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Original Article

SPATIAL PARTITION FOR HETEROGENEOUS CITY NETWORKS COMPOSED OF FACTORS THAT INFLUENCE THE DISTRIBUTION OF THE MACROSCOPIC FUNDAMENTAL DIAGRAM

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Highlights:

- evaluated the influencing factors that influence the MFD curve;
- developed an MST–Ncut sub-region partition method based on MFD influencing factors;
- proved the accuracy of the MST–Ncut method is better than the Ncut method.

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Abstract. Using a Macroscopic Fundamental Diagram (MFD) to implement partition control is effective in improving mobility in heterogeneous city networks. As one of the most complex issues in partition control, accurate sub-region partition is critical for control effectiveness. Current partition methods focus on the link density and precondition of existing MFD and disregard the factors that influence MFD distribution. To overcome this drawback, this study uses the characteristic value of the link and the intersection connected to the link as the analysis object and proposes an MFD sub-region partitioning method for large-scale networks. Firstly, the influences of road state parameters on MFD distribution are classified into traffic flow parameters, network physical properties, network operation mechanisms and emergencies. Simulation experiments are conducted to determine the degree to which these classifications affect MFD distribution. Secondly, a partition method combined with the link density and influence parameters of MFD is developed. The method is used for a preliminarily division of a road network through Minimum Spanning Tree (MST) and depth partition by the Normalised Cut (Ncut) algorithm. Finally, a case study is conducted in an actual city centre network, and results show that the developed method is superior to the single method based simply on link density.

Keywords: partition control, sub-region partition, macroscopic fundamental diagram, minimum spanning tree, normalised cut algorithm.

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Notations

Abbreviations:

GPS – global positioning system;
MFD – macroscopic fundamental diagram;
MST – minimum spanning tree;
MST–Ncut – MST and Ncut.
Ncut – normalised cut;
NP – nondeterministic polynomial.

Variables and functions:

A or B – symbol of sub-regions;
C – all sub-region sets contained in the road network, both A and $B \in C$;
C_m^n – mth cluster (contains n data objects, similar to a sub-region);
$cut(A, B)$ – similarity between sub-regions A and B;
d_i or d_j – density of link i or j;
$dif(C_m^n)$ – average difference degree of C_m^n;
$dif(l_i, l_j)$ – difference between l_i and l_j;
$\mathbf{G}$ – graph;
i or j – link symbol;
k – number of sub-regions;
l_i – ith edge of graph $\mathbf{G}$;
l_j – jth edge of graph $\mathbf{G}$;
MNS_k – discriminant value of reverse merger; that is, the average value of the NS of all sub-regions when reverse-merged into k sub-regions;
N_A or N_B – number of all links in the A or B sub-region;
$Nassoc(A, B)$ – internal similarity in A and B;
$Ncut(A, B)$ – difference between A and B;
$NS(A, B)$ – density difference between sub-regions A and B;

$NS(A)$ – discriminant value of sub-region A (determine if sub-region A achieves the partition goal);
$NS(A, A)$ – density difference within sub-region A;
NS_k – discriminant value of partition for road network; that is, the average value of the NS of all sub-regions when divided into k sub-regions;
o – number of factors influencing the MFD curve;
P – influence factor of the MFD curve;
$s(l_i, l_j)$ – coefficient of the difference between l_i and l_j;
$s_p(l_i, l_j)$ – difference of the 2 connected edges considering the Pth influence factor, $s_p \in [0, 1]$;
T – original tree;
u_A or u_B – mean of the link density of the A or B sub-region;
$Var(A)$ or $Var(B)$ – density variance of the link in the sub-region A or B;
$w(i, j)$ – similarity function between links i and j;
λ_p – influence degree of the Pth influence factor, $\lambda_p \in [0, 1]$.

Introduction

In a large-scale urban road network, different areas simultaneously show different traffic states (free, critical and congestion flows). The boundary control target is crucial in reducing the inflow to the congested area and relieving the congested status. The MFD was proposed by Godfrey (1969) and further verified by Daganzo & Geroliminis (2008), Gonzales *et al.* (2009), Geroliminis & Sun (2011a); it used data from real loop detectors in Yokohama (Japan) and simulation data in Nairobi (Kenya). Since the introduction of MFD, boundary control aimed at congested areas has considerably advanced because it does not need a complex origin-to-destination matrix, and it has become the core means to solve traffic congestion in large-scale networks. Geroliminis *et al.* (2013), Ding *et al.* (2018) and Haddad (2017) divided a road network into 2 sub-regions and prompted boundary control. Multi-region boundary control was also examined by Aboudolas & Geroliminis (2013), Ramezani *et al.* (2015), Keyvan-Ekbatani *et al.* (2015), Ampountolas *et al.* (2017), Haddad & Mirkin (2017) and Kouvelas *et al.* (2017). Previously developed control methods are effective in solving the congestion problem. However, the MFD sub-region partitions presented by Aboudolas & Geroliminis (2013), Hajiahmadi *et al.* (2013, 2015), Keyvan-Ekbatani *et al.* (2015), Ramezani *et al.* (2015), Zhou *et al.* (2016) and Ding *et al.* (2017) do not provide a precise partition process.

Sub-region partitioning considerably affects boundary control. Reasonable sub-region partitioning is the assignment of links to proper clusters with high intra-similarity and low inter-similarity; thus, each sub-region has a perfect MFD curve. On the contrary, unreasonable sub-region partitioning may cause the characteristic of the link in the sub-region to become diverse and cannot acquire a perfect MFD curve. Wang & Jia (2010), Geroliminis & Sun (2011b), Ji & Geroliminis (2012), Geroliminis *et al.* (2013) and Ji *et al.* (2013, 2014) used congestion characteristics in a given period to divide an entire traffic network and extended this procedure to the partitioning problem of a neighbourhood to obtain appropriate sub-region partitioning for boundary control. Haddad & Geroliminis (2012) proposed a boundary control model and a sub-region partition model to analyse the optimal control problem between 2 regions. Li & Zhao (2012), who proposed the standard for partitioning, also used MFD to study the sub-region partition method. Ma *et al.* (2010) used the correlation of adjacent intersections as a source and divided a macro network through the spectral method. The spatial–temporal distribution of the congestion area was analysed by Ji & Geroliminis (2012) and Geroliminis *et al.* (2013) based on the spatial features of congestion during a specific period, and 3 consecutive algorithms were developed to partition urban transportation networks. The preceding methods provided a theoretical basis for solving the network partition problem. Nevertheless, these methods use link density as the characteristic parameter, and their core is the Ncut algorithm (Wang, Jia 2010; Ji, Geroliminis 2012; Haddad, Geroliminis, 2012; Geroliminis *et al.* 2013). The methods exert a good effect on the network topology of link density and usually proceed in view of the total network. They cluster links with a similar density into one sub-region and only distinguish the most obvious main area. Other related methods, such as that of Ji *et al.* (2013, 2014), used the GPS dataset of 20000 taxis from Shenzhen and distinguished congestion areas from the rest of the network. Considering that congestion is spatially correlated in adjacent roads, Saeedmanesh & Geroliminis (2016) provided a 3 step clustering algorithm to partition heterogeneous networks into connected homogeneous regions. Saeedmanesh & Geroliminis (2017) also proposed a method of combining dynamic and static clustering to better understand the formation and dissolution of congestion.

Although all of these partition methods can directly divide large-scale networks, they can still be improved. Existing methods use only one factor (link density) as the basis of sub-region partition and disregard the influence of other factors, such as traffic management, signal control and traffic accident, on MFD (Daganzo, Geroliminis 2008; Ji, Geroliminis 2012; Gayah, Daganzo 2011; Zhang *et al.* 2013). A variation in these influencing factors will scatter the MFD data dots of outflow to traffic accumulation, thereby affecting the fitting accuracy of the MFD curve (Peng 2013).

This study provides an MFD sub-region partition method for large-scale networks that considers the influence of road state parameters on MFD distribution. The degree of influence of various factors on the shape of the MFD curve is analysed based on simulation data on road networks. Then, the MST algorithm is used to divide a road network into several sub-trees. In these divided sub-trees, a low-density sub-tree still can accommodate vehicles; thus, controlling the inflow of its boundary is unnecessary. Meanwhile, the inflow of high-density sub-trees should be metered by boundary control. However, the largest sub-tree may be too large to take boundary control (advised scale is shown in Geroliminis & Daganzo (2007)), and further partition is indispensable. In this situation, the Ncut algorithm is applied in consideration of link density.

The partition processes include MST–Ncut algorithms. The full text is organised as follows:

- MFD curve influencing factor analysis;
- criteria of sub-region partition;
- MST–Ncut sub-region partition;
- case analysis;
- discussion;
- conclusions.

1. Degree of factors' influence on the MFD curve

1.1. Influencing factors

MFD is an inherent attribute of traffic networks, but other factors also affect the curve distribution. Mazloumian *et al.* (2010), Wang & Jia (2010), Ji & Geroliminis (2012), Ji *et al.* (2014), Haddad & Geroliminis (2012), Li & Zhao (2012), Geroliminis *et al.* (2013) used simulation data and indicated that a sudden change in traffic demand exerts a considerable effect on the shape of MFD. Zhang *et al.* (2013) used the cellular automata model to study MFD in the same road network under different adaptive signal control systems and showed that different signal control methods produce different MFD shapes. In addition, Xu *et al.* (2013) found that traffic management measures affect the shape of MFD. Ding *et al.* (2017) and Zhao *et al.* (2014) discovered that traffic guidance changes path selection and has a considerable influence on MFD distribution of a network with congestion state.

In summary, the factors that affect the state of MFD can be divided into the following categories:

- traffic flow parameters: density distribution, traffic demand;
- road network physical properties: proportion of main road;
- road network signal control schemes: signal/no-signal control, signal cycle and split;
- emergencies: traffic accidents.

A 10×10 grid network (the distance between intersections is 500 m) is used in the next subsection to analyse the influencing degree of these factors.

1.2. Degree of influence of factors

The factors that affect MFD are simulated, and the results are shown in Figure 1. Figure 1a shows MFDs under heterogeneous densities, where traffic density considerably varies between links of the road network. Figure 1b presents MFDs under homogeneous densities, where the road sections in the road network are in the same state, and the density of traffic flow is close. Comparison of Figure 1a and Figure 1b indicates that MFDs under heterogeneous density exhibit a strong hysteresis phenomenon, whereas MFDs under homogeneous density are perfect. This result shows that the homogeneous density of a road network is the basis for the existence of ideal MFD; thus, all simulation experiments and resulting statistics for the influence of various factors on MFD are based on homogeneous regions.

During the simulation process, different traffic demands are inputted into the network at different periods, as shown in Figure 1c, the peak values of the curve are between 1700 and 1800 veh/h and obtained near the average density of the road network (80 veh/km). Thus, traffic demand nearly has no effect on road network MFD. Even after changing the proportion of the main road in the road network, although the difference between the peak and density values is relatively small, we can still determine the impact of the main road proportion on MFD (Figure 1d). Then, after changing the signal control scheme at the intersection of the road network (Figures 1e–g), the intersection signal control coverage rate in the road network exerts a serious impact on the MFD of the area, which is mainly reflected in the peak value of the average flow. Moreover, under different signal cycles, the MFD of the road network produces certain differences. The peak values and the density corresponding to the peak values exhibit several differences. Thus, the signal cycle has a small effect on the MFD curve, whereas split exerts a considerable influence on the MFD curve of the road network. The last factor, emergency, is dynamic, but the final embodiment of the emergency in the road network is the change in traffic density, traffic capacity and other parameters. Thus, these parameters can be dynamically set to simulate emergency and find the average traffic peak value of the road network. The average traffic peak value of the road network under emergency 1 can reach 1700 veh/h, that under emergency 2 cannot reach 1200 veh/h and that under emergency 3 can only reach approximately 1000 veh/h. An emergency can have a considerable impact on the MFD of the road network, and the influence degree of different emergency on MFD of the road network also varies (Figure 1h).

The MFD characteristic parameters obtained from a traffic simulation are shown in Table 1. Signal/no-signal control, split and emergency are the 3 major factors, and signal cycle and main road proportion of the network are the 2 secondary factors that affect MFD distribution. These factors change the location of the MFD critical point (critical density – maximum outflow flow). Nevertheless, the 2 endpoints of the MFD curve do not considerably change. The location of the critical point affects the state and outflow capacity of the road network. Hence, the influence degree of each factor on the MFD diagram is attributed to the influence degree on critical flow, which is regarded as a reference value. Given that the MFD data points of the

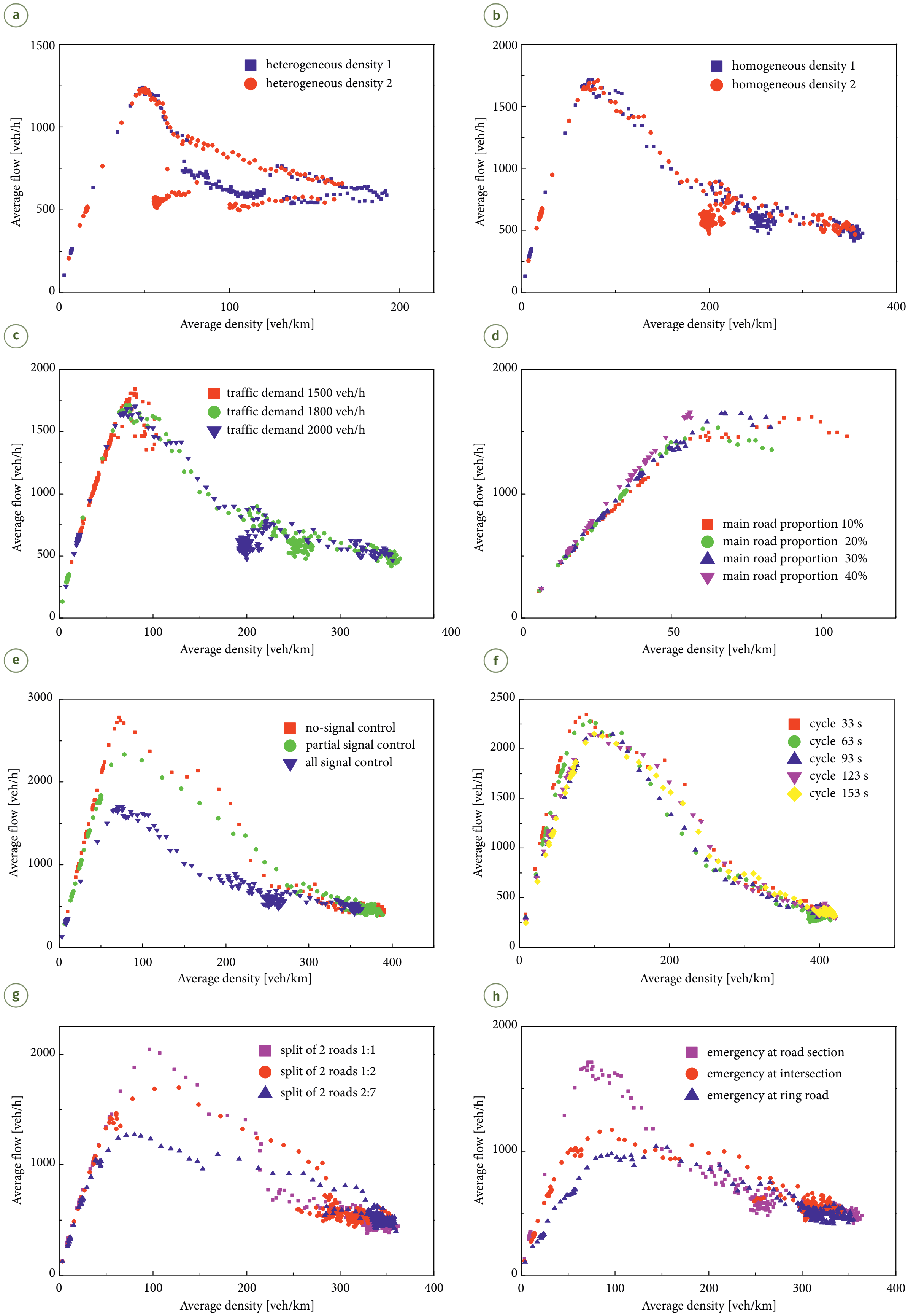

Figure 1. MFDs under different influencing factors:
a – heterogeneous densities; b – homogeneous densities; c – different traffic demands; d – different main road proportions; e – different signal control measures; f – different signal cycles; g – different splits; h – different locations of emergencies

accumulation in the region and trip completion flow are a range and not an exact value, directly obtaining the exact influence degree of each factor on MFD is difficult. Based on the test data (Table 1), this study determines whether these factors are major or secondary according to the degree of influence on the critical flow. The major influencing factors can affect the range of critical outflow (−800, 800), and the secondary influencing factors approximately account for 20% of the influence degree of the major factors, that is, in the range of (−160, 160). Several factors, such as signal cycles, are continuous variables, and defining the impact of different cycles on MFD is difficult. Therefore, when partitioning the sub-region, the influence degree of the major factors is defined as 1 and that of the secondary factors is recommended as 0.2.

2. Criteria of sub-region partition

Sub-region partitioning assigns links to proper clusters with high similarity within the sub-region and with low similarity between sub-regions for partition control. The discriminant model for the difference in traffic factors is:

$$NS(A, B) = \frac{\sum_{i \in A} \sum_{j \in B} \left(d_i - d_j\right)^2}{N_A \cdot N_B}. \tag{1}$$

To determine if sub-region A achieves the partition goal, the discriminant equation is:

$$NS(A) = \frac{NS(A, A)}{\min\left(NS(A, X) \mid X \text{ is the neighbor sub-region of } A\right)}, \tag{2}$$

when: $NS(A) < 1$, the partition of sub-region A meets the target; A small $NS(A)$ yields good results.

If B is the neighbour sub-region of A and $NS(A, B) = \min(NS(A, X))$, then Equations (1) and (2) can be transformed into an equation of variance and mean as follows:

$$NS(A, B) = Var(A) + Var(B) + \left(u_A - u_B\right)^2 \tag{3}$$

and

$$NS(A) = \frac{2 \cdot Var(A)}{Var(A) + Var(B) + \left(u_A - u_B\right)^2}. \tag{4}$$

In Equation (4), if density mean u_A is not close to u_B and $Var(A)$ and $Var(B)$ are relatively small, then the value of $NS(A)$ will be small, which is the most ideal effect. If the density mean of 2 sub-regions is similar, then the following 3 situations can be observed:

- when the internal variance is relatively small, the value of $NS(A)$ is around 1. This condition indicates that the 2 sub-regions have the potential to merge, and the partition plan may not be optimal;

Table 1. Influence of traffic factors on MFD

Influence factor	Classification	Critical flow [veh/h]	Critical density [veh/km]	Average flow [veh/h]	Average density [veh/km]	Hysteresis phenomenon	Influence degree
Traffic flow parameters	heterogeneous density 1	1239	48.80	630	88.65	yes	1
	heterogeneous density 2	1225	48.25	660	66.45	yes	
	homogeneous density 1	1709	71.49	664	197.97	no	0
	homogeneous density 2	1708	81.75	721	177.97	no	
	traffic demand 1500 veh/h	1841	80.76	1124	44.49	no	0
	traffic demand 1800 veh/h	1709	75.63	664	197.97	no	
	traffic demand 2000 veh/h	1708	81.75	721	177.97	no	
Physical property of the road network	main road proportion 10%	1607	84.36	1067	46.73	no	0.2
	main road proportion 20%	1528	65.69	976	36.68	no	
	main road proportion 30%	1651	67.96	1062	39.62	no	
	main road proportion 40%	1661	56.47	1031	32.447	no	
Signal control schemes of the road network	no-signal control	2774	72.12	812	278.34	no	1
	partial signal control	2331	78.85	687	284.58	no	
	all signal control	1709	71.50	664	197.98	no	
	cycle 33 s	2340	89.97	1180	189.80	no	0.2
	cycle 63 s	2252	101.90	587	325.36	no	
	cycle 93 s	2147	109.06	664	322.36	no	
	cycle 123 s	2142	95.43	565	349.79	no	
	cycle 153 s	2148	99.41	594	343.97	no	
	split of 2 roads 1:1	2039	96.73	644	251.30	no	1
	split of 2 roads 1:2	1692	128.02	646	246.36	no	
	split of 2 roads 2:7	1268	80.18	578	276.37	no	
Emergency	emergency at road section	1709	75.12	664	197.97	no	1
	emergency at intersection	1163	97.24	591	256.03	no	
	emergency at ring road	1036	143.81	536	265.42	no	

- when $Var(A) < Var(B)$, the value of $NS(A)$ is less than 1. At this time, the partition of sub-region A is optimal;
- when $Var(A) > Var(B)$, the value of $NS(A)$ is greater than 1. This condition indicates that the 2 sub-regions have the potential to continue to be divided.

In summary, the smaller the $NS(A)$ of sub-region A is, the better the partition scheme is. For $\forall A_m \in C$, m is the sub-region serial number, and the optimal criterion for the k sub-region partition is:

$$\min NS_k = \frac{1}{k} \cdot \sum_{m=1}^{k} NS\left(A_m\right), \tag{5}$$

where: NS_k is the discriminant value when the road network is divided into k sub-regions.

3. MST–Ncut partition method

3.1. Partition process

Among the factors that influence MFD, signal/no-signal control, split and cycle are intersection characteristics. Flow density and lane number are link characteristics. To merge all of the influence factors during sub-region partitioning, all characteristics of a link and its downstream intersection are combined as the attributes of an edge. Based on graph theory (Feng 2014), the entire road network is mapped into an undirected graph (Figure 2) comprising nodes and edges: $\mathbf{G}(V, E)$ is the adjacency matrix of the graph, where: V is the set of nodes, where $V = \left\{v_1, v_2, \ldots, v_i, \ldots\right\}$; v_i is the ith node; E is the edge set; $E = \left\{l_1, l_2, \ldots, l_i, \ldots\right\}$; l_i is the ith edge.

The steps of the MST–Ncut method are as follows:

- Step 1: the difference function between 2 edges is developed, and the difference value matrix of the undirected graph $\mathbf{G}$ of the entire road network is obtained;

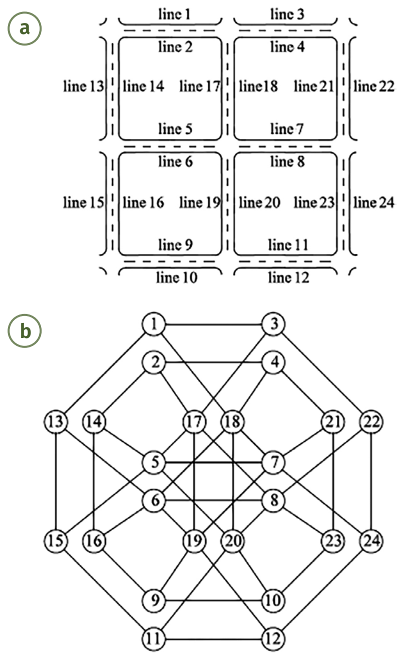

Figure 2. Schematic of network mapping: (a) – schematic of a 2-way road network; (b) – mapping graph of a 2-way road network

- Step 2: the MST algorithm is used to simplify the road network after initial clustering for network nodes. Consequently, several rough sub-trees (sub-regions) are obtained (Chen 2013; Knoop *et al.* 2015; Peng *et al.* 2013; Wu, Leahy 1993);
- Step 3: after the partition of the MST algorithm, large-scale sub-trees that are inconsistent with partition control effect are further divided using the Ncut algorithm, and the value NS_k of each partition step is calculated;
- Step 4: the partitioned sub-regions are reverse merged, and the discriminant value MNS_k of each reverse step is calculated;
- Step 5: the values of NS_k and MNS_k are compared, and the minimum value is the optimal partition scheme.

3.2. Partition method

3.2.1. Initial partition

Initial partition based on MST is the first step of the MST–Ncut method. It focuses on the various factors that affect MFD shape then divides the network. The purpose of this step is to classify large sub-trees representing the overall characteristics of the road network. Because the other scattered sections are smaller than the core sub-trees and different from the surrounding network, these small sub-trees can be used as an independent sub-region in partitioning control and do not need to be partitioned again. Small sub-trees that are not related to the eigenvalues of the large sub-trees are regarded as the guidance and acceptance region in the actual traffic control of the regional road network; they are not constrained by the saturation of the road network.

For $\forall l_i$ and $l_j \in C_m^n$, the difference between l_i and l_j is $dif\left(l_i, l_j\right) = s\left(l_i, l_j\right) \cdot \sqrt{\left(d_i - d_j\right)^2}$. The average difference degree of initial partition clustering C_m^n of MST is:

$$dif\left(C_m^n\right) = \frac{1}{n^2} \cdot \sum_{i=1}^{n} \sum_{j=1}^{n} dif\left(l_i, l_j\right). \tag{6}$$

The difference coefficient value $s(l_i, l_j)$ is:

$$s\left(l_i, l_j\right) = \sum_{p=1}^{o} \lambda_p \cdot S_p\left(l_i, l_j\right). \tag{7}$$

5 factors (signal/no-signal control, split, emergency, signal cycle and road gradation) of the edges are merged, and the difference coefficient value $s(l_i, l_j)$ of l_i and l_j is described as:

$$s\left(l_i, l_j\right) = \left[\lambda_1\ \lambda_2\ \lambda_3\ \lambda_4\ \lambda_5\right] \cdot \begin{bmatrix} \left(sc\left(l_i\right) - sc\left(l_j\right)\right)^2 \\ \left(ac\left(l_i\right) - ac\left(l_j\right)\right)^2 \\ 1 - \exp\left(-10 \cdot \sqrt{\left(gr\left(l_i\right) - gr\left(l_j\right)\right)^2}\right) \\ 1 - \exp\left(-0.2 \cdot \sqrt{\left(sp\left(l_i\right) - sp\left(l_j\right)\right)^2}\right) \\ 1 - \exp\left(-\sqrt{\left(lm\left(l_i\right) - lm\left(l_j\right)\right)^2}\right) \end{bmatrix}, \tag{8}$$

where: $\left(sc\left(l_i\right)-sc\left(l_j\right)\right)^2$ is the difference between signal and no-signal controls (if the 2 edges have the same control measures, then the value is 0; otherwise, it is 1); $\left(ac\left(l_i\right)-ac\left(l_j\right)\right)^2$ is the difference in emergency occurrence (if the 2 edges both are in the emergency state or not, then the weight is 0; otherwise, it is 1); $1-\exp\left(-10\cdot\sqrt{\left(gr\left(l_i\right)-gr\left(l_j\right)\right)^2}\right)$ is the difference in the split because the split is less than 1, and the difference between the split occupied by 2 segments (directions) is usually small (far less than 1); however, this influence factor seriously affects the shape of MFD; thus, the difference is multiplied by 10, and the function exp(–*x*) is used to dramatically change the difference from 0 to 1 – exp(–10) (approximately 1); $1-\exp\left(-0.2\cdot\sqrt{\left(sp\left(l_i\right)-sp\left(l_j\right)\right)^2}\right)$ is the difference in the signal cycle, and this factor is a minor one; it needs 10 s to see obvious changes; therefore, the cycle difference is multiplied by 1/5, and $1-\exp\left(-\sqrt{\left(lm\left(l_i\right)-lm\left(l_j\right)\right)^2}\right)$ is the difference of road gradation; λ represents the degree to which various factors affect MFD; according to the simulation results, the coefficients of the 3 main factors are $\lambda_1 = \lambda_2 = \lambda_3 = 1$, and the coefficients of the 2 secondary factors are $\lambda_4 = \lambda_5 = 0.2$.

The initial process of MST removes the edge with the maximum difference value. If the average difference between 2 sub-trees is smaller than the original one, then the partition operation is executed; otherwise, the 2nd maximum edge is removed, and this operation continues until all edges are traversed. The sub-trees are continuously divided until no new tree is generated. The rule combines the difference of the graph (compactness) and merges the methods for the factors (influencing MFD) of distance and density. Consequently, the local and overall characteristics are considered. Assuming that original tree *T* has *n* edges referred to as $l_1, l_2, ..., l_n$ and the 2 ends of l_i are connected with the branch nodes of *T*, the specific algorithm steps are as follows:

- Step 1: set the MST *T*;
- Step 2: the edges l_i of *T* are ranked in descending order based on the difference value $\frac{1}{n}\cdot\sum_{j=1}^{n} dif\left(l_i, l_j\right)$, $l_j \in C_m^n$;
- Step 3: delete the edge with the maximum difference value in *T* and obtain T_1 and T_2 sub-trees;
- Step 4: if $dif\left(T_1\right) < dif\left(T\right)$ and $dif\left(T_2\right) < dif\left(T\right)$ (refer to Equation (6)), then proceed to Step 5; otherwise, go to Step 6;
- Step 5: split T_1 and T_2 and go to Step 8;
- Step 6: if all edges have been traversed, then proceed to Step 7; otherwise, return to Step 3;
- Step 7: determine the number of sub-trees;
- Step 8: end.

3.2.2. Depth partition

On the basis of MST initial partition, the Ncut algorithm (Shi, Malik 2000; Ji, Geroliminis 2012) is further used to divide sub-trees with a scale inconsistent with the control demand. Ncut is an algorithm based on graph theory and is derived from the field of image segmentation.

Cluster sub-trees C_m^n, which need further partition after MST partition, can be partitioned into 2 parts, namely, *A* and *B*. If $i \in A$ and $j \in B$, then the similarity function between links *i* and *j* is:

$$w\left(i, j\right) = \begin{cases} \exp\left(-\left(d_i - d_j\right)\right)^2, & \text{the spatial distance between 2 links is the shortest path;} \\ 0, & \text{the spatial distance between 2 links is not the shortest path.} \end{cases} \tag{9}$$

The total similarity between *A* and *B* can be expressed as:

$$cut\left(A, B\right) = \sum_{i\in A,\, j\in B} w\left(i, j\right). \tag{10}$$

The discriminant value of the Ncut algorithm is defined as Equations (11) and (12):

$$Ncut\left(A, B\right) = \frac{cut\left(A, B\right)}{cut\left(A, C_m^n\right)} + \frac{cut\left(A, B\right)}{cut\left(B, C_m^n\right)}; \tag{11}$$

$$Nassoc\left(A, B\right) = \frac{cut\left(A, A\right)}{cut\left(A, C_m^n\right)} + \frac{cut\left(B, B\right)}{cut\left(B, C_m^n\right)}. \tag{12}$$

Equations (11) and (12) are converted to:

$$Ncut\left(A, B\right) = 2 - Nassoc\left(A, B\right). \tag{13}$$

In Equation (13), the minimum value of *Ncut*(*A*, *B*) is consistent with the maximum value of *Nassoc*(*A*, *B*). The partition model is expressed as:

$$\begin{aligned} &\min Ncut\left(A, B\right) \\ &\text{subject to } A, B \subset C_m^n. \end{aligned} \tag{14}$$

The solution of Equation (14) is an NP-hard problem, and the vast calculation amount involved in a large network is time consuming. However, an approximate solution can provide approximate results by solving the equivalent eigenvalue. The solving process involves obtaining the minimum characteristic values and subsequently selecting the eigenvector corresponding to the 2nd smallest eigenvalue to divide the entire undirected graph. Then, optimal judgment is implemented according to the criteria for each partition, and the discriminant value is recorded.

3.2.3. Reverse merger process

Depth partition divides a sub-region into 2 parts each time. Nevertheless, separating 2 parts that should have remained in a region is inevitable. Figure 3 shows a road network with 3 different density areas, which is divided into 4 sub-regions by the Ncut method. The 2 sub-regions on the right have the same average density, and they

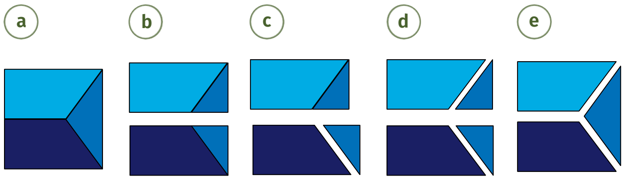

Figure 3. Deep partition and reverse merge:
a – initial state; b – 1st partition; c – 2nd partition; d – 3rd partition; e – 4rth partition (reverse merger)

should have been merged into a sub-region. Thus in this situation, a reverse merger process is essential to merge the areas with the same density and further enhance the rationality of the partition results. The reverse merger steps are as follows:

- Step 1: calculate the average density difference of all 2 adjacent sub-regions with Equation (11);
- Step 2: search for the smallest density difference of 2 sub-regions to merge and calculate the value of NS_k for the entire road network;
- Step 3: repeat Steps 1 and 2 until all sub-regions are merged into a sub-region.

4. Case analysis

4.1. Road network parameter calibration

Figure 4a shows a city centre network with 812 links and 367 intersections. In the network, the number of unsignalised intersections is 120 and that of signalised intersections is 247 (80 and 167 large and small signalised intersections, respectively). Early peak traffic data on the road network on 12 January 2016 (Tuesday) were obtained, and the density of each link is shown in Figure 4b.

4.2. Results of sub-region partitioning

Two experiments on Ncut and MST–Ncut are conducted on the network with detection data.

4.2.1. Ncut partition

The Ncut method is divided into 2 steps: partitioning and reverse merging. According to the scale of the optimal sub-regional square (5…10 km^2) proposed by Geroliminis & Daganzo (2007), the number of sub-regions is set to 8 (may be another number) in the division stage.

Depth partition. The number of the sub-regions is set to 8. The Ncut partition includes 7 steps; for each step, the road network has more sub-regions, and the partition discriminant value NS_k is calculated. The results of Ncut partition are shown in Figure 5, and each step is marked in different colours.

Reverse merger. The reverse merger process merges the 8 sub-regions one by one. For each step, the number of sub-regions is reduced by one until 2 sub-regions remain, and the reverse merger discriminant value MNS_k is calculated. The reverse merger comprises 5 steps, and the merge results of each step are shown in Figure 6.

The partition discriminant value NS_k and the reverse merger discriminant value MNS_k are shown in Table 2. The reverse merger discriminant value MNS_2 = 0.510 is the smallest when the number of sub-regions is 2 according to Equation (14). Figure 6f shows the optimal partition results of the Ncut method.

4.2.2. MST–Ncut method

Initial partition. According to the traffic characteristic parameters of the road network, the MST algorithm and Equation (6) are used for initial division. The largest sub-tree (Figure 7, described by the black line), which is a collection of most tightly connected links, represents the core of the entire road network. The other scattered small sub-trees daubed in yellow may adopt partition control for their respective regions. In the largest sub-tree, the MST algorithm effectively aggregates the factors that affect MFD, and the density difference between the largest sub-tree and the other sub-trees is large.

Depth partition. The scale of the largest sub-tree is extremely large, and further depth partition is implemented using the Ncut method. The division comprises 7 steps. The road network has more sub-regions at each step, and the partition discriminant value NS_k is calculated. The results of the depth partition are shown in Figure 8.

Reverse merger. The reverse merger process merges the 8 sub-regions one by one. For each step, the number of sub-regions is reduced by one until 2 sub-regions remain. The reverse merger discriminant value MNS_k is calculated, and the results of each step are shown in Figure 9.

The depth partition discriminant value NS_k and the reverse merger discriminant value MNS_k are shown in Table 3. The reverse merger discriminant value MNS_5 = 0.365 is the smallest when the number of sub-regions is 5. According to Equation (14), Figure 9c is the optimal partition result of the MST–Ncut method.

4.3. Result analysis

Comparison of Tables 2 and 3 (results are shown in Table 4) shows that the NS_k and MNS_k values of the MST–Ncut method are smaller than those of the Ncut method. The optimal partition result of the MST–Ncut method in Figure 9c is also more reasonable than that of the Ncut method in Figure 6f.

5. Discussion

This article proposed an MST–Ncut partition method that considers the basic properties of road networks when dividing a heterogeneous traffic network into multiple homogeneous sub-regions for perimeter control. Comparison of MST–Ncut and Ncut methods yielded the following results:

- from a macroscopic viewpoint, the Ncut method can divide a road network according to density. Several high-density areas that should be separately controlled are included within a sub-region, and the results are easily

Figure 4. Distribution of intersections and link density

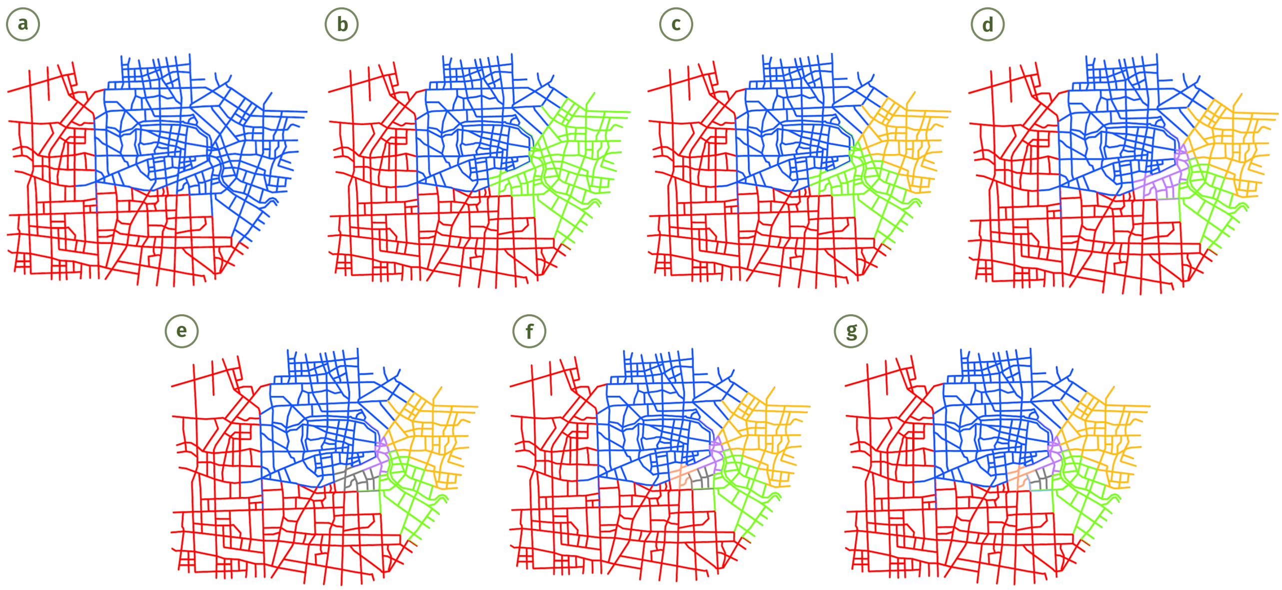

Figure 5. Partition result of Ncut:
a – 1st partition; b – 2nd partition; c – 3rd partition; d – 4th partition; e – 5th partition; f – 6th partition; g – 7th partition

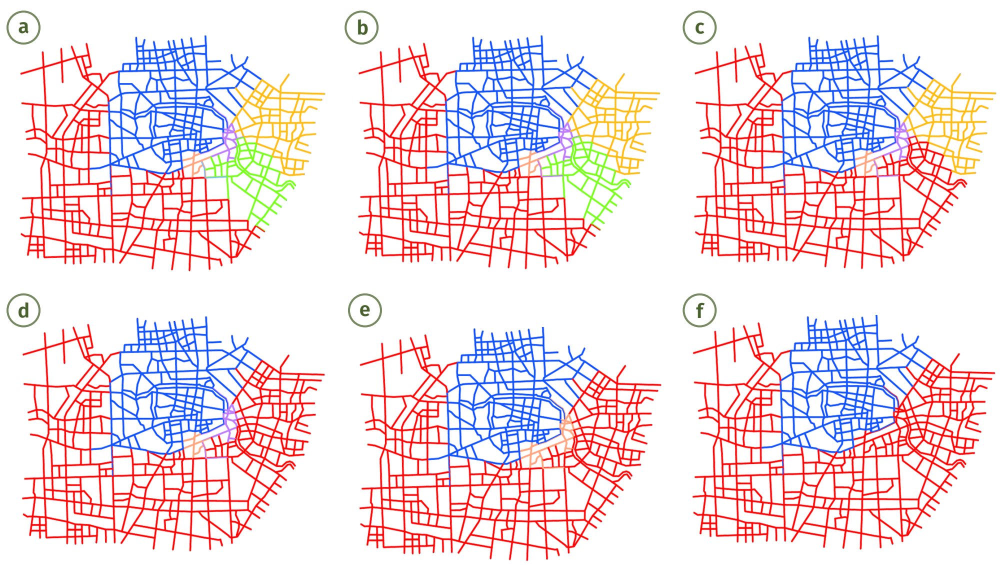

Figure 6. Result of reverse merger:
a – 1st merger; b – 2nd merger; c – 3rd merger; d – 4th merger; e – 5th merger; f – 6th merger

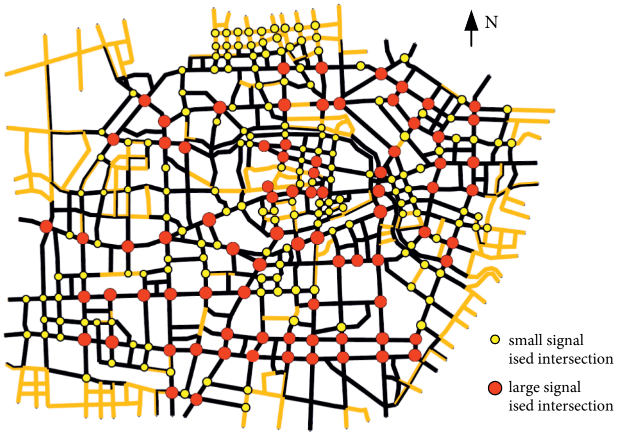

Figure 7. Initial partition based on MST (the black road network is the largest sub-tree, and the rest of the scattered sub-trees are marked as yellow)

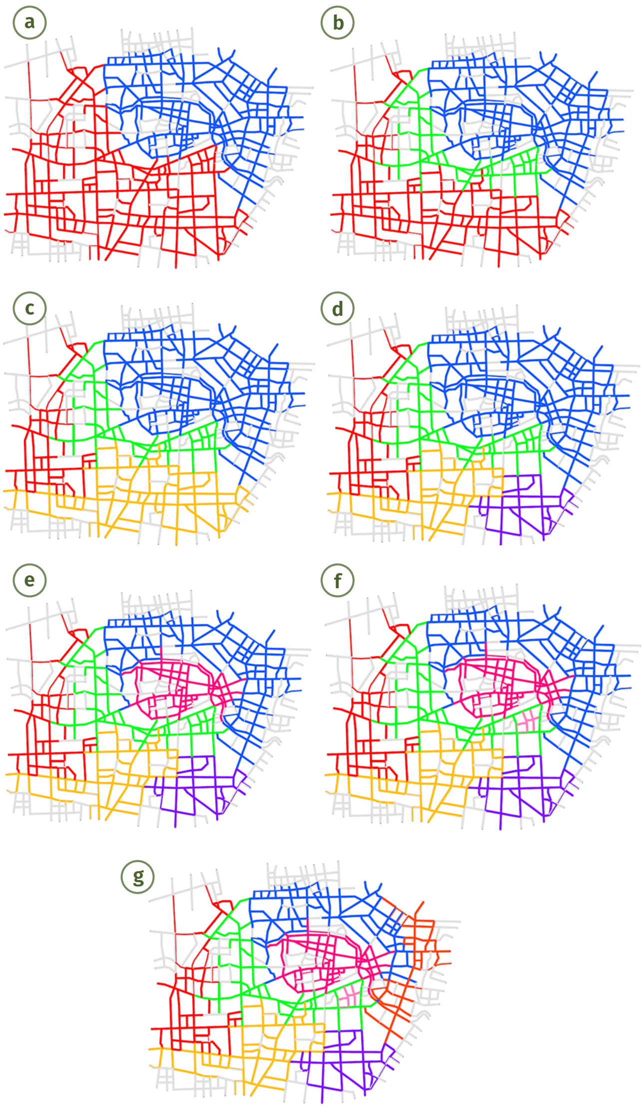

Figure 8. Depth partition results:
a – 1st partition; b – 2nd partition; c – 3rd partition; d – 4th partition; e – 5th partition; f – 6th partition; g – 7th partition (different colours represent different sub-regions; silver sections are removed in the sub-tree because they are outside the scope of the partition)

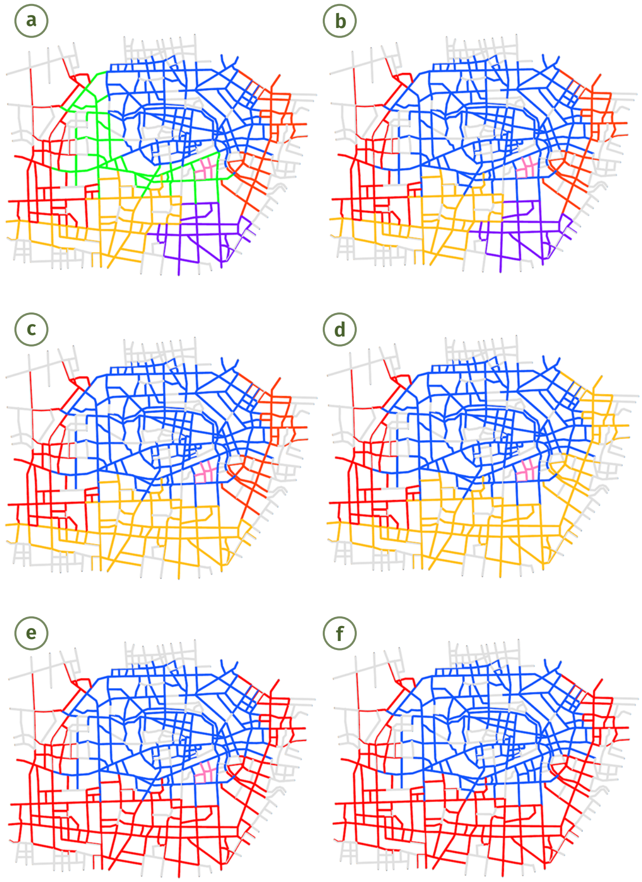

Figure 9. Result of reverse merger:
a – 1st merger; b – 2nd merger; c – 3rd merger; d – 4th merger; e – 5th merger; f – 6th merger

Table 2. Discrimination value of each stage

Number of sub-regions	1	2	3	4	5	6	7	8
NS_k	0.000	0.951	1.000	1.020	1.056	0.992	1.110	1.114
MNS_k	1.000	0.510	0.779	0.926	0.986	1.053	1.030	–

Table 3. Discriminant values of each stage

Number of sub-regions	1	2	3	4	5	6	7	8
NS_k	0.000	0.790	0.609	0.564	0.602	0.579	0.550	0.402
MNS_k	1.000	0.413	0.377	0.428	0.365	0.461	0.491	–

Table 4. Comparison results of 2 partition methods

Method	Maximum value	Minimum value	Standard deviation	Mean
Ncut	1.113	0.510	0.155	0.964
MST–Ncut	0.790	0.365	0.116	0.510

gathered in one sub-region along with the subsequent partition. Thus, the large sub-regions cannot be effectively divided. The case presented in this study showed that the Ncut method cannot clearly distinguish the fragmented density distribution of the road network and is not a good clustering approach. Therefore, although the Ncut method is often used in macro partition, it is unsuitable for networks with a mix of high- and low-density links;

- the initial partition based on MST in the MST–Ncut method has good treatment of the road network and can remove several separate small areas that disperse the overall density distribution and MFD influencing factors. In a sense, this partition removes 'impurities', and the rest of the core sub-trees have a high similarity in the link attribute, thus paving the way for subsequent partitioning based on density. The depth partition results based on the Ncut algorithm in the MST–Ncut method are more detailed than those of the 1st partition in the Ncut method based on regional discrimination. For example, a large area that is not previously involved may be divided, and more details of the network density distribution can be extracted. Meanwhile, the distribution of sub-regions, road network density, and MFD influencing factors is roughly the same, and the discriminant values NS_k and MNS_k are considerably better (lower) than those of the Ncut method. The optimal discriminant value even reaches 0.365, and the stability of the division is high.

Conclusions

A sub-region partition method based on MFD influencing factors is proposed. The following conclusions are obtained. Firstly, the factors that influence the MFD curve were determined through simulation experiments. The 3 main influencing factors are signal/no-signal control, split and emergency event. The 2 secondary factors are signal cycle and number of link lanes. The analysis results provided a database for the development of the sub-region partition method based on MFD. Secondly, the MST–Ncut method considers the characteristic parameters that influence MFD, and the method mainly involves initial partitioning based on MST and depth partitioning based on the Ncut algorithm. Lastly, a large-scale road network was tested, and the results showed that the accuracy of the proposed sub-region partition method is better than that of the Ncut method. The MST–Ncut method considers not only traffic density but also other related factors, making it suitable for road networks with mixed traffic density distribution.

We fully considered the factors that affect the MFD curve in traffic practice and explored the combination of partition methods. Nevertheless, several problems remain to be addressed. Firstly, in addition to the influencing factors of road network MFD described in this work, buses, bikes and pedestrians in different urban regions also affect MFD. Moreover, several micro parameters of the road network design exert an effect, and effective analysis of the effects of these factors on MFD distribution remains to be further studied. Secondly, the influence degree of the reference coefficient value was determined based on a single influencing factor. Hence, reference coefficient and quantitative results under the influence of comprehensive factors are still to be determined. Lastly, based on management requirements, this study only provides the MFD sub-region merging process based on the conservation of traffic flow, a fast acquisition method for each parameter in practical applications is yet to be established.

Author contributions

Heng Ding and *Hanyu Yuan* conceived the study and were responsible for the design and development of the data analysis.

Xiaoyan Zheng, *Hanyue Ma*, *Haijian Bai* were responsible for data collection and analysis.

Hanyue Ma was responsible for data interpretation.

Hanyu Yuan wrote the 1st draft of the article.

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References

Aboudolas, K.; Geroliminis, N. 2013. Perimeter and boundary flow control in multi-reservoir heterogeneous networks, *Transportation Research Part B: Methodological* 55: 265–281. https://doi.org/10.1016/j.trb.2013.07.003

Ampountolas, K.; Zheng, N.; Geroliminis, N. 2017. Macroscopic modelling and robust control of bi-modal multi-region urban road networks, *Transportation Research Part B: Methodological* 104: 616–637. https://doi.org/10.1016/j.trb.2017.05.007

Chen, X. 2013. *Research on Clustering Algorithm Based on Minimum Spanning Tree*. MSc Thesis. Chongqing University, China. 57 p. (in Chinese).

Daganzo, C. F.; Geroliminis, N. 2008. An analytical approximation for the macroscopic fundamental diagram of urban traffic, *Transportation Research Part B: Methodological* 42(9): 771–781. https://doi.org/10.1016/j.trb.2008.06.008

Ding, H.; Guo F.; Zheng X.; Zhang W. 2017. Traffic guidance–perimeter control coupled method for the congestion in a macro network, *Transportation Research Part C: Emerging Technologies* 81: 300–316. https://doi.org/10.1016/j.trc.2017.06.010

Ding, H.; Zhang, Y.; Zheng, X.; Yuan, H.; Zhang, W. 2018. Hybrid perimeter control for two-region urban cities with different states, *IEEE Transactions on Control Systems Technology* 26(6): 2049–2062. https://doi.org/10.1109/TCST.2017.2746061

Feng, A. R. 2014. *Research on Image Segmentation Based on Normalized Cuts*. MSC Thesis. Chongqing University, China. (in Chinese).

Gayah, V. V.; Daganzo, C. F. 2011. Clockwise hysteresis loops in the macroscopic fundamental diagram: an effect of network instability, *Transportation Research Part B: Methodological* 45(4): 643–655. https://doi.org/10.1016/j.trb.2010.11.006

Godfrey, J. W. 1969. The mechanism of a road network, *Traffic Engineering and Control* 11: 323–327.

Gonzales, E. J.; Chavis, C.; Li, Y.; Daganzo, C. F. 2009. *Multimodal Transport Modeling for Nairobi*, *Kenya: Insights and Recommendations with an Evidence-Based Model*. Working Paper UCB-ITS-VWP-2009-5. UC Berkeley Center for Future Urban Transport, Berkeley, CA, US. 41 p.

Geroliminis, N.; Daganzo, C. F. 2007. Macroscopic modeling of traffic in cities, in *TRB 86th Annual Meeting Compendium of Papers CD-ROM*, 21–25 January 2007, Washington DC, US. 21 p.

Geroliminis, N.; Haddad, J.; Ramezani, M. 2013. Optimal perimeter control for two urban regions with macroscopic fundamental diagrams: a model predictive approach, *IEEE Transactions on Intelligent Transportation Systems* 14(1): 348–359. https://doi.org/10.1109/TITS.2012.2216877

Geroliminis N.; Sun J. 2011a. Hysteresis phenomena of a Macroscopic Fundamental Diagram in freeway networks, *Transportation Research Part A: Policy and Practice* 45(9): 966–979. https://doi.org/10.1016/j.tra.2011.04.004

Geroliminis, N.; Sun, J. 2011b. Properties of a well-defined macroscopic fundamental diagram for urban traffic, *Transportation Research Part B: Methodological* 45(3): 605–617. https://doi.org/10.1016/j.trb.2010.11.004

Haddad, J. 2017. Optimal perimeter control synthesis for two urban regions with aggregate boundary queue dynamics, *Transportation Research Part B: Methodological* 96: 1–25. https://doi.org/10.1016/j.trb.2016.10.016

Haddad, J.; Geroliminis, N. 2012. On the stability of traffic perimeter control in two-region urban cities, *Transportation Research Part B: Methodological* 46(9): 1159–1176. https://doi.org/10.1016/j.trb.2012.04.004

Haddad, J.; Mirkin, B. 2017. Coordinated distributed adaptive perimeter control for large-scale urban road networks, *Transportation Research Part C: Emerging Technologies* 77: 495–515. https://doi.org/10.1016/j.trc.2016.12.002

Hajiahmadi, M.; Haddad J.; De Schutter, B.; Geroliminis, N. 2013. Optimal hybrid macroscopic traffic control for urban regions: perimeter and switching signal plans controllers, in *2013 European Control Conference* (*ECC*), 17–19 July 2013, Zurich, Switzerland, 3500–3505. https://doi.org/10.23919/ECC.2013.6669572

Hajiahmadi, M.; Haddad, J.; De Schutter, B.; Geroliminis, N. 2015. Optimal hybrid perimeter and switching plans control for urban traffic networks, *IEEE Transactions on Control Systems Technology* 23(2): 464–478. https://doi.org/10.1109/TCST.2014.2330997

Ji, Y.; Geroliminis, N. 2012. On the spatial partitioning of urban transportation networks, *Transportation Research Part B: Methodological* 46(10): 1639–1656. https://doi.org/10.1016/j.trb.2012.08.005

Ji, Y.; Luo, J.; Geroliminis, N. 2013. Dynamic partitioning of urban transportation networks, in *13th Swiss Transport Research Conference: STRC 2013*, 24–26 April 2013, Monte Verita, Ascona, Switzerland. 9 p. Available from Internet: https://www.strc.ch/2013/Ji_EtAl.pdf

Ji, Y.; Luo, J.; Geroliminis, N. 2014. Empirical observations of congestion propagation and dynamic partitioning with probe data for large-scale systems, *Transportation Research Record: Journal of the Transportation Research Board* 2422: 1–11. https://doi.org/10.3141/2422-01

Keyvan-Ekbatani, M.; Yildirimoglu, M.; Geroliminis, N.; Papageorgiou, M. 2015. Multiple concentric gating traffic control in large-scale urban networks, *IEEE Transactions on Intelligent Transportation Systems* 16(4): 2141–2154. https://doi.org/10.1109/TITS.2015.2399303

Knoop, V. L.; Van Lint, H.; Hoogendoorn, S. P. 2015. Traffic dynamics: its impact on the macroscopic fundamental diagram, *Physica A: Statistical Mechanics and its Applications* 438: 236–250. https://doi.org/10.1016/j.physa.2015.06.016

Kouvelas, A.; Saeedmanesh, M.; Geroliminis, N. 2017. Enhancing model-based feedback perimeter control with data-driven online adaptive optimization, *Transportation Research Part B: Methodological* 96: 26–45. https://doi.org/10.1016/j.trb.2016.10.011

Li, G.; Zhao, Y. 2012. Urban traffic signal control network partitioning based on macro-traffic flow theory, in *The 7th China Intelligent Transportation Conference Excellent Proceedings*, 26–28 September 2012, Beijing, China, 67–72. (in Chinese).

Ma, Y.-Y.; Yang, X.-G.; Zeng, Y. 2010. Urban traffic signal control network parting using spectral method, *Systems Engineering – Theory* & *Practice* 30(12): 2290–2296. (in Chinese).

Mazloumian, A.; Geroliminis, N.; Helbing D. 2010. The spatial variability of vehicle densities as determinant of urban network capacity, *Philosophical Transactions of the Royal Society A: Mathematical*, *Physical and Engineering Sciences* 368(1928): 4627–4647. https://doi.org/10.1098/rsta.2010.0099

Peng, B.; Zhang, L.; Zhang, D. 2013. A survey of graph theoretical approaches to image segmentation, *Pattern Recognition* 46(3): 1020–1038. https://doi.org/10.1016/j.patcog.2012.09.015

Peng, J. X. 2013. *Macroscopic Characteristics of Dense Road Networks*. MSC Thesis. University of Hong Kong, Pok Fu Lam, Hong Kong. 107 p. Available from Internet: https://hub.hku.hk/handle/10722/195994

Ramezani, M.; Haddad, J.; Geroliminis, N. 2015. Dynamics of heterogeneity in urban networks: aggregated traffic modeling and hierarchical control, *Transportation Research Part B: Methodological* 74: 1–19. https://doi.org/10.1016/j.trb.2014.12.010

Saeedmanesh, M.; Geroliminis, N. 2016.Clustering of heterogeneous networks with directional flows based on "snake" similarities, *Transportation Research Part B: Methodological* 91: 250–269. https://doi.org/10.1016/j.trb.2016.05.008

Saeedmanesh, M.; Geroliminis, N. 2017. Dynamic clustering and propagation of congestion in heterogeneously congested urban traffic networks, *Transportation Research Part B: Methodological* 105: 193–211. https://doi.org/10.1016/j.trb.2017.08.021

Shi, J.; Malik, J. 2000. Normalized cuts and image segmentation, *IEEE Transactions on Pattern Analysis and Machine Intelligence* 22(8): 888–905. https://doi.org/10.1109/34.868688

Wang, B.; Jia, K. 2010. Synthetic algorithm of vehicle flow detection based on video analysis, *Computer and Communications* (1): 20–25. (in Chinese).

Wu, Z.; Leahy, R. 1993. An optimal graph theoretic approach to data clustering: theory and its application to image segmentation, *IEEE Transactions on Pattern Analysis and Machine Intelligence* 15(11): 1101–1113. https://doi.org/10.1109/34.244673

Xu, F.-F.; He Z.-C.; Sha Z.-R. 2013. Impacts of traffic management measures on urban network microscopic fundamental diagram, *Journal of Transportation Systems Engineering and Information Technology* (2): 185–190. (in Chinese).

Zhang, L.; Garoni, T. M.; De Gier, J. 2013. A comparative study of macroscopic fundamental diagrams of arterial road networks governed by adaptive traffic signal systems, *Transportation Research Part B: Methodological* 49: 1–23. https://doi.org/10.1016/j.trb.2012.12.002

Zhao, T.-T.; Li, Z.-H.; Huang, B.-Y.; Mu, B.-P.; Zhang, Y. 2014. Exploring the influence of traveller information on macroscopic fundamental diagrams, *IET Intelligent Transport Systems* 8(1): 58–67. https://doi.org/10.1049/iet-its.2011.0234

Zhou, Z.; Lin, S.; Xi, Y.; Li, D.; Zhang, J. 2016. A hierarchical urban network control with integration of demand balance and traffic signal coordination, *IFAC-PapersOnLine* 49(3): 31–36. https://doi.org/10.1016/j.ifacol.2016.07.006