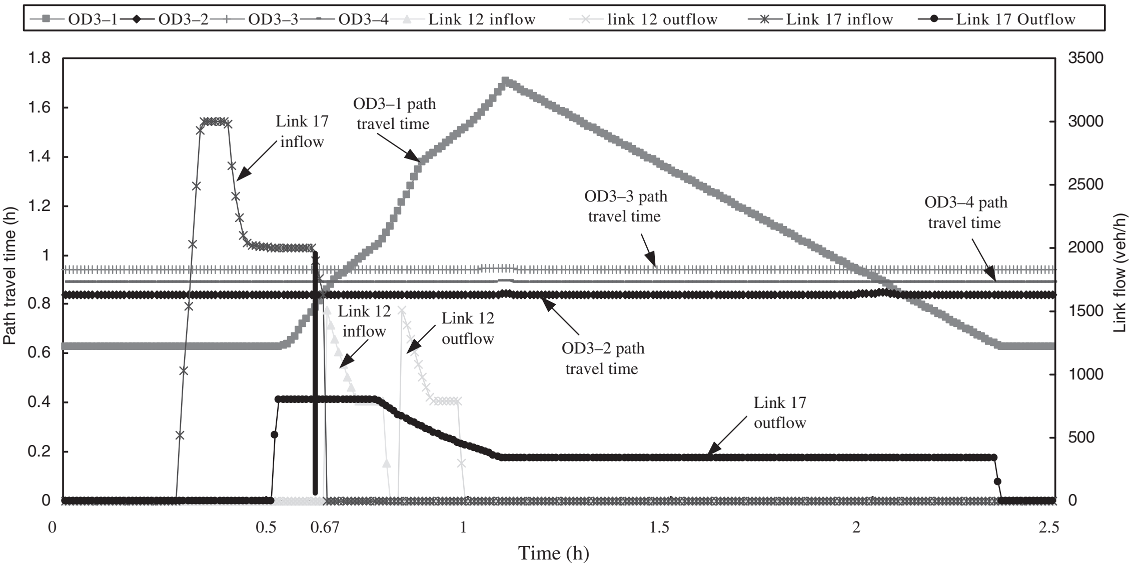

Fig. 5. Car flow and instantaneous travel time on Links 12 and 17

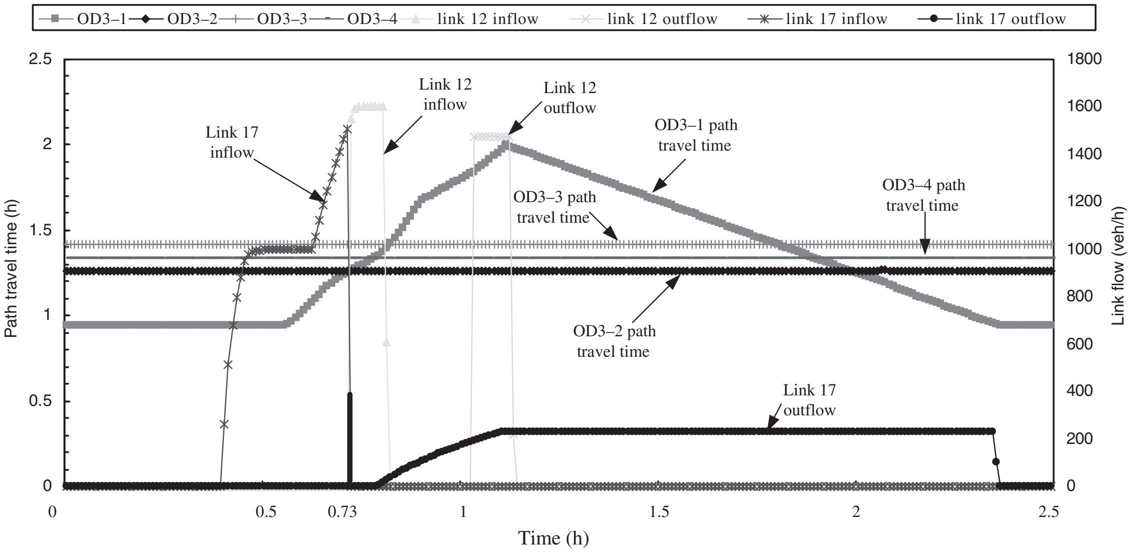

Fig. 6. Truck flow and instantaneous travel time on Links 12 and 17

always choose minimal instantaneous travel time path to travel at each time interval at decision node 2, on the other hand, the travelers of car and truck both follow the reactive dynamic user equilibrium condition at each time interval at the decision node.

Conclusions

The paper extends the single-vehicle type reactive dynamic user equilibrium with physical queue to multiple-vehicle type one so as to describe the multi-type dynamic queue backing up phenomena.

Based on the single-type cumulative curve given by Newell (1993), further the multiple-vehicle type kinematic wave theory is analyzed and multiple-vehicle type reactive dynamic user equilibrium condition with physical queue is presented.

Finally, a simple algorithm is proposed, and the results show dynamic traffic model based on the cumulative flow can reflect physical queue conditions. Further, car and truck travelers always choose minimal instantaneous travel time path to travel at each time interval at each decision node, and the multiple-vehicle type reactive dynamic user equilibrium condition can be satisfied at each time interval at each decision node.

Acknowledgments

Financial supports provided by the National Natural Science Foundation of China (No. 60804049) and the Special Fund for Basic Scientific Research of Central Colleges provided by Chang'an University (CHD

2010JC022, CHD2012DK09) are acknowledged. The paper was also partly supported by the National High-Technology Research and Development (863) Programme of China (grant no. 2011AA110404) in China.

References

Bliemer, M. C. J. 2007. Dynamic queuing and spillback in analytical multiclass dynamic network loading model, *Transportation Research Record* 2029: 14–21. http://dx.doi.org/10.3141/2029-02

Bliemer, M. C. J.; Bovy, P. H. L. 2003. Quasi-variational inequality formulation of the multiclass dynamic traffic assignment problem, *Transportation Research Part B: Methodological* 37(6): 501–519. http://dx.doi.org/10.1016/S0191-2615(02)00025-5

Gentile, G.; Meschini, L.; Papola, N. 2007. Spillback congestion in dynamic traffic assignment: a macroscopic flow model with time-varying bottlenecks, *Transportation Research Part B: Methodological* 41(10): 1114–1138. http://dx.doi.org/10.1016/j.trb.2007.04.011

Huang, H.-J.; Lam, W. H. K. 2002. Modeling and solving the dynamic user equilibrium route and departure time choice problem in network with queues, *Transportation Research Part B: Methodological* 36(3): 253–273. http://dx.doi.org/10.1016/S0191-2615(00)00049-7

Kuwahara, M.; Akamatsu, T. 1997. Decomposition of the reactive dynamic assignments with queues for a many-to-many origin-destination pattern, *Transportation Research Part B: Methodological* 31(1): 1–10. http://dx.doi.org/10.1016/S0191-2615(96)00020-3

Kuwahara, M.; Akamatsu, T. 2001. Dynamic user optimal assignment with physical queues for a many-to-many OD pattern, *Transportation Research Part B: Methodological* 35(5): 461–479. http://dx.doi.org/10.1016/S0191-2615(00)00005-9

Lebacque, J. P.; Lesort, J. B.; Giorgi, F. 1998. Introducing buses into first-order macroscopic traffic flow models, *Transportation Research Record* 1644: 70–79. http://dx.doi.org/10.3141/1644-08

Li, S. 2010. A simplified theory of multiple vehicle-type macroscopic kinematic waves based on the cumulative flow, in *Proceedings of the 29th Chinese Control Conference*, 29–31 July 2010, Beijing: Chinese Association of Automation, 5404–5407.

Li, J.; Fujiwara, O.; Kawakami, S. 2000. A reactive dynamic user equilibrium model in network with queues, *Transportation Research Part B: Methodological* 34(8): 605–624. http://dx.doi.org/10.1016/S0191-2615(99)00040-5

Li, S.; Ju, Y. 2009. Evaluation of bus-exclusive lanes, *IEEE Transactions on Intelligent Transportation Systems* 10(2): 236–245. http://dx.doi.org/10.1109/TITS.2009.2018326

Li, S.; Su, Y. 2005. Multimode stochastic dynamic simultaneous route/departure time equilibrium problem with queues, *Journal of the Eastern Asia Society for Transportation Studies* 6: 2092–2107.

Li, S.; Xu, H. 2009. Physical-queue discrete-time dynamic network loading with multiple vehicle types, *Journal of Transportation Systems Engineering and Information Technology* 9(1): 56–61. http://dx.doi.org/10.1016/S1570-6672(08)60045-8

Lo, H. K.; Szeto, W. Y. 2002. A cell-based variational inequality formulation of the dynamic user optimal assignment problem, *Transportation Research Part B: Methodological* 36(5): 421–443. http://dx.doi.org/10.1016/S0191-2615(01)00011-X

Logghe, S.; Immers, L. H. 2008. Multi-class kinematic wave theory of traffic flow, *Transportation Research Part B: Methodological* 42(6): 523–541. http://dx.doi.org/10.1016/j.trb.2007.11.001

Newell, G. F. 1993. A simplified theory of kinematic waves in highway traffic, part I: general theory, *Transportation Research Part B: Methodological* 27(4): 281–287. http://dx.doi.org/10.1016/0191-2615(93)90038-C

Ngoduy, D. 2008. Applicable filtering framework for online multiclass freeway network estimation, *Physica A: Statistical Mechanics and its Applications* 387(2–3): 599–616. http://dx.doi.org/10.1016/j.physa.2007.10.013

Ngoduy, D.; Liu, R. 2007. Multiclass first-order simulation model to explain non-linear traffic phenomena, *Physica A: Statistical Mechanics and its Applications* 385(2): 667–682. http://dx.doi.org/10.1016/j.physa.2007.07.041

Nie, Y. M.; Zhang, H. (M.) 2010. Solving the dynamic user optimal assignment problem considering queue spillback, *Networks and Spatial Economics* 10(1): 49–71. http://dx.doi.org/10.1007/s11067-007-9022-y

Ran, B.; Boyce, D. E. 1996. A link-based variational inequality formulation of ideal dynamic user-optimal route choice problem, *Transportation Research Part C: Emerging Technologies* 4(1): 1–12. http://dx.doi.org/10.1016/0968-090X(95)00017-D

Ran, B.; Boyce, D. E.; LeBlanc, L. J. 1993. A new class of instantaneous dynamic user-optimal traffic assignment models, *Operations Research* 41(1): 192–202. http://dx.doi.org/10.1287/opre.41.1.192

Szeto, W. Y.; Lo, H. K. 2004. A cell-based simultaneous route and departure time choice model with elastic demand, *Transportation Research Part B: Methodological* 38(7): 593–612. http://dx.doi.org/10.1016/j.trb.2003.05.001

Wong, G. C. K.; Wong, S. C. 2002. A multi-class traffic flow model – an extension of LWR model with heterogeneous drivers, *Transportation Research Part A: Policy and Practice* 36(9): 827–841. http://dx.doi.org/10.1016/S0965-8564(01)00042-8

Zhang, H. M.; Jin, W. L. 2002. Kinematic wave traffic flow model for mixed traffic, *Transportation Research Record* 1802: 197–204. http://dx.doi.org/10.3141/1802-22

Zhu, Z.; Chang, G.-L.; Wu, T. 2003. Numerical analysis of freeway traffic flow dynamics for multiclass drivers, *Transportation Research Record* 1852: 201–208. http://dx.doi.org/10.3141/1852-25