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Original Article

A MULTI-OBJECTIVE FUZZY OPTIMIZATION MODEL FOR MULTI-TYPE AIRCRAFT FLIGHT SCHEDULING PROBLEM

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Highlights:
- a multi-objective optimization model for AFSP is presented to assign a set of aircrafts located at different airports to perform all flight trips;
- the proposed model features each flight trip with its own special aircraft type and fuzzy flight time as well as flight trip with a small aircraft being covered by a big one;
- a novel heuristic algorithm based on NSGA-II is further designed to efficiently yield meta-optimal solutions for such NP problem;
- a real airline scheduling example in China is conducted by CPLEX and heuristic algorithm to compare the difference between proposed and traditional model.

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Abstract. This study proposes a multi-objective optimization model for an Aircraft Flight Scheduling Problem (AFSP) for assigning a set of aircraft located at different airports to conduct all flight trips. The proposed model features each flight trip with its own special aircraft type and fuzzy flight time. Moreover, a flight trip with a small aircraft being covered by a larger one is fully accounted for in the model. The model can effectively reduce the number of aircraft and achieve the minimum total idle time for adjacent flight trips covered by an aircraft. A novel heuristic algorithm based on the Non-dominated Sorting Genetic Algorithm (NSGA-II) is further designed to yield meta-optimal solutions efficiently for such a Non-deterministic Polynomial (NP) problem. Finally, a real airline scheduling example in China is conducted using CPLEX and the proposed heuristic algorithm to evaluate the difference between the proposed and traditional models. The results show that the given scheduling problem effectively enhances the operational efficiency of the aircraft fleet.

Keywords: aircraft flight scheduling, fuzzy flight time, heuristic algorithm, multiple aircraft type, multi-objective.

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Notations

AFSP – aircraft flight scheduling problem;
CNY – Chinese yuan;
CPLEX – IBM ILOG CPLEX Optimization Studio (*https://www.ibm.com/products/ilog-cplex-optimization-studio*);
GA – genetic algorithm;
MAT – multiple aircraft type;
NP – nondeterministic polynomial;
NSGA-II – non-dominated sorting GA;
SAT – single aircraft type;
TSPM – 2-stage stochastic programming model;
VRP – vehicle routing problem.

1. Introduction

An AFSP, which assigns a set of aircraft located at different airports to conduct all flight trips, is one of the core aspects of daily airline management. If the scheduling plan is not reasonable, the fleet size would not only be increased, but the total idle time for all aircraft trips is also very high, which may increase the operating costs of airline companies. At present, AFSPs have gradually drawn the widespread attention of domestic and foreign scholars (Badi, Abdulshahed 2019; Bardenhagen, Rakov 2019; Petrović *et al.* 2018). On the one hand, many variations of AFSPs have been studied due to many factors affecting the problem, such as fleet assignment, flight timetabling, and crew scheduling with consideration of delays and maintenance requirements, etc. (Kenan *et al.* 2018a, 2018b). On the other hand, since AFSP is an extension of the VRP, which is an NP-hard problem, it is without a polynomial-time algorithm. Therefore, several intelligent, heuristic, or approximate algorithms have been designed for this to quickly find a good solution in a short time (Lalla-Ruiz, Voß 2020).

In an AFSP, each flight trip has its own special aircraft type, mainly depending on the number of passengers and the mileage. Existing research, i.e., an AFSP with a SAT (AFSP-SAT), divides all trips into multiple groups on the basis of special aircraft types, and some trips in a group with the same aircraft type are only assigned to an aircraft with the corresponding type (Huang *et al.* 2011; Listes, Dekker 2005). In reality, the supply and demand of different types of aircraft may not be balanced in time and space, i.e., some small planes might be missing at one point while larger ones are redundant over some time period. In this case, AFSPs with a MAT (AFSP-MAT), by allowing a flight trip with a small aircraft to be covered by a larger one, can dramatically reduce the aircraft fleet and improve the operational efficiency compared with AFSP-SAT. Further, various perspectives in passengers and airline companies of the optimization process for AFSPs typically involve 2 or more conflicting objectives, including fleet size, idle time, operating cost, and so forth. How to make a tradeoff between them to obtain a set of Pareto solutions is also very important (Sherali *et al.* 2013; Salazar-González 2014). However, multi-objective AFSP-MAT has received much less attention in the literature.

Another purpose of this study is to present an AFSP with uncertain flight times. When adjacent flight trips are covered by an aircraft, some delays, caused by a traffic accident and weather, should be considered in the process of designing the scheduling scheme. These disturbances may lead to the failure of the scheduling scheme (Jamili 2017; Lan *et al.* 2006). They can be quantified by using stochastic approaches, fuzzy sets, grey sets and language, etc. Each has its own advantages and disadvantages. Compared with other uncertain techniques, fuzzy sets can be used to describe uncertainty more accurately by drawing on the advice and expertise of experts and stakeholders, in the absence of statistical data (Yang *et al.* 2011; Wei *et al.* 2015). Hence, it is very important to investigate the optimal relationship between the robustness and reliability of aircraft scheduling and the fuzzy flight time so as not to negatively affect operational efficiency.

The main contribution of this study is the development of a multi-objective fuzzy optimization framework for AFSP-MAT. It focuses on 2 key tasks: (1) the coordination of the aircraft route and its construction to meet the requirements of all trips with their own special aircraft types and fuzzy flight times, and (2) the development of the NSGA-II-based heuristic algorithm to yield a set of Pareto solutions efficiently for such a large-scale problem. In this study, the fuzzy flight time is modelled by a triangular fuzzy number, including the expected value, optimistic value and pessimistic value; this is one of the most used approaches to fuzzy numbers. Based on this, a fuzzy expected model of AFSP-MAT is built and converted to an equivalent deterministic model (Wei *et al.* 2015). NSGA-II, an intelligent algorithm for solving multi-objective problems, is used to resolve the model (Kaucic *et al.* 2019). The proposed NSGA-II algorithm with fuzzy measures is more realistic and practical by taking vague and imprecise data into consideration. Finally, a real-world case study is used to prove the validity and correctness of the proposed methodology.

The rest of the manuscript is as follows:

- the introduction is provided in the current Section 1;
- the research status of AFSPs is summarized in Section 2;
- the framework of the proposed methodology and its mathematical model are described in Section 3;
- the Section 4 gives the details for a multi-objective heuristic algorithm based on NSGA-II;
- a numerical example is used to illustrate the validity of the proposed model and algorithm, which is discussed in Section 5;
- future work and concluding remarks are given in Section 6.

2. Literature review

In general, flight scheduling, fleet assignment, aircraft routing, and crew scheduling are 4 main planning processes that airline companies should accurately implement in reducing the operating cost. Although these 4 decision problems are typically solved sequentially and independently, an integration of some of these decisions in airline planning, which attracts the attention of many researchers, can further improve efficiency (Kenan *et al.* 2018a, 2018b). The main objective of this study is to present an approach that solves the integrated fleet assignment and aircraft routing problems as these are the key factors to impact profits.

A flight schedule is the input to airline fleet assignment and aircraft routing. Abara (1989) presented a fleet assignment model based on a connection-based network structure, in which some constraints, such as cover and flow balance and the number of available aircraft, were considered. Hane *et al.* (1995) 1st used a time-space network structure to build a large-scale integer program model for fleet assignment to reduce the problem complexity. Clarke *et al.* (1996) further extended this work by considering aircraft maintenance and crew considerations. Rushmeier & Kontogiorgis (1997) proposed a mixed-integer multicommodity flow model for the fleet assignment problem to maximize the profit and minimize violations from the preplanned schedule. Faust *et al.* (2017) presented an integrated model for a flight schedule and maintenance routing problem with a homogeneous fleet. Salazar-González (2014) also presented an integrated model for fleet assignment, aircraft routing, and crew-pairing problems. These works were extended by Cacchiani & Salazar-González (2017) to include 2 enhanced models: the path–path model and the arc–path model.

The premise of the fleet assignment problem is an assumption of a deterministic demand. The output regarding fleet assignment is the input of crew scheduling. The assignment is generally made 10...12 weeks in advance, and, therefore, the demand is highly uncertain at the time. Hence, the fleet assignment problem with an itinerary-based demand is closer to more detailed and reliable demand information. A demand-driven fleet assignment model, 1st proposed by Berge & Hopperstad (1993) and extended by Fry (2015) and Sherali *et al.* (2005), attempted to use a predicted demand to dynamically reassign aircraft to cover trips. Jiang & Barnhart (2009) also further integrated these fleet assignment models with dynamic scheduling by considering flight retiming. Sherali *et al.* (2013) presented an integrated model for flight scheduling, fleet assignment, and aircraft routing. Jamili (2017) presented an integrated model for flight scheduling, fleet assignment, and aircraft routing with uncertain traveling time. However, the demand-driven models only considered the deterministic demand. A TSPM is generally used to deal with demand uncertainty for fleet assignment. In this case, the aircraft family and aircraft type are assigned to each flight leg respectively (Kenan *et al.* 2018a; Sherali, Zhu 2008; Cadarso, De Celis, 2017). Listes and Dekker (2005) 1st studied a TSPM model for the fleet assignment problem with an uncertain demand. Cadarso & De Celis (2017) proposed a nonlinear TSPM to solve such uncertain fleet assignment problems. Kenan *et al.* (2018a) further proposed a TSPM for the integrated flight scheduling and fleet assignment problem.

The algorithms for solving AFSPs can be divided into exact methods and heuristic algorithms (Jamil 2017), where the former can be further classified into Lagrange relaxation-based methods (Lan *et al*. 2006; Cacchiani, Salazar-González 2017), column generation (Cacchiani, Salazar-González 2017; Faust *et al*. 2017), and dynamic programming (Sherali *et al*. 2005). The latter can also be further classified into route-building heuristics and metaheuristics. Since this problem belongs to the class of NP-hard problems, exact methods often perform very poorly in terms of computational efficiency and are unable to solve large-scale instances (Jamil 2017; Sherali, Zhu 2008; Kenan *et al*. 2018a). Heuristic algorithms are therefore more appropriate. In route-building methods, a set of feasible routes is generated initially and then there is further search and fine-tuning of initial solutions according to the proposed constraints at a reasonable computational cost. For meta-heuristic methods, simulated annealing (Jamil 2017), tabu search (Gui *et al*. 2024; Lahooti Eshkevari *et al.* 2025), GAs (Deb *et al*. 2002; Kaucic *et al*. 2019; Abualigah, Hanandeh 2015), and ant colony algorithm approaches (Wei *et al*. 2015; Zhou *et al*. 2020) are widely used.

Although existing research has successfully handled a variety of AFSPs, 3 issues deserve further study:

- traditional AFSPs consider only a specified type of scheduling plan in the implementation of the specific aircraft for each flight trip. Most studies have neglected the integrated operation of aircraft routing (a set of trips covered by an aircraft) and aircraft type guidance (type determination of each aircraft). Since each flight trip with its own special aircraft type can be covered by a larger aircraft (Huang *et al.* 2011), the ignorance of this integrated model can easily increase the operating cost;
- most existing studies have used definite flight times as the basic assumption in the methodology of AFSPs. However, the flight time changes dramatically under different conditions, such as the weather. This can be quantified by the use of uncertain data techniques with their own advantages and disadvantages. In the absence of statistical data, AFSPs with fuzzy flight times have been widely regarded as having a vital role in the reliability of the result (Jamili 2017; Lan *et al.* 2006; Wei *et al.* 2015);
- AFSPs involve some conflicting objectives, such as the number of aircraft, operating cost, and idle time. In this case, building a multi-objective programming model to find a set of Pareto solutions to balance them is important (Kenan *et al.* 2018a; Jiang, Barnhart 2009; Sherali, Zhu 2008). Hence, it is necessary to apply NSGA-II to solve this problem efficiently.

3. Methodology

3.1. Research framework

This study explores a multi-type AFSP that assigns a set of various types of aircraft located at different airports to conduct all flight trips with their special aircraft types. Each flight trip is covered by an aircraft from the departure airport at the start time to the arrival airport at the end time. A flight trip with a small aircraft can be covered by a larger one. If adjacent flight trips are covered by an aircraft, the arrival time of the 1st flight trip plus the aircraft's maintenance time should not be greater than the departure time of the 2nd flight trip, except for the arrival airport of the former being the same as the departure airport of the latter. A special aircraft type 1st leaves the docked airport to begin working, perform a sequence of flight trips, and return to the original one in the end. Obviously, the uncertain flight time of each trip, caused by traffic accidents and weather, also plays an important role in the process of designing a scheduling plan. Triangular fuzzy numbers are used to characterize the changes in flight time due to the lack of data. To reveal the optimal relationship between operational costs, multiple types of aircraft, and uncertain flight times to maximize the efficiency in the AFSP, a fuzzy multi-objective mixed-integer programming model is formulated to find a set of Pareto solutions so as to balance the number of aircraft and the total idle time.

Figure 1 provides a simple explanation of the proposed methodology. This includes 3 airports (A1...A3) and 6 flight trips (F1...F6) with their arrival and departure times. The aircraft type for F1 and F2 is T1, while that for F3, F4, F5, and F6 is T2. In such a small example, the optimization process yielded 2 aircraft routes as follows. A T1 aircraft is illustrated with a solid line (F1→F2→F4→F6) and a T2 aircraft with a solid line (F3→F5). For example, aircraft 2 departs from D2 to perform F3 at 7:00 and arrives at D3 to complete the task at 9:30. After waiting for 20 minutes, it starts to cover F5 and finally returns back to D2 at 11:30. Obviously, an AFSP with a SAT needs more than one T2 aircraft, which further proves the validity of the model.

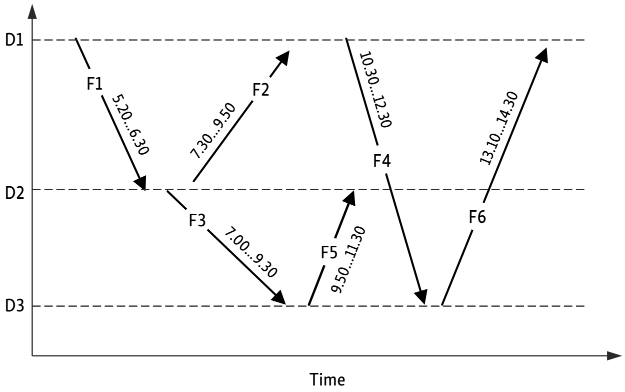


Figure 1. Graphical representation of the AFSP

The objective of the present study is to find an aircraft flight scheduling plan that simultaneously minimizes the operational costs of different types of aircraft performing all trips and the penalty costs of the total idle time for adjacent flight trips covered by these aircraft. To ensure that the proposed methodology can be applied to real-world situations, the present study is based on the following assumptions:

- each flight trip cannot be cancelled and must be covered by an aircraft;
- the fuzzy flight time of each trip can be estimated through civil aviation big data analysis;
- the fixed and operating costs of each aircraft type for a trip can be obtained in advance.

3.2. Model formulation

3.2.1. Notation

To facilitate the development of the AFSP, Table 1 summarizes all definitions and notations used throughout this work.

3.2.2. Formulation

Definition 1. For a discourse domain Γ, let *P*(Γ) be a power set of Γ. If a set function *Pos* in *P*(Γ) satisfies the following conditions:

- $Pos(\phi) = 0$, $Pos(\Gamma) = 1$;
- $Pos\left(\bigcup_{i \in I} A_i\right) = sup_{i \in I} Pos(A_i)$, then *Pos* is a possibility measure.

Definition 2. For a possibility space of the triple $\{\Gamma, \mathrm{P}(\Gamma), Pos\}$, let the set function:

$$Cr(A) = 0.5 \cdot \left(1 + Pos(A) - Pos(A^c)\right)$$

be the credibility measure of the event *A*, in which A^c denotes the complementary set of *A*.

Definition 3. For a credibility space of the triple $\{\Gamma, \mathrm{P}(\Gamma), Cr\}$, the expected value of the fuzzy variable ξ is denoted as:

$$\mathrm{E}(\xi) = \int_0^{+\infty} Cr\{\xi \ge r\}\mathrm{d}r - \int_{-\infty}^{0} Cr\{\xi \le r\}\mathrm{d}r.$$

Table 1. Parameters and variables in the mathematical model

Indices	
i, j	flight trip index
0	virtual flight
k	aircraft index
t	aircraft type index
d	airport index
Sets	
F	set of trips
T	set of aircraft types
D	set of airports
K_d^t	set of aircraft belonging to the special type t located at airport d
Parameters	
sp_i	starting airport of trip i
dp_i	ending airport of trip i
st_i	departure time of trip i
T_i	fuzzy flight time of trip i
b_i	capacity related to special aircraft type for trip i
B_t	capacity related to aircraft type t
T_s	minimum safe time
c_t^1	fixed cost of aircraft type t
c_t^2	cost of idle time of aircraft type t
c_t^3	operational cost of aircraft type t
H	a constant
Decision variables	
x_{ij}^k	whether trip i precedes trip j on aircraft k or not
y_i^k	whether trip i is covered by aircraft k or not
U_{ik}	an auxiliary (real) variable for the sub-tour elimination constraint in aircraft k

Because of the lack of data to analyse changes in flight time, a triangular fuzzy variable $T_i = \left(T_i^1, T_i^2, T_i^3\right)$ is used to describe the uncertain flight time, in which T_i^1, T_i^2, and T_i^3 are the minimum value, the most probable value, and the maximum value, respectively. Since the fuzzy flight time could not be directly handled by a computer, its expected value $\mathrm{E}(T_i) = 0.25 \cdot \left(T_i^1 + 2T_i^2 + T_i^3\right)$, as shown in Figure 2, is obtained based on definitions 1–3.

The proposed problem is formulated as follows:

$$\min f_1 = E\left(\sum_{\forall d \in D} \sum_{\forall t \in T} \sum_{\forall k \in K_d^t} \sum_{\forall i,\, j \in F} x_{ij}^k \cdot \left(st_j - et_i - T_i - T_s\right) \cdot c_t^2\right); \quad (1)$$

$$\min f_2 = E\left(\sum_{\forall d \in D} \sum_{\forall t \in T} \sum_{\forall k \in K_d^t} \left(c_t^1 + \sum\nolimits_{\forall i \in F} T_i \cdot y_i^k \cdot c_t^3\right)\right), \quad (2)$$

which is subject to:

$$\sum_{\forall i \in F} y_i^k = 1,$$

$$\forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T; \tag{3}$$

$$b_i \le \left(1 - y_i^k\right) \cdot H \le B_t,$$
$$\forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T; \tag{4}$$

$$2 \cdot x_{ij}^k \le y_i^k + y_j^k,$$
$$\forall i,\ j \in F,\ \forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T; \tag{5}$$

$$\sum_{\forall j \in F \cup \{0\}} x_{ij}^k = \sum_{\forall j \in F \cup \{0\}} x_{ji}^k = y_i^k,$$
$$\forall i \in F,\ \forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T; \tag{6}$$

$$U_{ik} - U_{jk} + \left|F \cup \{0\}\right| \cdot x_{ij}^k \ge \left|F \cup \{0\}\right| - 1,$$
$$\forall i,\ j \in F,\ \forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T; \tag{7}$$

$$et_i + E\left(T_i\right) + \left(1 - x_{ij}^k\right) \cdot H + T_s \le st_j,$$
$$\forall i,\ j \in F,\ \forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T; \tag{8}$$

$$dp_i + \left(1 - x_{ij}^k\right) \cdot H = sp_j,$$
$$\forall i,\ j \in F,\ \forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T,\ \forall k \in K; \tag{9}$$

$$d + \left(1 - x_{0i}^k\right) \cdot H = sp_j,$$
$$\forall i,\ j \in F,\ \forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T,\ \forall k \in K; \tag{10}$$

$$d + \left(1 - x_{i0}^k\right) \cdot H = dp_j,$$
$$\forall i \in F,\ \forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T,\ \forall k \in K. \tag{11}$$

The primary Objective function (1) is used to minimize the cost of the total idle time for adjacent flight trips covered by an aircraft. The secondary Objective function (2) is used to minimize the fixed cost of the flight fleet and operational cost, which is the total travel time of the designed aircraft routes. Constraint (3) indicates that each flight trip must be assigned to an aircraft. Constraint (4) guarantees that a flight trip with a small aircraft is covered by a larger one. Constraints (5) and (6) set all flight trips served by the aircraft to have the same incoming and outgoing arcs. Constraint (7) is used for the sub-tour elimination in aircraft routing. Constraint (8) guarantees that the arrival time of the 1st flight trip plus the aircraft's maintenance time should not be greater than the departure time of the 2nd flight trip, if adjacent flight trips are covered by an aircraft. Constraint (9) guarantees the arrival airport of the former being the same as the departure airport of the latter, if adjacent flight trips are covered by an aircraft.

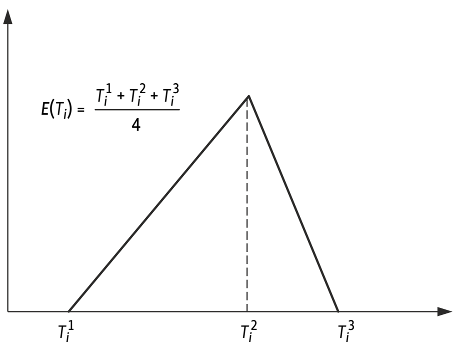


Figure 2. Triangular fuzzy numbers for uncertain flight time

Constraints (10) and (11) guarantee that a special type of aircraft 1st leaves the docked airport to begin working, perform a sequence of flight trips, and return to the original airport at the end.

Theorem. Let ξ and η be 2 fuzzy variables in:

$$\left\{\Gamma, \mathrm{P}\left(\Gamma\right), Pos\right\};$$
$$\mathrm{E}\left(a \cdot \xi + b \cdot \eta\right) = a \cdot \mathrm{E}\left(\xi\right) + b \cdot \mathrm{E}\left(\eta\right)$$

holds true for any arbitrary real numbers a and b.

As stated earlier, since Equations (1), (2), and (8) include fuzzy parameters, the fuzzy expected value model is transformed into the following equivalent linear regression model with certainty:

$$\min f_1 =$$
$$\sum_{\forall d \in D} \sum_{\forall t \in T} \sum_{\forall k \in K_d^t} \sum_{\forall i,\ j \in F} x_{ij}^k \times$$
$$\left(st_j - et_i - 0.25 \cdot \left(T_i^1 + 2T_i^2 + T_i^3\right) - T_s\right) \cdot c_t^2; \tag{12}$$

$$\min f_2 =$$
$$\sum_{\forall d \in D} \sum_{\forall t \in T} \sum_{\forall k \in K_d^t} \left(c_t^0 + \sum_{\forall i \in F} 0.25 \times \right.$$
$$\left. \left(T_i^1 + 2 \cdot T_i^2 + T_i^3\right) \cdot y_i^k \cdot c_t^3 \right), \tag{13}$$

which is subject to:

$$et_i + 0.25 \cdot \left(T_i^1 + 2 \cdot T_i^2 + T_i^3\right) + \left(1 - x_{ij}^k\right) \cdot H + T_s \le st_j,$$
$$\forall i,\ j \in F,\ \forall k \in K_d^k,\ \forall d \in D,\ \forall t \in T. \tag{14}$$

The other constraint conditions remain unchanged.

4. A heuristic algorithm based on NSGA-II for resolving an AFSP

NSGA-II is a kind of rapid and dominant multi-objective optimization algorithm with an elite retention strategy to find a set of Pareto-optimal solutions. NSGA-II has been successfully applied to solve multi-objective problems such as logistics distribution and site selection (Deb *et al.* 2002; Abualigah, Hanandeh 2015; Li *et al.* 2019; Roy *et al.* 2019). As an extension of the VRP, NSGA-II was designed to yield acceptable solutions efficiently in a reasonable amount of time because the exact algorithm could not solve efficiently large-scale problems (Pamucar, Ćirović 2018; Barma *et al.* 2019). The detailed process of the NSGA-II algorithm is discussed next.

4.1. Chromosome coding and fitness function

Efficient coding of GA chromosomes, which can capture the characteristics of the solution structure, plays a key role in the process of GA search. In the present study, y_i^k determines x_{ij}^k. Once the value of y_i^k is determined, trip i preceding trip j for an aircraft (i.e., x_{ij}^k) is easily obtained after sorting the departure time of trip i for aircraft k.

Therefore, if vectors $U=\left(u_1,u_2,\ldots,u_F\right)$ are adopted to represent solutions for this model, in which the element u_i (the vector of integer variables) is used to assign trip i to different aircraft $\forall k\in K_d^t\left(\forall d\in D,\forall t\in T\right)$, then u_i ranges from 1 to $\bigcup_{\forall d\in D,\ \forall t\in T}K_d^t$. For example, a chromosome vector $U=\left\{112212\right\}$ of 2 aircraft and 6 trips could be coded as follows: aircraft 1 covers trips 1, 2, and 5; aircraft 2 covers trips 3, 4, and 6.

During the process of chromosome decoding, a randomly generated solution might violate Constraint (8), except for the arrival airport of the previous trip being the same as the departure airport of the next one, and the departure airport of the 1st trip being the same as the arrival airport of the last trip. To evaluate solutions, this constraint was taken as a penalty term into the objective function, given as follows:

$$F_1=f_1+M_1\times \sum_{\forall d\in D}\sum_{\forall t\in T}\sum_{\forall k\in K_d^t}\sum_{\forall i,\,j\in F}\max\left(et_i+E\left(T_i\right)+\left(1-x_{ij}^k\right)\times H+T_s-st_j,0\right); \tag{15}$$

$$F_2=f_2+M_1\times \sum_{\forall d\in D}\sum_{\forall t\in T}\sum_{\forall k\in K_d^t}\sum_{\forall i,\,j\in F}\max\left(et_i+E\left(T_i\right)+\left(1-x_{ij}^k\right)\times H+T_s-st_j,0\right), \tag{16}$$

where: f_1 and f_2 are the objective functions (Equations (1) and (2)) of the proposed model; F_1 and F_2 are the functions used in fitness evaluation; M_1 is a large, positive penalty coefficient.

4.2. Fast non-dominant rule for Pareto-optimal solutions

4.2.1. Fast non-dominant sorting operators

The core of solving a multi-objective optimization problem is to find a set of Pareto-optimal solutions. In this section, a fast non-dominant sorting operator is used to classify the population according to the non-inferior solution level of the individual, so as to guide the search in the direction of finding a Pareto-optimal solution:

fast-non-dominated-sort P

for each $p\in P$

  $S_p\leftarrow\varnothing$ and $n_p\leftarrow 0$ initialize individual dominated and the number of dominating

  for each $q\in P$ update set of solutions dominated

    if $\left(p\prec q\right)$ then $S_p\leftarrow S_p\cup\left\{q\right\}$

    else if $\left(q\prec p\right)$ then $n_p\leftarrow n_p+1$

  if $n_p=0$ then

    $p_{rank}\leftarrow 1$ and $F_1\leftarrow F_1\cup\left\{p\right\}$

while $F_i\neq\varnothing$ and $i\leftarrow i+1$

  $Q\leftarrow\varnothing$

  for each $p\in F_i$ rank assignment of individuals

    for each $q\in S_p$

      $n_q\leftarrow n_q-1$

      if $n_q=0$ then $q_{rank}\leftarrow i+1$ and $Q\leftarrow Q\cup\left\{q\right\}$

  $F_i\leftarrow Q$

4.2.2. An individual crowding distance calculating operator

To converge to Pareto-optimal solutions, the crowding distance, which denotes the sum of the 2 front and rear solutions in the direction of each objective function, is used to maintain the diversity of solutions:

crowding-distance-assignment I

for each i, set $I\left[i\right]_{distance}\leftarrow 0$ initialize distance

for each objective m

  $I\leftarrow sort\left(I,m\right)$ and $I\left[1\right]_{distance}=I\left[l\right]_{distance}\leftarrow 0$

  for $i=2$ to $\left(\left|I\right|-1\right)$

$$I\left[i\right]_{distance}\leftarrow I\left[i\right]_{distance}+\frac{I\left[i+1\right]\cdot m-I\left[i-1\right]\cdot m}{f_m^{\max}-f_m^{\min}}$$

4.2.3. A crowded comparison operator

Individuals are selected based on the non-dominant comparison operation area. In this case, after the non-domination rank i_{rank} and the crowding distance $i_{distance}$ are obtained, a partial order $\prec_n$ is found by satisfying one of the 3 conditions as follows:

- $i\prec_n j$, if $i_{rank}\langle j_{rank}$;
- $i_{rank}=j_{rank}$;
- $i_{distance}\rangle j_{distance}$.

4.3. Implementation of NSGA-II

2 new populations are 1st generated by performing selection (roulette and elite strategy), crossover (single-point strategy), and mutation (single-point strategy) operations on their parent populations. As can be seen in Figure 3, these genetic operators do not break the solution structure of the proposed model. These 2 populations are merged and ordered based on the fast non-dominant rule. In this case, good individuals are selected to enter the next-generation population from large to small accord-

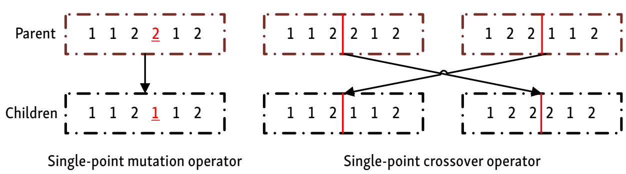


Figure 3. Crossover and mutation operators in the proposed NSGA-II

ing to the crowding distance, and a set of Pareto-optimal solutions are found:

main loop

$R_t \leftarrow P_t \cup Q_t$

$F \leftarrow$ fast-non-dominated-sort$\left(R_t\right)$

$P_{t+1} \leftarrow \varnothing$ and $i \leftarrow 1$

until $\left|P_{t+1}\right| + \left|F_i\right| \le N$

crowding-distance-assignment$\left(F_i\right)$

$P_{t+1} \leftarrow P_{t+1} \cup F_i$

$i \leftarrow i+1$

sort$\left(F_i, \prec_n\right)$

$P_{t+1} \leftarrow P_{t+1} \cup F_i\left[1:\left(N - \left|P_{t+1}\right|\right)\right]$

$Q_{t+1} \leftarrow$ make-new-pop$\left(P_{t+1}\right)$ use selection, crossover, and mutation

$t \leftarrow t+1$

5. Numerical example

5.1. Example description

To illustrate the validity and the wide applicability of the proposed models in designing an aircraft flight scheduling plan for an airline, 22 flight (F1…F22) trips between 11 airports (D1…D11) in China were selected for a case study. 2 types of aircraft were used for these trips (T1 and T2), as well as a certain number of different types of aircraft (A1…A6) initially located at 2 base airports, that is, Xian (D1) and Yinchuan (D8), as shown in Table 2. Moreover, the departure and arrival times, the starting and ending airports, and aircraft type of each flight trip are shown in Table 3. The key parameters used in this study are described as follow:

- fixed cost of aircraft type t: c^1_{T1} = 10000 CNY/aircraft and c^1_{T2} = 11000 CNY/aircraft;
- idle time cost of aircraft type t: c^2_{T1} = 1.7 and CNY/min c^2_{T2} = 2.5 CNY/min;
- operational cost of aircraft type t: c^3_{T1} = 1.9 and CNY/min c^3_{T2} = 3 CNY/min;
- minimum safe time: T_s = 30 min.

5.2. Results

As explained previously, the proposed model could yield 12 feasible Pareto-optimal solutions in 2 dimensions, including the assignment of the flight trip to the aircraft and the order of tasks for each aircraft. Figure 4 shows changes in the 2 goals. The upper and lower bounds of objective function 1 are 75083 CNY and 74980 CNY, and the upper and lower bounds of objective function 2 are 1683 CNY and 1617.5 CNY, respectively. As the value of objective function 1 becomes larger, that of objective function 2 becomes smaller. This was because the reduction in the total idle time inevitably led to the need for more aircraft or larger aircraft covering flight trips for a smaller one, thus increasing operating costs.

Take the Pareto-optimal solution (74, 995, 1669) as an example. Table 3 summarizes the assignment results, which include departure and arrival times of the aircraft at the

Table 2. Basic information regarding flight trips

Flight trip No	Origin airport	Destination airport	Departure time	Fuzzy flight time	Aircraft type
F1	Xian (D1)	Sanya (D2)	07:00	(300, 310, 320)	T1
F2	Sanya (D2)	Xian (D1)	13:00	(260, 275, 290)	T1
F3	Xian (D1)	Sanya (D2)	07:00	(305, 310, 315)	T1
F4	Sanya (D2)	Xian (D1)	13:00	(270, 275, 280)	T1
F5	Xian (D1)	Zhuhai (D3)	18:35	(165, 175, 185)	T2
F6	Zhuhai (D3)	Xian (D1)	22:20	(145, 155, 165)	T2
F7	Xian (D1)	Dunhuang (D4)	06:15	(160, 165, 170)	T2
F8	Dunhuang (D4)	Xian (D1)	09:50	(125, 135, 145)	T2
F9	Xian (D1)	Guilin (D5)	06:15	(130, 135, 140)	T2
F10	Guilin (D5)	Xian (D1)	10:05	(110, 120, 130)	T2
F11	Xian (D1)	Xiamen (D6)	13:15	(310, 320, 330)	T1
F12	Xiamen (D6)	Xian (D1)	19:35	(250, 260, 270)	T1
F13	Xian (D1)	Songyuan (D7)	13:15	(270, 280, 290)	T1
F14	Songyuan (D7)	Xian (D1)	18:50	(295, 305, 315)	T1
F15	Yinchuan (D8)	Nanchang (D9)	07:00	(120, 130, 140)	T2
F16	Nanchang (D9)	Yinchuan (D8)	10:05	(145, 160, 175)	T2
F17	Yinchuan (D8)	Dalian (D10)	17:10	(120, 130, 140)	T1
F18	Dalian (D10)	Yinchuan (D8)	20:15	(140, 145, 150)	T1
F19	Yinchuan (D8)	Yantai (D11)	08:00	(130, 140, 150)	T2
F20	Yantai (D11)	Yinchuan (D8)	11:20	(145, 150, 155)	T2
F21	Yinchuan (D8)	Nanchang (D9)	16:55	(125, 135, 150)	T1
F22	Nanchang (D9)	Yinchuan (D8)	20:00	(140, 155, 170)	T1

starting airport of each flight trip and the compatibility between the trip's special aircraft type and the type of assigned aircraft. Taking a larger aircraft A2 visiting the flight trip of F1 as an example, the aircraft leaves the origin airport D1 at 07:00 and arrives at the destination airport D2 between 12:00 and 12:20. The expected running time of F1 covered by an aircraft is about 310 minutes. Before conducting the flight trip F4, it stays at the airport for about an expected idle time of 20 minutes. Table 4 shows the routing plans of all aircraft, in which the 2 base airports D1 and D8 are selected for one T1 aircraft and 5 T2 aircraft, and their total idle and work times are also obtained. Taking the route of A1 as an example, the aircraft leaves the base airport of D1, covers trips F9, F10, F11, and F12, and terminates at the base airport D1.

Table 3. Assignment result of the flight trip covered by the aircraft

Flight trip No	Expected idle time [min]	Aircraft	Aircraft type
F9	95	A1	T2
F10	70		
F11	60		
F12	–		
F7	50	A2	T2
F8	70		
F13	55		
F14	–		
F1	50	A3	T2
F4	60		
F5	50		
F6	–		
F2	50	A4	T1
F3	–		
F15	55	A5	T2
F16	265		
F17	55		
F18	–		
F19	60	A6	T2
F20	185		
F21	50		
F22	–		

Table 4. Routing and scheduling plan for all aircrafts

Aircraft	Flights taken by the plane	Running time [min]	Idle time [min]
A1	D1→F9→F10→F11→F12→D1	835	225
A2	D1→F7→F8→F13→F14→D1	885	175
A3	D1→F1→F4→F5→F6→D1	915	160
A4	D1→F3→F2→D1	585	50
A5	D8→F15→F16→F17→F18→D8	565	375
A6	D8→F19→F20→F21→F22→D8	580	295

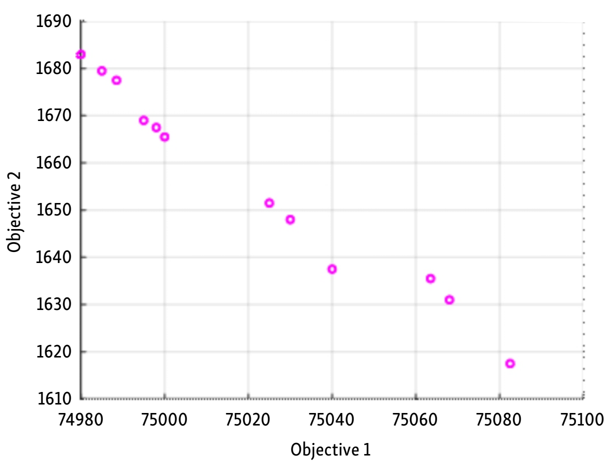
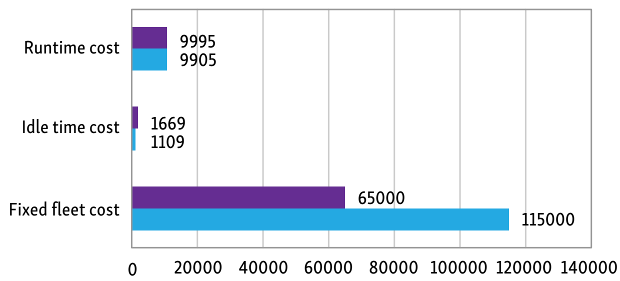


Figure 4. Pareto-optimal solutions for an AFSP

Furthermore, the proposed method (AFSP-MAT) has unique features compared with an AFSP-SAT. Figure 5 shows the difference between AFSP-SAT and AFSP-MAT. The cost of the fleet of the proposed model is reduced by 43.5%. However, the total idle time and operational cost for running flight trips for the proposed model are increased by 50.5% and 0.9%, respectively, compared with those for AFSP-SAT. This was due to a flight trip with a small aircraft being covered by a large one, which reduced the number of needed aircraft and increased operating costs for running flight trips using a more expensive aircraft type. The increase in the number of flight trips per aircraft also increased the total idle time for 2 adjacent flight trips covered by the same aircraft. As shown earlier, the increase in the operating cost was far less than the decrease in the fixed cost for fleets, confirming that AFSP-MAT is better than AFSP-SAT.

Figure 5 Comparison of results of the proposed and traditional models Moreover, the solutions for NSGA-II are compared with CPLEX to analyse the robustness, solution quality and calculation time of the proposed algorithm. The results are shown in Table 5. As the number of trips became larger, CPLEX can always find the best solution, but the time taken is larger. With increases in the number of the trips, the quality of the solutions of NSGA-II worsened and this may not find the best solution. The difference in optimal solutions between the heuristic algorithm and the use of CPLEX is acceptable, but the proposed algorithm requires less computational time.



	Fixed fleet cost	Idle time cost	Runtime cost
AFSP-MAT	65000	1669	9995
AFSP-SAT	115000	1109	9905

Figure 5. Comparison of results of the proposed and traditional models

Table 5. Comparison of the different algorithms

Scale of problem	NSGA-II				CPLEX	
	Best solution	Worst solution	Average solution	Computation time	Best solution	Computation time
22	76664	76722	76679	8 s	76664	>1 h
30	116400	116500	116452	11 s	116400	>1 h
50	172000	172170	172129	13 s	172000	>1 h
100	246000	246250	246178	16 s	233700	>1 h
150	377000	377490	377230	21 s	361920	>1 h

6. Conclusions

This study presented a multi-objective optimization model for AFSP-MAT. The significance of this model lies in revealing the relationship between temporal and spatial distributions of flight trips, uncertain trip times, and scheduling schemes. Once the path of an aircraft taking a set of flight trips is determined, a specific type for each aircraft is known by taking each flight trip covered by itself or a larger one into account. Since a commercial solver like CPLEX fails to solve large instances, a novel heuristic algorithm is necessary. NSGA-II is superior to CPLEX by solving different scales of problem instances with an optimality gap of less than 6% in less than one hour. A real-world case study validated the feasibility and applicability of the proposed framework. The results showed that the cost for the fleets of the proposed model was reduced by 43.5%; while the total idle time and operational cost for running the flight trips of the proposed model were increased by 50.5% and 0.9%, respectively, compared with those for AFSP-SAT. This proved that the proposed framework could be used as an effective tool for transit authorities to design flight scheduling plans. Further, a sensitivity analysis between the total idle time and operating costs was performed. A reduction in total idle time would have a positive influence on the operating costs. This is mainly due to having more aircraft or larger ones being needed to cover flight trips.

However, the proposed model and algorithm can be used by small and medium-sized airline companies. This work should be extended to resolve AFSP-MAT for large companies. There are many factors that affect AFSP. Note that this study neglected the integration of crew scheduling with consideration of delays and maintenance requirements. Another limitation is not having an interactive operational process of aircraft flight timetabling and scheduling. As a result, extending the possibilities of a fully integrated AFSP in different ways is worthy of further investigation. Although this full integration may reduce aircraft fleets and improve operational efficiency significantly, it makes models more complex, and sophisticated solution methods are needed to solve large-scale problems.

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Author contributions

Ming Wei and *Shangwen Yang* conceived the study and were responsible for the design and development of the data analysis.

Bo Sun, *Ming Wei*, and *Wei Wu* were responsible for data collection and analysis.

Shangwen Yang was responsible for data interpretation.

Ming Wei and *Shangwen Yang* wrote the 1st draft of the article.

Disclosure statement

All authors declare we have no competing financial, professional, or personal interests from other parties.

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