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Original Article

STABILITY ANALYSIS FOR HETEROGENEOUS TRAFFIC FLOW WITH LANE-CHANGE DISTURBANCE

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Highlights:

- mixed traffic flow with heterogenous driving behaviours is discussed in the model;
- the model focuses on the instability of the CACC platoon caused by en-route lane-changes;
- a group of stability recognition parameters, influencing factors and description indexes are established;
- a numerical analysis method is proposed to evaluate the instability with diversity influencing factors including vehicles' type, CV's penetration, Backward gap and Lane-change spot;
- the models and findings can be applied effectively in practice to determine the optimal time and location for en-route lane change and to assist with traffic management and lane selection at the entrance.

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Abstract. Stability analysis and benefit estimation have substantial implications for lane-change decision-making to reduce delay and variation. Connected platoons drive with minor headway to increase capacity, whereas dividing or reforming platoons significantly impacts traveling efficiency. Therefore, this article focuses on the instability of the platoon caused by an en-route lane-change. Construction of platoon forming, combination rules, and car-following models for various vehicle types are presented to describe driving behaviours. Then, a velocity adjustment and a model for lane-change preparation and recovery are proposed. In addition, a group of stability recognition indexes and related stability evaluation factors are presented. Experiments involving numerical comparisons of the proposed factors are conducted to demonstrate the propagation properties of the instability and reveal the fluctuation degree. The variation duration, velocity variation range, and total delay are the primary indicators for evaluating lane-change feasibility. The models and findings can be applied effectively in practice to determine the optimal time and location for en-route lane-change and to assist with traffic management and lane selection at the entrance.

Keywords: heterogeneous traffic flow, lane-change, cooperative adaptive cruise control platoons, stability analysis, disturbance propagation, fluctuation degree.

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Notations

ACC – adaptive cruise control;
CACC – cooperative ACC;
CAV – connected and automated vehicle;
CV – connected vehicle;
HDV – human-driven vehicle;
IDM – intelligent driver model;
PATH – partners for advanced transpotation technology;
V2V – vehicle-to-vehicle.

1. Introduction

CVs can obtain real-time driving information from preceding vehicles via V2V communication to implement CACC. CVs can form CACC platoons and drive at a constant time interval, which is shorter than the time interval between HDVs. Therefore, CVs can reduce traffic congestion and increase the effectiveness of road traffic (Marsden *et al.* 2001; Shladover *et al.* 2012). In the future, heterogeneous traffic flow constituted with CVs and HDVs will be popular for a long period. Analysing the driving characteristics of CVs and CACC platoons in heterogeneous traffic flow is crucial. Additionally, stable platoons without division and reforming are extremely to avoid speed fluctuation and larger headway, which decreases travel time, fuel assumption and emission. Among the various stability influencing factors including communication delay, vehicles' kinematic failure, etc., en-route lane-change is also a frequent phenomenon to generate platoon's variation. Driving in or out of a platoon is accompanied with acceleration/deceleration and platoon reconstruction in lane-change preparation and recovery process and has significant influence on platoon's stability. Therefore, this article aims to analyse the stability with lane-change disturbance.

Some researchers in the existing literature adopt the stability theory and analytical method to analyse the effect of CVs on the instability caused by a single vehicle's delay (Liu *et al.* 2018a; Cao *et al.* 2021; Zhou *et al.* 2021). Lyaplov's stability method is applied to the stability analysis of heterogeneous traffic flow of mixed ACC vehicular flow, and the basic stability criteria are established (Ward 2009). When the CV follows the HDV, the function of the CACC system cannot be fully realized, and only the basic driving information of the preceding vehicle can be obtained, that is, it degenerates into the ACC system (Ploeg *et al.* 2015). Therefore, the change in the penetration rates of CVs makes the types of vehicles in the traffic flow diverse. In view of this phenomenon, the criteria for determining the stability of heterogeneous traffic flow with CACC, ACC and ordinary vehicles are proposed, which can be used to analyse the impact of the penetration rates of CVs on the stability of the traffic flow and obtain the critical value of stable speed (Wang *et al.* 2019; Yao *et al.* 2020; Qin, Wang 2023; Chang *et al.* 2020). Increasing the stability of individual vehicles can effectively reduce platoon oscillation, and adjusting the expected time interval is the most effective factor for reducing traffic oscillation (Sun *et al.* 2020).

The transfer function theory is mainly used to analyse the transfer law of the traffic flow oscillation (Konishi *et al.* 2000). The transmission process of the traffic flow oscillation is study to determine the stability limit of the traffic flow, obtain the conditions to determine the steady state, and verify the stability of the traffic flow under different conditions, such as the impact of the network coupling signal delay, and the effectiveness of the improved car-following model (Davis 2013; Ge, Orosz 2014; Zhang *et al.* 2021a).

In addition, simulation is another method for analysing instability (Wang *et al.* 2022b). The research finds that the position of the CACC platoon affects traffic flow oscillation. By simulating different positions of the CACC platoon in traffic flow, the influence of the CACC platoon on traffic flow is analysed, and the results show that when the CACC platoon drives at the front of the flow, the oscillation will be reduced (Sharma *et al.* 2021). In order to improve the vehicle stability, the speed adjustment strategy that is formulated by analysing and predicting the preceding vehicle (Tian *et al.* 2021; Gregurić *et al.* 2022) and an extended car-following model of CVs by considering the driving state and expected speed of 2 preceding vehicles (Wang *et al.* 2022c), are used to reduce the CVs' reaction time and braking frequency, which are verified by simulation. A dynamic route choice model for CVs can also be established to simulate the running conditions of CVs at various headway gaps, so as to analyse the stability of the platoons (Lee *et al.* 2018). In addition, information sharing between vehicles also affects the stability of the CACC platoons, and simulation results suggest that the greater amount of shared information corresponded to improve stability (Zhang *et al.* 2021b).

The real-car experiment is also a focused issue to discuss stability. In the real car test, a group of continuous ACCs will amplify the speed disturbance of the preceding vehicle, whereas CACC can overcome this issue (Milanés, Shladover 2014). In actual traffic flow, the degradation of CACC vehicles is difficult to avoid completely, but the impact of degradation can be mitigated by sharing information with the more advanced CVs (Lee *et al.* 2021). By improving the control algorithm, the car-following performance and platoon stability can be improved, and the negative impact of communication delay can be reduced (Flores, Milanés 2018). Through the questionnaire, the driver's acceptance and performance requirements for the CV can be examined to adjust the parameters of the car-following model (Chen *et al.* 2022).

Most existing literature focuses on stability, but few studies investigate the disturbance caused by en-route lane-change, which is a common phenomenon that may cause deceleration, congestion, and instability. An entire lane-change process includes preparation, execution and recovery corresponding with a continuous speed adjustment of several related vehicles and different from the perturbance in the existing literatures. Drivers often choose lane-changing spots with large gaps to improve safety in the Internet environment (Ali *et al.* 2018); In addition, the communication delay has a greater effect on mandatory lane-change than discretionary lane-change (Ali *et al.* 2021). It is obvious that the characteristics of CVs will change the lane-changing behaviour of vehicles, and more suitable lane-changing rules need to be established, including discretionary lane-change, mandatory lane-change, and expected lane-change (Liu *et al.* 2018b). The CVs can realize collaborative lane-change in the case of traffic congestion (Wang *et al.* 2022a). Moreover, lane-changing detection and prediction models can be established by utilizing the shared information, and then the CVs make better decisions (Guo *et al.* 2022). By establishing a virtual preceding vehicle of adjacent lane to guide the vehicle to change lanes, it can effectively reduce vehicle speed oscillation and enhance driving comfort (Schmidt 2017).

These literatures focus primarily on coordinated lane-change preparation and car-following strategies to improve efficiency. Nonetheless, there is a lack of research on the effect of lane-change on flow stability, which is significant to provide the basis for lane-change intention, speed adjustment rules, selected intervals, and spots. Therefore, the primary objective is to evaluate the degree of influence under various factors and to provide a reference for lane-change decision-making. 1st, a framework for platoon formation, driving modes, lane-change rules, and a car-following model is developed to determine the vehicle's type relative to the preceding one and to describe the driving characteristics of connected and HDVs. The platoon forming and car-following models described in this article are used to dispatch vehicles and determine their lane-preserving trajectories. 2nd, a lane-change model constituted by velocity adjustment and platoon recombination rules is presented. Different gaps correspond to various acceleration/decoration manoeuvre. The preparation and recovery behaviours before and after lane-change are all modelled in this section. 3rd, the stability indexes and instability factors are analysed in depth and simulated to determine the lane-change law.

The remainder of this article is organized as follows. Section 1 is an introduction. Section 2 constructs models to describe platoon forming, combination rules and car-following behaviours on an identical lane for various kinds of vehicles. Section 3 proposes a velocity adjustment and lane-change model of CVs and HDVs for lane-change preparation and recovery. In Section 4, a group of indexes for stability recognition and related factors are explained for stability evaluation. Section 5 presents the results of comparison experiments and indicates the perturbation propagation rules under various factors. Finally, Section 6 summarizes the findings, implications, and limitations and suggests some future research directions.

2. Car-following behaviours for heterogeneous traffic

The mixed traffic flow is consisted of HDVs and CVs with a market penetration. CVs can transmit real-time driving data to each other and form a platoon to drive at a closer gap as a result of the evolution of communication and the increase in CV penetration. To distinguish between CVs in a platoon and those not in a platoon, a more granular classification is defined, and CACC platoon formation rules combined with a car-following model is proposed to describe the driving behaviours of heterogeneous traffic.

Different from the existing literatures, communication or reaction delay is not the essential element to induce instability. The focus is paid to the lane-change disturbance. Consequently, several hypotheses are predetermined: within the communication range, CVs can receive in real-time without delay, interference, or failure, allowing for a timely response. In addition, it is assumed that all vehicles have the same kinetic parameters, including maximum acceleration, deceleration, velocity, and vehicle length.

2.1. Formation rules for CACC platoons

HDVs decide on acceleration, speed, and other behaviours based on the drivers' observations and consistent rules for following other vehicles. In contrast, the driving conditions for CVs will be more complex when multiple preceding vehicles are present. As depicted in Figure 1, a variety

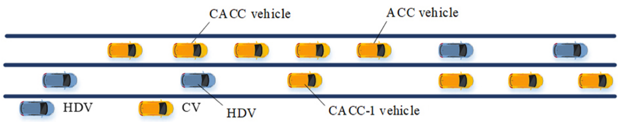

Figure 1. The driving states of vehicles

of vehicles following states, including CV following HDV, CV following CACC platoon, the following vehicle in the platoon, and HDV will appear. Therefore, it is necessary to identify the vehicle types and apply the corresponding driving behaviour models. Especially as the penetration of CVs increases, the platoon length can reach its maximum length, and 2 consecutive CACC platoons will appear. Meanwhile, the CV can communicate with the preceding vehicle while driving as the platoon leader. Consequently, vehicles will be categorized into 4 categories, and the applicable models will be discussed below:

- ***ACC vehicle: a CV following an HDV.*** When CVs drive after HDVs, the real-time information of the preceding vehicle is not acquired, but only the data transmitted from the on-board sensing system installed roadside can be obtained. Therefore, compared to driving after CVs, a longer reaction time is required and this phenomenon is known as internet-CV degradation. Notably, a platoon's leader vehicle degrades to an ACC vehicle when it follows an HDV or the preceding vehicle is out of communication range in low-density traffic flow;
- ***CACC vehicle: a CV in the CACC platoon.*** The CV is part of a platoon of 2 or more CVs. The leading vehicle will share real-time driving information with the following CACC vehicles without considering communication delays. When a stable state has been reached, the following vehicles in a platoon will maintain a constant gap and speed. If a disturbance occurs, subsequent vehicles implement the CACC following model quickly;
- ***CACC-1 vehicle: a CV following a CACC platoon for a maximum length.*** When the preceding CACC platoon reaches the maximum length and the subsequent vehicle will act as a new platoon's leader. Distinguished from Case (1) and Case (2), the CACC-1 vehicle still performs CACC following model but the communication and reaction time between 2 CVs will be larger than the value in platoons;
- ***HDV: HDV maintains a constant following gap and reaction time to the preceding vehicle.*** The validated IDM model will be utilized to characterize the driving behaviours of HDVs.

The details of each of the preceding models will be covered in Section 2.2, and the platoon formation rules and vehicle classification will be presented in the remainder of this section. Considering limited communication range, the subsequent vehicle can activate the free-driving mode to reduce the gap and reach a higher velocity. This phenomenon will be observed in low-density traffic flows with sufficient driving-in time intervals. Clearly, different vehicles have different safety requirements, and the marginal gap for free-driving mode is set as a distinct value.

As depicted in Figure 2, the desired gaps of 4 types of vehicles are represented as S_1, S_2, S_3 and S_4 (the detailed expression is shown in Section 2.2). The specific platoon formation rules and corresponding appropriate driving models are described to characterize the various driving states based on the types and gaps between preceding vehicles.

2.2. Driving models for heterogeneous vehicles

The different driving models are described to correspond with various states as follows.

ACC and CACC mode. In this article, the PATH laboratory's car-following model is introduced to describe the driving behaviour of CVs (Milanés, Shladover 2014). When CVs degrade to ACCs due to a communication failure, longer response time and greater safety gaps are necessary. The PATH laboratory's proposed ACC model can objectively represent the driving characteristics of this mode (Milanés, Shladover 2014). The following expressions are presented:

$$a_i(t) = k_1 \cdot \left(g_{i-1,i}(t) - T_1 \cdot v_i(t) - l - s_0\right) + k_2 \cdot \Delta v(t); \tag{1}$$

$$S_1(t) = v_i(t) \cdot T_1 + l + s_0 - \frac{k_2}{k_1} \cdot \Delta v(t), \tag{2}$$

where: T_1 is the desired time gap of ACC vehicles; $\Delta v(t)$ is the speed difference between vehicle i and $i-1$; k_1, k_2 are the calibrated parameters; s_0 is an extra gap for stopping; l is vehicle's length; $g_{i-1,i}(t)$ is gap between i and $i-1$; $S_1(t)$ is desired gap of ACC vehicle in Figure 2.

Furthermore, the driving model for CACCs is expressed as follows:

$$v_i(t+\Delta t) = v_i(t) + k_3 \cdot e_i(t) + k_4 \cdot \dot{e}_i(t); \tag{3}$$

$$e_i(t) = g_{i-1,i}(t) - s_0 - l - T_2 \cdot v_i(t); \tag{4}$$

$$S_2(t) = v_i(t) \cdot T_2 + l + s_0; \tag{5}$$

$$S_3(t) = v_i(t) \cdot T_3 + l + s_0, \tag{6}$$

where: $e_i(t)$ is time gap error; $\dot{e}_i(t)$ is derivative of $e_i(t)$; T_2, T_3 are desired time gap of CACC vehicle and CACC-1 vehicle; k_3, k_4 are the calibrated parameter; $S_2(t)$, $S_3(t)$ are the desired gap of CACC vehicle and CACC-1 vehicle.

HDV model. Existing models for HDVs, including Newell's model (Newell 2002), Gipps' model (Gipps 1981; Ciuffo *et al.* 2012), and IDM model (Treiber *et al.* 2000), have been proposed to characterize HDV driving behaviours. In current opinion, IDM has better fitness in crowded or free traffic flow, and the model parameters can reflect the personalized driving habits and decision-making of ordinary travellers. This article uses the IDM model as the car-following model for HDVs. The detailed expressions are as follows:

$$a_i(t) = a_{\max} \cdot \left(1 - \left(\frac{v_i(t)}{v_0}\right)^{\theta} - \left(\frac{S_i(t)}{g_{i-1,i}(t)}\right)^2\right); \tag{7}$$

$$S_4(t) = s_0 + v_i(t) \cdot T_4 + \frac{v_i(t) \cdot \Delta v(t)}{2 \cdot \sqrt{a_{\max} \cdot b}}, \tag{8}$$

where: $S_4(t)$ is the desired gap of HDV; v_0 is the free flow velocity; T_4 is desired time gap of HDV; θ is acceleration component; b is desired deceleration; $a_{\max}$ is the maximum acceleration.

Acceleration mode. This mode in Figure 2 is applied to describe the driving behaviours with sufficient gaps to obtain a higher velocity and a smaller headway:

if $v_i(t) = v_{\max}$, then

$$a_i(t) = 0, \; v_i(t+\Delta t) = v_i(t); \tag{9}$$

if $v_i(t) < v_{\max}$, then

$$a_i(t) = \min\left(a_{\max}, \delta \cdot \left(v_{\max} - v_i(t)\right)\right),$$

$$v_i(t+\Delta t) = v_i(t) + a_i(t) \cdot \Delta t, \tag{10}$$

where: $v_{\max}$ is the maximum speed; $\delta = 0.4$ is acceleration coefficient (Liu *et al.* 2018a, 2018b).

The other parameters in the Equations (1)–(10) are valued as in Table 1, which are calibrated by PATH laboratory and the ones in IDM model are determined according to the articles (Milanés, Shladover 2014; Treiber *et al.* 2000; Kesting *et al.* 2008).

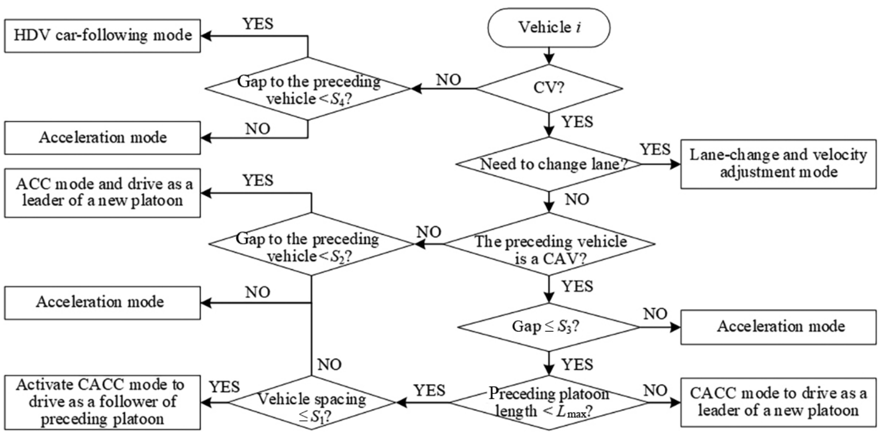

Figure 2. The platoon formation rules

Table 1. Parameter value of model

Parameter	Value
v_0, v_{max}	33.3 m/s
θ	4
a_{max}	1.8 m/s^2
b	2 m/s^2
s_0	2 m
l	4 m
T_1	1.1 s
T_2	0.6 s
T_3	2 s
T_4	1.5 s
k_1, k_2	0.230, 0.007
k_3, k_4	0.45, 0.25

3. Lane-change model for mixed traffic

En-route lane-change behaviour is common due to high-speed chase or objective lane's differences. Many existing models have been proposed to describe the motivation, preparation on current lanes, execution, and car-following on target lanes. Nevertheless, for heterogeneous traffic, some complex driving patterns and unique characteristics must be discussed in greater detail. Specifically, lane-change behaviour will have a significant impact when entering or exiting a CACC platoon. The process of platoon recombination is capable of changing the roles of a platoon's vehicles and causes significant disruptions. In general, the motivation for lane-change is the benefit, such as reduced travel time, higher speed, or driving into an exact exit lane. In the literatures for HDVs, the integrated benefit is computed and applied to evaluate the lane-change intention at a given time interval. Otherwise, stability and recovery are not considered, which is crucial for CACC platoons. Therefore, this article focuses heavily on evaluating stability for various lane-change locations.

As depicted in Figure 3, the subject vehicle i is preparing to enter the target lane at the correct location. A safe gap must be maintained between 2 adjacent vehicles:

$$g_{j-1,i}(t) \ge g_i(t),\; g_i(t) = v_i(t)\cdot T_i + l; \tag{11}$$

$$g_{i,j}(t) \ge g_j(t),\; g_j(t) = v_j(t)\cdot T_j + l. \tag{12}$$

At time t, if the Equations (11) and (12) can be satisfied, a direct inserting will be executed. There will be no speed adjustments and minor oscillations for vehicles driven by humans. Alternately, for heterogeneous traffic, platoon recombination and subsequent instability continue to exist. Especially when the safe gap is inadequate, vehicles must adjust their speeds to provide larger gaps. The subject vehicle i will transmit a lane-change signal, and the vehicle j will assist in enlarging the space until all safety constraints are met. The whole process for various conditions is detailed in Figure 4 as follows:

- if Equations (11) and (12) are both satisfied, the subject vehicle i drives into the mapping position directly;

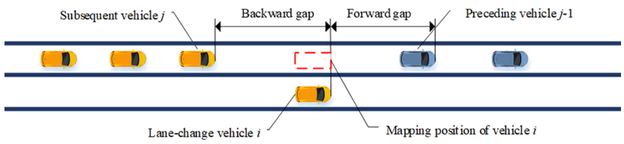

Figure 3. Sample for vehicles' distance

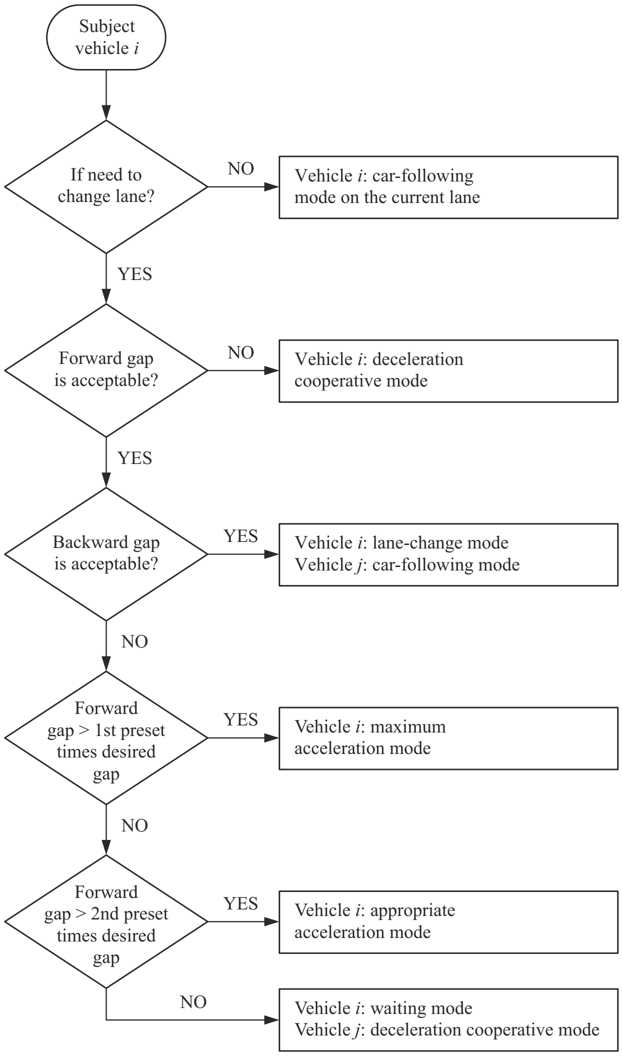

Figure 4. Lane-change preparation process

- if $g_{j-1,i}(t) < g_i(t)$, i.e., the forward gap is not sufficient, vehicle i activates deceleration cooperative mode as shown in Figure 4; then, vehicle i will speed down at a proper deceleration. A deduction is constructed as in Equations (13)–(15). The preceding vehicle j – 1 on the target lane remains at the original speed, and the subsequent vesicle j follows with vehicle j – 1 and i simultaneously.

If vehicle i speeds down and the forward gap in constraint (Equation (11)) can be satisfied during one single time interval Δt, a constraint for this situation is presented in Equations (13) and (14), otherwise, a maximum deceleration is executed to larger gap quickly; therefore, a corresponding deduction for a comfortable deceleration is computed as in Equation (15):

$$g_i(t)=x_{j-1}(t)+v_{j-1}(t)\cdot\Delta t-\left(x_i(t)+v_i(t)\cdot\Delta t+\frac{1}{2}\cdot a_i'(t)\cdot(\Delta t)^2\right); \quad (13)$$

$$a_i'(t)=\frac{2\cdot\left(g_i(t)-\left(x_{j-1}(t)-x_i(t)-\left(v_{j-1}(t)-v_i(t)\right)\cdot\Delta t\right)\right)}{(\Delta t)^2}; \quad (14)$$

$$a_i(t)=\max\left(d_{\max},\frac{v_{\min}-v_i(t)}{\Delta t},a_i'(t)\right), \quad (15)$$

where: $x_i(t)$ is the position of vehicle i, obviously, the feasible deceleration $a_i(t)$ is the maximum of the 3 values considering the vehicles' kinematics;

- if $g_{j-1,i}(t)\geq g_i(t)$, $g_{i,j}(t)<g_j(t)$, i.e., the forward gap is sufficient, but the backward gap is not. For this case, 2 strategies, including an acceleration of vehicle i and a deceleration of vehicle j can be verified the feasibility successively. Naturally, acceleration has the priority to be tested due to the advantage of achieving higher speed and lower time cost:
  - if $g_{j-1,i}(t)\geq\alpha\cdot g_i(t)$, $\alpha>1$, $g_{i,j}(t)<g_j(t)$, corresponding to the maximum acceleration mode in Figure 4, then the gap between vehicle i and $j-1$ is allowed to accelerate for a long time. To quickly close a sufficient gap, vehicle i will accelerate at its maximum rate, as shown in Equations (16). In this situation, frequent acceleration/deceleration and drastic speed fluctuation are avoided due to the long gap. Meanwhile, subsequent vesicle j follows with vehicle $j-1$ and i simultaneously.

    Similarly, if acceleration is capable of ensuring a safe gap in a given amount of time, the following restriction is necessary:

    $$a_i(t)=\min\left(a_{\max},\frac{v_{\max}-v_i(t)}{\Delta t}\right); \quad (16)$$

  - if $\beta\cdot g_i(t)\leq g_{j-1,i}(t)<\alpha\cdot g_i(t)$, $g_{i,j}(t)<g_j(t)$, corresponding to the appropriate acceleration mode in Figure 4. Different from the above cases, a maximum acceleration of vehicle i increases the possibility of frequent velocity oscillation, and vehicle i will speed up with a proper acceleration to close vehicle $j-1$ and then decelerate to follow with an identical velocity. Meanwhile, subsequent vesicle j follows with vehicle $j-1$ and i simultaneously:

    $$g_i(t)=x_{j-1}(t)+2\cdot v_{j-1}(t)\cdot\Delta t-\left(x_i(t)+2\cdot v_i(t)\cdot\Delta t+a_i'(t)\cdot(\Delta t)^2\right); \quad (17)$$

    $$a_i(t)=\min\left(a_i'(t),\frac{v_{\max}-v_i(t)}{\Delta t}\right); \quad (18)$$

  - if $g_i(t)\leq g_{j-1,i}(t)<\beta\cdot g_i(t)$, $g_{i,j}(t)<g_j(t)$, corresponds to waiting mode, vehicle i will stick on the original driving mode to remain a desired gap; meanwhile, the subsequent vehicle$j$ on the target lane activates the deceleration cooperative mode described in Equations (13)–(15).

Noticeably, for CVs and HDVs, different safety gap, minimum reaction time and driving models are involved in the above speed adjustment expressions. Otherwise, the optimum lane-change trajectories organization aided by a central controller for CVs is out of consideration.

4. Stability analysis after lane-change disturbance

4.1. Stability recognition and descriptive index

Traffic flow stability has a significant impact on capacity, speed fluctuations, safety, and fuel consumption. In the existing literatures on stability, much focus is paid to the instability generated by a little speed perturbance in an instant time of an individual vehicle $\Delta v_i(t)$. Methodology analytical methods are used to estimate disturbance propagation based on driving models of vehicles. In this situation, $\Delta v_i(t)$ is a single influence factor and the subsequent vehicles just change velocity to follow with the preceding vehicle. Otherwise, stability analysis due to lane-change disturbance has many differences. More parameters will be affected by lane-change behaviour, including the vehicles' roles in a platoon (leader vehicle, following vehicle, or degrading to an ACC vehicle), order, and velocity. Meanwhile, the speed disturbance $\Delta v_i(t)$ will continue until the lane-change is completed. Therefore, multiple influence factors exist, and the platoons or driving organization of vehicles on each lane are reconstructed.

For the stability analysis, defining the flow stability indexes is a primary task. After entering the research zone, the vehicle will typically execute the car-following model or speed adjustment model presented explicitly in Section 2 to match the speed of the preceding vehicle and maintain a predetermined gap. Finally, the vehicles on an identical lane will reach a stable balance after a period of speed fluctuation without sudden interference. For this state, the vehicles' acceleration, gap error, and velocity difference are zero. Namely, all the vehicles drive at the desired velocity, and the gaps between adjacent vehicles remain constant.

Consequently, it can be concluded that similar indexes can be selected to characterize the stability following disturbance. When reaching a steady state, there is no acceleration and all vehicles travel at the same speed. For heterogeneous traffic flow, time gaps vary based on reaction time, and the driving model of adjacent vehicles can be used to calculate the gap between them. Therefore, for saturated flow, the stable driving state can be described as: $a_i(t)=0$, $v_i(t)=v_{i-1}(t)$, $g_{i,i-1}(t)=v_i(t)\cdot T_j+l+s_0$, j = 1, 2, 3, 4 corresponds to different vehicle types.

If the stability is disturbed by lane-change interference, a local disturbance of velocity and vehicle gap will propagate from the lane-change location and disappear after a predetermined time. A car-following and speed adjustment process will be executed and a steady traffic flow is maintained for a sufficient amount of time.

4.2. The influencing factors and analysed characteristic parameters

As discussed in the preceding sections, the minimum safe gap is a prerequisite for lane-change acceptance, and the selected gap corresponds directly to the velocity adjustment duration. The driving models are related to several parameters, including the selected lane-change spot, the vehicles' gap $g_{i,j}(t)$, the type of vehicles i, j and the saturated and unsaturated traffic flow on objective lanes. Therefore, a group of influencing factors will be simulated and compared to determine the rules of instability. The factors discussed and simulated comparison scenes in the subsequent experiments are described in Table 2:

- ***type of vehicle** i, j**:*** closely related to the required gap, the platoon recombination, and the speed adjustment process;
- ***CV's penetration:*** closely related to platoon's length, occupation ratio;
- ***lane-change spot:*** closely related to platoon recombination, type of vehicle $j-1$, j;
- the minimum safety gap, velocity adjustment mode, and car-following behaviour;
- ***the backward gap** $g_{i,j}(t)$**:*** direct factor that determines the duration of velocity adjustment;
- ***the stable traffic flow state on the target lane:*** closely related to headway.

Throughout the entire transition from oscillation to stability, vehicles within a certain range will be affected by deceleration and time delay. The primary objective is to evaluate the degree of effect, calculate the benefits of lane-change, and determine the lane-change interval and location. Therefore, it is essential to develop a framework for evaluation indexes and a computation method to analyse stability after disturbance for various traffic flow states:

- ***the fluctuation range of velocity:*** this factor is crucial in determining driving comfort, fuel consumption, and greenhouse gas emissions. It has great significance as a metric for evaluating lane-change benefits. In the following experiment, a velocity fluctuation will be displayed spatial-temporally, and the variation process for each vehicle will be observed along with the timeline;
- ***the length of perturbance duration time:*** this index is a direct indicator of instability. The mean variation time for all affected vehicles will be calculated to reflect the recovery cost;
- ***the accumulated travel time delay:*** travel delay is chosen to evaluate the decrease in efficiency, and the total driving time on the setting lane for all vehicles is calculated by combining the performance with or without lane-change. It is applicable for evaluating the validity of the lane-change decision.

5. Simulation for various traffic flow scenes

In this section, multiple experiments with different initial conditions are conducted to analyse the proposed stability indexes and evaluate their impact on various lane-change decisions. The heterogeneous traffic flow is consisted of HVs, ACCs, and CACCs and distributed randomly according to a setting penetration to illustrate the diversity. In addition, detailed descriptions of the stability region, time delay, fluctuation degree, and additional indexes are investigated.

5.1. Data for example

The proposed analysis of vehicle stability focuses on the vehicles' entry into the target lane from the original lane. To make a significant contract after lane-change disturbance, the distributed flow is based on the following assumptions:

Vehicles follow a Poisson distribution to be dispatched on lanes, and the traffic flow reaches a steady driving state

Table 2. Discussed factors and simulated scenes

Vehicle i	Factors	Simulation setting	Comparison experiments
CV	subsequent vehicles' type	▪ traffic a: a saturated platoon; ▪ traffic b: a HDV; ▪ traffic c: an unsaturated platoon	▪ lane-change point: head of a platoon; ▪ backward gap: half of the stable gap
	the CV's penetration	different penetrations from 10 to 100%	▪ lane-change point: head of a platoon; ▪ backward gap: half of the stable gap; ▪ CV's penetration: from 10 to 100%
	backward gap	from the minimum gap to the maximum gap with an interval 5 m	lane-change point: head of a platoon; CV's penetration: 40%
	saturated or unsaturated flow	▪ saturated flow: with minimum safety gap; ▪ unsaturated flow: larger than minimum safety gap	the gap is larger than the stable gap, the other parameters keep identical with the above experiments
	lane-change spot	head, middle and rear of the target platoon	▪ backward gap: from the minimum to the maximum gap; ▪ CV's penetration: 40%
HDV	▪ subsequent vehicles' type; ▪ lane-change spot; ▪ backward gap	same as the settings of CVs	identical with the CVs

with the same suitable speed and no acceleration when disturbances occur. This initial condition ensures the striking contract with velocity, gap, and acceleration fluctuations and is reasonable for forming a stable flow in a short time period, which can be observed to verify the hypothesis after running the driving models presented in Section 2.

The specific parameter values for the proposed model are displayed in Table 3.

Table 3. The value of models' parameters

Parameter	Description	Value
L_{max}	maximum length of platoon	10 veh
v_{max}	maximum velocity	33.3 m/s
v_{min}	minimum velocity	27.7 m/s
α	coefficient	2
T	total simulation time	800 s
δ	acceleration coefficient	0.4
d_{max}	maximum deceleration	3 m/s^2
β	coefficient	1.3

5.2. Analysis of simulation results

This section will present some contract simulations to reveal the rules with different preconditions. In the preceding section, the most significant influencing factors are outlined. To ensure the validity of the comparison, the foundation data, excluding contract parameters, will be set to fixed values. The detailed results are shown as follows.

5.2.1. A CV changes into the target lane

Case 1: the 1st factor: subsequent vehicles' type. As shown in Section 2, when the traffic flow is saturated and drives as a stable state, the vehicle's time gap is at its minimum value. The gap $g_{i,\,j-1}(t)$ is insufficient $g_{i,\,j-1}(t) < g_{safe}(t)$, vehicle i, j will decelerate to reach a comfortable gap, and the manoeuvres have an impact on flow stability. A group of results are illustrated in the following figures to display the characteristics:

As shown in Figures 5–8, a description of velocities' variation can be observed. The mean variation time is 132.01, 149.99, 17.61 s for 3 kinds of traffic flow shown in Figure 8. If the subsequent vehicle is an HDV (illustrated as *traffic flow* (*b*)), a long-time deceleration will occur to obtain a sufficient safe gap and a drastic velocity oscillation appears. Inversely, if the subsequent vehicle is a platoon and $L < L_{max}$ (illustrated as *traffic flow* (*c*)), then vehicle i will join into the platoon and drives as a head vehicle to require a minor gap. Therefore, a relatively flat vibration of less influenced vehicles is presented. Notably, when $L = L_{max}$ (illustrated as *traffic flow* (*a*)), and the rear vehicle $j + L_{max} - 1$ will drive out of the current platoon to drive as ACC mode. At the same time, a more drastic fluctuation appears from the rear spot to the upstream traffic, as shown in *traffic flow* (*a*) in Figure 5 and Figure 6. Obviously, for *traffic flow* (*c*), the vehicles' gap recovers to the former value after oscillation, but a larger distance appears after lane-changing in *traffic flow* (*a*) and *traffic flow* (*b*).

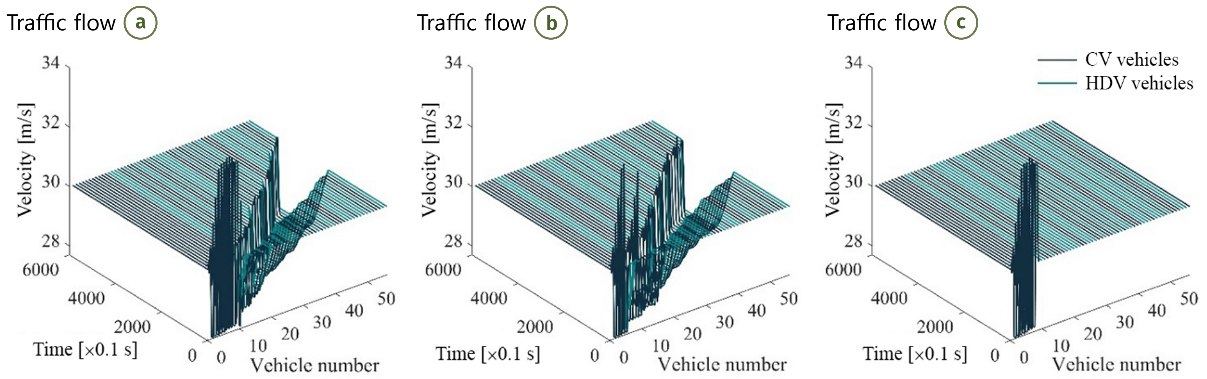

Figure 5. The velocity variation with different subsequent vehicles

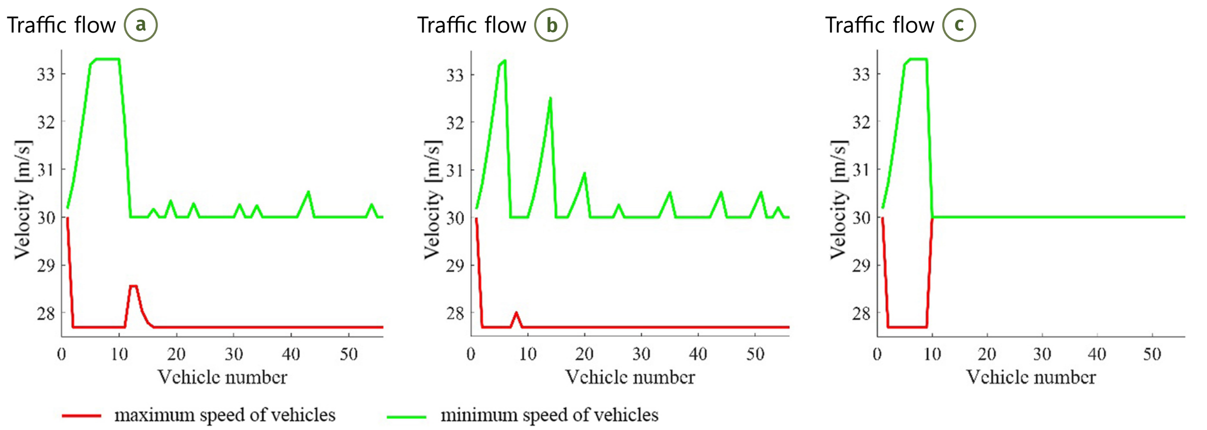

Figure 6. The velocity variation range

Case 2: the 2nd factor: CVs' penetration. The vehicles' dispatching obeys the Poisson distribution with a preset CV's penetration. In addition, the vehicles' gap is the minimum value at the stable state. To highlight the focused factors, the lane-change spot to the subsequent vehicle $g_{i,\,j}(t) = \frac{1}{2} \cdot g_{safe}(t)$ and vehicle j is required to decelerate to provide sufficient gap. Some other parameters are similar to Case (1). Figures 9 and 10 have separately displayed the velocity disturbance's propagation, and the average oscillation time under different penetration rates.

In the simulation results, the mean variation times are 195.17, 173.56, 146.23, 116.91, and 142.30 s corresponding to the penetration 10, 20, 40, 80 and 100%, respectively. A trend of decreasing at the early stage and then increasing with the rising penetration can be observed. As shown in Figure 10, the disturbance propagates much faster with a deeper penetration; the starting time slots for the variation are 18.99 s (p = 100%) and 177.03 s (p = 10%), respectively. These phenomena are feasible and can be explained: the advantage of a short reaction time, the CV's faster response to a disturbance, and a reduction in recovery time. For extreme penetrations, however, an inserting vehicle may cause frequent platoon reconfiguration or division, as well as a significant velocity perturbance and long-term instability.

Case 3: the 3rd factor: the gap $g_{i,\,j}(t)$. This discussed factor determines whether the following vehicle or lane-change vehicle will accelerate or decelerate and has a significant impact on the propagation of the disturbance. The primary objective is to determine the boundary value that distinguishes the instability range:

- for saturated traffic flow, the headway in the platoon is 24 m and the gap $G_{j-1,\,j}(t)$ = 39 m. If vehicle i changes lane to perform as a new leading vehicle, the minimum sufficient gap $G_{i,\,j}(t)$ = 22 m, $G_{j-1,\,i}(t)$ = 37 m is larger than the provided gap, and speed adjustment is required. Obviously, if $g_{i,\,j}(t) \in (0, 2]$ m, the gap to vehicle $j - 1$ is sufficient, vehicle i drives at the constant velocity, and vehicle j decelerates to larger the gap; if $g_{i,\,j}(t) \in (2, 39]$ m, vehicles i and j both decelerate. The experiment results, including mean variation time and total time delay are presented in Figures 11 and 12.

  The observed phenomena in the figures include the following: for the 3 preset types of traffic flow, a great deal of variation appears with multiple breakpoints and demonstrates a decreasing trend at the beginning, followed by an increasing trend as the gap increases. The instability in *traffic flow* (*b*) is the most extreme and the least in *traffic flow* (*c*). The detailed maximum and minimum values for perturbance time and mean travel delay are displayed in Table 4;

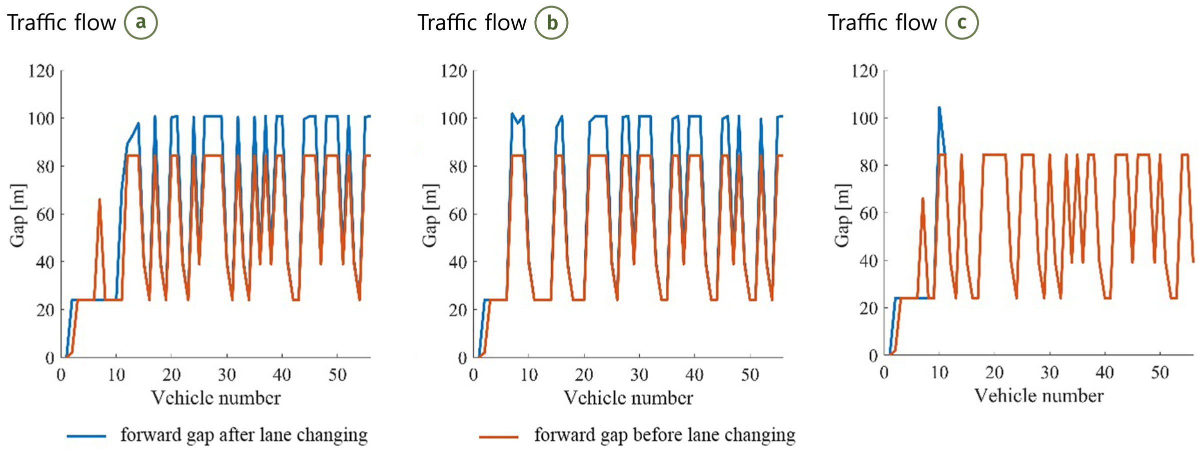

Figure 7. Vehicles' gap before or after lane-change

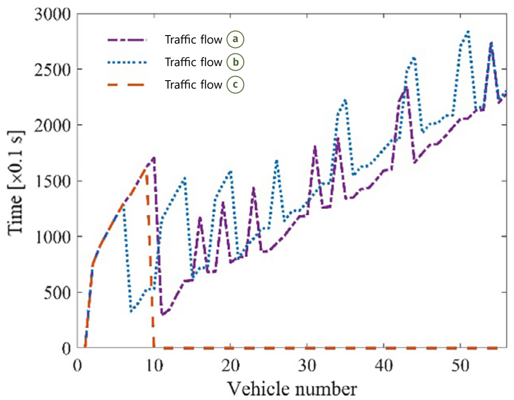

Figure 8. Vehicles' variation time

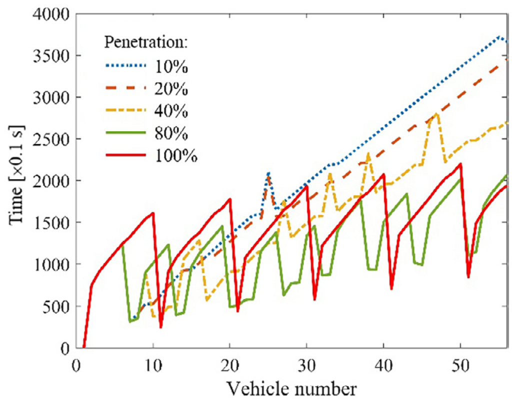

Figure 9. The variation time with different penetrations

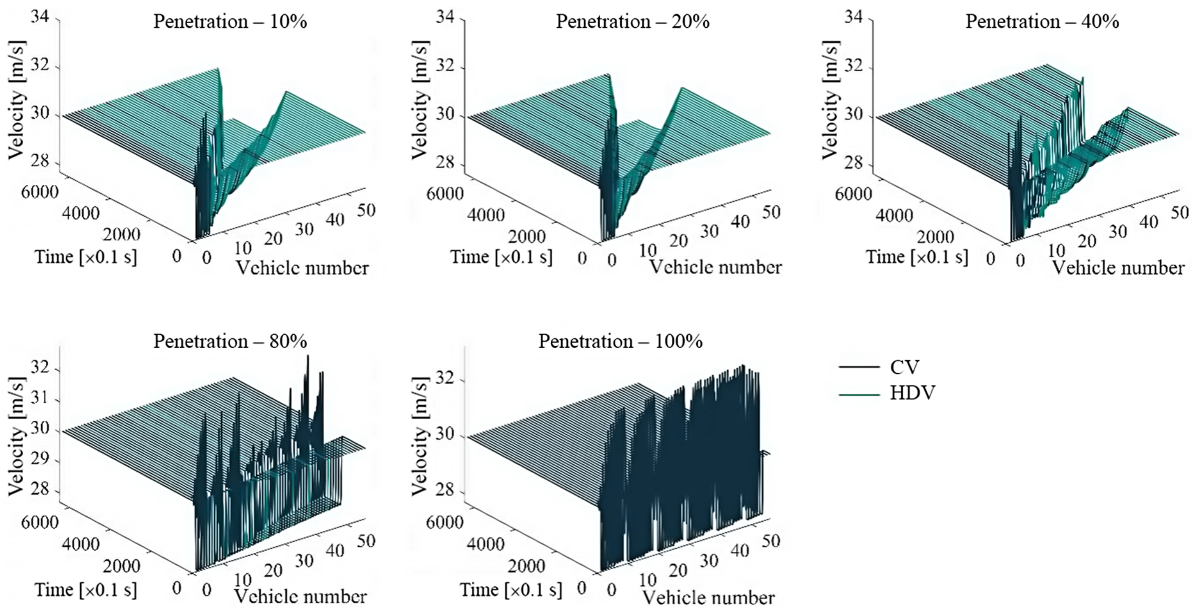

Figure 10. The velocity variation with different penetrations

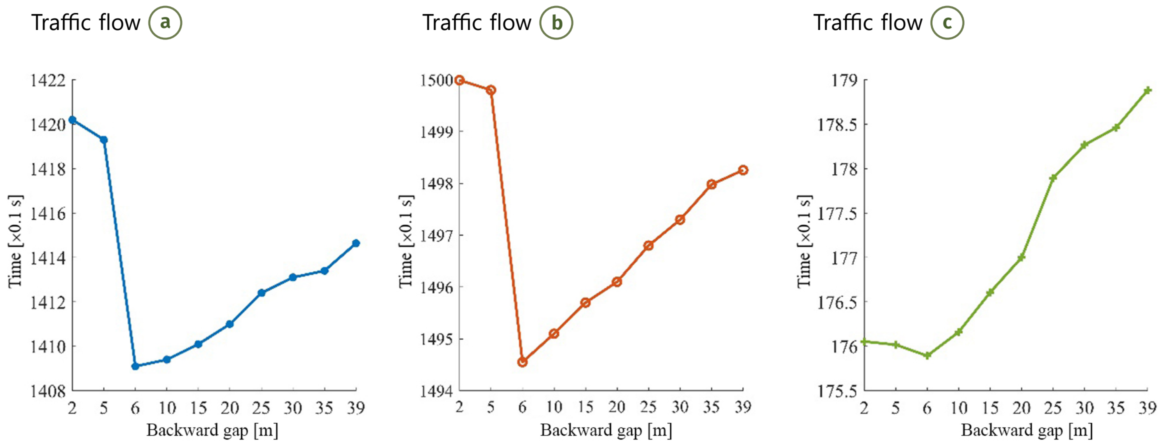

Figure 11. The mean variation time with different gaps

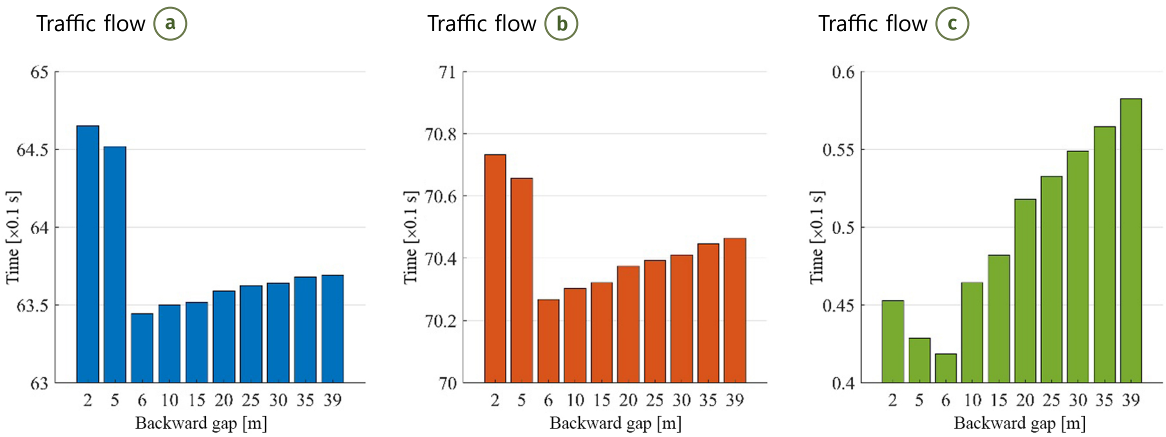

Figure 12. The mean delay with different gaps

- for unsaturated flows, the gap $g_{j-1,\,j}(t) > 2 \cdot g_{safe}(t)$ is possible to be sufficient for direct lane-change, the front gap $g_{j-1,i}(t)$ or the back gap $g_{i,\,j}(t)$ is enough to avoid velocity variation and change gap enlarging mode. The following experiments compute the mean value of a group of results for different values $g_{j-1,\,j}(t)$ to conclude the common rules, the detailed results are presented in Figures 13–16.

The results are displayed in 2 separated groups, $g_{i,\,j}(t) \in [2, 24]$ m in Figures 13 and 14, and $g_{i,\,j}(t) \in [24, 66]$ m in Figures 15 and 16. The phenomenon observed in the figures includes: when $g_{i,\,j}(t) \in [2, 24]$ m, for the 3 kinds of traffic flow, many variations appear with several breakpoints and present a trend that it decreases at the early stage, then increases and finally decreases with the larger gap. The instability in case *traffic flow* (*b*) presents the most drastic and the slightest in case *traffic flow* (*c*). The detailed maximum and minimum values for perturbance time and mean travel delay are displayed in Table 1. The variation peaks and valleys for the 3 cases all occur at $g_{i,\,j}(t) = 8$ m and $g_{i,\,j}(t) = 20.5$ m. Notably, the delays in case *traffic flow* (*c*) are negative, the lane-change vehicle will act as a communication connection and promote the vehicles' acceleration to join the platoon and reduce travel time. When $g_{i,\,j}(t) \in [24, 66]$, the variation presents a longer duration with a larger gap and disappears when the $g_{i,\,j}(t) \geq 66$ m.

Table 4. The detailed data for the head of the platoon

Traffic flow	Mean variation time [s]		Mean delay [s]	
	maximum value	minimum value	maximum value	minimum value
a	142.02	137.29	6.47	5.53
b	158.99	146.51	7.57	6.34
c	17.85	13.59	0.46	0.15

Case 4: the 4th factor: different lane-change spots in a platoon. The following experiments are conducted to present the comparative results by simulating the lane-change behaviours at the rear or the middle of the platoon. Different gaps $g_{i,\,j}(t)$ similar to Case (3) are simulated under a saturated stable traffic flow, and a mean value is recorded to be compared with other lane-change spots. The detailed data are presented in Tables 5 and 6, and the comparisons are shown in Figures 17 and 18.

The mean variation time and delay of changing into middle or rear of a platoon present a decrease at the early stage, then increase and finally decrease with the larger gap for *traffic flow* (*a*), *traffic flow* (*b*) and a continuous increase for *traffic flow* (*c*), which is similar to Case (3).

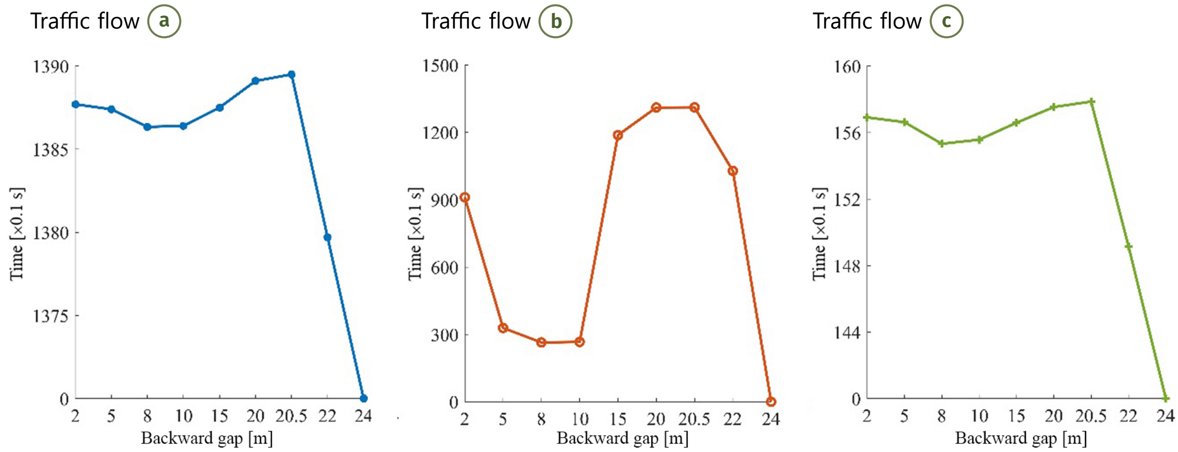

Figure 13. The mean variation in gaps range [2, 24] m

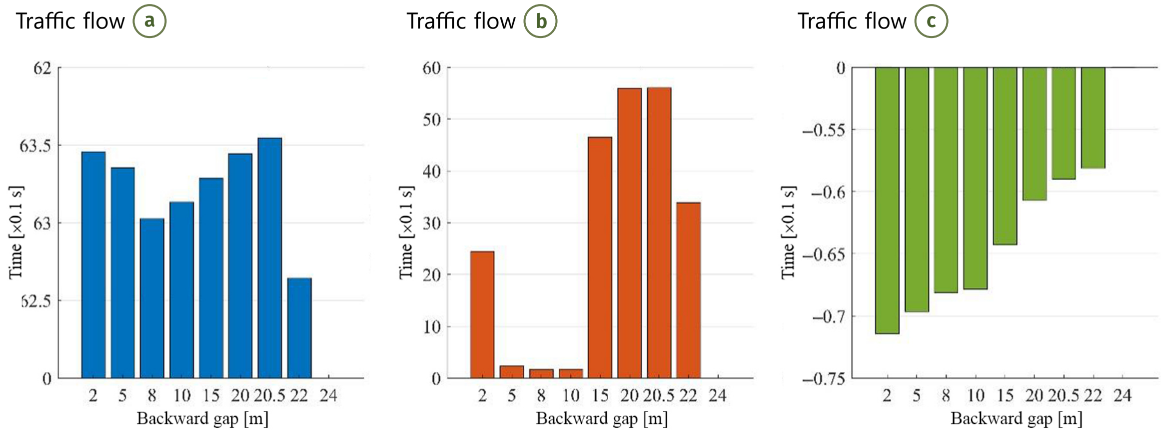

Figure 14. The mean delay in gaps range [2, 24] m

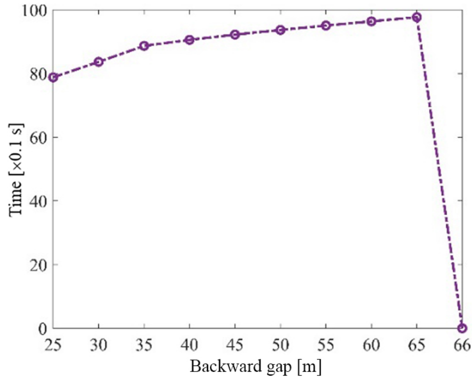

Figure 15. Mean variation time in gaps range [25, 66] m

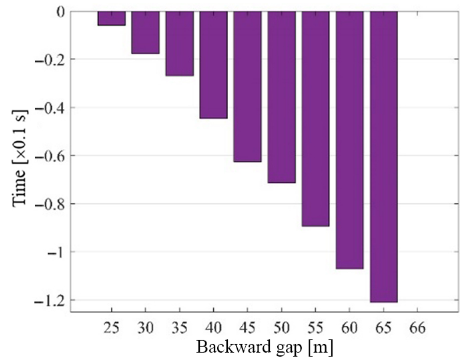

Figure 16. Mean delay in gaps range [25, 66] m

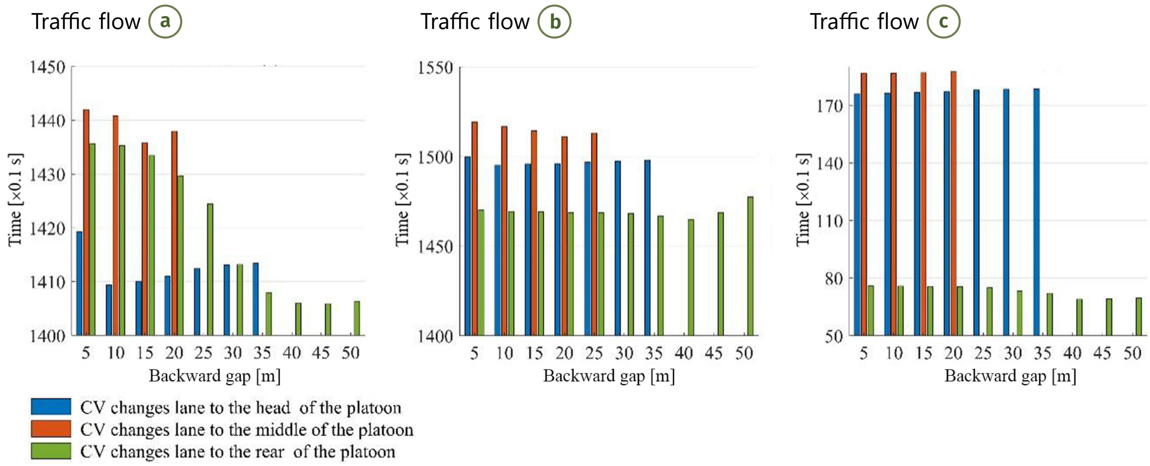

Figure 17. The mean variation time for different spots

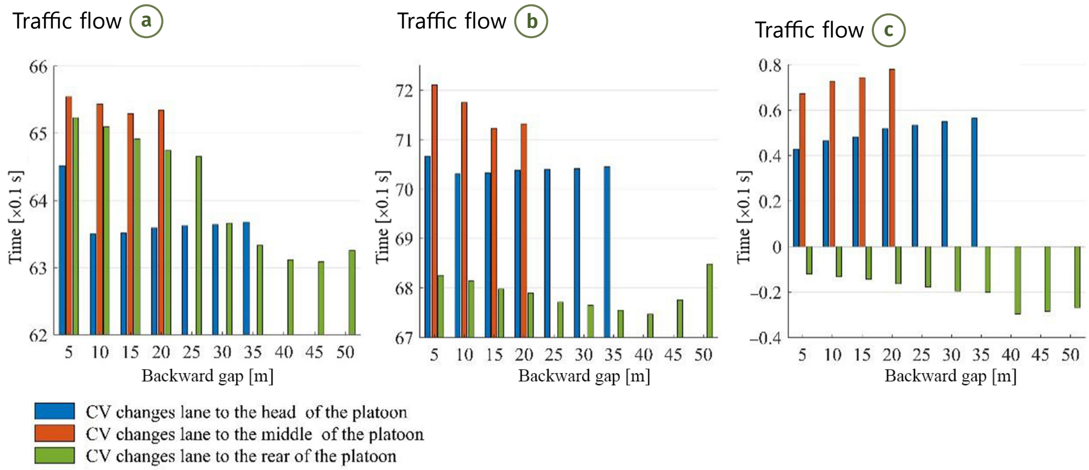

Figure 18. The mean delay for different spots

Table 5. The detailed data for the middle of the platoon

Traffic flow	Mean variation duration [s]		Mean delay [s]	
	maximum value	minimum value	maximum value	minimum value
a	144.57	143.46	6.57	6.49
b	151.94	151.00	7.25	7.10
c	18.76	18.60	0.08	0.06

Table 6. The detailed data for the rear of the platoon

Traffic flow	Mean variation duration (value/corresponding gap) [s]		Mean delay (value/ corresponding gap) [s]	
	maximum value	minimum value	maximum value	minimum value
a	143.56	140.19	6.52	6.29
b	148.44	146.43	6.74	7.02
c	7.61	6.85	–0.01	–0.03

Observed from the comparison of different lane-change spots depicted in Figures 17 and 18, the perturbance presents the most serious deduced by inserting into the middle and the slightest for the rear except a special situation that gap $g_{i,j}(t)$ is in the range [5, 30] m for *traffic flow* (*a*).

5.2.2. A HDV changes into the target lane

The experiments in this section aim to reveal the influence difference between HDVs and CVs. In accordance with the discussions in Section 4.2.1, the factors, including lane-change spots, gaps, and different following vehicles are considered, and the other experimental parameters are maintained to ensure the consistency of comparison.

A HDV changes lane to the head of a platoon. This experiment is conducted with the same parameters as CVs and a group of gaps $g_{i,j}(t)$ are simulated to compute the mean values of the indexes. The mean variation times for 3 kinds of flows are depicted in Figure 19. Compared with the results in Figure 11, it can be observed that the fluctuation caused by the lane-change of a HDV is significantly greater. Notably, for *traffic flow* (c), the differences are particularly obvious. Because for an HDV, a larger safe gap is required to force the reformed platoon to reduce velocity for a longer period of time, whereas for a CV, the subsequent platoon can recombine with the preceding platoon and drive at a constant velocity.

Furthermore, different from the trends of CVs, the variation time and delay show a continuous increase with the gap $g_{i,j}(t)$. The reason can be explained that the minimum safe gap for HDV is 49 m, larger than the headway of a platoon 39 m, and deceleration is required. With the increase of $g_{i,j}(t)$, a longer time is required to decelerate for vehicle i to larger the gap and a minimum velocity difference $\Delta v_{i,j}(t+\Delta t)$ is obtained. Therefore, a longer perturbance time is the cost to reach the safe gap.

A HDV changes lane to the middle of a platoon. The results in Figure 20 display the variation time of inserting into the middle of a platoon. Similar rules can be concluded that the fluctuation deduced by HDV is more serious than CV and the variation lasts longer with the gap increase due to the insufficient gap 24 m in a platoon.

Meanwhile, the variation time and delay show a continuous increase with the gap $g_{i,j}(t)$ because the minimum safe gap 49 m is larger than the headway 24 m. Larger gap $g_{i,j}(t)$ corresponds to a longer deceleration time of vehicle i and serious perturbance.

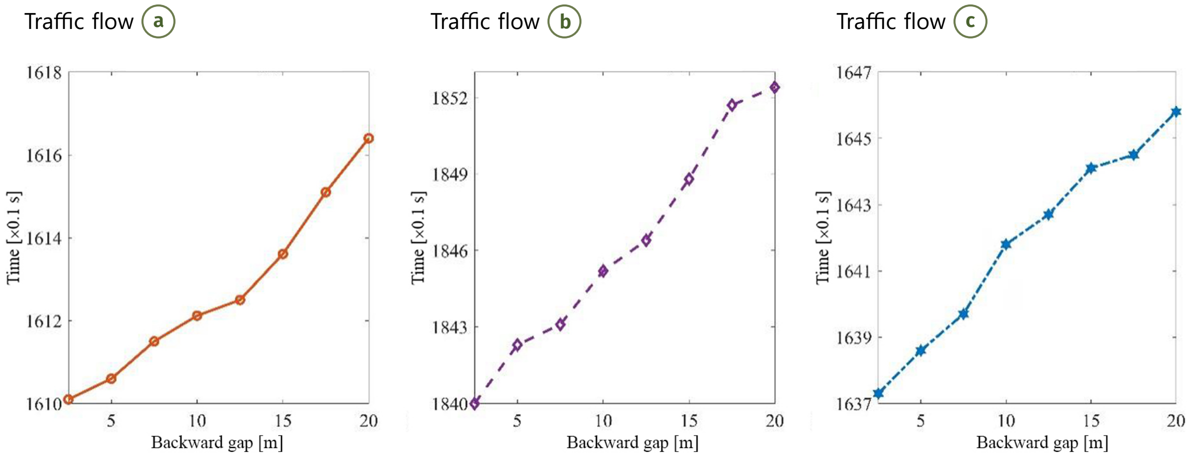

Figure 19. Variation time for HDV changing into the head of a platoon

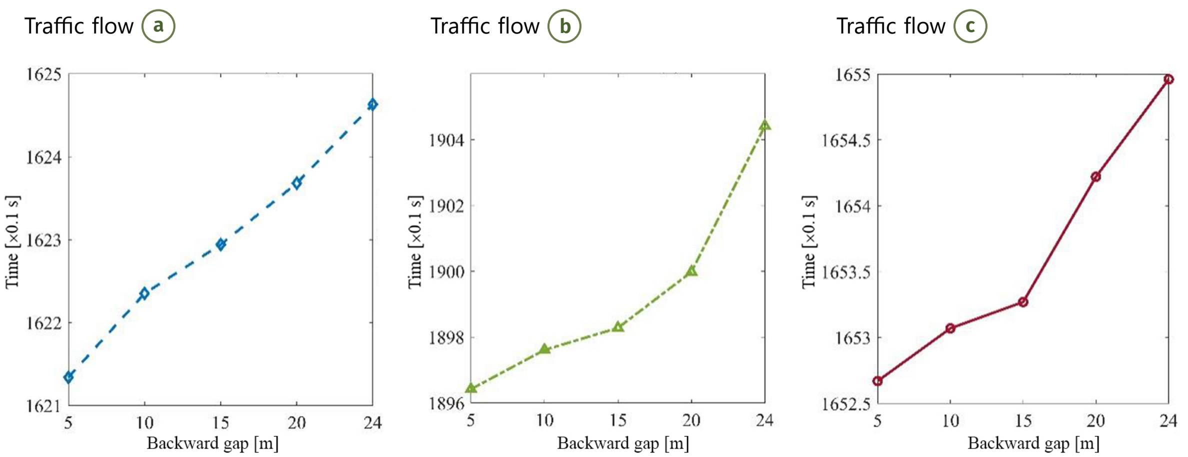

Figure 20. Variation time for HDV changing into the middle of a platoon

A HDV changes lane to the rear of a platoon. The results in Figure 21 display the variation time of inserting into the rear of a platoon. The mean variation times and delay are still more serious than CV. For *traffic flow* (*a*) and *traffic flow* (*c*), the variation presents a continuously increasing trend with the larger gaps due to the insufficient gap of 39 m between 2 adjacent platoons. Remarkably, for *traffic flow* (*b*), the variation time and delay decrease to the minimum values 188.30 and 9.67 s in the gap range of [10, 31.5] m and increases along with the larger gap. For *traffic flow* (*b*), the subsequent vehicle is an HDV, and the provided vehicle gap $g_{j-1,j}(t)$ = 99 m. The gap $g_{j-1,j}(t)$ to the preceding vehicle $j-1$ is satisfied to the minimum lane-change gap when $g_{i,j}(t) \le 30$ m and vehicle i will accelerate to enlarge $g_{i,j}(t)$ 1st and then decelerate to acquaint a stable velocity. During this process, a larger gap corresponds to shorter acceleration and deceleration to the comfortable velocity. When $g_{i,j}(t) \ge 30$ m, vehicles i and j both decelerate, and large gap forces vehicle i to speed down for a longer time.

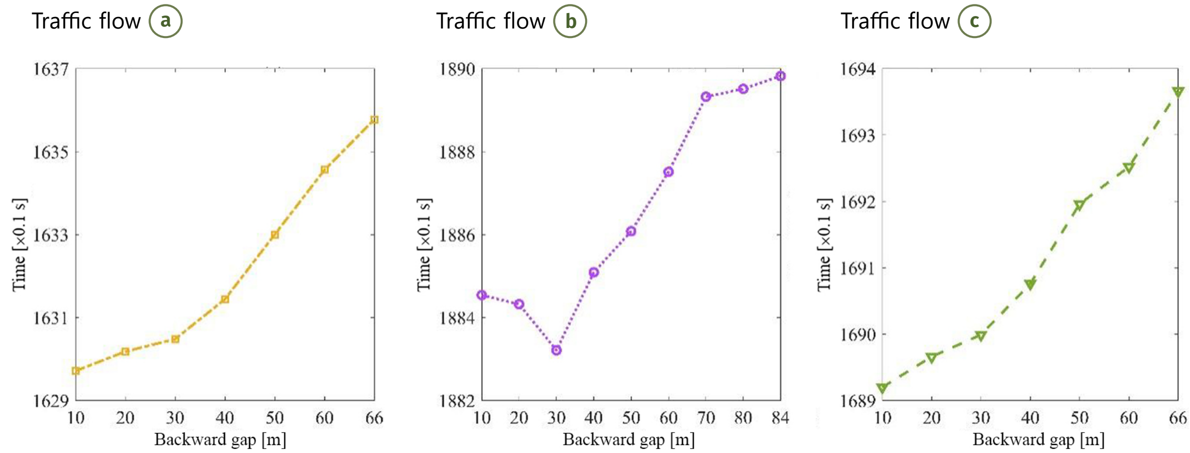

Figure 21. Variation time for HDV changing into the rear of a platoon

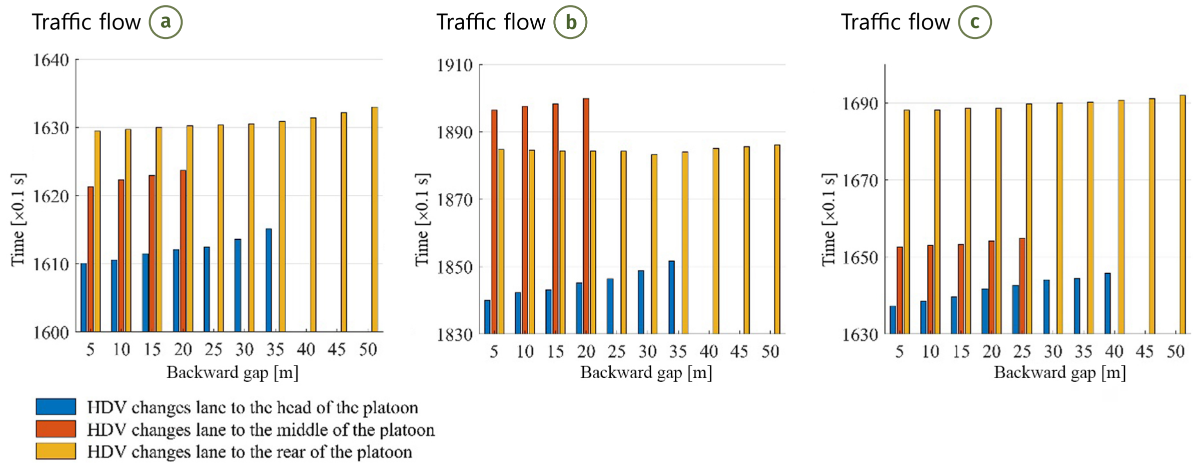

Figure 22. The mean variation for different spots

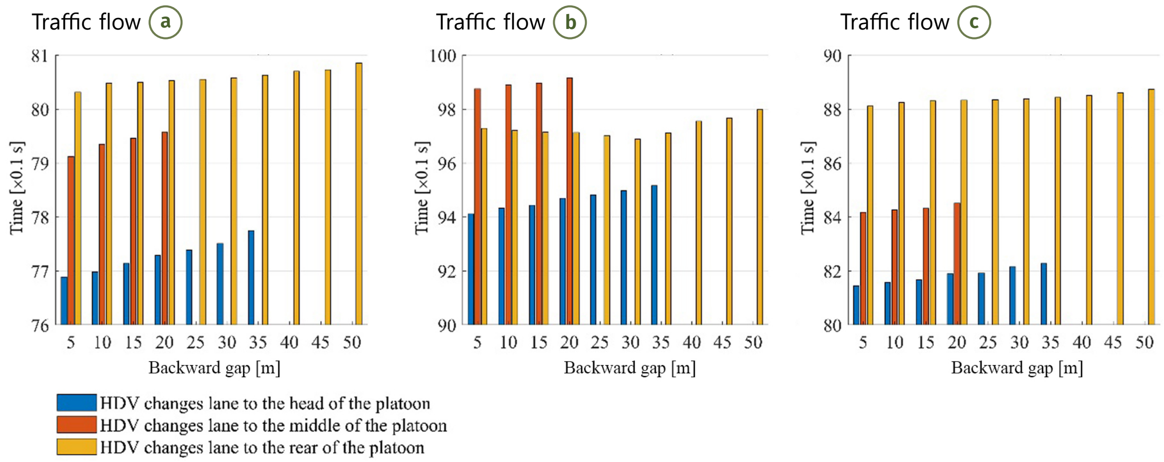

Figure 23. The mean delay for different spots

The comparison results of mean variation time and delay for different lane-change spots are shown in Figures 22 and 23. A disparate characteristics can be found from the CVs for 3 kinds of flows. Noticeably, for *traffic flow* (*a*) and *traffic flow* (*c*), the perturbance presents the most serious deduced by change into the rear and the slightest for the head. For *traffic flow* (*b*), inserting to the middle variations more drastic, which is distinct from the phenomenon of CVs. The divided platoon combines with the subsequent platoon is benefit to reform a longer platoon and reduce instability, therefore a sighter fluctuation and less delay is displayed. But for *traffic flow* (*b*), dividing the platoon deduces larger gap and serious variation.

6. Conclusions

This study presents a car-following and lane-changing model for describing the flow driving states; furthermore, a simulation method was established to evaluate the stability of a vehicle platoon in heterogeneous traffic flow. A final state with no acceleration and a constant speed for all vehicles was defined to recognize the stable flow. A serious of influencing factors including subsequent vehicles type, CV penetration, backward gap, saturated or unsaturated flow and lane-change spot are preset to discuss the fluctuation range of velocity, the length of perturbation duration time and the accumulated travel time delay. Comparison results for various simulation scenes are summarized as follows:

- the upstream traffic flow has significant influence on stability. Inserting into an unsaturated platoon will result in minor fluctuation and the mean variation time will be reduced to 17.61 s and less than a tenth of that occurred in saturated platoon or HDVs. Simultaneously, the vehicles' gap recovers to the former value after oscillation for unsaturated flow but presents a larger one for the other 2 cases;
- higher penetration corresponds to shorter recovery time and slighter fluctuation when penetration is less than 80%; otherwise, saturated penetration will deduce drastic instability due to frequent platoon division; the minimum mean variation time is 116.91 s obtained with the penetration 80%. Meanwhile, the disturbance propagates much faster with deeper penetrations due to the shorter reaction time (initial variation time 18.99 s with p = 100%, 177.03 s with p = 10%);
- in the case that a CV inserting into a saturated flow, a great deal of variation demonstrates a decreasing trend at the beginning, followed by an increasing trend as the gap enlargement and the minimum value appears at $g_{i,j}(t)$ = 22 m. Otherwise for unsaturated flow, the phenomenon presents more complicated. When $g_{i,j}(t) \in [2, 24]$ m, the variation peak and valley occur at $g_{i,j}(t)$ = 8 m and $g_{i,j}(t)$ = 20.5 m, separately. When $g_{i,j}(t) \in [24, 66]$ m, the variation presents longer duration with a larger gap and disappears at $g_{i,j}(t) \in [24, 66]$ m;
- differently, for HDV changing lane, the variation time and delay show a continuous increase with the gap except for a special situation that HDV drives into the rear of a platoon followed by a HDV. For the special case, a slightest variation can be found at $g_{i,j}(t)$ = 31.5 m;
- lane-change spot has great influence on stability, due to the type of the subsequent vehicle. Different characteristics are found for HDVs and CVs. For CVs, dividing a platoon should be avoid to diminish instability, but for HDVs, it is possible to promote the subsequent vehicles to reform a longer platoon and reduce delay.
- for CVs, the perturbation presents the most serious deduced by inserting into the middle and the slightest for the rear. Noticeably, different results can be observed for HDVs that the most serious and slightest perturbation is deduced by change into the rear and the head separately.

The above simulation results are applicable to lane-change management, with the main contribution being that the instability is useful in evaluating the reasonableness of lane-change behaviour and quantifying lane-change intention. For mandatory or discretionary lane-change, selecting the optimal time and location is necessary to ensure the greatest travel utility, and the index in this article serves as a decision-making resource.

This study can be extended to address some unsolved issues:

- a further development is to combine the en-route lane-change with the pre-trip lane choice to reduce the lane-change frequency by estimating the instability of en-route lane-change and the congestion at the entrance;
- the driving mode through the instability simulation in this article is based on the CVs making self-decisions with V2V information. A deeper thought is to manage the vehicle's trajectories with some proper objectives to reduce variation and promote stability;
- since all the results were obtained by simulating fixed car-following and lane-changing models, the next step will be to develop an analytical expression that fits various models, which is a challenging task.

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We have confirmed the final authorship for this manuscript, ensure that all listed authors have made a significant scientific contribution and the authorship needs no further changes.

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Author contributions

Hao Li conceived the study and wrote the 1st draft of the paper.

Yun Pu was responsible for design of the model.
Lingjuan Chen was responsible for data analysis.
Xiaoyu Luo was responsible for data collection.
All authors reviewed, edited and approved the paper.

Disclosure statement

This paper does not have any competing financial, professional, or personal interests from other parties.

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