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Original Article

REAL-TIME INFORMATION-BASED COMBINED CONTROL METHOD FOR BUS DELAY

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Highlights:
- article proposes a composite control strategy integrating speed control and spare bus replacement to address bus delays;
- article establishes a bi-objective control optimization model to minimize passengers' travel time costs and the energy consumption costs of bus companies;
- article verifies the performance of the composite strategy-based control optimization model under both off-peak and peak scenarios;
- article compares and confirms the composite strategy-based control optimization model to evaluate the advantages of the composite control strategy.

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Abstract. As environmental pollution and energy consumption become increasingly serious concerns, more cities are opting for electric buses over traditional fuel buses. However, the stability and reliability of electric buses during operation are challenged by the unpredictability of traffic flow and passenger demand. Factors such as traffic congestion, weather conditions, and fluctuations in passenger numbers can compromise the punctuality of electric bus services, often resulting in delays. To address these challenges, a real-time dual-objective bus control model for mixed traffic scenarios has been proposed. This model aims to minimize both passenger time costs and company operational costs. Factors such as intersections and traffic flow are also considered. A combined control strategy, including speed control and a backup bus replacement strategy, has been proposed. Speed control is specifically aimed at managing intersection delays, allowing buses to adjust their speeds to pass through intersections optimally between queue dissipation and the end of the green-light period. The backup bus replacement strategy, on the other hand, is implemented at bus terminals, where a backup bus replaces a delayed one to maintain the schedule. A heuristic algorithm based on Particle Swarm Optimization (PSO) is incorporated into the model, enhancing its effectiveness by iteratively updating the positions and velocities of particles in the search space. Harbin City Road 96 was selected as a case study for model validation. In the off-peak case scenario, schedule deviation was reduced by 89% through the implementation of the proposed speed control strategy. Additionally, passenger waiting time was reduced by 8%, and passenger travel time was reduced by 14%. In the peak case scenario, the proposed control strategy effectively eliminated bus departure delays originating within the bus system. These results demonstrate the potential of the proposed model to significantly enhance the reliability and stability of public transportation systems, thereby improving the overall quality of public transport services.

Keywords: electric bus, speed control, backup bus replacement strategy, dual-objective model, heuristic algorithm.

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Notations

Abbreviations:

BTR – bus rapid transit;
PSO – particle swarm optimization.

Symbols:

A – windward area of a bus;
$A_{i,j}$ – planned arrival time of bus i at stop j;
$a_{i,j}$ – actual arrival time of bus i at stop j;
$a_{i,J}$ – arrival time of bus i at the terminal predicted using the prediction algorithm;
a_{ij} – acceleration of bus i traveling between stop j and stop $j+1$ when there is no intersection between adjacent stops;
a_{ij}^{n} – acceleration of bus i in the nth segment between stop j and stop $j+1$;
a_n – number of signal cycles when bus i arrives at the nth intersection between stop j and stop $j+1$;

b_n – number of signal cycles that are expected to be experienced by the bus through the nth signalized intersection in the case of traveling at the maximum allowable speed to compensate for the delay;
C_D – air drag coefficient;
c_1 – learning factor, representing the influence weight of individual cognitive part;
c_2 – learning factor, representing the influence weight of global cognitive part;
$D'_{i,J}$ – planned departure time of bus i on the lower line;
$d_{i,j}$ – actual departure time of bus i from stop j;
$d'_{i,J}$ – actual departure time of bus i on the lower line.
E_A – energy consumption from bus acceleration;
E_U – energy consumption generated by a bus at constant speed;
F_1 – passenger time cost, including passenger waiting time and in-vehicle travel time;
F_2 – total energy consumption of the bus system, including acceleration process energy and constant-speed process energy;
f – rolling resistance coefficient of buses;
g – gravitational acceleration;
$gbest_j(t)$ – global optimal position found by all particles in the space;
$h_{i,j}$ – service time of bus i at stop j;
K_i – traffic flow densities before intersection;
K_j – traffic flow densities after intersection;
L_Q – queue length;
l_{ij} – distance between stop j and stop j + 1 when there are no intersections between adjacent stops;
l_{ij}^n – distance of bus i in the nth segment between stop j and stop j + 1;
M – the total weight of a bus;
m_p – the average passenger mass of a bus;
m_v – the curb weight of a bus;
$N_{i,j}$ – number of passengers on bus i when it leaves stop j;
n_p – population size of the PSO algorithm;
$O_{i,j}$ – number of passengers boarding to bus i when it stops at stop j;
$pbest_{ij}(t)$ – individual optimal position of the particle in the space;
Q_E – rated number of passengers on bus i;
$Q_{i,j}$ – number of passengers in bus i at bus stop j;
$q_{i,j}^d$ – number of passengers alighting from bus i at bus stop j;
$q_{i,j}^u$ – number of passengers boarding to bus i at bus stop j;
r_1 – random number within the interval [0,1];
r_2 – random number within the interval [0,1];
$r_{i,j}$ – running time of bus i between stops j and j + 1;
T_D – queue dissipation time at a green-light at an intersection;
T_{ij}^n – theoretical time required for bus i to travel through the nth segment between stop j and stop j + 1 at the maximum allowable speed v_{max};
T_{max} – maximum number of iterations of the PSO algorithm;
T_T – passenger travel time on a bus;
T_W – passenger waiting time at a stop;
t_0^n – residual time in the nth signal cycle when bus i departs from stop j;
t_c^n – cycle times of traffic signal;
t_g^n – duration of green-light of traffic signal;
t_r^n – duration of red-light of traffic signal;
t_h – threshold values for control parameter settings;
t_{ij} – travel time of bus i in the segment between stop j and stop j + 1;
t_{ij}^n – travel time of bus i in the nth segment between stop j and stop j + 1;
t_{ij}^N – time of bus i in the last segment between stop j and stop j + 1;
t_{ij}^{acc} – acceleration travel time of bus i between stop j and stop j + 1;
t_{ij}^{cru} – uniform travel time of bus i between stop j and stop j + 1;
$t_{i,j}^d$ – time required for passengers to alight bus i at stop j;
$t_{i,j}^u$ – time required for passengers to board bus i at stop j;
V_f – speed at which a bus can reach in ideal conditions on the road;
V_{max} – maximum velocity of the particles;
V_{min} – minimum velocity of the particles;
V_{ra} – rated bus speed;
V_{ro} – urban road design speed;
V_{tr} – traffic flow speed;
v_{ij} – travel speed of bus i in the segment between stop j and stop j + 1;
$v_{i,j}$ – actual cruising speed of bus i in the segment between stop j and stop j + 1;
$v_{ij}(t)$ – velocity of the jth dimension of particle i in the PSO algorithm at the tth iteration;
$v_{ij}(t + 1)$ – the updated velocity of the particle;
v_{ij}^n – speed of bus i in the nth segment between stops j and stop j + 1;
$v_{ij}^n\left(t\right)$ – function of speed with respect to time when bus i accelerates between stop j and stop j + 1;
v_{max} – maximum speed that can be achieved under current conditions;
W – traffic wave speed;
w – the inertia weight controls the amount of particle velocity retention;
w_{max} – maximum value of the inertia weight in the PSO algorithm;
w_{min} – minimum value of the inertia weight in the PSO algorithm;
x_{ij} – binary variable indicating whether the speed control strategy is triggered for bus i at stop j;

$x_{ij}(t)$ – current position of the particle in the PSO algorithm;
$x_{m,p}$ – initial position of particle in the PSO algorithm, used to encode the speed of buses in the optimization process;
y_{ij} – binary variable indicating whether the backup bus replacement strategy is triggered for bus i at stop j;
α – road gradient;
α_1 – weighting factor for passenger waiting time;
α_2 – weighting factor for passenger travel time;
δ – rotating mass conversion factor;
η_i – vehicle density after density normalization;
η_1 – normalized traffic density coefficient during red-light phase (congestion propagation);
η_2 – normalized traffic density coefficient during green-light phase (queue dissipation);
η_m – motor efficiency of buses;
η_{ness} – inverter efficiency of buses;
η_t – mechanical transmission efficiency of buses;
$\iota_J(t)$ – number of spare cars in the yard at time t.

1. Introduction

This article is structured as follows: Section 1 introduces the research problem. Section 2 provides a review of the relevant literature. Section 3 describes a control scenario and develops a combined control model for buses. The algorithm underpinning the model is detailed in Section 4. Section 5 presents 2 real-world case studies to validate the model; these case studies highlight different levels of delay scenarios and demonstrate the effectiveness of the approach. Finally, Section 6 concludes the article, summarizing the findings and implications of the research.

As the global economy continues to develop, city sizes and populations are expanding, leading to a marked increase in the number of private cars owned by residents (Han *et al.* 2018). However, infrastructure development has not kept pace with the growth in transportation needs. There is a significant gap between the supply of urban transport and the travel demands of residents. Traffic congestion has become particularly severe in urban areas. In response, many cities are placing a greater emphasis on public transportation. Additionally, concerns about air quality, greenhouse gas emissions, and energy demand are intensifying. The electrification of buses has been identified as a significant measure to reduce environmental impacts and decrease the exploitation of natural resources (Alamatsaz *et al.* 2022).

However, the operation of buses has revealed several issues, including low stability and reliability, which can negatively influence residents' choices of public transport. Common operational instabilities in buses, such as "bunching" and large intervals between adjacent buses, often lead to untimely arrivals and prolonged waiting times for passengers. These issues can diminish passenger travel efficiency and reduce the level of service provided by public transport. Therefore, improving the stability and reliability of bus operations is crucial for enhancing the travel experience for passengers and increasing the attractiveness of public transport as a preferred mode of travel.

Many scholars have extensively studied bus scheduling and control, designing various models suited to different scenarios. The goal is to minimize bus operation delays and enhance the reliability of the public transit system. This article proposes a composite control model that combines speed control based on traffic signals with the spare bus replacement strategy at the terminal station. This model is specifically aimed at addressing bus delays, a critical factor in improving the reliability of public transportation systems and enhancing passenger travel efficiency.

2. Literature review

Recently, several bus control models have been developed. These models employ various strategies such as speed control, stop skipping, and bus signal priority to improve the reliability of bus operations. Muñoz *et al.* (2013) classified different control strategies into 3 categories:

- station control, including bus holding and stop skipping;
- interstation control, including operating speed control, bus overtaking, and traffic signal priority mechanisms;
- other control measures such as adding vehicles.

In this study, based on their work, control methods were classified into 3 categories of:

- interval control;
- station control;
- combined control.

The speed control method is widely used in public transport to maintain stable bus headways, proving effective in various studies. Zhao *et al.* (2006) investigated the determination of optimal slack time based on a timetable, aiming to minimize passenger expected waiting time. They provided conditions for implementing interval journey speed control. Daganzo & Pilachowski (2011) proposed an adaptive control scheme that adjusts the cruising speed of a bus in real-time according to its front-to-back spacing. This scheme achieves a normal frequency faster than previous control methods. Yang *et al.* (2016) developed an optimization control model for adjusting operating speed and scheduling based on the timetable, which reduced the total energy consumption of the system. Deng *et al.* (2020) proposed a real-time speed control modelling approach by setting up 3 typical infrastructure scenarios on a single route, effectively suppressing deviations in departure intervals. Ampountolas & Kring (2021) used a follow-the-leader 2-bus system to obtain the position and speed of the leading bus on a route. They controlled the speed of the following bus, alleviating the bus-bunching problem and improving service regularity.

In addition, as a supplementary means, the bus signal priority strategy plays a key role in dealing with actual operation delays. This method helps buses recover their scheduled timetable faster by dynamically adjusting the timing of traffic signals, thereby improving the efficiency and punctuality of the entire public transportation system. Janos & Furth (2002) proposed a conditional public transit signal priority strategy, including green extension, green early, and red early. This strategy improves the operational efficiency of public transit while minimizing the impact on traffic flow. Ma *et al.* (2010) established an optimal combination of intersection group priority strategies that results in the actual delay time of buses approaching the allowable delay specified by the bus operation system. He *et al.* (2016) developed an adaptive control algorithm suitable for pre-signals tailored to real-time private and public transportation demands, thereby minimizing the average pedestrian delay at intersections. Chow *et al.* (2017) developed an optimization control formula to maximize the reliability of bus services by optimizing the adjustment of signal timings. Anderson & Daganzo (2019) proposed a novel form of signal priority called conditional signal priority. They developed a model and conducted simulation experiments to validate the effectiveness of their model at improving the reliability of bus systems. Seman *et al.* (2020) integrated bus headways and signal controls using real-time dynamic control methods. They also coordinated the movement of opposing buses in bidirectional lanes, resulting in a reduction in total passenger waiting time.

The stopping strategy is one of the most widely studied strategies at present. With the goal of maintaining a predetermined driving schedule, it stabilizes the operation order of buses on an entire line by controlling the departure times of public transit vehicles at specific bus stops or other set control points. Cats *et al.* (2011) and others established a dynamic bus simulation model by adjusting the stopping strategy to select stopping points and the dwell time at stops, thereby reducing passengers' waiting time. Van Oort *et al.* (2012) and others set the travel time and stopping time of buses based on historical data, which can reduce the additional travel time by up to 60%. Based on the stopping strategy, the optimal equation is derived through stochastic decision-making, and the optimal holding strategy is found using backward induction (Berrebi *et al.* 2015). This strategy requires less information and is very close to the optimal scheduling strategy. Liang *et al.* (2016) proposed a bus stopping control strategy that effectively adjusts the headway by comparing the operational data of buses before and after the control point.

Skip-stop control, as another important method to improve the stability of public transportation operations, is suitable for situations where passenger flow is highly concentrated during peak hours on weekdays. By selectively skipping certain stops, the congestion inside the bus can be effectively relieved. The operational stability and punctuality of the public transportation system are ensured. Passengers can enjoy more reliable and efficient travel services. Fu *et al.* (2003) proposed a new transit operation strategy that allows bus rapid service to skip some stops and improve the stability of the bus system, while Sun & Hickman (2005) studied skip control for different lengths of service interruptions and proposed the possibility of controlling vehicles to still let passengers off at stops on skipped sections as an alternative to improve the reliability of the bus system based on consideration of passenger benefits. Liu *et al.* (2013) used skip as an optimization model to minimize the weighted sum of the total waiting time, total on-board travel time, and total operating costs by considering random transit travel times. Chen *et al.* (2015b) optimized the operational efficiency and quality of the BRT system through a skip-stop strategy. Chen *et al.* (2015a) proposed a control strategy in which all buses can skip stops, but no stop can be skipped by 2 consecutive buses, which improves the stability of the transit system.

In addition to stopping control and skip-stop strategies, there are some other stop control methods. Daganzo (2009) dynamically adjusted the service time at stops based on real-time information to maintain a stable headway on the route. Compared to traditional methods, this approach allows for faster travel speeds, improving the operational efficiency of buses. Petit *et al.* (2018, 2019) maintained the punctuality of bus services by setting up spare buses to avoid bus-bunching, thereby improving the operational stability of the transit system. However, this requires the establishment of spare stations and vehicles, which increases operating costs. Wang *et al.* (2021) proposed a proactive real-time control method based on data-driven multi-source traffic data for bus demand prediction. By predicting potential future disruptions and optimizing dispatching methods, the reliability of service is improved. Gkiotsalitis (2021) considered scheduled charging time in the objective function to establish a bus waiting problem model for electric buses. With a slight increase in passenger time, the charging time is greatly reduced, improving the operational efficiency of buses.

In order to further enhance the stability and operational efficiency of the public transportation system, in addition to adopting a single control method, many scholars and researchers have begun to explore the possibility of combining multiple control strategies. This comprehensive approach aims to fully leverage the advantages of various control strategies in order to achieve a better overall effect. Koehler & Kraus (2010) combined bus signal priority with departure control to avoid queuing time for buses at intersections. Ma *et al.* (2012) proposed an auxiliary operation system that can adjust the bus speed and stop time in real-time to reduce system energy consumption and improve the service level of the bus. Estrada *et al.* (2016) proposed a novel dynamic bus control strategy that considered the capacity constraints of buses; this strategy controls the bus speed and combines it with signal control to reduce the overall system cost. Nesheli *et al.* (2016) established an operational strategy "library" based on real-time information, which reduces bus-bunching; Koehler *et al.* (2019) proposed a real-time holding and priority control strategy for BRT and high-frequency segregated transit systems. This strategy aims to minimize the total delay experienced by passengers onboard and at bus stations. Cao *et al.* (2019) built on their work and focused on autonomous public transport vehicles as a control objective. They established discrete event modelling using a time-space matrix within a simulation framework, and aimed to minimize the weighted sum of schedule deviation, passenger travel time, and energy consumption of buses through bus holding and speed control. Bie *et al.* (2020) reduced the headway deviation of buses by combining adjustments to the overall traffic signal timing and speed guidance. Zhang *et al.* (2021) proposed a real-time hybrid control strategy that reduces passenger waiting times and increases the reliability of bus services through real-time skip control and the bus holding method.

The broader concept of bus operation stability is a key focus in the study of urban bus operation control. However, the issue of bus operation delays has not been deeply studied. When bus delays are too long, existing control strategies may struggle to effectively address the issue, leading to problems such as chaotic bus operations and a decline in service quality. Therefore, this article aims to explore the scenario of bus operation delays and propose corresponding control strategies to enhance the overall operational efficiency of the bus system and improve passenger satisfaction. At present, the method of adjusting the interval average speed is used as a speed control strategy. However, when traffic signals are present within a section, the red-light phase may further increase the delay time. This article takes into account the phases of traffic signals and bus queuing times in the model, aiming to accurately reduce delay times. In addition, by parking backup bus at the terminal station, the overall operating costs of the bus system can be effectively reduced.

3. Control strategies and model construction

3.1. Problem description

In the daily operation of public transportation, the bus-bunching problem is caused by several factors, including traffic congestion, intersection delays, and fluctuations in passenger demand. This results in buses being unable to operate on schedule, leading to issues such as delays and early arrivals, thereby increasing passenger time costs, irritating passengers, and diminishing the attractiveness of and reducing trust in public transportation. Therefore, this study focuses on the problem of bus delays. Different studies have defined bus delay differently (Huo *et al.* 2018). In this study, buses arriving at stops later than scheduled are defined as delayed buses. 3 common delay scenarios are illustrated in Figure 1. Bus delays caused by traffic congestion on the road section are represented in Figure 1a. Bus delays caused by red-light phases at intersections are represented in Figure 1b. Bus delays caused by excessive passenger demand at stops are represented in Figure 1c.

Consider a high-frequency bus route operating with I buses and J bus stops. Bus i is ahead of bus j + 1 and the bus line sequentially passes through stops j and j + 1. The operation of these buses is influenced by other vehicles and intersections. Access to basic traffic flow information, weather conditions, and passenger flow data is available. Moreover, buses can be controlled in real-time through a comprehensive traffic management system. This setup allows for adjustments to be made dynamically to enhance service efficiency and reliability.

3.2. Control strategies

3.2.1. Speed control

This control strategy is triggered when a bus is behind schedule. The relationship between bus arrival time and speed control is intuitively represented by the binary variable x_{ij}, this strategy is used to hold a bus to its planned schedule:

$$x_{ij} = \begin{cases} 0, & a_{i,j} - A_{i,j} \le t_h; \\ 1, & a_{i,j} - A_{i,j} > t_h. \end{cases} \quad (1)$$

The implementation of dedicated bus lanes and bus signal priority is limited in extent and scale. Therefore, bus lanes and bus signal priority are not considered in the model. In contrast, the key factor that limits urban road traffic capacity, namely intersections – are focused on. In the absence of bus signal priority, delays experienced by buses at intersections are a major cause of bus delays. As a result, addressing the issue of intersection delays was considered as the main problem in the research. For analytical purposes, the road sections between adjacent bus stops were divided into N segments with intersections acting as nodes. The relationship between the distance, theoretical travel time, and theoretical speed can be determined using Equation (2):

$$v_{ij}^1 : v_{ij}^2 : \ldots : v_{ij}^n : \ldots : v_{ij}^N = \frac{l_{ij}^1}{t_{ij}^1} : \frac{l_{ij}^2}{t_{ij}^2} : \ldots : \frac{l_{ij}^n}{t_{ij}^n} : \ldots : \frac{l_{ij}^N}{t_{ij}^N}. \quad (2)$$

The arrival time in relation to travel time can be calculated using Equation (3):

$$a_{i,j+1} = d_{i,j} + t_{ij}^1 + t_{ij}^2 + \ldots + t_{ij}^n + \ldots + t_{ij}^N. \quad (3)$$

The relationship between travel time and the number of signal cycles a_n is expressed by Equation (4):

$$a_n = \frac{t_{ij}^1 + t_{ij}^2 + \ldots + t_{ij}^n - t_0^n}{t_c^n}. \quad (4)$$

To conserve energy, it is essential to minimize the idle running times of buses. Therefore, buses must pass through intersections without coming to a complete stop. According to traffic wave theory, the process of a vehicle passing through an intersection involves 2 stages:

- the 1st stage, the accumulation process, occurs when vehicles queue up and form a platoon;
- the 2nd stage, the dissipation process, involves the dispersal of the queue, creating a dissipating wave.

The speed of this wave can be calculated using traffic wave theory. By utilizing Greenhill's linear model and nor-

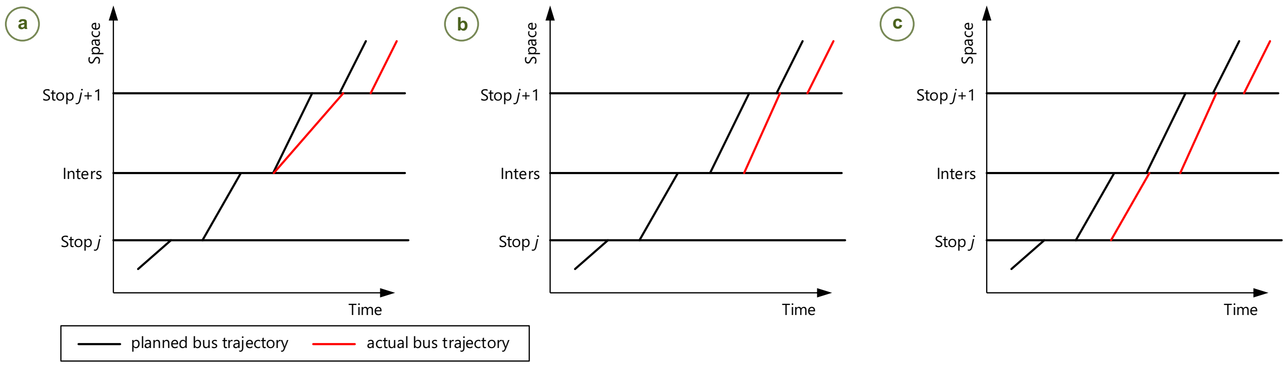


Figure 1. Visualization of bus delay scenarios (inters = intersection)

malizing the density, Equation (6) is derived. The process of buses passing through the intersection is demonstrated in Figure 2. Buses pass through the intersection after the dissipation of queuing time during the green phase.

$$\eta_i = \frac{K_i}{K_j}; \tag{5}$$

$$W = V_f \cdot \left(1 - \eta_1 - \eta_2\right). \tag{6}$$

The traffic flow speed and traffic flow density in front of an intersection are represented by V_1 and K_1, respectively. After the intersection, the traffic flow speed and density are represented by V_2 and K_2, respectively. When the signal is red, vehicles stop at the stop line. At this point, where $K_2 = K_j$ and $\eta_2 = 1$, the congestion wave propagates backward at the speed of $V_f \cdot \eta_1$ and it takes t_r^n to travel. Similarly, when the signal turns green, the dispersion wave propagates backward at a speed of $V_f \cdot \eta_2$, which is equivalent to $V_f - V_2$. Because of the initially small value of V_2, the wave propagates backward at a speed of V_f. The queue length L_Q, dissipation time T_D, and ideal time t_{ij}^n for bus i to reach intersection n are defined in Equations (7)–(9):

$$L_Q = V_f \cdot \eta_1 \cdot t_r^n; \tag{7}$$

$$T_D = \frac{V_f \cdot \eta_1 \cdot t_r^n}{V_f} = \eta_1 \cdot t_r^n; \tag{8}$$

$$t_0^n + a_n \cdot t_c^n + T_D \le t_{ij}^n < t_0^n + a_n \cdot t_c^n + t_g^n. \tag{9}$$

As the delay continues to increase, buses become unable to correct the delay at the next stop, even when traveling at their maximum speed. Therefore, the delay time of the bus should be minimized. $v_{\max}$ represents the maximum speed that can be achieved under current conditions. The theoretical time for bus i to pass through intersection n is expressed in Equation (10). The theoretical number of signal cycles required for bus i to pass through intersection n is defined in Equation (11). The ideal time range for bus i to reach intersection n is defined in Equation (12).

$$T_{ij}^n = \frac{l_{ij}^n}{v_{\max}}; \tag{10}$$

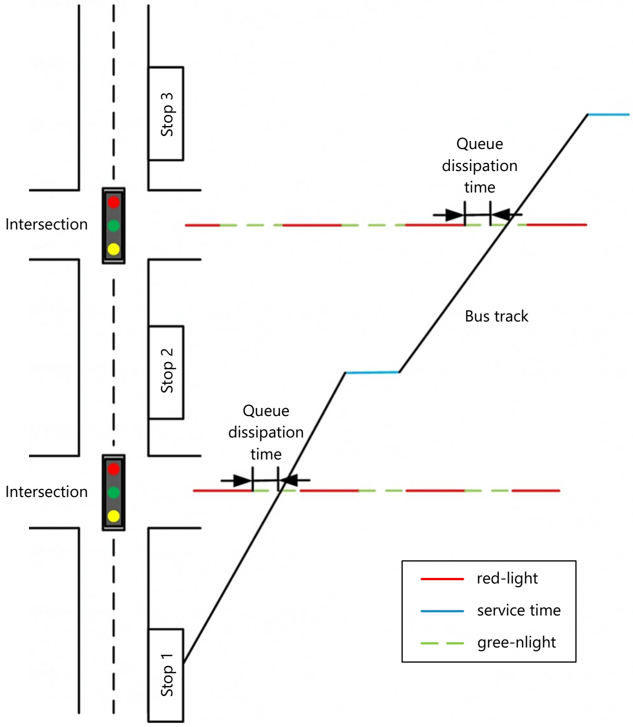


Figure 2. Schematic diagram of a bus passing through an intersection

$$b_n = \begin{cases} a_n, & t_{ij}^n + t_{ij}^{n-1} + \ldots + t_{ij}^1 - t_0^n - a_n \cdot t_c^n - t_g^n \le 0; \\ a_n + 1, & t_{ij}^n + t_{ij}^{n-1} + \ldots + t_{ij}^1 - t_0^n - a_n \cdot t_c^n - t_g^n > 0; \end{cases} \tag{11}$$

$$t_0^n + b_n \cdot t_c^n + T_D \le t_{ij}^{n'} < t_0^n + b_n \cdot t_c^n + t_g^n. \tag{12}$$

Equations (9) and (12) define the ideal time range for bus i to reach intersection n at stops j and $j + 1$. Equations (13)–(15) are the expressions for the relationship between speed v_{ij}^n, time t_{ij}^n, acceleration a_{ij}^n, and distance l_{ij}^n for bus i at stop j and stop $j + 1$ in the 1st, nth, and Nth segments, respectively:

$$t_{ij}^1 = \frac{v_{ij}^1}{a_{ij}^1} + \frac{l_{ij}^1 - \dfrac{\left(v_{ij}^1\right)^2}{2 \cdot a_{ij}^1}}{v_{ij}^1}; \tag{13}$$

$$t_{ij}^n = \frac{v_{ij}^n - v_{ij}^{n-1}}{a_{ij}^n} + \frac{l_{ij}^n - \dfrac{\left(v_{ij}^n\right)^2 - \left(v_{ij}^{n-1}\right)^2}{2 \cdot a_{ij}^n}}{v_{ij}^n}; \tag{14}$$

$$t_{ij}^N = \frac{l_{ij}^N + \dfrac{\left(v_{ij}^N\right)^2}{2 \cdot a_{ij}^N}}{v_{ij}^N} - \frac{v_{ij}^N}{a_{ij}^N}. \quad (15)$$

When there are no intersections between adjacent stops, the speed v_{ij} can be calculated from the distance l_{ij}, acceleration a_{ij}, and time t_{ij} if the delay is corrected at the next station. This process is defined by Equation (16). If the delay cannot be corrected at the next stop, then the bus travels at the maximum speed $v_{\max}$:

$$v_{ij} = \frac{l_{ij} - \dfrac{\left(v_{ij}\right)^2}{a_{ij}}}{t_{ij} - 2 \cdot \dfrac{v_{ij}}{a_{ij}}}. \quad (16)$$

3.2.2. Backup bus replacement strategy

In the previous subsection, the possibility of using speed control to correct terminal delays was explored. This section examines scenarios where terminal delays remain uncorrected despite speed control measures. Under these circumstances, a backup bus replacement strategy is implemented. This strategy involves substituting a delayed bus with a backup bus stationed at the terminal, which then continues the scheduled trip. While some scholars have suggested placing a backup station midway along the bus route to facilitate bus replacements, this method introduces additional operational costs related to energy consumption, station setup, and requirements for drivers and buses. To mitigate these costs, the backup station is strategically located at the terminal. This arrangement allows a backup bus to seamlessly continue the journey of a delayed bus, eliminating the need for additional stations and reducing energy expenses. By concentrating resources at a single station, the strategy significantly lowers operational costs and ensures that bus services proceed as scheduled. To further minimize costs, only one backup bus is stationed at the terminal. Given the dynamic nature of public transport systems, bus service intervals and running times vary. Running times for buses are predicted using historical data, and arrival time $a_{i,J}$ for bus i is predicted using a prediction algorithm. When bus operations do not deviate significantly, buses can follow the planned schedule, and spare buses rest at the station, as shown in Figure 3a. When there are significant delays in bus operations and the timetable cannot be followed normally, the delayed bus is replaced by a spare bus to execute the trip, as shown in Figure 3b.

The binary variable y_{ij} is the activation parameter of the spare vehicle replacement strategy. When y_{ij} is 0, even though the bus has a delay at the terminal, it does not affect the execution of the planned schedule, so the backup bus does not need to be involved in the schedule. When y_{ij} is 1, the delay of bus i affects the normal operation of the schedule. Therefore, a backup bus is used to execute the schedule to ensure the normal operation of the bus system. The departure time of the schedule is related to the expected arrival time. When the expected arrival time is less than $D'_{i+1,J}$, the planned schedule is executed. When the expected arrival time is greater than $D'_{i+1,J}$, the arrival time is predicted by a short-term prediction algorithm, and the departure time is solved based on the arrival time, as in Equation (18).

$$y_{ij} = \begin{cases} 0, & a_{i,J} - D'_{i,J} \le 0; \\ 1, & a_{i,J} - D'_{i,J} > 0; \end{cases} \quad (17)$$

$$d'_{i,J} = \frac{d'_{i-1,J} + a_{i,J}}{2}. \quad (18)$$

3.3. Model construction

3.3.1. Objective function

The primary goals of the proposed model are to eliminate bus delays, reduce travel expenses for passengers, and reduce operational expenses for bus companies. To achieve these objectives, the model must possess several key characteristics:

- 1st, the punctuality of public transport should be maximized to ensure reliability;
- 2nd, from the perspective of passengers, minimizing travel time cost is crucial, making bus attractiveness one of the most important metrics for evaluating bus operations;
- 3rd, the model must consider both passenger and enterprise perspectives in its target functions.

An economic measure that reflects the productivity and quality of service of these businesses is the operating cost. Therefore, controlling operating costs is a critical objective for any bus company and should be a core consideration in the model.

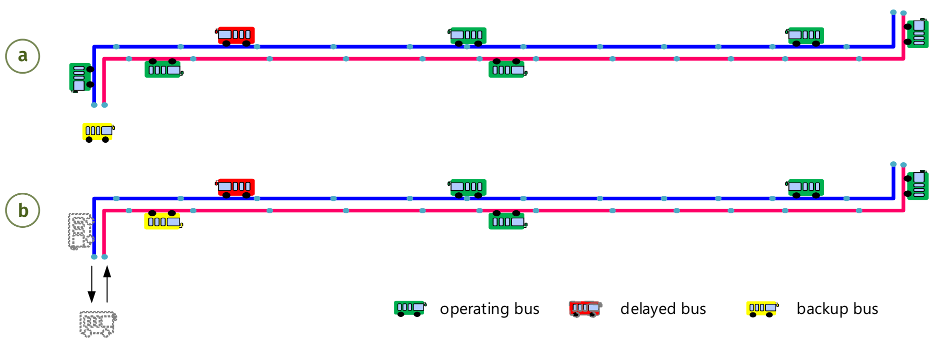


Figure 3. Diagram of bus operations: (a) – general conditions; (b) – combined control

The objective function F_1 is defined as the passenger time cost because it has a significant impact on appealing to passengers. This objective function is shown in Equation (19). Passenger time cost has 2 components, namely the passenger waiting time at stops and bus travel time, which are weighted differently owing to user experience differences.

$$\min F_1 = \alpha_1 \cdot T_W + \alpha_2 \cdot T_T , \tag{19}$$

where:

$$T_W = \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{1}{2} \cdot N_{i,j} \cdot \left(d_{i,j} - d_{i-1,j}\right); \tag{20}$$

$$T_T = \sum_{i=1}^{I} \sum_{j=1}^{J} \Big(N_{i,j-1} \cdot \left(a_{i,j} - d_{i,j-1}\right) + \left(O_{i,j} - N_{i,j}\right) \cdot \left(d_{i,j} - a_{i,j}\right) \Big). \tag{21}$$

To reduce the operating costs of bus companies, the energy consumption generated during bus operations should be considered. This energy consumption consists of the energy consumption during the acceleration process and that during the constant-speed process. Therefore, optimization goal F_2 is established to minimize energy consumption:

$$\min F_2 = E_A + E_U , \tag{22}$$

where:

$$E_A = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{n=1}^{N} \int_{t=0}^{t=t_{ij}^{acc}} \frac{v_{ij}^{n}(t)}{3600 \cdot \eta_t \cdot \eta_m \cdot \eta_{ness}} \times \left(M \cdot g \cdot f \cdot \cos\alpha + M \cdot g \cdot \sin\alpha + \frac{C_D \cdot A \cdot \left(v_{ij}^{n}(t)\right)^2}{21.15} + \delta \cdot M \cdot a_{ij}^{n} \right) dt; \tag{23}$$

$$E_U = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{n=1}^{N} \int_{t=t_{ij}^{acc}}^{t=t_{ij}^{acc}+t_{ij}^{cru}} \frac{v_{ij}^{n}}{3600 \cdot \eta_t \cdot \eta_m \cdot \eta_{ness}} \times \left(M \cdot g \cdot f \cdot \cos\alpha + M \cdot g \cdot \sin\alpha + \frac{C_D \cdot A \cdot \left(v_{ij}^{n}\right)^2}{21.15} \right) dt; \tag{24}$$

$$M = m_v + m_p \cdot N_{i,j} . \tag{25}$$

3.3.2. Constraints

In addition to the time constraints in Equations (9) and (12), a bus is subject to the following constraints during its operation. Equations (26) and (27) describe the actual arrival, service, and departure times of bus *i*. Equation (29) describes the capacity constraint during bus operations. Equation (31) describes the number of backup bus constraints. $v_{\max}$ is the maximum speed allowed for buses limited by equipment, roads, and other factors. Equation (33) indicates that the bus cruising speed is limited by road conditions. Equation (34) indicates that the traffic volume limits the bus cruising speed and Equation (35) indicates that bus speed is also limited by equipment characteristics such as the motor and mass.

$$d_{i,j} = a_{i,j} + h_{i,j}; \tag{26}$$

$$a_{i,j+1} = d_{i,j} + r_{i,j}; \tag{27}$$

$$h_{i,j} = \max\left(t_{i,j}^{u}, t_{i,j}^{d}\right); \tag{28}$$

$$Q_{i,j} \le Q_E; \tag{29}$$

$$Q_{i,j} = q_{i,1}^{u} + q_{i,2}^{u} - q_{i,2}^{d} + \ldots q_{i,j}^{u} - q_{i,j}^{d}; \tag{30}$$

$$\iota_J(t) \ge 0; \tag{31}$$

$$v_{\max} = \min\left(V_{ro}, V_{tr}, V_{ra}\right); \tag{32}$$

$$0 \le v_{i,j} \le V_{ro}; \tag{33}$$

$$0 \le v_{i,j} \le V_{tr}; \tag{34}$$

$$0 \le v_{i,j} \le V_{ra} . \tag{35}$$

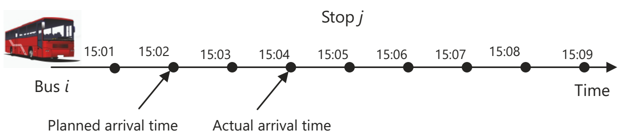
4. Solution approach

To address the bus delay issue and enhance the stability of the bus system, a bi-objective composite control model is established. Minimizing passenger travel time and company operating costs are considered as the main objectives of the model. The model is considered as a discrete system and is represented by a spatiotemporal matrix graph. The operating speed of a bus is derived using the PSO algorithm. The bus running times are predicted through neural network fitting and the timetable for spare buses is derived similarly.

4.1. Spatiotemporal solution matrix

On a single bus route, several discrete events occur, such as arriving at a stop, leaving a stop, and stopping at a stop. In this study, each bus stop is defined as a node. Once a bus arrives at a node, 3 possible control strategies can be implemented, either separately or in combination: no control, speed control strategy, or backup bus replacement strategy. The speed control strategy adjusts based on the current bus running time relative to the planned schedule. In contrast, the backup bus replacement strategy is activated based on the difference between the actual and planned arrival times at the terminal stop. A visualization of the control strategies applied to a single bus is presented in Figure 4.

The binary variables that determine the control strategy are associated with the bus stop at which a bus is located and its time attributes. At node 1, x = 0 and y = 0,



t	0	1	2	3	4	5	6	7	8	9	10	...
x_{ij}	0	0	1	1	1	1	1	1	1	1	1	...
y_{ij}	0	0	0	0	0	0	0	0	0	1	1	...

Figure 4. An example of vehicle *i* operating at stop *j*

so neither strategy needs to be used. At node 2, $x = 1$ and $y = 0$, meaning that bus i activates the speed control strategy at stop 2. At node 9, $x = 1$ and $y = 1$, meaning both control strategies are activated simultaneously.

4.2. Bus operating speed optimization based on PSO

4.2.1. Particle encoding method

In the PSO algorithm, potential solutions within a model's solution space are depicted as particles. The positions of these particles signify specific parameter values that constitute a solution. The method of encoding particles differs across various problems. In the study at hand, particles are encoded as a series of real numbers, each representing a parameter within the solution. To accurately reflect bus operating conditions, the speeds of buses along different route segments are encoded as positive real numbers. Furthermore, a critical speed value is established based on model constraints and criteria. The operating speed for each route segment must surpass the minimum speed acceptable to passengers, but not exceed the road's free flow speed or the bus's maximum rated speed. Within each route segment, particles represent the operating speeds, which can either increase or decrease: increases in bus speed are represented by positive values, decreases by negative values, and a value of zero indicates no change in speed. An example of this particle encoding is illustrated in Figure 5.

4.2.2. Initialization

The initialization phase is the 1st step in the PSO process. The goal is to generate an initial population of particles, providing a starting point for the algorithm. The population size is set to n_p. The maximum velocity of the particles is set to $V_{\max}$. The minimum velocity is set to $V_{\min}$. The interval defined in the model is defined as the search space. The positions and velocities of all particles are initialized.

4.2.3. Calculate the fitness of particles

Calculating the fitness of a particle is the process of evaluating the quality of the solution represented by the particle. The fitness function is defined based on the target optimization problem and used to measure the quality of solutions. The objective function was utilized as the fitness function in this study. The goal of each particle is to minimize the value of its fitness function by searching the solution space.

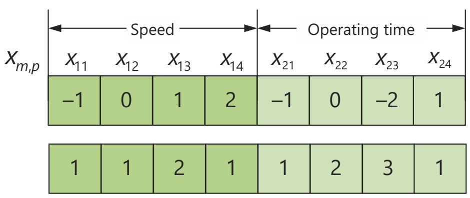


Figure 5. Particle encoding example

4.2.4. Update the velocities and positions of individuals

Updating the velocities and positions of particles is a core step in the PSO process. The search efficiency of the algorithm and quality of the final solution found are determined by the updating of the velocities and positions of the particles. The next velocity and position of each particle are calculated based on the current state of each particle, individual historical best position *pbest*, and global best position *gbest*. The velocity updating formula is defined in Equation (36). The position updating formula is defined in Equation (38).

$$\begin{aligned} v_{ij}(t+1) &= w \cdot v_{ij}(t) + \\ & c_1 \cdot r_1(t) \cdot \left(pbest_{ij}(t) - x_{ij}(t)\right) + \\ & c_2 \cdot r_2(t) \cdot \left(gbest_j(t) - x_{ij}(t)\right), \end{aligned} \tag{36}$$

where:

$$w = w_{\max} - \frac{\left(w_{\max} - w_{\min}\right) \cdot t}{T_{\max}}; \tag{37}$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1). \tag{38}$$

4.2.5. Set the termination condition

The terminal condition defines the end of the PSO process. When a particle reaches the terminal condition, the iteration process is stopped. Terminal conditions include the maximum number of iterations and fitness threshold of solutions.

4.3. Derivation of a bus timetable with a backup bus based on neural network fitting

Neural network fitting is a powerful machine learning technique that utilizes neural networks to model and predict data. These networks are capable of automatically extracting features and patterns from data, continuously refining their parameters during training to enhance their fitting accuracy. Neural networks boast strong generalization capabilities, allowing them to make reliable predictions on new, unseen data. They also exhibit a degree of fault tolerance, maintaining functionality even when some neurons fail.

To train a neural network model for bus operation optimization, one can start by collecting historical data on bus operations along with real-time traffic data. This model can then learn the relationships between bus running times and various influencing factors from the data. By adjusting the weights of the neural network, it works to minimize the prediction error. Additionally, tweaking parameters within the model can further refine its performance and enhance its predictive capabilities. The specific steps for scheduling are as follows:

- step 1: when a bus is delayed in the public transportation system, determine whether to activate the backup bus replacement strategy;

- step 2: use neural network fitting to predict the arrival time of the delayed bus at the terminal station;
- step 3: based on the schedule and predicted arrival time of the delayed bus at the terminal station, develop an optimal schedule.

5. Case study

5.1. Description of the case study

To validate the proposed combined control strategy, the model and algorithm were applied to the No 96 bus route in Harbin, a city with a well-established public transportation system. The effectiveness and feasibility of implementing the proposed model on this specific route were assessed using real-world data gathered through field surveys. The results of the proposed combined control method were compared with those observed under general conditions for the target bus route. The No 96 bus route in Harbin includes 14 stops, starting at the East Square of the Harbin West Railway Station and ending at the Harbin Railway Station. The total length of the route is 8.2 km, as depicted in Figure 6.

The key parameters of the bus operation control model in the simulation are detailed in Table 1. The table includes the mass of the bus, the mass of the passenger, the parameters of the bus operation, etc. The dynamic characteristics of the bus during operation can be more accurately simulated. The energy consumption of the bus during its operation process is accurately calculated. The number of passengers boarding at each stop for buses 1–5 in the off-peak scenario is shown in Figure 7a. The number of passengers boarding buses 1–7 at each stop in the peak scenario is shown in Figure 7b.

The detailed geographic and operational information of the No 96 bus route in Harbin is listed in Tables 2–4. The station names are displayed in Table 2. The distances between stations are displayed in Table 3. The traffic signal phase information is displayed in Table 4. This information provides precise parameters for the control model, enabling the complexity and dynamics of the actual bus operating environment to be accurately reflected. The distance information between stops in Table 3 provides basic data for the calculation of bus travel time in the model. The traffic signal information in Table 4 enables a more realistic simulation of the traffic light phases encountered by buses at different intersections. The accuracy of the model in real-time control applications is improved.

Table 1. Bus parameters at case calculation

Parameter	Notation	Value
Bus mass	m_v	13300 kg
Mass per passenger	m_p	65 kg
Mechanical drive efficiency	η_t	0.9
Efficiency of inverter	η_{ness}	0.95
Efficiency of motor	η_m	0.96
Rolling resistance coefficient	f	0.01
Air drag coefficient	C_D	0.34
Windward area of a bus	A	8.0325
Rotating mass conversion factor	d	1.1
Weighting factor for passenger waiting time	α_1	0.6
Weighting factor for passenger travel time	α_2	0.4

Figure 6. Map of No 96 bus route in Harbin

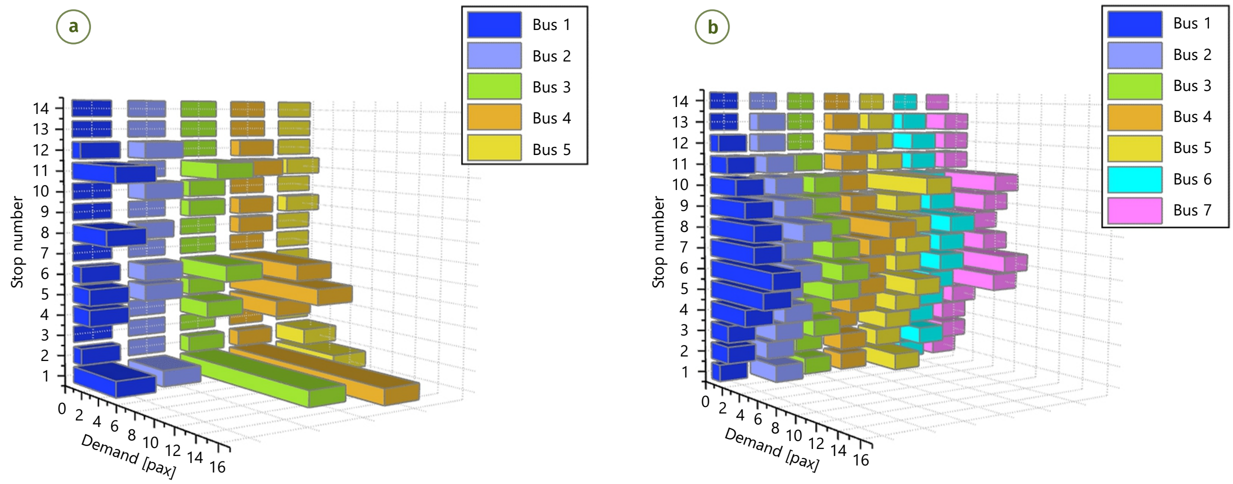


Figure 7. Number of passengers waiting at each stop: a – off-peak situation; b – peak situation

Table 2. Bus route information

Stop number	Name of stop
Stop 1	Harbin West railway station
Stop 2	Harbin street
Stop 3	Harbin street (exit of Hexie avenue)
Stop 4	Harbin street (Wuwei junction)
Stop 5	Harbin street (exit of Haxi street) (Golden City)
Stop 6	Railway street (Exit of Qinghua street)
Stop 7	Railway street (Kangning junction)
Stop 8	Railway street (junction of Hexing 11th street)
Stop 9	Railway street (junction of Hexing 3rd street)
Stop 10	Railway street (junction of Qingming 2nd street)
Stop 11	Railway street (junction of Qiaobei street)
Stop 12	Railway street (junction of Manzhouli street)
Stop 13	Railway street (junction of Shangfang street)
Stop 14	Harbin railway station

Table 3. Distance between bus stops

Stop numbers	Distance [m]
Stop 1 – Stop 2	449
Stop 2 – Stop 3	478
Stop 3 – Stop 4	531
Stop 4 – Stop 5	370
Stop 5 – Stop 6	525
Stop 6 – Stop 7	655
Stop 7 – Stop 8	772
Stop 8 – Stop 9	398
Stop 9 – Stop 10	1100
Stop 10 – Stop 11	545
Stop 11 – Stop 12	1100
Stop 12 – Stop 13	393
Stop 13 – Stop 14	955

Table 4. Signal light information

Signal number	Green-light [s]	Cycle [s]
Signal 1	18	114
Signal 2	43	125
Signal 3	39	119
Signal 4	43	120
Signal 5	31	207
Signal 6	43	77
Signal 7	54	79
Signal 8	105	133
Signal 9	56	96
Signal 10	50	94
Signal 11	59	94
Signal 12	64	94

5.2. Analysis of the results

In the 1st target scenario, road traffic volume was relatively low, providing a broad range for speed adjustments. It should be noted that t_h was set to 30 s, as deviations below this are considered negligible and can be adjusted immediately without surpassing human perceptual abilities. The proposed model was validated by monitoring the statuses of 5 buses over a one-hour timeframe on weekdays. The spatiotemporal trajectories of the bus operations are presented in Figure 8. In the figure, blue lines represent the trajectories of the buses after implementing control measures, and red lines depict the actual trajectories of the buses. Dashed lines indicate the planned bus trajectories. It is evident that following the implementation of the speed control strategy, the operating trajectories of the buses gradually aligned with the originally planned trajectories. These results demonstrate that effective adjustments to driving speed can successfully guide buses to operate according to the established timetable, thereby ensuring the reliability of operations.

Table 5 presents a comparison of 4 evaluation metrics between the speed control scenario, which utilizes real-time information, and general conditions. Under speed control, there is a slight increase in energy consumption; however, the overall schedule deviation is significantly reduced by 198.59 min. Additionally, the total passenger travel time decreases by 258.70 min, and the total passenger waiting time is reduced by 65.72 min. The value of objective function F_1 is reduced by 12.2%. These results indicate that the speed control approach can significantly improve the service quality of the bus system by reducing schedule deviation, passenger travel, and waiting time.

Figures 9 and 10 present the schedule deviations under general conditions and the scenario with speed control, respectively. Under general conditions, schedule deviation gradually increases over time. However, in the speed control scenario, schedule deviation is corrected at the next stop or several stops, thereby preventing a continuous increase in schedule deviation. The proposed control model reduces the schedule deviation by 89% compared with the general conditions, with the maximum schedule deviation being reduced from 10.18 to 2.50 min. This reduction in schedule deviation significantly improves the stability and reliability of the bus system. Overall, the speed control approach is effective at correcting schedule deviations and maintaining a more reliable bus schedule.

Figures 11 and 12 illustrate the average passenger waiting time and average passenger travel time under general conditions and with speed control, respectively. The control model, when compared with general conditions, reduces the average passenger waiting time by 8%, decreases maximum schedule deviation from 18.95 to 15.50 min, and cuts average passenger travel time by 14%.

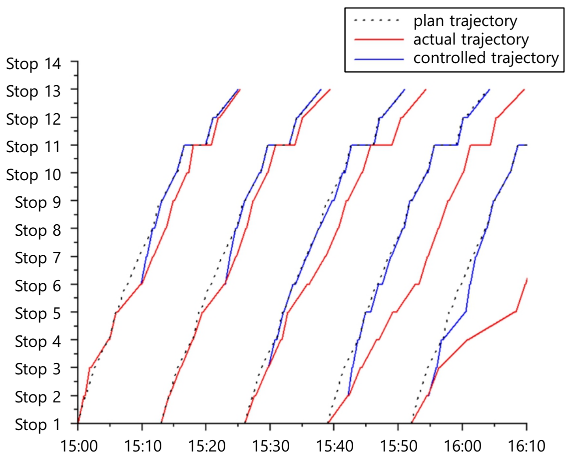


Figure 8. Spatial-temporal trajectory map of bus operation

Table 5. Comparison of the speed control with general conditions

Performances	General conditions	Speed control
Energy consumption [kW·h]	6.25	12.00
Schedule deviation [min]	222.20	23.61
Passenger travel time [min]	1941.60	1682.90
Passenger waiting time [min]	836.75	771.03

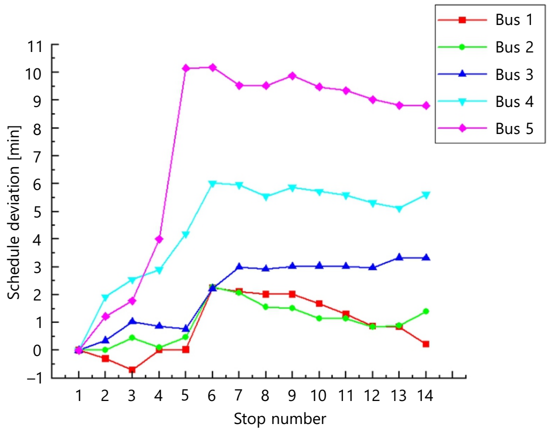


Figure 9. General conditions schedule deviation

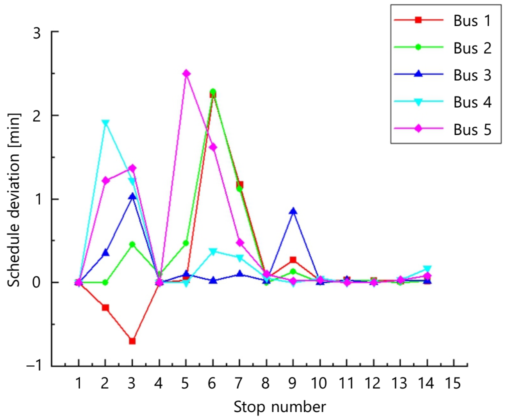


Figure 10. Schedule deviation after speed control

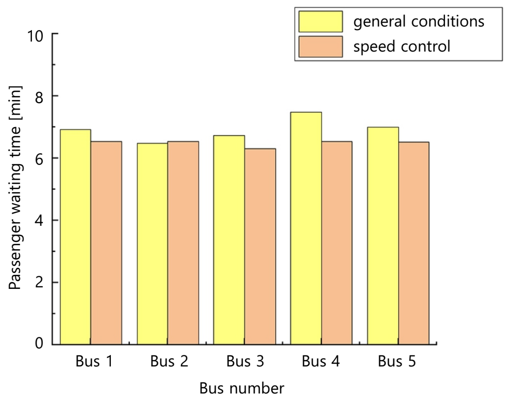


Figure 11. Per capita passenger waiting time

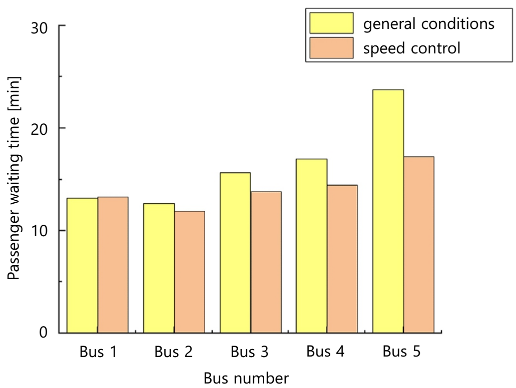


Figure 12. Per capita passenger travel time

Overall, the control model positively impacts both passenger waiting and travel times, enhancing the attractiveness of the bus service and improving its overall quality.

In the scenario described above, through speed control alone, buses could arrive at the terminal normally. However, during peak hours, traffic flow is high and bus speed cannot be adjusted or can only be adjusted within a small range. Therefore, buses are unable to arrive at terminals on time when delays or unexpected events occur. In such cases, the backup bus replacement strategy is activated.

Figures 13 and 14 depict the schedule deviations under general conditions and with speed control, respectively. It is apparent that the effectiveness of speed control diminishes during peak hours due to a narrower range of achievable speeds for buses. Despite this, speed control still manages to reduce the average delay in reaching the destination terminal by 6.50 min compared to general conditions. Nonetheless, buses 4 and 6 continue to exhibit increasing schedule deviations even with speed control measures in place. Additionally, buses 5 and 7 experience significant delays near the terminal, preventing them from arriving on time and disrupting the regular schedule. To address these challenges and enhance the stability of the bus system, a backup bus replacement strategy is implemented. This strategy involves a backup bus stationed at the terminal, ready to take over the routes of delayed buses, thus reducing waiting times and ensuring smoother operations. This approach significantly enhances the overall stability of the bus system and mitigates disruptions caused by delays near the terminal.

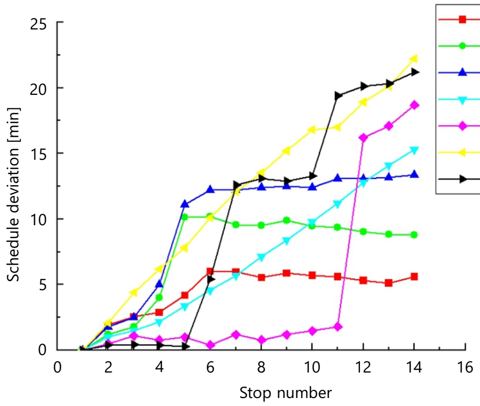


Figure 13. General conditions schedule deviation

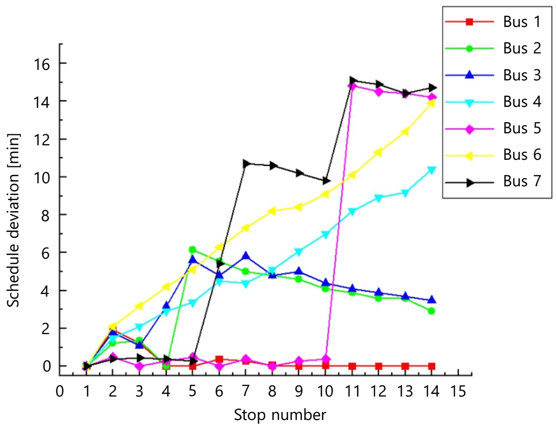


Figure 14. Schedule deviation after speed control

The bus schedule under delayed operation conditions is shown in Table 6. The bus schedule is displayed on the top of the table, and the buses are represented on the left side of the table. It can be seen that although buses 1–3 experience delays, they can follow the planned schedule after implementing speed control. However, even after adopting speed control, buses 4–7 still have a delay time greater than the reserved rest time, which affects the operation of the planned schedule. By adding spare buses to the schedule, the schedule of buses 4–7 can proceed on time, and the issue of bus delay can be effectively alleviated by the control strategy. When the bus cannot follow the schedule, the spare bus can replace the delayed bus to execute the schedule. The service of the route is not affected, and the stability of the public transport system is ensured. The reliability of the public transport system and passenger satisfaction have improved.

Table 7 presents the delays in departure times at the terminal caused by delayed buses under 3 different conditions: general conditions, speed control, and combined control. In the combined control scenario, the delay time of the buses is reduced through speed control. Additionally, the backup bus replacement strategy further reduces the departure delay at the terminal. Therefore, by using the combined control approach, departure delays at the terminal can be further reduced or even eliminated compared with the use of speed control alone. Thus, the combined control approach can significantly reduce or even eliminate departure delays at the terminal compared to using speed control alone. This strategy is also effective at minimizing the variance in headways between buses, re-

Table 6. Bus schedule operating plan table

	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6	Bus 7	Backup bus
Backup bus				operation				
Bus 1	operation							
Bus 2		operation						
Bus 3			operation					
Bus 4					operation			
Bus 5						operation		
Bus 6							operation	
Bus 7								operation

Table 7. Bus departure delays

Bus number	General conditions [min]	Speed control [min]	Combined control [min]
Bus 1	0	0	0
Bus 2	0	0	0
Bus 3	3.4	0	0
Bus 4	5.3	0.1	0
Bus 5	8.7	4.2	0
Bus 6	12.2	3.9	0
Bus 7	11.2	4.7	0

ducing passenger waiting times, and mitigating additional energy consumption that arises from correcting schedule deviations. Ultimately, these improvements enhance the stability and reliability of the bus system.

6. Conclusions

In this study, a real-time bus control model was established to improve the reliability of bus systems under heterogeneous traffic conditions. Combined control strategies were proposed to reduce passenger waiting and travel times, and enhance the attractiveness of public transportation. The model adjusts the running times of buses between adjacent stops using real-time speed control strategies and considers factors such as the intervals of traffic signals. The phenomenon of bus delays was addressed, and bus departure delays were eliminated through the implementation of a backup bus replacement strategy. The effectiveness of this approach was demonstrated through a real case study on No 96 bus route in Harbin. Experimental results indicate that, compared to typical conditions, the proposed model significantly reduces schedule deviations, effectively eliminates departure delays, and decreases passenger waiting and travel times. Furthermore, a heuristic algorithm based on the principles of PSO was designed. This algorithm aims to minimize energy consumption while reducing schedule deviations and time costs for passengers. By implementing a backup bus replacement strategy, the stability and reliability of the bus system were improved, and departure delays at the terminus were reduced.

There are 2 promising directions for future research:

- 1st, while this study concentrated on a single bus route, expanding the scope to include an entire bus network could provide insights into broader urban bus system management, encompassing operational efficiency, route optimization, and passenger demand;
- 2nd, in line with smart city development initiatives, the evolution of autonomous vehicles presents a significant area of interest.

Although this study was limited to control models for regular electric buses, future research could extend these models to autonomous buses, potentially enhancing the efficiency and reliability of bus systems in smart cities.

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Author contributions

Baoyu Hu provides research ideas.

Models were built by *Hu Baoyu* and *Xinlei Song*.

Xinlei Song and *Yangyang Fu* performed model validation and data analysis.

Xinlei Song wrote the manuscript.

Disclosure statement

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References

Alamatsaz, K.; Hussain, S.; Lai, C.; Eicker, U. 2022. Electric bus scheduling and timetabling, fast charging infrastructure planning, and their impact on the grid: a review, *Energies* 15(21): 7919. https://doi.org/10.3390/en15217919

Ampountolas, K.; Kring, M. 2021. Mitigating bunching with bus-following models and bus-to-bus cooperation, *IEEE Transactions on Intelligent Transportation Systems* 22(5): 2637–2646. https://doi.org/10.1109/TITS.2020.2973585

Anderson, P.; Daganzo, C. F. 2019. Effect of transit signal priority on bus service reliability, *Transportation Research Procedia* 38: 2–19. https://doi.org/10.1016/j.trpro.2019.05.002

Berrebi, S. J.; Watkins, K. E.; Laval, J. A. 2015. A real-time bus dispatching policy to minimize passenger wait on a high frequency route, *Transportation Research Part B: Methodological* 81: 377–389. https://doi.org/10.1016/j.trb.2015.05.012

Bie, Y.; Xiong, X.; Yan, Y.; Qu, X. 2020. Dynamic headway control for high-frequency bus line based on speed guidance and intersection signal adjustment, *Computer-Aided Civil and Infrastructure Engineering* 35(1): 4–25. https://doi.org/10.1111/mice.12446

Cao, Z.; Ceder, A.; Zhang, S. 2019. Real-time schedule adjustments for autonomous public transport vehicles, *Transportation Research Part C: Emerging Technologies* 109: 60–78. https://doi.org/10.1016/j.trc.2019.10.004

Cats, O.; Larijani, A. N.; Koutsopoulos, H. N.; Burghout, W. 2011. Impacts of holding control strategies on transit performance: bus simulation model analysis, *Transportation Research Record*: *Journal of the Transportation Research Board* 2216: 51–58. https://doi.org/10.3141/2216-06

Chen, J.; Liu, Z.; Zhu, S.; Wang, W. 2015a. Design of limited-stop bus service with capacity constraint and stochastic travel time, *Transportation Research Part E*: *Logistics and Transportation Review* 83: 1–15. https://doi.org/10.1016/j.tre.2015.08.007

Chen, X.; Hellinga, B.; Chang, C.; Fu, L. 2015b. Optimization of headways with stop-skipping control: a case study of bus rapid transit system, *Journal of Advanced Transportation* 49(3): 385–401. https://doi.org/10.1002/atr.1278

Chow, A. H. F.; Li, S.; Zhong, R. 2017. Multi-objective optimal control formulations for bus service reliability with traffic signals, *Transportation Research Part B*: *Methodological* 103: 248–268. https://doi.org/10.1016/j.trb.2017.02.006

Daganzo, C. F. 2009. A headway-based approach to eliminate bus bunching: systematic analysis and comparisons, *Transportation Research Part B*: *Methodological* 43(10): 913–921. https://doi.org/10.1016/j.trb.2009.04.002

Daganzo, C. F.; Pilachowski, J. 2011. Reducing bunching with bus-to-bus cooperation, *Transportation Research Part B*: *Methodological* 45(1): 267–277. https://doi.org/10.1016/j.trb.2010.06.005

Deng, Y.-J.; Liu, X.-H.; Hu, X.; Zhang, M. 2020. Reduce bus bunching with a real-time speed control algorithm considering heterogeneous roadway conditions and intersection delays, *Journal of Transportation Engineering*, *Part A*: *Systems* 146(7): 04020048. https://doi.org/10.1061/JTEPBS.0000358

Estrada, M.; Mensión, J.; Aymamí, J. M.; Torres, L. 2016. Bus control strategies in corridors with signalized intersections, *Transportation Research Part C*: *Emerging Technologies* 71: 500–520. https://doi.org/10.1016/j.trc.2016.08.013

Fu, L.; Liu, Q.; Calamai, P. 2003. Real-time optimization model for dynamic scheduling of transit operations, *Transportation Research Record*: *Journal of the Transportation Research Board* 1857: 48–55. https://doi.org/10.3141/1857-06

Gkiotsalitis, K. 2021. Bus holding of electric buses with scheduled charging times, *IEEE Transactions on Intelligent Transportation Systems* 22(11): 6760–6771. https://doi.org/10.1109/TITS.2020.2994538

Han, F.; Xie, R.; Lai, M. 2018. Traffic density, congestion externalities, and urbanization in China, *Spatial Economic Analysis 13*(4): 400–421. https://doi.org/10.1080/17421772.2018.1459045

He, H.; Guler, S. I.; Menendez, M. 2016. Adaptive control algorithm to provide bus priority with a pre-signal, *Transportation Research Part C*: *Emerging Technologies* 64: 28–44. https://doi.org/10.1016/j.trc.2016.01.009

Huo, Y.; Li, W.; Zhao, J.; Zhu, S. 2018. Modelling bus delay at bus stop, *Transport* 33(1): 12–21. https://doi.org/10.3846/16484142.2014.1003324

Janos, M.; Furth, P. G. 2002. Bus priority with highly interruptible traffic signal control: simulation of San Juan's Avenida Ponce de Leon, *Transportation Research Record*: *Journal of the Transportation Research Board* 1811: 157–165. https://doi.org/10.3141/1811-19

Koehler, L. A.; Kraus, W. 2010. Simultaneous control of traffic lights and bus departure for priority operation, *Transportation Research Part C*: *Emerging Technologies* 18(3): 288–298. https://doi.org/10.1016/j.trc.2009.01.007

Koehler, L. A.; Seman, L. O.; Kraus, W.; Camponogara, E. 2019. Real-time integrated holding and priority control strategy for transit systems, *IEEE Transactions on Intelligent Transportation Systems* 20(9): 3459–3469. https://doi.org/10.1109/TITS.2018.2876868

Liang, S.; Zhao, S.; Lu, C.; Ma, M. 2016. A self-adaptive method to equalize headways: numerical analysis and comparison, *Transportation Research Part B*: *Methodological* 87: 33–43. https://doi.org/10.1016/j.trb.2016.02.008

Liu, Z.; Yan, Y.; Qu, X.; Zhang, Y. 2013. Bus stop-skipping scheme with random travel time, *Transportation Research Part C*: *Emerging Technologies* 35: 46–56. https://doi.org/10.1016/j.trc.2013.06.004

Ma, W.; Xie, H.; Han, B. 2012. Development and evaluation of an economic-driving assistance program for transit vehicles, *Energies* 5(2): 371–385. https://doi.org/10.3390/en5020371

Ma, W.; Yang, X.; Liu, Y. 2010. Development and evaluation of a coordinated and conditional bus priority approach, *Transportation Research Record*: *Journal of the Transportation Research Board* 2145: 49–58. https://doi.org/10.3141/2145-06

Muñoz, J. C.; Cortés, C. E.; Giesen, R.; Sáez, D.; Delgado, F.; Valencia, F.; Cipriano, A. 2013. Comparison of dynamic control strategies for transit operations, *Transportation Research Part C*: *Emerging Technologies* 28: 101–113. https://doi.org/10.1016/j.trc.2012.12.010

Nesheli, M. M.; Ceder, A.; Gonzalez, V. A. 2016. Real-time public-transport operational tactics using synchronized transfers to eliminate vehicle bunching, *IEEE Transactions on Intelligent Transportation Systems* 17(11): 3220–3229. https://doi.org/10.1109/TITS.2016.2542268

Petit, A.; Lei, C.; Ouyang, Y. 2019. Multiline bus bunching control via vehicle substitution, *Transportation Research Part B*: *Methodological* 126: 68–86. https://doi.org/10.1016/j.trb.2019.05.009

Petit, A.; Ouyang, Y.; Lei, C. 2018. Dynamic bus substitution strategy for bunching intervention, *Transportation Research Part B*: *Methodological* 115: 1–16. https://doi.org/10.1016/j.trb.2018.06.001

Seman, L. O.; Koehler, L. A.; Camponogara, E.; Kraus, W. 2020. Integrated headway and bus priority control in transit corridors with bidirectional lane segments, *Transportation Research Part C*: *Emerging Technologies* 111: 114–134. https://doi.org/10.1016/j.trc.2019.12.001

Sun, A.; Hickman, M. 2005. The real-time stop-skipping problem, *Journal of Intelligent Transportation Systems*: *Technology*, *Planning, and Operations* 9(2): 91–109. https://doi.org/10.1080/15472450590934642

Van Oort, N.; Boterman, J. W.; Van Nes, R. 2012. The impact of scheduling on service reliability: trip-time determination and holding points in long-headway services, *Public Transport* 4(1): 39–56. https://doi.org/10.1007/s12469-012-0054-4

Wang, W.; Zong, F.; Yao, B. 2021. A proactive real-time control strategy based on data-driven transit demand prediction, *IEEE Transactions on Intelligent Transportation Systems* 22(4): 2404–2416. https://doi.org/10.1109/TITS.2020.3028415

Yang, X.; Chen, A.; Ning, B.; Tang, T. 2016. A stochastic model for the integrated optimization on metro timetable and speed profile with uncertain train mass, *Transportation Research Part B*: *Methodological* 91: 424–445. https://doi.org/10.1016/j.trb.2016.06.006

Zhang, H.; Liang, S.; Zhao, S.; Shang, Q. 2021. Advantages of bus stop skipping and holding control in reducing schedule deviation, *Proceedings of the Institution of Civil Engineers – Municipal Engineer* 174(1): 14–23. https://doi.org/10.1680/jmuen.18.00036

Zhao, J.; Dessouky, M.; Bukkapatnam, S. 2006. Optimal slack time for schedule-based transit operations, *Transportation Science* 40(4): 529–539. https://doi.org/10.1287/trsc.1060.0170