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Original Article

A CALCULATION METHOD FOR HIGH-SPEED RAILWAY CAPACITY BASED ON IMPROVED TRAIN DEDUCTION METHOD

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Highlights:

- a capacity calculation method for high-speed railways is proposed, taking into account train stops and train speeds through a train deduction method;
- an optimized capacity calculation model, based on the minimum number of deduction trains in a parallel timetable, is constructed for scenarios involving high-speed trains with the same-speed;
- an optimized capacity calculation model is developed to minimize the total moving time caused by inserting lower-speed trains in scenarios with different-speeds of high-speed trains;
- an algorithm based on the train deduction method and optimized capacity calculation models is proposed, which availably improves the solution efficiency of solving the large-scale computations.

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Abstract. A reasonable calculation of railway capacity is very important for research. With the rapid development of high-speed railways, more and more differences between traditional and modern railways have emerged in the transportation organization and capacity calculation methods. In this article, the calculation methods used under different conditions with different train types are studied based on the train deduction method. Train deduction is a method that calculates the number of trains that cannot pass through the line when other trains change their stop plan or operation speed based on the coefficient of deduction method, which is widely used in China. Then, optimized models for trains with the same-speed and different-speeds are built to determine the maximum number of trains. These models are built based on the constraints of passenger service quality and overtaking times. In addition, the models are solved based on a train operation plan. Hence, the capacity calculated by these methods is more reasonable for the actual condition. Finally, an actual case of Beijing–Shanghai high-speed railway is implemented and tested with the model. The optimal capacity scheme is simulated and analysed, and the result agreed well with the railway transport enterprise.

Keywords: high-speed railway, capacity, calculation method, train deduction, train speed, buffer time.

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Definitions

Carrying capacity – the number of the trains that can operation in a section in 1 h.
Parallel timetable – all trains operate with the same-speed and no stops.
Train deduction – the number of trains that cannot operate in a section based on the parallel timetable.
Minimal headway – the minimal time between 2 adjacent trains.
Interval time – the actual time between 2 adjacent trains; this time is not less than the minimal headway.
Buffer time – the difference between the actual interval time and the minimal headway.

Introduction

Capacity is one of the important factors of railway transportation. For the classification of railway capacity, both theoretical capacity and practical capacity have been defined (UIC 2013; Chu, Oetting 2013). The practical capacity of railway is studied in this article. The practical capacity is defined as the maximal number of trains that can pass through a line within a period of time based on the practical demand calculated using the train operation plan. Recently, high-speed railways have been developed in many parts of the world. Especially, in China, the total mileage of high-speed railway has increased to more than 45000 km (till end of 2023). However, the calculation method for the capacity of high-speed railway is different from the approach used for conventional railways, and this difference is mainly manifested in the following 2 aspects:

- there are only passenger trains on the high-speed railway lines in China. Also, there are at most 2 kinds of train speeds on one line, and usually all the trains operate at the same-speed on a line;
- due to the fluctuation in the travel patterns of high-speed railway passengers, the capacity of high-speed railway is calculated by taking the operation plan as an input for a different time period. Our method involves studying the basic train operation plan to satisfy the actual operation conditions, and the capacity is evaluated in 1 h units. A different train operation plan may have a different capacity result for a different hour.

Many studies have addressed the method for railway capacity calculation and analysis, including analytical and simulation methods (Pouryousef, Lautala 2015). Based on their method, we summarize the subdivided method as follows (the 1st 3 methods are analytical methods):

- ***blocking time and timetable compression method:***

  The International Union of Railway (UIC 2013) had described the method of blocking time and timetable compression for railway capacity in detail. The following articles analysed the calculation method for railway capacity based on the blocking time model. Hansen & Pachl (2008) and Klabes (2010) analysed the railway capacity using the blocking time method. This method is well associated with the signal system and contributes to a reasonable timetable. At the same time, a graphical representation was used to display the train operation process and the line capacity more clearly and intuitively. Later on, Lindner (2011) studied the capacity calculation and assessment method used for UIC406 and analysed the impact of 4 factors, namely the number of trains, average speed, stability, and heterogeneity, on capacity. A timetable compression method was studied to analyse the influence of factors such as average speed and section length on capacity. Jamili (2018) proposed new methods to define the exact amount of practical capacity based on the computation of the minimum buffer times using the compression process in single-track railway lines. A new approach was proposed to compute the practical capacity using 2 methods: one is adding supplementary times to the running and dwell times, and the other is inserting necessary buffer times in the timetable. In this way, the available capacity was obtained.

  These articles studied railway capacity based on the blocking time and timetable compression method, which are effective methods to calculate the capacity. However, these methods require the timetable as the input data. We use the theoretical approach for timetable compression in Section 2.2;
- ***optimization analysis method:***

  Huisman *et al.* (2002) applied the queuing model to describe the train operation status within the railway network. The railway network is divided into 3 main parts: the station, the line intersection, and the section. The queuing model is used to describe the state of each part of the train. The capacity and delays can then be calculated without a timetable. Pachl & White (2004) calculated the occupancy time of each section based on the train operation routes, and then calculated the minimum headway for all adjacent trains. They calculated the average minimum headway based on the minimum headway and different combinations of train operations. Then, the optimal calculation method for capacity utilization was studied based on the average minimum headway. In research by Burdett (2015a), models for the total absolute capacity of railway networks with different competing objectives including train services, different network corridors, and train types were proposed. Then, Burdett (2015b) built a mathematical model for determining the theoretical capacity of a railway network with more complex train paths. Burdett (2016) researched the optimal model for expanding the theoretical capacity of railways. This model can remove the physical bottlenecks in the current railway system. Isobe *et al.* (2012) redefined the method for capacity analysis. A modelling method based on the "time-CSP" (CSP – Communicating Sequential Processes) method was proposed, which can comprehensively consider the railway capacity and train operation safety. Mussone & Calvo (2013) defined capacity as the maximum number of trains that can operate on the railway network, when considering the constraints of connection point capacity, station track capacity, line capacity, and the specified train operation delay rate within a given time period. An optimal model and an algorithm for railway capacity were built. Yaghini *et al.* (2014a, 2014b) evaluated the effect of train type interactions on railway line capacity and presented an integer-programming model for both line and line section. The problem was formulated as a multi-commodity network design model on a space-discrete time network. This model considered the impact of train types on capacity and waiting time, and the main output of the proposed model was the saturated timetable. Riejos *et al.* (2016) presented a method for scheduling railway service on networks with a radial-backbone topology. This method calculated the structure of service (number of trains between stations) needed to meet the required travel demand. The aim was to achieve maximum occupancy of trains, while eliminating the number of transfers and minimizing the total fleet size in both the main and the radial lines. Zhang & Nie (2016) proposed a Minimum Cycle Time Calculation (MCTC) model based on the Periodic Event Scheduling Problem (PESP) for a given train line plan, which is a promising method for macroscopic train timetabling and capacity analysis. Weik *et al.* (2016) and Weik & Nießen (2017) studied a quasi-birth–death process approach for the integrated modelling of capacity and reliability. Liang *et al.* (2017) analysed the influence of dispatching on the relationship between capacity and operation quality. 3 dispatching algorithms were considered: 1st come 1st serve, the state-dependent dispatching algorithm, and a state-independent dispatching algorithm. Jensen *et al.* (2017) developed a new framework for strategic planning to calculate railway infrastructure occupation and capacity consumption in networks, independent of the timetable. Further, a model implementing the framework was presented. In this model, different train sequences were generated and assessed to obtain timetable independence.

These articles used some optimized approaches to build models for railway capacity, and they provided us significant guidance to build the models shown in Section 4.1;

- ***coefficient of deduction method:***

  In China, railway capacity is commonly calculated using the coefficient of deduction method. The coefficient of deduction refers to the number of ordinary freight trains lost from the parallel timetable due to the increase in number of passenger trains, high-speed freight trains, or pick-up trains (Ma 2005). Hu & Zhao (1998) studied the coefficient of deduction method for trains with different-speeds at the same time and quantified the formulas for calculating the coefficient of deduction, thus determining a calculation method for the capacity. The theoretical idea underlying this method is also used in calculating the capacity of high-speed railway lines. However, unlike the traditional method, the coefficient of deduction method for high-speed railway considers the number of trains that need to be deducted from the parallel timetable when increasing the number of trains with a different-speed. Calculation methods for the coefficient of deduction and capacity have been proposed for different conditions, including different stop schemes with the same-speed and different stop schemes with different-speed grades. In researches by Tian *et al.* (2002) and Zheng & Liu (2012), and many other studies, a series of analyses and examples for the capacity of high-speed railway based on the coefficient of deduction method were studied. These applications refined the coefficient of deduction method conveniently and effectively for railway capacity calculations. Lv *et al.* (2016) proposed a calculation method for high-speed railway carrying capacity based on the transportation organization mode of high-speed railways with trains having different-speeds. Based on the method of train overtaking and train stop group, a specific value method of low-speed train deduction coefficient and train stop deduction coefficient was proposed. Based on a combination of the coefficient of deduction method and the average minimum headway method, Peng (2018) proposed a calculation method for the available capacity of high-speed railway by considering the occupancy of the train stop and overtaking. Chen (2018) established a carrying capacity calculation model for high-speed railway based on the coefficient of deduction method. The target of the model is to ensure that only a minimum number of trains are deducted, and the constraints include the effective running time, running time in the section, headway, stop-time, overtaking, and the difference in runtime. A genetic algorithm was designed to solve the model.

  These articles studied railway capacity in China based on the coefficient of deduction method, which formed the theoretical basis for this article. In Sections 2 and 3, a train deduction method suitable for high-speed railways is researched using their methods;

- ***simulation method:***

  Pouryousef & Lautala (2015) summarized the capacity simulation methods used in the US. and Europe and applied a hybrid simulation approach to research the method for improving railway capacity. Many simulation tools (e.g., Rail Traffic Controller (RTC), *OpenTrack* and *RailSys*) were used by the authors. In China, some researchers calculated the railway capacity by using some simulation tools they developed themselves using *Visual Studio* (*https://visualstudio.microsoft.com*). Most of the theoretical researches and simulations of railway transportation operations and capacity are based on the station or section. Few researches have been conducted for the capacity of the overall railway system. There are many studies based on static and deterministic models, but only a few studies on the overall research and macroscopic simulation of network capacity (Shan 2011).

The research results of the existing literature are summarized as follows:

- the timetable compression method can consider the actual demands well and uses a graphical method to explain the utilization of capacity more intuitively; moreover, the value of the capacity obtained is relatively accurate. However, in the process of capacity calculation, it is necessary to use the timetable as the main input data. In addition, the capacity is the actual value, and an expansion method for carrying capacity cannot be studied. Most of the authors focus on analysing the factors that influence capacity;
- the optimization analysis method takes the maximum number of trains as the objective function, and takes into account the passenger travel demand, train delay, and other factors. It uses different solving algorithms to solve the model. Therefore, the railway capacity can be obtained under different demand conditions. However, the model is usually complex and difficult to solve, and the solving process and results are not intuitive enough;

- the capacity deduction method is the main method for calculating the carrying capacity in China. A graphical method is used to represent the carrying capacity based on a combination of the basic train operations, and the solving process is relatively simple. However, the traditional methods usually only use a fixed deduction coefficient for the calculation; therefore, the calculation results are not accurate enough to provide constraints during the transport organization process. Also, the traditional deduction coefficient method is no longer applicable to the calculation of high-speed railway capacity; therefore, the train deduction method needs to be improved;
- the calculation process of the simulation method is clear, which can restore the actual train operation state to the greatest extent possible. However, the input parameters are relatively complex, and the calculation process and the time required for the calculations are long.

Therefore, we propose a calculation method for the carrying capacity, which combines the advantages of 1st 3 above methods. Firstly, the train deduction method is improved by eliminating the need to calculate a fixed deduction coefficient. We take the train group as the unit of calculation, and a train deduction method adapted to different conditions is proposed based on specific train combination modes. Then, we use the idea of timetable compression method to solve the problem of repeated deduction between different train units. Finally, under the 2 transport organization conditions of same-speed trains and different-speed trains, calculation models of capacity, which maximize the number of the trains, are established. In the models, the passenger demand and buffer time are considered. The calculation process and results of this article can be visualized graphically and adapted to the problem of capacity optimization under different conditions.

1. Methodology

In this article, a capacity calculation approach is proposed for high-speed railway. Our approach considers the train operation plan of a line in 1 h. Therefore, in this article, capacity is defined as the maximal number of trains that pass through a line within 1 h based on a train operation plan. Our approach is based on the coefficient of deduction method for calculating the capacity of high-speed railway. Many studies have researched railway capacity based on the train deduction method, but the capacity is usually calculated by the stable deduction coefficient. Therefore, the calculation results are not accurate. Based on the idea of the deduction method, a train deduction method is proposed for trains with the same-speed but different stops. Unlike the traditional coefficient of deduction method, this approach considers 2 conditions to calculate the capacity.

Firstly, when the trains have different stop-times, the capacity can be maximized by using a reasonable combination of operations. The basic principles of the combination of trains with different stops are as follows. These principles can make the trains operate in an optimal sequence.

As shown in Figure 1, the green lines indicate the operation lines of the parallel timetable. The red line indicates the trains with increased stop-times, which are going to remain in the service. The green dotted lines indicate trains that would be deducted and will no longer in the service. The horizontal axis represents the time and the vertical axis represents the distance. These are applicable all the figures in this article.

According to the "train unit" principle, the trains are divided into small groups with a certain number for calculations. As shown in Figure 1, there are 3 train groups for the 6 trains (the red operation lines). The calculation for the 1st train of each group begins with the train of the parallel timetable, which has no effect at all from the group in front of it. The group is the train unit. This principle divides the trains into many small groups, and the trains in the same train unit can be operated in a fixed optimal mode to obtain maximal capacity.

According to the "stop farther 1st" principle, there are 2 adjacent trains with the same stop-time. The train that makes the 1st additional stop below the other is the train in front, as shown by the train unit "*b*" in Figure 2.

According to the "stop less 1st" principle, when there are 2 adjacent trains with different stop-times, the train that has less stop-times compared to the other will be the train in front, as shown be the train unit "*b*" in Figure 3.

Based on the principles, a staged calculation method of railway capacity is studied. The 1st phase includes the calculation method based on same train speed; the 2nd phase includes the calculation method based on different train speeds; the 3rd phase involves comprehensive optimal models:

- ***the calculation method for the 1st phase:***

  Contribution: Unification of the train speed is the most effective method to improve the carrying capacity of railway lines. Therefore, it is necessary to study the capacity calculation method of trains with the same-speed. In this article, the traditional deduction coefficient method *i* improved. The train unit is taken as the minimum calculation unit, and the maximum capacity scheme is calculated and analysed by setting the train optimal operation order principle in advance. Then, the carrying capacity is calculated by combining the train deduction method and timetable compression method. Unlike the traditional calculation methods, the proposed method can calculate the capacity more accurately without considering the timetable.

  Method: For the 1st phase, the maximal capacity of the parallel timetable for all high-speed trains in 1 h is calculated. The parallel timetable is a train diagram in which all trains have the same-speed and do not have stops. Based on the parallel timetable, the approach analyses the train deduction method for different combined conditions of trains with different stops. In this article, the train deduction is the number of deducted trains on a parallel timetable when one or more trains change their stop plan. Then, we can get an optimal train operation scheme (with only the time structure and not the specific time of the trains at each station) with the minimal deduction of trains. The maximal capacity can be calculated by this scheme;

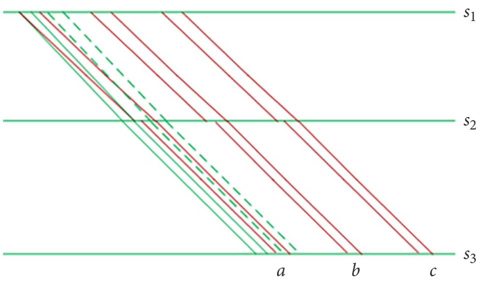

Figure 1. "Train unit" rule

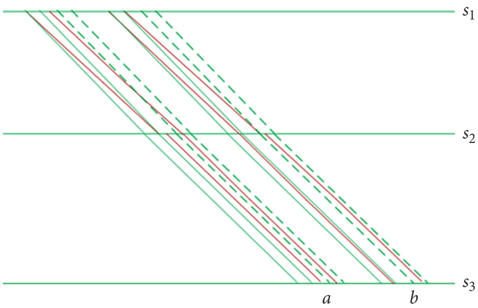

Figure 2. "Stop farther 1st" principle

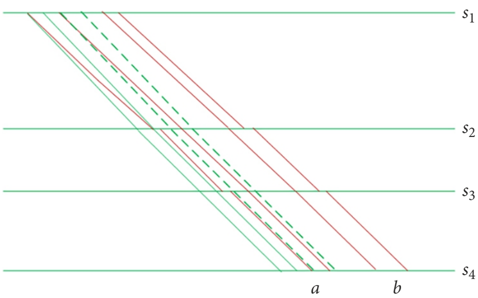

Figure 3. "Stop less 1st" principle

- ***the calculation method for the 2nd phase:***

  Contribution: The existing literatures calculated the capacity based on a fixed deduction coefficient value for different-speed trains, which makes it difficult to accurately calculate the number of trains that need to be deducted due to the speed difference and different stop modes. In this stage, the method of train deduction is transformed into the method of moving train operation time, which can more intuitively and accurately express the operation relationship and interval time of the trains.

  Method: For the 2nd phase, based on the optimal train operation scheme of the 1st phase, a low-speed train inserting method is obtained by analysing the different operation relationships among the different-speed trains. By calculating the minimal moving time of the trains, the influence of trains with different-speed classes on the capacity can be analysed, and the maximum capacity value of the mixed operation of trains with different-speed classes can be obtained;
- ***the calculation method for the 3rd phase:***

  Contribution: Based on the methods of the 1st 2 phases, the capacity optimal models are established, and the constraints are considered. We can get the optimal capacity scheme under different conditions with different passenger demands and timetable quality by using these models. According to the comprehensive optimization of the 2-phase optimization model, the corresponding solving algorithm is designed, and the maximum capacity scheme and the optimal capacity scheme satisfying certain constraints can be obtained.

  Method: We built 2 models separately for these 2 phases, in order to reduce the calculation complexity of the coefficient of deduction method. This approach can calculate the capacity of the high-speed railway and analyse the relationship among the trains and the influence factors such as buffer time and train service quality at each station. The EC (2013) and EPCEU (2004) indicated that the delays, disturbances, and re-routing of traffic are indicators related to the consequences of accidents, and these indicators have a certain impact on traffic safety assessment. Zieger *et al.* (2018) analysed the influence of buffer time distributions in the delay propagation of the railway. Therefore, the buffer time constraint is used in this article to limit and analyse the impact of train operation safety and reliability on capacity. The results of this approach can provide the theoretical basis for the railway enterprise to formulate the transportation plans.

2. Calculation method for the capacity of high-speed trains with the same-speed

2.1. Train deduction with different combinations of stops

The capacity of parallel timetable is defined as C_p, which can be calculated using Equation (1) (Su *et al.* 2008). This capacity is the basic value for the train deduction method:

$$C_p = \frac{60}{t^l_{\min}}, \quad (1)$$

where: $t^l_{\min}$ is the minimal headway and the unit is min. The train deduction is calculated by the parallel timetable. The train deduction of a train with increasing f stops is defined as ε^f_{one}. The additional arrival time, additional departure time, and stop-time at the station are denoted by t^a_{add}, t^d_{add}, and $t^{stop}_{l,s}$. Then, the train deduction ε^f_{one} can be calculated using equation (Su *et al.* 2008), where the unit is the number of trains:

$$\varepsilon^f_{one} = \frac{f \cdot \left(t^a_{add} + t^d_{add} + t^s_{stop}\right)}{t^l_{\min}}. \quad (2)$$

Then, the train deduction method for a combination of different train stop-times is studied. In this article, train deduction is defined as the number of trains that could be cancelled because of the other trains increasing their stop-times.

If all the trains have the same stop modes, including the same stop-times and the same stop stations in the train unit x, as shown in Figure 4, the train deduction is equal to the deduction of one train. The train deduction of the train unit x is defined as ε_x.

Hence, the train deduction of the train unit x can be calculated by using equation:

$$\varepsilon_x = \varepsilon^f_{one}. \quad (3)$$

If the trains have different stop modes in the train unit x, there are 2 kinds of combinations, namely same stop-times and different stop-times.

For the same stop-times condition, the "stop farther 1st" principle is used for the calculation. Because the number of deducted trains for the 2 adjacent trains are the same in this condition, the "stop farther 1st" principle can ensure that the interval time of the trains in the 1st section is the minimal headway. The train deduction due to the 1st train is the same as the train deduction due to the 2nd train at the same time Hence, the train deduction of the train unit x can also be calculated by using Equation (3).

For the different stop-times condition, the "stop less 1st" principle is used for the calculation. The section with the minimum headway of these 2 adjacent trains is the 1st section, as shown in Figure 5. Moreover, the interval time between these trains gradually becomes larger with the rear sections. Therefore, the total train deduction is equal to the train deduction of the train with the maximum stop-times. Then, the train deduction in the train unit x under this condition can be calculated by using equation:

$$\varepsilon_x = \max_f \varepsilon^f_{one}. \quad (4)$$

2.2. Compression method between 2 adjacent train units

There would be repeated train deduction between 2 adjacent train units as shown in Figure 6, and if the interval time is bigger than the minimal headway between 2 adjacent train units, this method should be used.

As shown in Figure 8, there is a buffer time between the last train of the front capacity unit and the 1st train of the rear adjacent capacity unit. The buffer time should be compressed by calculating the interval time of these 2 trains at each station and section to find the bottleneck interval time with the shortest buffer time. If the last train and the 1st train in 2 adjacent units are l_j and l_{j+1}, respectively, the arrival interval time and departure interval time between train l_j and train l_{j+1} at each station can be calculated based on the train operation time (including the running time, stop-time, additional departure time, and additional arrival time) and train stop plan. The section with the minimum interval time is the bottleneck section, and the maximum extra deduction time is $t_{de} = t^l_{l_j, l_{j+1}} - t^l_{\min}$, where l_{de} is the extra time needed for compression and the $t^l_{l_j, l_{j+1}}$ is the actual interval time between trains l_j and l_{j+1} in the bottleneck section. Then, the number of the trains that need to be deducted repeatedly can be calculated using the extra time and the minimal headway. The timetable structure after compression is shown in Figure 7.

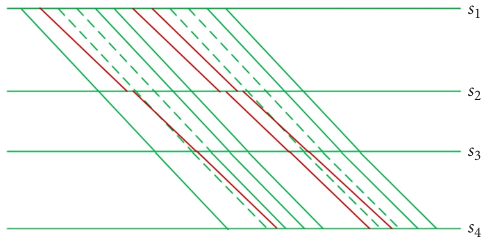

Figure 4. Train deduction of trains with the same stop-times

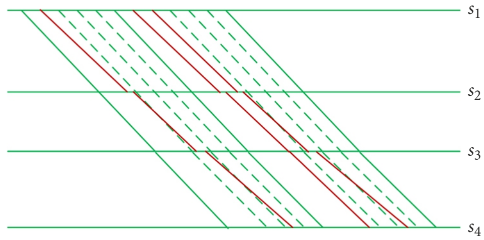

Figure 5. Train deduction of the trains with different stop-times

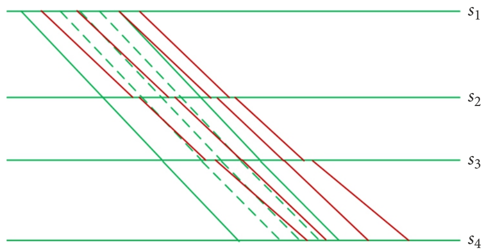

Figure 6. Repeated train deduction between 2 train units

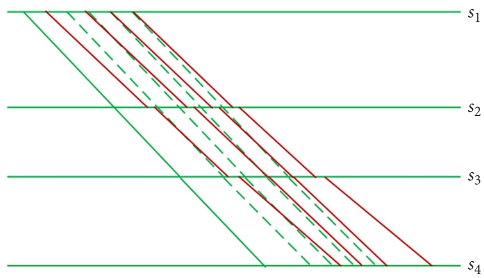

Figure 7. Timetable of 2 capacity units after removing the repeatedly deducted trains

Therefore, the carrying capacity for the same-speed trains based on the train deduction method can be calculated by using equation:

$$C_{sp} = C_p - \sum_{x=1}^{N_x} \varepsilon_x + \sum_{x=1}^{N_x - 1} \frac{t^{x,x+1}_{de}}{t^l_{\min}}. \quad (5)$$

3. Calculation method for the capacity of high-speed trains with different-speeds

Based on the optimal capacity scheme of the same-speed trains in Section 2.2, an inserting method is studied for different-speed trains. When a low-speed train is inserted, the operation time of some high-speed trains at the stations should be moved. The moving time includes the moving time in the section and the moving time at the station. The moving time in the section is caused by the speed difference between the trains, the additional arrival time, and the additional departure time of the train stop. The moving time at the station is caused by the different stop-times at the station.

3.1. Moving time in the section

The moving time due to the insertion of the low-speed train l_j when the train needs to be moved in section w is defined as $t_{l_j}^{move,w}$, the difference in time between 2 different-speed trains in section w is defined as $t_{l_j,l_{j+1}}^{ds,w}$, the departure interval time and the arrival interval time of the trains at station s_i are defined as $t_{l_j,l_{j+1}}^{dd,s_i}$ and $t_{l_j,l_{j+1}}^{da,s_i}$, respectively, the original interval time between the 2 adjacent trains l_{j-1} and l_{j+1} at station s_i before inserting the low-speed trains is defined as $t_{l_{j-1},l_{j+1}}^{in,s_i}$, and $\sigma_{l_j}^{s_i}$ (a variable 0 or 1) indicates whether train l_j stops at station s_i (when it is equal to 1, l_j stops at station s_i, and when it is equal to 0, l_j does not stop at station s_i). The distance of section w is defined as L_w^{se}, the speeds of the high-speed train and low-speed train are denoted by v_h and v_m, respectively, and the running times of the high-speed train and low-speed train are denoted by t_{H,l_j}^w and t_{L,l_j}^w, respectively.

As shown in Figure 8, the moving time can be divided into 3 different conditions (section w includes the station s_1 and station s_2):

- as shown by "*a*" in Figure 10, the departure time of the low-speed train is earlier than the departure time of the rear high-speed train at station s_1. The difference in time between trains of 2 different-speeds can be calculated by using 2 methods. One is the sum of the difference in departure times at station s_1 and the difference in arrival times at station s_2, expressed by Equation (6); the other is the difference in times of these 2 trains running in the same section with different-speeds, expressed by Equation (9):

$$t_{l_j,l_{j+1}}^{ds,w} = t_{l_j,l_{j+1}}^{dd,s_1} + t_{l_j,l_{j+1}}^{da,s_2}; \tag{6}$$

$$t_{L,l_j}^{w} = \frac{L_w^{se}}{v_m} + \sigma_{l_j}^{s_1} \cdot t_{add}^{a} + \sigma_{l_j}^{s_2} \cdot t_{add}^{d}; \tag{7}$$

$$t_{H,l_{j+1}}^{w} = \frac{L_w^{se}}{v_h} + \sigma_{l_{j+1}}^{s_1} \cdot t_{add}^{a} + \sigma_{l_{j+1}}^{s_2} \cdot t_{add}^{d}; \tag{8}$$

$$t_{l_j,l_{j+1}}^{ds,w} = t_{L,l_j}^{w} - t_{H,l_{j+1}}^{w}. \tag{9}$$

The difference in departure times at the former station depends on the actual interval time and the minimum headway between the 2 high-speed trains before inserting the low-speed train; therefore, the difference in time can be expressed using equation:

$$t_{l_j,l_{j+1}}^{dd,s_1} = t_{l_{j-1},l_{j+1}}^{in,w} - t_{\min}^{l}. \tag{10}$$

As shown in Figure 10, the moving time in section w can be calculated using equation:

$$t_{l_j}^{move,w} = t_{l_j,l_{j+1}}^{da,s_2} + t_{\min}^{l}. \tag{11}$$

By Equations (6)–(11), the total moving time in section w due to the insertion of the low-speed train l_j can be expressed by equation:

$$t_{l_j}^{move,w} = t_{L,l_j}^{w} - t_{H,l_{j+1}}^{w} - t_{l_{j-1},l_{j+1}}^{in,s_1} + 2 \cdot t_{\min}^{l}; \tag{12}$$

- as shown by "*b*" in Figure 8, before inserting the low-speed train l_j, the interval time between 2 high-speed trains (l_{j-1} and l_{j+1}) is equal to the minimum headway; therefore, the difference in time between the 2 different-speed trains running in the section is equal to the difference in the arrival times of these 2 trains at station s_2:

$$t_{l_j,l_{j+1}}^{da,s_2} = t_{l_j,l_{j+1}}^{ds,w} = t_{L,l_j}^{w} - t_{H,l_{j+1}}^{w}. \tag{13}$$

The total moving time in section w due to the insertion of the low-speed train l_j can be expressed by equation:

$$t_{l_j}^{move,w} = t_{L,l_j}^{w} - t_{H,l_{j+1}}^{w} + t_{\min}^{l}; \tag{14}$$

- as shown by **c** in Figure 8, before inserting the low-speed train l_j, the interval time between 2 high-speed trains l_{j-1} and l_{j+1} is greater than the sum of the time difference between 2 different-speed trains and the minimum headway, which can be expressed as $t_{l_j,l_{j+1}}^{dd,s_1} > t_{l_j,l_{j+1}}^{ds,w}$. Then, the values can be calculated by equation:

$$t_{l_j,l_{j+1}}^{da,s_2} = t_{l_j,l_{j+1}}^{dd,s_1} - t_{l_j,l_{j+1}}^{ds,w}. \tag{15}$$

The total moving time in section w due to the insertion of the low-speed train l_j can be expressed by equation:

$$t_{l_j}^{move,w} = 2 \cdot t_{\min}^{l} - t_{l_{j-1},l_{j+1}}^{in,s_1} + t_{L,l_j}^{w} - t_{H,l_{j+1}}^{w}. \tag{16}$$

Then, the rear sections would appear due to the condition $t_{l_j,l_{j+1}}^{dd,s_1} = t_{\min}^{l}$, as shown in Figure 11. Therefore, the total moving time in section w due to the insertion of the low-speed train l_j can be expressed by Equation (17). As shown in Figure 9, the moving time of the trains is only affected by the time difference between 2 different-speed trains.

$$t_{l_j}^{move,w} = t_{l_j,l_{j+1}}^{ds,w} = t_{L,l_j}^{w} - t_{H,l_{j+1}}^{w}. \tag{17}$$

3.2. Moving time at the station

The moving time due to the insertion of the low-speed train l_j at station s_i is defined as $t_{l_j}^{move,s_i}$. As shown in Figure 10, the former is the condition when the train is not overtaking, and the latter is the condition when the low-speed train is overtaken by the high-speed train.

Figure 10 shows that there are 2 conditions: overtaking and not overtaking. The moving time of the train at the station can be expressed as follows:

- ***condition 1:*** overtaking:

$$t_{l_j}^{move,s_i} = t_{\min}^{l} + t_{l_j,s_i}^{stop} - t_{l_j,l_{j+1}}^{da,s_i}; \tag{18}$$

- ***condition 2:*** not overtaking:

$$t_{l_j}^{move,s_i} = t_{\min}^{l} + t_{l_j,l_{j+1}}^{da,s_i} + t_{l_j,l_{j+1}}^{dd,s_i} - 2 \cdot t_{\min}^{l} = 3 \cdot t_{\min}^{l} - t_{l_j,l_{j+1}}^{da,s_i} - t_2. \tag{19}$$

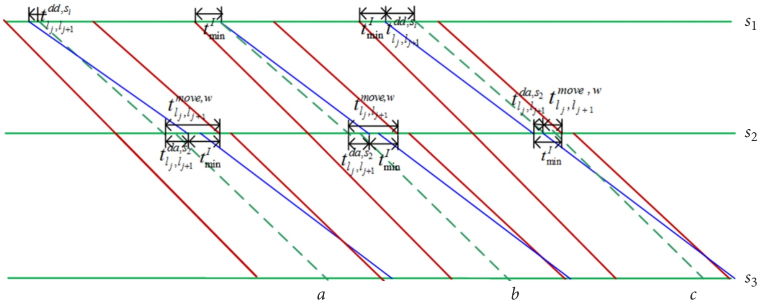

Figure 8. Different conditions by inserting a low-speed train

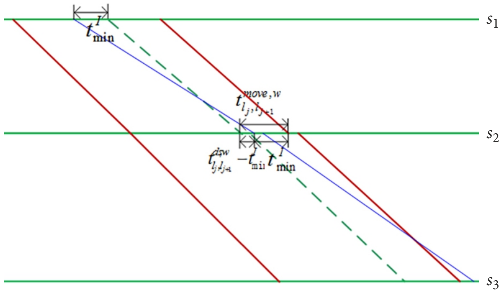

Figure 9. Different conditions by inserting a low-speed train

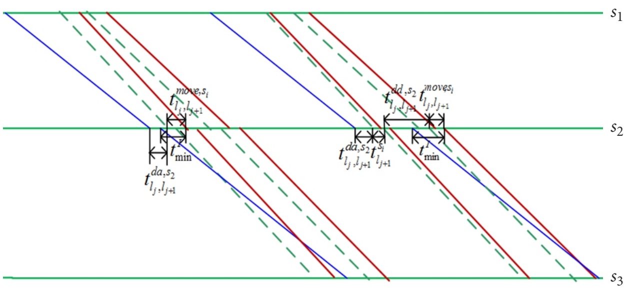

Figure 10. Moving time of the train at the station

Considering these 2 conditions, the high-speed train that is behind the low-speed train should overtake at the station when equation is satisfied:

$$t^{dd,\,s_1}_{l_j,\,l_{j+1}} > 2 \cdot t^{l}_{\min} - t^{stop}_{l_j,\,s_i}. \quad (20)$$

Therefore, the train deduction due to the insertion of the low-speed trains is the ratio of the total moving time of the sections and stations and the minimal headway. The carrying capacity of the railway line with different-speed trains can be calculated by equation:

$$C_{dp} = C_{sp} - \frac{\sum_{j=1}^{N_{low}} \sum_{w=1}^{N_{se}} t^{move,\,w}_{l_j} + \sum_{j=1}^{N_{low}} \sum_{i=1}^{N_{se}+1} t^{move,\,s_i}_{l_j}}{t^{l}_{\min}}. \quad (21)$$

4. Calculation method for high-speed railway capacity

4.1. Mathematical model

4.1.1. Model I: the calculation model for same-speed trains

The model is built based on the train operation plan. The capacity calculation model takes the maximal number of trains as the target function in Equation (22). The train operation sequence lq_y is obtained under the conditions of optimal target value, and $LQ = \left\{ lq_y \mid y = 1, 2, \ldots, N_{lq} \right\}$ is the set of train operation sequences. The model considers the following constraints:

- the minimum headway at the station and in the sections can expressed by Equations (23)–(25). The constraint indicates that the headway of any 2 adjacent trains should be not less than the minimum headway at the station and in the section;
- the buffer time in the sections can be expressed by Equation (26). This constraint indicates that the buffer time of any 2 adjacent trains should not be less than the minimum buffer time in the section;
- the service quality of trains at each station can be expressed by Equation (27). The service quality of trains at each station in this constraint refers to the ratio between the maximal period of time in which there are no train services and the total period of time. Because the time of train operation is not calculated in the solving process, the index turns to the ratio between the maximum number of trains between 2 stops and the total number of trains;
- the total usable time can be expressed by Equation (28). This constraint indicates the time period outside, which the trains cannot operate.

In Equations (22)–(28), $t^{da,\,s_i}_{l_j,\,l_{j+1}}$ and $t^{dd,\,s_i}_{l_j,\,l_{j+1}}$ are the actual departure interval time and arrival interval time between train l_j and train l_{j+1} at station s_i, $t^{d}_{\min}$ and $t^{a}_{\min}$ are the minimal departure headway time and arrival headway time, respectively, $t^{l,\,w}_{l_j,\,l_{j+1}}$ is the actual interval time between train l_j and train l_{j+1} in section w, N_{se} is the number of sections, N_l is the number of trains, $t^{b,\,w}_{l_j,\,l_{j+1}}$ is the buffer time between train l_j and train l_{j+1} in section w, $t^{b}_{\min}$ is the minimum buffer time in the section, $n^{s_i}_{l_j,\,l_{j+k}}$ is the maximum number of trains between train l_j and train l_{j+k} at station s_i. Both train l_j and train l_{j+k} stop at station s_i. Besides, there are no trains between train l_j and train l_{j+k} at station s_i. μ_{SE} is the minimum quality service parameters of the trains in the section, and $T^{s_i,\,sq(y)}_{l_j}$ is the arrival time of train l_j at station s_i.

Hence, the calculation model for the capacity of the same-speed trains is as follows:

$$\max_{lq_y \in LQ} C_{sp}\left(lq_y\right); \quad (22)$$

$$t^{da,\,s_i}_{l_j,\,l_{j+1}} \geq t^{a}_{\min} \quad \forall l_j,\, s_i; \quad (23)$$

$$t^{dd,\,s_i}_{l_j,\,l_{j+1}} \geq t^{d}_{\min} \quad \forall l_j,\, s_i; \quad (24)$$

$$t^{l,\,w}_{l_j,\,l_{j+1}} \geq t^{l}_{\min} \quad \forall l_j,\, w; \quad (25)$$

$$t^{b,w}_{l_j,l_{j+1}} \ge t^{b}_{\min} \quad \forall l_j, w; \tag{26}$$

$$\frac{n^{s_i}_{l_j,l_{j+k}}}{N^{SE}_{l}} \le \mu_{SE} \quad \forall l_j, s_i; \tag{27}$$

$$T^{s_{N_{se}},lq(y)}_{l_{N_l}} - T^{s_0,lq(y)}_{l_1} \le 60 - t^{l}_{\min} + \sum_{w=1}^{N_{se}} \frac{L^{se}_{w}}{v_h}. \tag{28}$$

4.1.2. Model II: the model for different-speed trains

The target function of this model is to maximize the total number of trains by inserting the low-speed trains based on the method in Section 3.2 expressed by Equation (29). The model should also satisfy the constraints of Equations (23)–(28). Then, in order to ensure the total operation time, it is necessary to limit the number of trains overtaken by other trains at the same station and the total overtaking number of a train. Therefore, this model needs to increase 2 constraints. The 1st one is the constraint for the number of trains overtaken by other trains at the same station, which is expressed by Equation (30). Here, $n^{ot,s_i}_{l_j}$ is the number of overtaking trains that are overtaken by other trains at station s_i, $n^{s_i}_{\max,ot}$ is the maximum number of trains overtaken by other trains at the same station, which can be expressed by Equation (31), and N_{low} is the number of low-speed trains that are inserted. The other one is the total number of times a train is overtaken by other trains on the line. Here, the maximum overtaking number is defined as $n^{se}_{\max,ot}$:

$$\max_{lq_y \in LQ} C_{dp}\left(lq_y\right); \tag{29}$$

$$n^{ot,s_i}_{l_j} \le n^{s_i}_{\max,ot}; \tag{30}$$

$$\sum_{i=1}^{N_{se}+1} n^{ot,s_i}_{l_j} \le n^{se}_{\max,ot}. \tag{31}$$

4.2. Algorithm

Step 1: Build the set for the operation sequence of the high-speed trains LQ, where $lq(y)$ is the operation sequence scheme.

Step 2: Choose the train operation sequence $lq(y)$ of set LQ, and $y = 1$.

Step 2.1: Judge whether the constraints Equations (23)–(28) are satisfied. If they are satisfied, go to Step 2.2, otherwise return to Step 2, and set $y = y + 1$ to choose the next train operation sequence;

Step2.2: The high-speed trains are divided into different train units x ($1 \le x \le N_x$). The number of trains in each train unit is N_{cl}. The initial state is $x = 1$;

Step 2.3: Judge the stop plan of all high-speed trains in the x-th train unit to obtain the combination type *flag* = (1, 2, 3, 4). Different calculation methods can be selected according to the *flag*:

- if *flag* = 1, it means that a single train should be calculated; then, ε_x = the number of deductions for this single train by Equation (2);
- if *flag* = 2, it means that the stop-times and the stop schemes of the trains are the same; then, ε_x = the number of deductions calculated by Equation (3);
- if *flag* = 3, it means that the stop schemes of the trains are different but the stop-times are the same; then, ε_x = the number of deductions calculated by Equation (3);
- if *flag* = 4, it means that both the stop schemes and stop-times of the trains are different; then, ε_x = the number of deductions calculated by Equation (4);

Step 2.4: Judge whether $x > N_x$. If it is, turn to Step 2.5, otherwise, set $x = x + 1$ and return to Step 2.3;

Step 2.5: Compress the interval time between any adjacent train units by using the "compression method between 2 adjacent train units" given in Section 2.2.

Step 3: Judge whether $y > N_{lg}$. If it is, turn to Step 4, otherwise, set $y = y + 1$ and return to Step 2.

Step 4: Based on the optimal scheme of the high-speed train operation sequence with the objective of Model I in Section 4.1.1, the low-speed train is inserted, and the moving time of each section through which the low-speed trains pass are calculated. The interval time between any 2 adjacent trains, including the interval time before the 1st train and the interval time after the last train, is defined as l ($1 < N_l + 2$), and $l = 1$, $t^{move}_{l_j} = 0$.

Step 4.1: Initialize the station $s = 1$ and the section $w = 1$;

Step 4.2: Calculate the moving time in section w based on the method in Section 3.1 and $t^{move}_{l_j} = t^{move}_{l_j} + t^{move,w}_{l_j}$. Set $i = i + 1$ and go to Step 4.3;

Step 4.3: Calculate the moving time at station s based on the method in Section 3.2 and $t^{move}_{l_j} = t^{move}_{l_j} + t^{move,s_i}_{l_j}$. Judge whether s is the last station of the line. If it is, go to Step 4.4, otherwise, set $w = w + 1$ and return to Step 4.2;

Step 4.4: Judge whether the constraints Equations (23)–(31) are satisfied. If they are, save the result of this scheme. Judge whether $1 < N_l + 2$. If it is, set $l = l + 1$ and return to Step 4.1, otherwise go to Step 4.5;

Step 4.5: Choose the scheme with the minimal $t^{move}_{l_j}$ as the optimal scheme, and calculate the maximal capacity and the optimal capacity utilization.

5. Case analysis

5.1. Parameter settings

The section from Beijing South to Jinan West of the Beijing–Shanghai High-Speed Railway line is analysed, as shown in the Figure 15. The line is 428 km long and includes Beijing South, Langfang, Tianjin South, Cangzhou West, Dezhou East and, Jinan West (6 stations and 5 sections). The distance of the 5 sections are 89, 62, 84, 104 and 89 km.

In Figure 11, the green lines and blue lines indicate the operation lines of the high-speed train and low-speed train, respectively. The speed of the high-speed train is 300 km/h, while the speed of the low-speed train is 250 km/h. The operation times of different-speed trains in different sections are shown in Figure 15.

The effective time period of the train operation is 60 min. The minimum section headway of train is 3 min. The stop-time at the station is 2 min. The arrival and departure headways and the additional time of the stops at each station are shown in Table 1. According to the calculation, the final average headway is 3.3 min. Therefore, the parallel timetable is calculated by a headway of 3.3 min.

In this case, the minimum buffer time is 0 min, the maximum value of the station service quality index is 0.5 in the same station, the maximum number of low-speed trains overtaken by high-speed trains is 1, the maximum total number of low-speed trains overtaken by high-speed trains is 2, and the maximum number of continuous departure low-speed trains is 1.

Based on the real train operation plan from Beijing South to Jinan West in March 2015, we select 1 h (from 7:00 pm to 8:00 pm) operating case for the analysis. As shown in Figure 12, in this train operating case, 8 trains are run in 1 h, and all of them are higher speed level trains. The service frequency in Langfang station, Tianjin station, Cangzhou station, and Dezhou station are 2, 5, 3 and 3 times, respectively. On this basic train operation plan, a low-speed train is added (the blue line). The speed of the added low-speed train is 250 km/h, with stops at Langfang, Tianjin South, and Cangzhou West, as shown in Figure 12. The solid points in Figure 12 express the train stops at the station and the numbers behind the lines express the number of trains with corresponding stop modes.

5.2. Capacity calculation of the high-speed trains

The number of trains for each train unit is 2. All the results are shown in Figure 13. The train deduction is between 4.2 trains/h and 9.13 trains/h. In the ascending order of the results for these solutions as shown in Figure 14, most of the results for these solutions are between 5 trains and 9 trains/h. Figures 15 and 16 show the capacity and capacity ratio for these solutions. The maximal average capacity would reach 13.4 trains/h based on the current time occupation condition, and the worst capacity is only less than 9 trains/h. This indicates that only one other train can be added based on the train operation plan. Therefore, the operation sequence of the trains is a critical factor for the capacity.

Using permutations of the 6 train units, a total of 720 operational sequence schemes were generated. The optimal solution is the 447th scheme. We calculate and analyse the capacity and optimal train operation sequence as shown in Figure 22. The capacity of the parallel timetable is 18 trains/h. Based on the optimal train operation sequence and calculation method, the train deduction is 4.2 trains/h, as shown in Figure 17. There are 8 trains in total, which occupy a total of 25.7 min at the departure station.

Hence, the trains can be added according to the original train operation structure as shown in Figure 18. For this scheme, we can still increase the number of trains by 3, which increases the final capacity to 12 trains/h. When in-

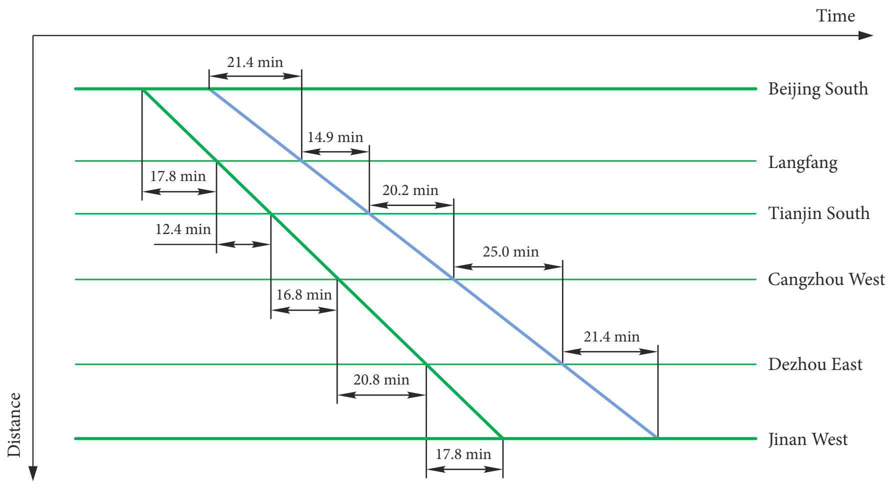

Figure 11. The Beijing South – Jinan West high-speed railway

Table 1. The minimal headway and additional time [min] based on the train stops at each section

Section	Departure headway	Arrival headway	Section headway	Final headway	Departure additional time	Arrival additional time
Beijing South – Langfang	3.2	3.1	3	3.2	2.5	2.5
Langfang – Tianjin South	2.9	3.2	3	3.2	1.6	2.8
Tianjin South – Cangzhou West	2.8	3.3	3	3.3	1.6	2.8
Cangzhou West – Dezhou East	2.9	3.2	3	3.2	1.6	2.6
Dezhou East – Jinan West	2.4	3.3	3	3.3	2.2	2.7

creasing the capacity to the 5th train, the remaining time is not enough to ensure normal operation; therefore, there is some buffer time for each section during the time period and the capacity ratio of each section is not large. The service quality of each station (Langfang, Tianjin South, Cangzhou West, and Dezhou East) are 0.33, 0.25, 0.33, and 0.42.

According to the optimal combination, if the train is added directly behind a train that has the same stops, the capacity can effectively be increased in this time period. As shown in Figure 19, according to this method, the capacity can reach 14 trains/h. At the same time, the original train structure is maintained. The service quality of each station (Langfang, Tianjin South, Cangzhou West, and Dezhou East) are 0.50, 0.42, 0.50, and 0.43. The service quality of each station had an effective increase, and the service quality is obviously reduced.

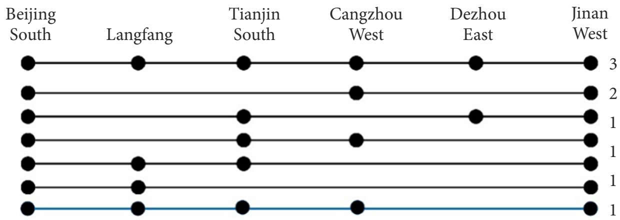

Figure 12. Train operation plan

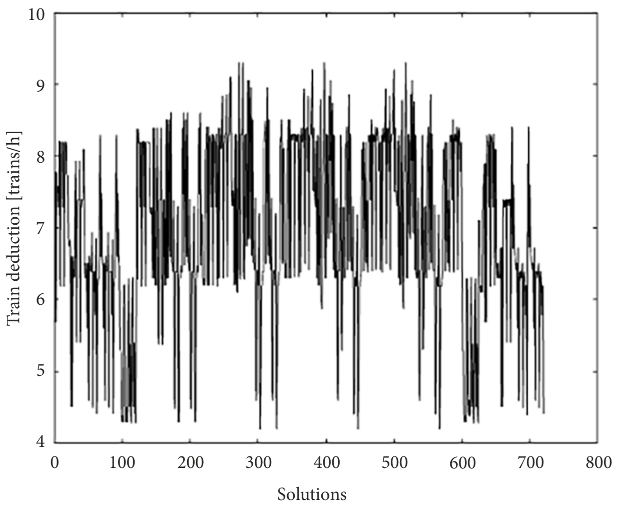

Figure 13. Results of the solutions

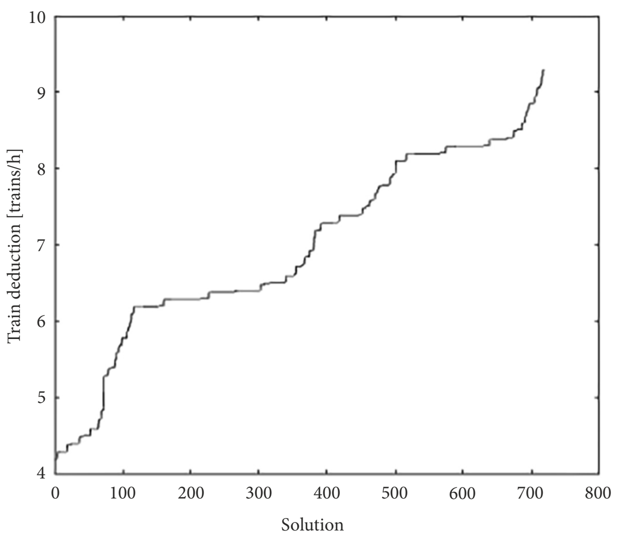

Figure 14. Results of the solutions in ascending order

5.3. Inserting a low-speed train

Due to the order of the low-speed trains, different train sequences may result in different schemes, which can lead to different results. In this case, the optimal scheme is as shown in Figure 20, where the low-speed train had be inserted into the rear of the last high-speed train. The arrival time of each train at each station is calculated based on the distance of the section, train speed, additional departure time, additional arrival time, and stop-time, and the train stop plan is as shown in Table 2. Train 9 is the low-speed train. Therefore, the capacity is 9 trains/h, and the service quality of each station (Langfang, Tianjin South, Cangzhou West, and Dezhou East) are 0.33, 0.33, 0.44, and 0.33.

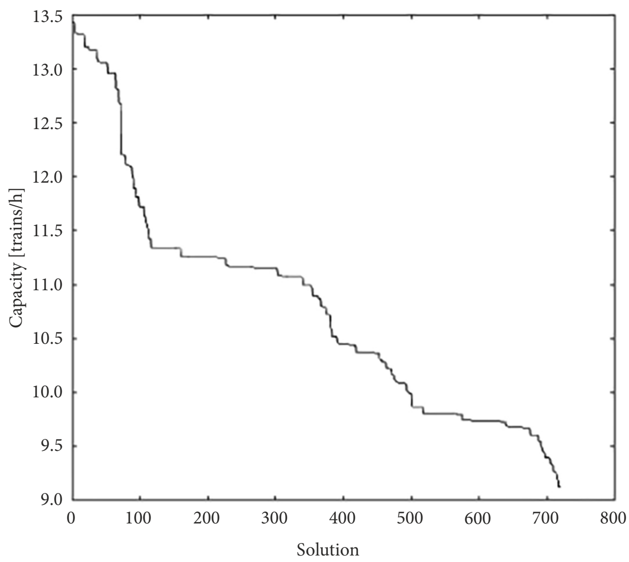

Figure 15. Maximal capacity for these solutions

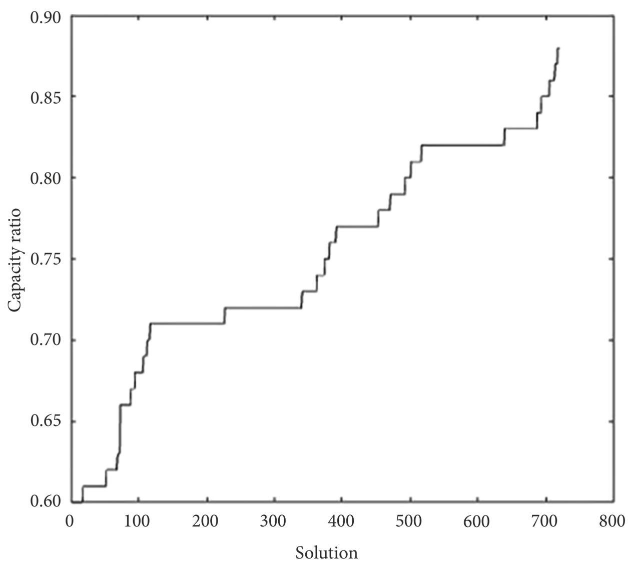

Figure 16. Capacity ratio for these solutions

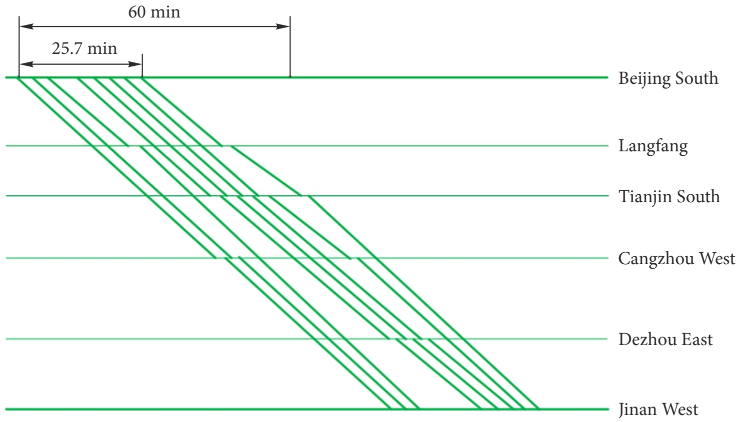

Figure 17. The optimal train operation sequence scheme based on the train operation plan

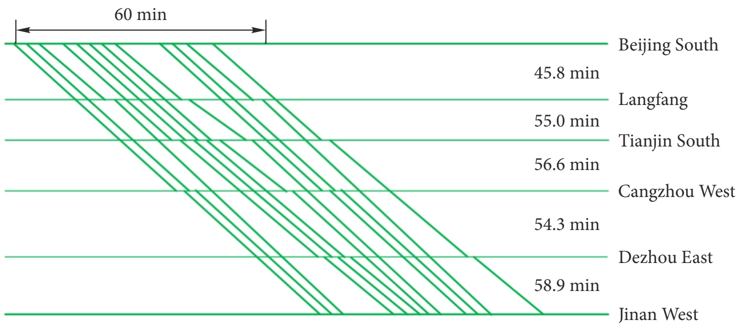

Figure 18. The capacity of the optimal train operation sequence

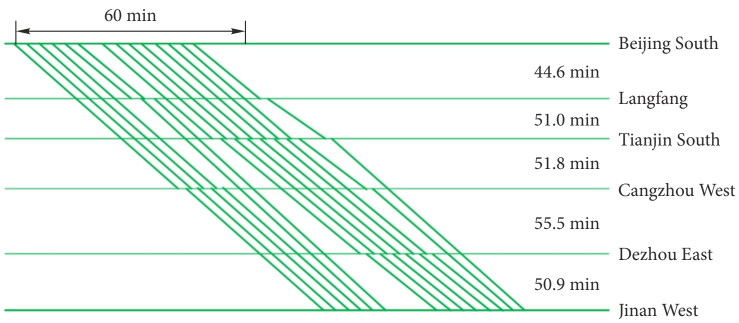

Figure 19. The capacity of the maximal trains

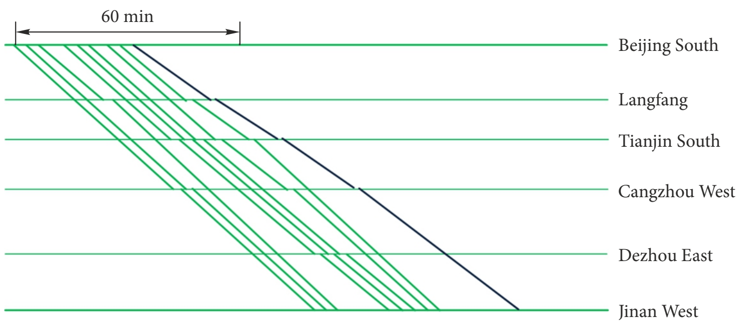

Figure 20. The optimal solution with different train speeds

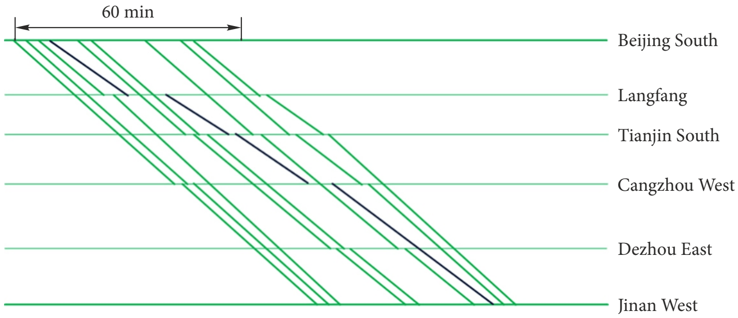

Figure 21. The suboptimal solution with different train speeds

Table 2. The arrival time [min] of each train at each station

Station	Train 1	Train 2	Train 3	Train 4	Train 5	Train 6	Train 7	Train 8	Train 9
Beijing South	0	3.3	6.6	12.4	15.7	19.0	22.3	25.6	31.4
Langfang	20.3	23.6	29.4	34.7	38.0	41.3	42.6	48.4	55.3
Tianjin South	32.7	36	45.4	49.9	53.2	56.5	59.8	67.2	76.3
Cangzhou West	52.3	55.6	62.2	70.3	73.6	76.9	83.0	87.6	103.2
Dezhou East	76.7	80	83.5	93.7	98	100.3	107.4	110.7	131.8
Jinan West	97.2	100.5	104	118.4	121.7	125	128.3	131.6	155.9

Table 3. The arrival time [min] of each train at each station

Station	Train 1	Train 2	Train 3	Train 4	Train 5	Train 6	Train 7	Train 8	Train 9
Beijing South	0	3.3	6.6	9.9	19.3	22.6	37.5	46.3	49.6
Langfang	20.3	23.6	29.4	36.3	39.6	42.9	59.8	66.6	72.4
Tianjin South	32.7	36.0	45.4	65.1	54.8	58.1	75.0	83.8	91.2
Cangzhou West	52.3	55.6	62.2	91.7	75.2	78.5	95.4	107.0	111.6
Dezhou East	76.7	80.0	83.5	124.9	98.6	102.9	118.8	131.4	134.7
Jinan West	97.2	100.5	104	149	123.3	126.6	143.5	152.3	155.6

However, this optimal solution would cause an imbalance in the capacity of each section, and there are no trains in the 2nd half of 1 h that can be selected by the passenger. Hence, we get a suboptimal solution in which the low-speed train is among the high-speed trains, as shown in Figure 21. The arrival time of each train at each station is shown in Table 3. Train4 is the low-speed train.

In this solution, the low-speed train is added behind the 3rd high-speed train. It will be overtaken by 2 trains at Langfang station and by one train at the Cangzhou West station. Based on the parallel timetable, 8.5 high-speed trains are deducted. Therefore, the capacity is 9 trains/h. The service quality of each station (Langfang, Tianjin South, Cangzhou West, and Dezhou East) are 0.22, 0.33, 0.33, and 0.33.

6. Result and discussion

According to the analysis of the data in Table 4, the following conclusions can be obtained:

- according to the 2 schemes of the same-speed train, the capacity of Scheme #2 is 18% larger than Scheme #1, and the occupying time of each section is larger. This indicates that the density of the train operation of Scheme #2 is very high, and most of the trains operate based on the minimal headway. This condition may cause a delay for most of the trains when any train is delayed. At the same time, the service quality at each station is worse. Therefore, the capacity can be increased by reducing the service quality, such as punctuality, which is inconvenient for passengers;
- according to the 2 schemes for different train speeds, the capacities of Scheme #3 and Scheme #4 are the same. However, the occupation time in each section of Scheme #4 is more balanced, and the service quality at each station is better. At the same time, the distribution for buffer time in each section of Scheme #4 is more balanced and can decrease the influence of the "knock-on delays". Therefore, a suitable operation sequence of the trains can increase the service quality by not reducing the capacity;
- according to the schemes for the same-speed trains and different-speed trains, the capacity would decrease by 3…5 trains/h. Therefore, the operation of low-speed trains has a significant influence on the capacity.

The results are compared with those of the existing methods under the same calculation conditions. In the literatures on calculating the carrying capacity for the entire day, the total effective calculation time of the section is 996 min, about 16.6 h. The results of different methods are compared in Table 5.

From the results in the table, the maximum capacity value calculated in this article is the largest. However, for the high-speed railway, we cannot simply calculate the maximum capacity. At the same time, the passenger demand should be considered under different conditions. Therefore, the maximum capacity calculation method in this article is mainly applicable to the calculation of the carrying capacity under the condition with large passenger flow and tense capacity of the railway line. The optimal capacity scheme can be analysed by adjusting the service quality constraint and buffer time constraint.

Then, the buffer time is analysed with the maximal capacity scheme, and the reliability of passenger travel can be expressed by the buffer time. Figure 22 shows the influence of different average buffer times of different solutions on the results. When the average buffer time was increased to 3.2 min, a part of the results of the solutions were larger than 60 min and became invalid solutions. The relationship between the buffer time and actual capacity value is shown in Figure 23. The larger the buffer time, the greater the loss of capacity.

Figure 24 shows the occupied time of each section for the scheme with all high-speed trains and the 2 schemes with different-speed trains (expressed as Scheme #1, Scheme #2, and Scheme #3). Figures 25–27 show the buffer times of each section between 2 adjacent trains for these 3 schemes, respectively. We find that the occupied time of each section of Scheme #3 is larger than that of Scheme #2, but the distribution of the buffer time is more uniform. When there are some delays among the trains, Scheme #3 can absorb the delay time more effectively.

Table 4. Comparison of different schemes

Index	Scheme #1 of the same train speed	Scheme #2 of the same train speed	Scheme #3 of different train speed	Scheme #4 of different train speed
Maximal capacity [trains/h]	12	14	9	9
Departure time at the 1st station [min]	41.3	42.1	31.4	49.6
Longest occupied time of the section [min]	58.9	51.8	58.7	59.3
Shortest occupied time of the section [min]	45.8	44.6	35.0	52
Best service quality value [–]	0.25	0.42	0.33	0.22
Worst service quality value [–]	0.42	0.50	0.44	0.33

Table 5. The results of different methods

Reference	Method	Maximal capacity [trains/h]	
		Same-speed scheme	Different-speed scheme
Zheng, Liu (2012)	train deduction method	12.0	6.6
Ma (2017)	timetable compression method	10.4	7.5
Zhao, Hu (2018)	optimization analysis method	12.65	–
This article	comprehensive optimization method	14	9

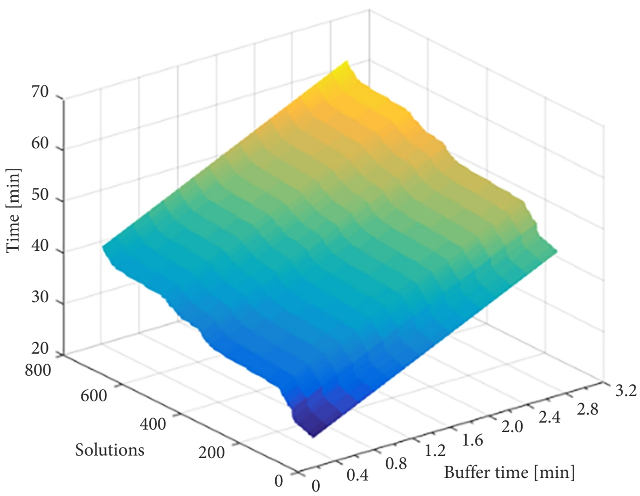

Figure 22. Influence of different average buffer times of different solutions on the results

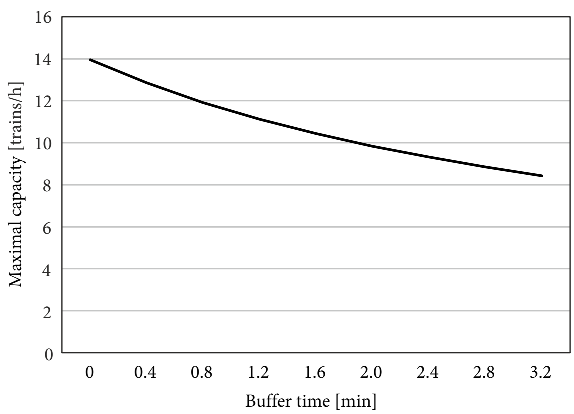

Figure 23. Influence of different average buffer times on maximal capacity

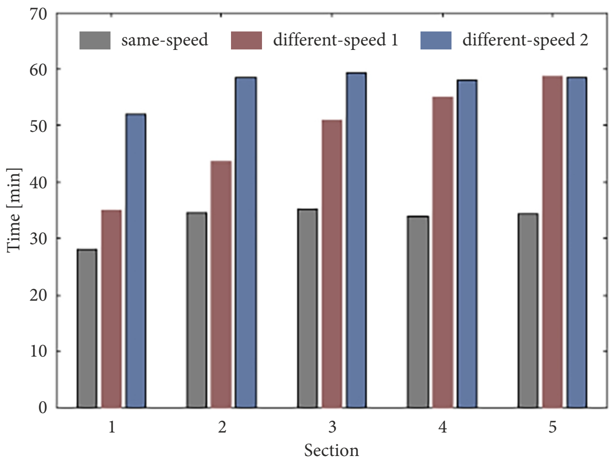

Figure 24. The occupied time of each section for the schemes

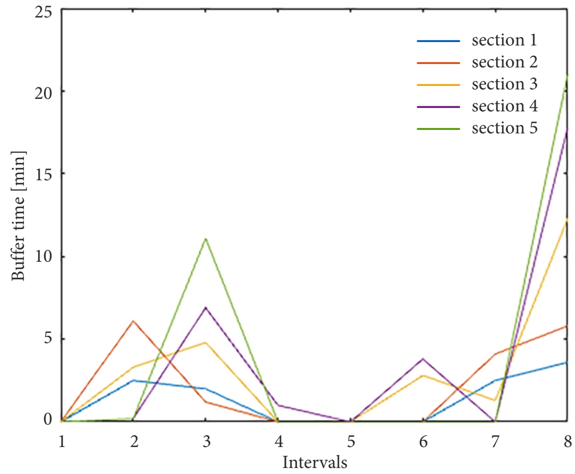

Figure 26. The buffer time of each section between the 2 adjacent trains of Scheme #2

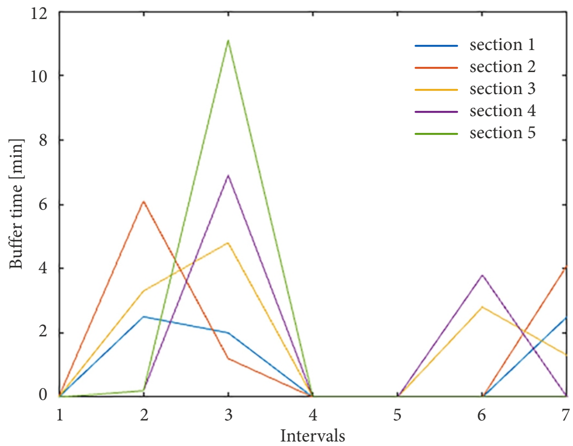

Figure 25. The buffer time of each section between the 2 adjacent trains of Scheme #1

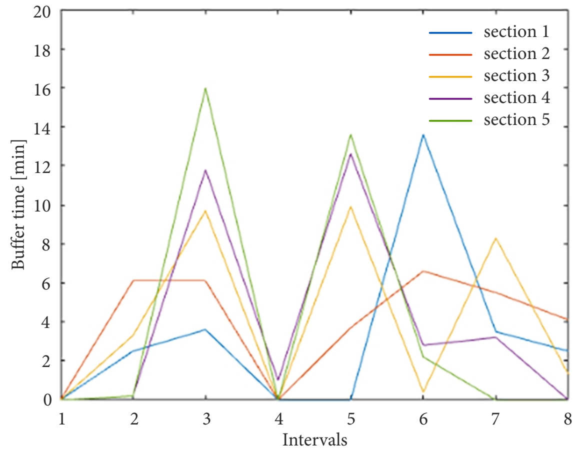

Figure 27. The buffer time of each section between the 2 adjacent trains of Scheme #3

Conclusions

This article proposes a calculation method for the capacity of a high-speed railway. 1st, based on the parallel timetable, and according to the rules of train combination, we research the capacity for trains with the same-speed based on the train deduction method. 2nd, by choosing the low-speed train's inserting position according to the actual headway, we research the time moving method in the section and at the station. The capacity is obtained under optimal train operation conditions.

Unlike the existing literatures, this article calculates the train speed classes according to the same-speed condition and different-speed condition. At the same time, the 2 methods are combined to obtain the final scheme. This method not only embodies the advantages of the train deduction method in visualizing the calculation process, but also combines the ideas of the timetable compression method and the minimum interval time method, which increases the accuracy of the traditional deduction coefficient method.

Based on the above method, the calculation model for the capacity of the high-speed railway considering the service quality of the passengers is established, and the corresponding algorithm is studied. The model takes the maximum capacity value as the optimization objective, and reflects passenger travel convenience with train service frequency and reliability with buffer time constraints. Therefore, the model in this article can be used to optimize the capacity under different conditions by adjusting the values of constraints. It can not only get the maximum capacity scheme, but also get the capacity optimization scheme under different passenger demand conditions.

According to the analysis of the cases in this article, we can obtain the following conclusions. When the trains have the same-speed, a reasonable combination of trains with different stop schemes expand the railways capacity. When the trains have different-speeds, there is a greater impact on capacity. However, by analysing the reasonable operating positions of low-speed trains, their impact on the capacity can be reduced. The operation sequence with different-speeds and different stop schemes has a greater influence on capacity. However, sequence optimization for train operation can effectively increase the practical capacity. Different capacity schemes would cause different service qualities that affect the convenience of passengers and have different influences depending on the sensitivity to train delays. The maximal capacity scheme usually has a poor service quality and recovering capability when there is a delay. Therefore, for meeting the passenger demand, the optimal capacity is of more significance than maximal capacity.

This method has a strong practical applicability and feasibility. Under the condition of not depending on the existing timetable, the carrying capacity is calculated according to certain operation parameters. The results obtained can be optimized on the premise of reliability and provide a theoretical basis for the actual operation and organization process.

In this article, the capacity limitation of the station and hub is not considered in the calculation method. The station capacity is taken as a prerequisite to be satisfied. In future research, the capacity of stations, sections, and lines will be considered and calculated in a unified way.

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Disclosure statement

The authors declare that there is no conflict of interest regarding the publication of this article. And we certify that we have participated sufficiently in the work to take public responsibility for the appropriateness of the experimental design and method, and the collection, analysis, and interpretation of the data.

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