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PROPOSING A MATHEMATICAL MODEL OF BALANCING THE INVENTORY OF MULTI-ZONE BICYCLE SHARING SYSTEMS WITH MOBILE STATIONS AND APPLYING MAINTENANCE CONSTRAINTS

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Abstract. Regarding the necessity of developing transportation infrastructures and its increasing importance in urban issues, nowadays in different cities, bicycles are considered the main and sustainable vehicle along with walking and drawing more attention day by day. The case considers highly paramount since preservation of the environment, natural resources, and energy is one of the significant pillars of sustainable development, and urban transportation intensively influences it. Thus, Bicycle Sharing System (BSS) is recognized as an innovative urban transportation option that meets the citizens' demand for commuting during the day. The BSS can highly affect the level of citizens' health, and it can be counted as one of the leading health programs whether it's added to the public transportation system, it can help the culture to be created to use bicycles instead of cars in most of the internal trips, and also it can be so influential in decreasing the air pollution and in the following its harmful effects on health issues. The mathematical model of rebalancing multi-zone BSS with mobile stations and applying maintenance constraints in a static status is considered in this research. The objective function of this research is a single-objective one, which is modeled with the aims of reducing the costs of traveled distances by the tracks within and outside the zones, reducing the costs of intact and defective bicycles transportation within and outside the zones, and eventually, reducing the costs of surplus bicycles depot at the stations. This issue is a multi-product one that includes different types of bicycles and balancing tracks. Computational results confirm the model's efficiency. Also, sensitivity analysis has been done to prove that the model is affected by both parameters of storage costs of surplus bicycles and transportation costs within and outside the zones.

Keywords: mathematical modelling, balancing the inventory, bicycle sharing system (BSS), mobile station, multi-zone, routing, maintenance.

Notations

BSRP – bike-sharing re-positioning problem;
BSS – bicycle sharing system;
GAMS – general algebraic modelling system;
NE 1 – numerical example 1;
NE 2 – numerical example 2.

Introduction

Regarding the necessity of developing transportation infrastructures and its increasing importance in urban issues, nowadays in different cities of the world, bicycles are considered the main and sustainable vehicle along with the walking and draw more attention day by day. It counts even more critical to protect the environment, natural resources, and energy as the paramount pillars of sustainable development affected by urban transportation. Public bicycle systems, also known as the BSS, are introduced as part of the urban public transportation system, expanding public transportation's availability to the final destinations. To integrate this system with the public transportation system and to provide accessible or affordable bicycles for intercity travel leads to build the culture to use the cars only for long trips. Thus, this action causes to decrease traffic, noise pollution, and air pollution.

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Regarding BSSs, some researchers have made efforts to introduce the issue as an optimization challenge (Lin, Yang 2011). For instance, Di Gaspero *et al.* (2013) introduced BSS as a bicycle distribution issue among citizens through a simple and affordable process. The idea is to set up different stations in the city, which a user can simply take a bicycle from a specific location and, after meeting the requirement, return it to any defined station (Dell'Amico *et al.* 2014). BSSs have been introduced to urban transportation as an innovative choice that covers citizens' transportation requirements and demands. These systems effectively gain sustainable transport, reduce air and noise pollution, decrease travel costs, decline traffic volume and accident risk, and improve citizens' health levels.

This research aims to facilitate the processes related to the vital issue of establishing a BSS. The importance of this issue in the current conditions of the world is not hidden from anyone, and any effort to develop these systems is commendable. Therefore, in recent years, the issue of managing BSS has been the subject of research in most universities in different parts of the world. In this study, researchers tried to develop realistic conditions in practice in the form of a mathematical model in a way that eliminates the concerns of decision-makers in the development and optimization of current systems. In the following sections of the paper, the proposed model's hypotheses, limitations, and features will be presented. In general, the simultaneous consideration of several factors, including the simultaneous balancing of several zones and maintenance systems and communication between zones in order to balance the inventory of several types of bicycles in the form of the heterogeneous transport fleet, are among the main differences of this research. This issue is considered a multi-zone and multi-product problem that includes different bicycles, and stations' demand for each bicycle is assumed definite. The instruction of loading/unloading bicycles is determined by a fleet of different types of heterogeneous trucks and unique features. Due to the multi-zone space of the defined problem, in the model, a zone is selected as a headquarters to simplify decision-making and be a center of communication between designated zones. In addition to optimizing the routing of the fleet, the proposed model aims to balance the maintenance process, reduce the trucks' transportation costs within and outside the zones, and finally, reduce the surplus bicycle holding costs in the stations.

In the following and the Section 1 of the paper, relevant research and research innovation will be considered. In the Section 2, the developed mathematical model for rebalancing BSS will be proposed. The Section 3 presents the numerical examples and results. Sensitivity analysis is examined in the Section 4, and in the last section, the finding will be stated.

1. Literature review

Theoretical and practical researches in the field of BSS optimization have explicitly been started since 2009. Papers related to this field are very diverse. For example, research on estimating the demand for bicycles at stations, persuading citizens to use the BSSs, locating bicycle stations, designing a revenue-generating system from BSSs, etc. Since the field of this paper is limited to the static rebalancing of BSSs, in this section, only research related to the mentioned features is pointed. For instance, Lin and Yang (2011) determined the number and location of stations, the network structure of bicycles' routes, and the users' travel routes by proposing a mathematical model. Chelma *et al.* (2013) presented a single-period mathematical model for the BSS rebalancing problem in the static state. Each station can be visited several times in the model, and a truck with a limited capacity is responsible for implementing the distribution strategy. Rainer-Harbach *et al.* (2013) designed a BSS whose 1st part is for vehicle routing and the 2nd part of the loading/unloading instruction. This model is a single-goal, single-phase, single-period, and single-product model whose objective function aims to minimize rebalancing deviations, loading time, and process time. Di Gaspero *et al.* (2013) proposed a combined meta-heuristic algorithm to solve the rebalancing problem between stations. This model was a single-objective model, and the objective function aims to minimize the process time deviations. Dell'Amico *et al.* (2014) designed a model to rebalance BSS based on the traveling salesman model. This balance is achieved using a fleet of homogeneous trucks with a specific capacity and a static environment. Ho and Szeto (2014) developed a one-period mathematical model in which the BSS is statically rebalanced. The stations are selected to be rebalanced, the sequence between the stations is signified, and the loading/unloading plan is applied according to the constraints. This single-objective problem aims to minimize the total fines, which occur for all the stations. Bortner *et al.* (2015) developed a single-objective single-period model to lessen the driving distance during BSS' rebalancing process. Erdoğan *et al.* (2015) proposed a single-period model to lessen the transportation's final cost by applying a precise algorithm to solve the BSS rebalancing problem. This model has specific features, such as a free number of stations that are visited by the fleets. Brinkmann *et al.* (2016) presented a multi-period model to rebalance the BSS. This model predicts the future demand of the stations, which are provided by tactical information. Di Gaspero *et al.* (2016) designed an optimal route for the vehicles along with the loading/unloading instructions among the stations to increase the conformity level with the future demands. They then applied two programming models to navigate according to the classic vehicles routing problem, and also, a step model that presents a programming perspective has been used. Liu *et al.* (2016) considered BSS databases of New York (US) and predicted the bicycles' receiving and delivery patterns. After that, using a mathematical model, stations have been rebalanced. The rebalancing model is a nonlinear-mixed-integer-programming model. The model is a single-objective one that aims to decrease the total amount of the traveled distance. The model's transportation fleet includes several types of trucks. Li and Yang (2016) proposed a single-objective single-period mathematical model to reduce the costs of static bicycle station rebalancing problems. Szeto *et al.* (2016) also presented a mathematical model to solve the shared bicycle station rebalancing problem, which is static and contains a truck and bicycle type and aims to minimize unsatisfied demands and operation time. Arabzad *et al.* (2016) proposed a mathematical model to rebalance the heterogeneous trucks station applying the traveling salesman modeling approach. This research belongs to the static models' group that does the rebalancing operation based on stations' demand in a specific period. Dell'Amico *et al.* (2018) designed a model to rebalance the BSS based on random demand. This study considered stations' demand, randomly and with a probability distribution, and the rebalancing operation has been assumed to be done using a transportation fleet with a specific capacity. Arabzad *et al.* (2018) designed a rebalancing model for multi-depot BSS with 2 periods and a static model. In this research, integer linear programming has been used to develop the routing problem of BSS. This study's objective function is a single-objective one and aims to decrease the costs of using trucks and optimal routing. Cavagnini *et al.* (2018) researched and designed two-stages random model based on allocating and rebalancing a BSS with one storage and multi-capacity station. In the 1st stage, the model allots and simulates bicycles using BSSs' real data of San Francisco (US). Then, in the 2nd stage, the model decides the way of rebalancing. Maggioni *et al.* (2019) proposed a two-stage mathematical model to optimize the BSS under uncertain conditions. So, in the 1st stage, the optimized number of bicycles in each of the mobile stations has been determined and then in the 2nd stage, their transportation has been optimized. The transportation's time and demands are considered random. Then this model has been solved in Bergamo (Italy) company, and the numerical results have been reported. Tang *et al.* (2020) studied a bike re-positioning problem with stochastic demand. The problem was formulated as a two-stage stochastic programming model to optimize the routing and loading/unloading decisions of the re-positioning truck at each station and depot under stochastic demands. The goal of the model is to minimize the expected total sum of the transportation costs, the expected penalty costs at all stations, and the holding cost of the depot. A simulated annealing algorithm was developed to solve the model. Soroushnia and Shirouyehzad (2020) also researched to mathematically model the inventory routing in a bike-sharing distribution network's transportation fleet. The proposed model was an integer nonlinear programming that can rebalance BSS by considering the appropriate inventory policies. This model aimed to find optimal routes and minimize the costs of rebalancing operations in multiple periods. The efficiency of the model has been validated by solving 2 numerical examples by the GAMS software (*https://www.gams.com*). Du *et al.* (2020) formulated an integer linear programming model for rebalancing static BSS problems, and a greedy-genetic heuristic was developed to solve it. They considered multiple depots, heterogeneous trucks, and multiple visits with malfunctioning bikes in free-floating BSS. Divvy BSS was utilized to test a large-scale instance in real life. Lv *et al.* (2020) studied the BSRP frequently encountered in modern BSSs. The mathematical model was 1st given, detailing the considerations of multiple depots available for re-positioning vehicles and the extra objective of inventory cost minimization. An effective clustering strategy was then proposed to put BSS into self-sufficient groups. A destroy-and-repair algorithm was developed to improve the clusters, and an adaptive variable neighborhood search algorithm was designed to conduct intra-cluster and inter-cluster vehicle routing optimization. Ma *et al.* (2021) developed an integer-programming model to consider multiple rebalancing vehicles with time-varying rental costs to alleviate the imbalanced bike distribution while also analyzing the intrinsic properties of such a model. They further proposed a chance constraint programming model, optimizing a bike-sharing network by implementing various genetic algorithms. Jia *et al.* (2021) considered a mixed fleet of electric vehicles and internal combustion vehicles as well as the traffic restrictions to the traditional vehicles in some metropolises. The mixed-integer-programming model was 1st established to minimize the total rebalancing cost of the mixed fleet. Then, a simulated annealing algorithm enhanced with variable neighborhood structures was designed and applied to a set of randomly generated test instances.

To better perceive the difference between some reviewed research and the current research, a summary has been presented in Table 1.

Considering the previous research in the BSS problem, a limited number of studies have simultaneously considered several types of bicycles and heterogeneous transportation systems (different types of rebalancing trucks) during static rebalancing operations. None of the previous researches have done the rebalancing operation by applying the rebalancing inventory levels and using the mobile station to execute the maintenance policy to increase customer satisfaction and optimize this system. Also, in few papers, mathematical modeling is such that it is permissible to allow for shortages or surplus while rebalancing due to demand and inventory of stations. On the other hand, in the case of having multi-zones for BSS in the real world, there is a big challenge in rebalancing the BSS. In addition to the above aspects, this issue has been addressed in this paper novelty.

2. Mathematical model

In this research, the rebalancing problem of the BSS is considered in a static status, and it is solved in a multi-zone mode, applying the maintenance constraints and considering the station as a mobile one. The conceptual model of research is presented in Figure 1 accordingly to increase transparency and perception of the problem. In addition, to optimize the fleet routing of rebalancing and

Table 1. A review of the most important researches in the field of BSS

Specifications Researchers	Transportation fleet		Mobile station		Maintenance attitude		Product		Demand covering		Demand		Balancing level		Objective	
	in-homogeneous	homogenous	not considered	considered	not considered	considered	multi-product	single-product	allow deficiency and surplus	definitive	uncertain	definitive	multi-zone	single zone	multi objective	single-objective
Di Gaspero *et al.* (2013)	✓			✓		✓		✓		✓		✓		✓		✓
Dell'Amico *et al.* (2014)	✓			✓		✓		✓		✓		✓		✓		✓
Bortner *et al.* (2015)		✓		✓		✓		✓		✓		✓		✓		✓
Erdoğan *et al.* (2015)		✓		✓		✓		✓		✓		✓		✓		✓
Di Gaspero *et al.* (2016)	✓			✓		✓		✓		✓		✓		✓		✓
Liu *et al.* (2016)	✓			✓		✓		✓		✓		✓		✓		✓
Li, Yang (2016)		✓		✓		✓	✓		✓			✓		✓		✓
Szeto *et al.* (2016)		✓		✓		✓		✓	✓			✓		✓		✓
Cruz *et al.* (2017)		✓		✓		✓		✓		✓		✓		✓		✓
Arabzad *et al.* (2016)	✓			✓		✓	✓			✓		✓		✓		✓
Dell'Amico *et al.* (2018)		✓		✓		✓		✓		✓	✓			✓		✓
Arabzad *et al.* (2018)	✓			✓		✓	✓			✓		✓		✓		✓
Cavagnini *et al.* (2018)		✓		✓		✓		✓	✓		✓			✓	✓	
Maggioni *et al.* (2019)		✓		✓		✓	✓		✓		✓		✓			✓
Soroushnia, Shirouyehzad (2020)				✓		✓	✓		✓			✓	✓			✓
Current research	✓		✓		✓		✓		✓			✓	✓			✓

its maintenance, this model aims to decline the trucks' transportation costs in and outside the zones and ultimately aims to decrease the holding costs of the surplus bicycles.

The central headquarters do the rebalancing operation based on data such as safety stock, station's inventory, number of the predicted demands, and the number of bicycles that require maintenance service and transmitted by the zones. Indeed, what is signified by the research model includes: (1) choosing the appropriate number and type of the trucks to carry the bicycles, (2) choosing the sequence of visiting zones' stations for the rebalancing and maintenance operation in order to decline the costs and routing optimally, (3) choosing the proper mobile station to collect and send bicycles that need maintenance service to the central workshop. For instance, yellow, orange, green, and blue trucks are selected and sent to the zones for rebalancing operation in this figure. As shown in Figure 1, bicycles are placed in transport trucks separated into tacit and defective ones, and defective bicycles are also separated into b types. According to Figure 1, the model solution space has z zones. Each zone contains $StaQue_z$ stations, b types of bicycles, and a transportation fleet including k heterogeneous trucks. 2 zones have also allocated one as a central headquarters and one as a maintenance center. The central headquarter is responsible for controlling and communicating to rebalance operations and sending the mobile maintenance station. In a way, this center contains a verity of bicycle types to meet the zones' requirements if shortage happens and tucks with different capacities to be utilized in the rebalancing and maintenance operation. Regarding applying maintenance

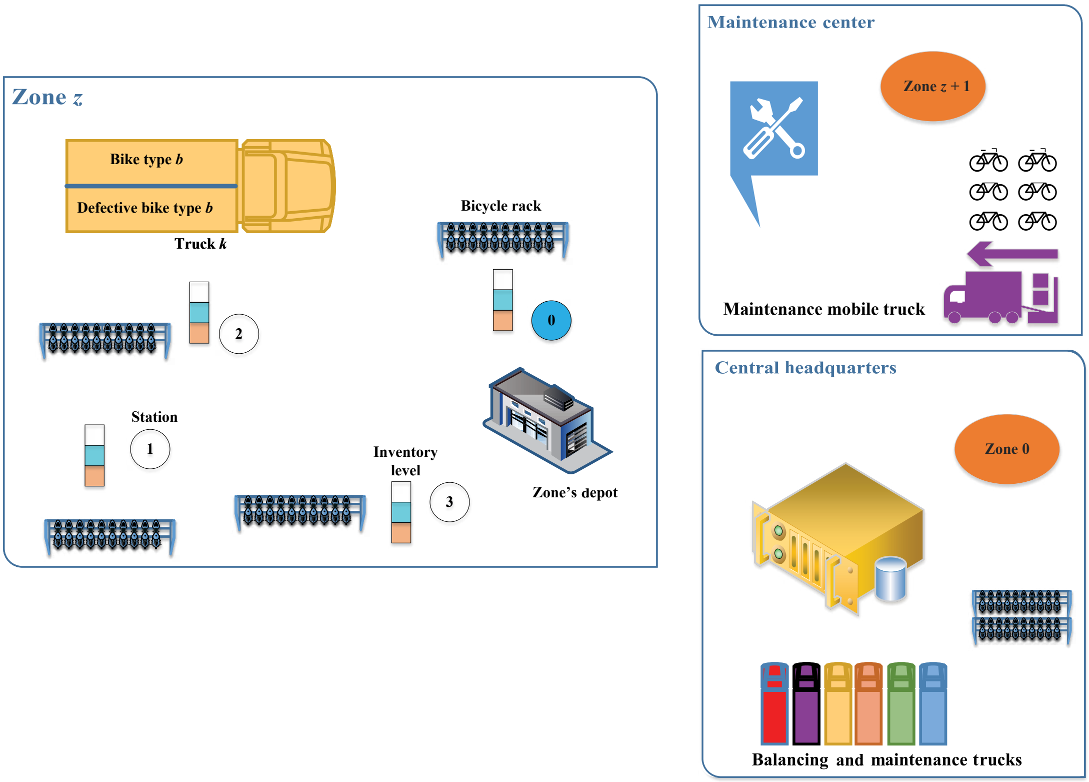

Figure 1. Conceptual model of the research

constraints, it's been required to mention that after the bicycles that need the maintenance service are signified, the headquarters has installed a mobile station to collect these bicycles from the zones' depots to send them to the maintenance center.

The assumptions of the mathematical research model are as below:

- the central warehouse is assigned to supply zones' depots among the defined zones, and a spot is dedicated as parking to rebalancing and maintenance trucks;
- regarding this research's maintenance attitude, there is a spot to do the maintenance service;
- trucks are assumed to be heterogeneous;
- bicycles are considered as 2 various types;
- the location of the stations is known and constant;
- an adequate number of vehicles is available;
- the distance between the stations is used as an estimation of the route passing costs;
- due to the static type of the problem's model thus, during the rebalancing time, the station will have no bicycle exchange between the stations and customers;
- each of the stations must be covered exactly for each type of bicycle;
- stations should be met just once by the truck;
- each station has a specific capacity, current inventory, and safety stock;
- during the rebalancing operation, bicycles that require maintenance due to service requirements or failure occurrence are collected by the rebalancing truck and transported to each zone's central depot;
- a truck is dedicated to carrying bicycles that require maintenance service; this truck takes the bicycles from the central depots and delivers them to the workshop center;
- a specific lot is devoted to different types of tacit and defective bicycles in local rebalancing trucks to be maintained.

The symbols, parameters, and variables of the problem are as follows.

Symbols:

i, j – stations' counters ($i = 0$ shows the main station);
I_1 – sub-set of i contain all station except the origin;
I_2 – sub-set of i contain only the origin;
i' – refers to each station i except the current station i while expanding the equation;
j' – refers to each station j except the current station j while expanding the equation;
o, p – counter of the central depot of all the zones;
o_1 – sub-set of o except the origin and destination ones;
o_2 – sub-set of o except the destination one;
o_3 – sub-set of o except the origin one;
o_4 – sub-set of o contain only the origin and destination ones;
o' – refers to each central depot of zone except the current point o while expanding the equation;
b – counter of different types of bicycles;
k – counter of different types of trucks;
k' – refers to each truck j except the current truck j while expanding the equation;
z – counter of the zones ($z = 0$ shows the central depot; $z + 1$ shows the maintenance depot).

Parameters:

$StaInv_{zib}$ – different types of bicycles' current inventory at the station i of the zones z;
$StaSS_{zib}$ – different types of bicycles' safety stock at the station i of the zones z;
$StaCap_{zib}$ – different types of bicycles' maintenance capacity at the station i of the zones z;
$StaPre_{zib}$ – demand prediction of different types of bicycles at the station i of the zones z;
$StaBiFai_{zib}$, $StaBiFai_{zjb}$ – number of defective bicycles at the station i or j of the zones z;
$StaDis_{zij}$ – distance from route i to j in zone z;
$StaQue_z$ – number of zones' stations;
$ZoneQue$ – number of zones;
$TruDis_k$ – the allowed distance to be driven by truck k;
$ZoneDis_{op}$ – the traveling cost between main stations;
$TCap_{kb}$ – the maximum capacity of the truck to carry different types of the bicycles;
$TruImp_k$ – the constant cost of the truck k employment;
α – the cost of bicycles transportation between the zones;
β – the cost of bicycles transportation within the zones;
γ – the cost of surplus bicycles holding.

Decision variables:

$NewStaInv_{zib}$ – the inventory of the station i in zone z after rebalancing;
FPM_{opkb}, FPM_{pokb} – different types of defective bicycles' number being transported by the truck between zone o and zone p;
FIZ_{zijkb}, FIZ_{zjikb} – number of different types of the tacit bicycles being transported by the truck k between stations i and zone j;
FPZ_{zijkb}, FPZ_{zjikb} – number of different types of defective bicycles being transported by the truck k between stations i and zone j;
FOZ_{opkb}, FOZ_{pokb} – number of different types of the tacit bicycles being transported by the truck k between zone o and zone p;
X_{zijk} – binary variable; taking value one if route $i - j$ is met by the truck k;
$ZoBiFai_{ob}$, $ZoBiFai_{pb}$ – number of defective bicycles of type b in zone o or zone p;
$TruIZ_k$ – binary variable; taking value one if tuck k is employed within zones;
$TruBZ_k$ – binary variable; taking value one if truck k is used between zones;
$StaPri_{zi}$, $StaPri_{zj}$ – auxiliary variable related with the sub-tours and prioritization of the within zone visiting;
$ZonePri_o$, $ZonePri_p$ – auxiliary variable related with the sub-tours and prioritization of the among zone visiting;
Y_{opk} – binary variable; taking value one if route $o - p$ is met by truck k;
W_{opk}, W_{pok} – binary variable; taking value one if route $o - p$ or $p - o$ is met by truck k between the headquarters and zones' depots;
$StaDem_{zib}$ – demand of the station i of the bicycle type b in zone z;
$DemTOT_{zb}$ – total demand of different bicycles type with in zones.

In the following, the mathematical relationships are presented and stated separately.

Equation (1) shows the objective function of the study. The objective function is modeled as a single-objective, and it consists of 6 parts in order to reduce the rebalancing operation costs. The 1st part is related to the driven distance by the trucks within the zones. The 2nd part points to calculate the distance traveled by the trucks to rebalance from the headquarters to the zones, collect defective bicycles, and send them to the maintenance center. The 3rd part considers the cost of employing the trucks within and outside the zones. In other words, using each truck has some constant costs such as driver payment or invisible costs that may depend on the use of the truck. The 4th and 5th parts are related to the tacit and defective bicycles' transportation cost within and outside the zones. The 6th part is about the surplus bicycles' maintenance cost in zones' stations and after the rebalancing operation. Equations (2) and (3) are applied by a truck to visit each zone's stations. In other words, a station might be visited several times by a truck and finally results in cots to be highly increased. Thus applying this constraint, the incident is prevented.

$$\begin{aligned}\min Z = &\sum_z\sum_i\sum_j\sum_k\left(StaDis_{zij}\cdot X_{zijk}\right)+\\ &\sum_o\sum_p\sum_k ZoneDis_{op}\cdot\left(W_{opk}+Y_{opk}\right)+\\ &\sum_k TruImp_k\cdot\left(TruIZ_k+TruBZ_k\right)+\\ &\sum_o\sum_p\sum_k\sum_b\alpha\cdot\left(FPM_{opkb}+FOZ_{opkb}\right)+\\ &\sum_z\sum_i\sum_j\sum_k\beta\cdot\left(FIZ_{zijkb}+FPZ_{zijkb}\right)+\\ &\sum_z\sum_i\sum_b\gamma\cdot NewStaInv_{zib}\end{aligned}\qquad(1)$$

subject to:

$$\sum_i \sum_k X_{zijk} = 1, \ \forall j, z; \quad (2)$$

$$\sum_i \sum_k X_{zjik} = 1, \ \forall j, z. \quad (3)$$

Equation (4) choose the optimal truck based on the rebalancing operation requirement within the zones; since, in this problem, there are different trucks in terms of capacity, employment cost and distances to be traveled and also it controls the number of routes that have to be visited and shouldn't exceed the number of stations:

$$\sum_i \sum_j X_{zijk} \le StaQue_z \cdot TruIZ_k, \ \forall z, k. \quad (4)$$

Equation (5) states that the rebalancing operation within the zones is just done by the truck that has started the 1st route. Also, the truck doesn't return to the station. To put it differently, given the trucks' multiplicity in this problem, as this constraint is considered, it won't happen that 2 trucks are doing the rebalancing operation in one zone. Otherwise, the problems cots are increased.

$$X_{zijk} + \sum_k X_{zjik} + \sum_{i'} \sum_{k'} X_{zji'k'} \le 1, \quad (5)$$

$$\forall i, j, z, k; \ i' \ne i; \ j' \ne j,$$

for more detail, $X_{zji'k'}$ points to routs conducted from station to each destination station (except i) with each truck (except k) in zone.

Equation (6) signifies the visiting priority or the consequence of visiting stations within the zones; it also prevents the sub-tours to be formed during the rebalancing operation:

$$StaPri_{zi} - StaPri_{zj} +$$

$$StaQue_z \cdot \sum_k X_{zijk} \le StaQue_z - 1,$$

$$\forall z, \forall i, j \in I_1. \quad (6)$$

Equation (7) aims to control the maximum distance that a truck can travel during the rebalancing operation; given the trucks' existence with different costs of being employed and the routes with different costs of being traveled, this constraint aims to decrease the costs:

$$\sum_z \sum_i \sum_j \left(StaDis_{zij} \cdot X_{zijk} \right) +$$

$$\sum_o \sum_p \left(ZoneDis_{op} \cdot Y_{opk} \right) \le TruDis_k, \ \forall k. \quad (7)$$

Equation (8) expresses that each truck just applied once in each rebalancing period, returns to the depot after accomplishing the mission:

$$\sum_j x_{zojk} + \sum_j x_{zjok} \le 2, \ \forall z, k, \quad (8)$$

for more explain, x_{zojk} points to routs starting the depots in each zone and x_{zjok} refers to routs ending to depots.

Equation (9) states that a truck that collects and sends defective bicycles from station number zero to the maintenance headquarters should use only one truck in all the routes to do the maintenance job:

$$\sum_k Y_{opk} = 1, \ p \in o_1. \quad (9)$$

Equation (10) indicates that each station should be visited just by one truck in rebalancing operation and is visited in collecting operation of defective bicycles:

$$\sum_p \left(Y_{opk} + W_{opk} \right) \le 1, \ \forall k. \quad (10)$$

Equations (11) and (12) are set to ensure the relationship between headquarters' and zones' trucks; in other words, it states that the truck sent from the headquarters to the zones should be exactly the same truck that returns after the mission is accomplished:

$$\sum_j X_{zojk} \le Y_{opk}, \ \forall z, k, p; \quad (11)$$

$$\forall k, p \in o_1, \quad (12)$$

for more explain, x_{zojk} points to routs starting the depots in each zone.

Equations (13) and (14) are applied in order to visit and defective collet bicycles from number zero stations (warehouses) of all the zones and ultimately send them to the repair center by truck:

$$\sum_k \sum_{o \in o_2} W_{opk} = 1, \ \forall p \in o_3; \quad (13)$$

$$\sum_k \sum_{o \in o_3} W_{pok} = 1, \ \forall p \in o_1. \quad (14)$$

Equations (15) and (16) indicate that each truck should only visit one station during the rebalancing operation:

$$\sum_k W_{opk} \le 1, \ \forall o, p; \quad (15)$$

$$\sum_k W_{pok} \le 1, \ \forall o, p. \quad (16)$$

Equation (17) selects the optimal truck based on the rebalancing operation's requirement type between the zones. Since there are different trucks in terms of capacity, employment cost, and distance traveled in this problem. Also, it controls the number of routes that should be visited, not to exceed the number of the station.

$$\sum_o \sum_p W_{opk} \le ZoneQue \cdot TruBZ_k, \ \forall k. \quad (17)$$

Equation (18) expresses that the whole rebalancing operation among the zones is just done by the truck that has started the initial route, and also, this truck doesn't return to the station; to put it differently, this constraint eliminates the unjustified states and obtains the optimal answer:

$$W_{opk} + \sum_k W_{pok} + \sum_{o'} \sum_{k'} W_{po'k'} \le 1, \ \forall o, p, k, \quad (18)$$

for more explain, $W_{po'k'}$ refers to the routes starts from p ending not to the destination o of previous summation and with other truck k implemented from previous summation of this equation.

Equation (19) signifies the priority or the sequence of visiting stations between zones and prevents the sub-tours from being formed during the rebalancing operation:

$$ZonePri_o - ZonePri_p + ZoneQue \cdot \sum_k W_{opk} \le ZoneQue - 1. \ \forall o, p \in o_1. \quad (19)$$

Equations (20) and (21) are applied to optimize the time of the problem to be solved; in other words, it shows that considering the rebalancing operation aspect, there is no connection between the headquarters and maintenance center:

$$\sum_k \left(W_{ook} + Y_{ook} \right) = 0, \ \forall o \in o_4; \quad (20)$$

$$\sum_k \left(W_{ook} + W_{pok} \right) = 0, \ \forall o \in o_4, \quad (21)$$

for more explain, W_{ook} and Y_{ook} point to the routes between each center with itself.

Equation (22) is set in order to control the inventory of the defective bicycles being transported between zones; to put it differently, the number of defective bicycles that are entered minus the number of defective bicycles removed equals the number of bicycles taken by the truck:

$$\sum_k \sum_o \left(FPM_{pokb} - FPM_{opkb} \right) = ZoBiFai_{pb}, \quad (22)$$
$$\forall b, p \in o_1.$$

Equation (23) is applied as an approach to calculate the number of defective bicycles, which are counted by station zero:

$$ZoBiFai_{pb} = \sum_j \sum_k FPZ_{zjikb} - StaBiFai_{zib}, \quad (23)$$
$$\forall z, p, b, i \in I_2.$$

Equations (24) and (25) are set to control the inventory of the defective bicycles being transported between stations and complement each other; in other words, the number of faulty bicycles that are entered minus the number of defective bicycles removed equals the number of bicycles taken by the truck:

$$\max\left(0, ZoBiFai_{ob}, - ZoBiFai_{pb}\right) \times W_{opk} \le FPM_{opkb}, \ \forall o, p, k, b; \quad (24)$$

$$\min\left(TCap_{kb}, TCap_{kb} + ZoBiFai_{ob}, TCap_{kb} - ZoBiFai_{pb}\right) \cdot W_{opk} \ge FPM_{opkb}, \quad (25)$$
$$\forall o, p, k, b.$$

Equation (26) is applied in order to control the inventory of the defective bicycles being transported within the zones' stations; to put it differently, the number of defective bicycles that are entered minus the number of defective bicycles removed equals the number of bicycles taken by the truck:

$$\sum_k \sum_i \left(FPZ_{zjikb} - FPZ_{zijkb} \right) = -StaBiFai_{zjb}, \quad (26)$$
$$\forall z, j, b \in I_1.$$

Equations (27) and (28) are considered rebalancing equations of the demand, station's capacity, safety stock, and stations' inventory; the relationships themself can reflect the calculation method:

$$StaDem_{zib} \le \min\left(StaCap_{zib} - StaInv_{zib}, StaCap_{zib} - StaInv_{zib} - StaPre_{zib} \right), \ \forall z, i, b; \quad (27)$$

$$StaDem_{zib} \ge StaSS_{zib} - StaInv_{zib} + StaPre_{zib}, \quad (28)$$
$$\forall z, i, b.$$

Equation (29) is set to perform a rebalancing operation or the number of bicycles that should be placed or removed within the zones' stations:

$$\sum_j \sum_k \left(FIZ_{zjikb} - FIZ_{zijkb} \right) = -StaDem_{zib}, \quad (29)$$
$$\forall z, b, j \in I_1.$$

Equations (30) and (31) are applied to control the inventory of the tacit bicycles being transported within stations and complement each other; in other words, the number of tacit bicycles that are entered minus the number of tacit bicycles removed equals the number of bicycles taken by the truck:

$$FIZ_{zijkb} \ge \max\left(StaDem_{zib} \cdot X_{zijk}, - StaDem_{zib} \cdot X_{zijk} \right), \ \forall z, i, j, k, b; \quad (30)$$

$$FIZ_{zijkb} \le TCap_{kb} \cdot X_{zijk}, \ \forall z, i, j, k, b. \quad (31)$$

Equation (32) indicates that the number of defective and tacit bicycles of the zone should be less than the truck's capacity to be carried; this constraint affects the truck's capacity to be chosen based on the required capacity, to put it differently:

$$FIZ_{zijkb} + FPZ_{zijkb} \le TCap_{kb} \cdot X_{zijk}, \ \forall z, i, j, k, b. \quad (32)$$

Equation (33) is set to calculate each zone's total demand according to the safety stock, current inventory, and demand prediction. In other words, the headquarters apply this constraint to decide whether this zone needs to do the rebalancing operation. Thus, it is so paramount.

$$DemTOT_{zb} \ge \sum_i StaSS_{zib} - \sum_i StaInv_{zib} - \sum_i StaPre_{zib}, \ \forall z, b. \quad (33)$$

Equation (34) states that the total number of bicycles that are taken from the stations within the zones must never be negative. In addition, whether the $DemTOT_{zb}$ takes a negative value. It shouldn't be less than this value. Same as the above constraint, constraint (Equation (35)) says that the total number of bicycles removed from the network must never be negative and shouldn't also be less than the total number of demands (whether the total number of bicycles is positive). Hence, considering the bicycle type b, $DemTOT_{zb}$ equals the total number of demands of all the zones' stations.

$$\sum_k \sum_j FIZ_{zijkb} \geq \max\left(0, -DemTOT_{zb}\right),$$
$$\forall z, b, \forall i \in I_2, j \in I_1; \quad (34)$$

$$\sum_k \sum_j FIZ_{zijkb} \geq \max\left(0, DemTOT_{zb}\right), \quad (35)$$
$$\forall z, b, \forall i \in I_2, j \in I_1.$$

Equations (36) and (37) are set to control the tacit bicycles' inventory being transported between stations; in other words, the number of tacit bicycles that are entered minus the number of tacit bicycles removed equals the number of bicycles taken by the truck:

$$FOZ_{opkb} = \sum_j FIZ_{zijkb} \cdot Y_{opk}, \quad (36)$$
$$\forall z, p, k, b, \forall i \in I_2, o \in o_3;$$
$$FOZ_{pokb} = \sum_j FIZ_{zjokb} - StaDem_{zob} . \, Y_{opk}, \quad (37)$$
$$\forall z, p, k, b, \forall i \in I_2, o \in o_3.$$

Equation (38) is applied to calculate the inventory of the stations within the zones and after the rebalancing operation. This constraint considers the station's current inventory, number of bicycles entering or leaving the station, demand prediction, and safety stock of the station to calculate the inventory after the rebalancing operation. The purpose of applying this constraint is to determine the surplus bicycles' holding cost based on parameter γ. Equation (39) also has a similar mechanism of action to the above constraint, except that it calculates the depot's inventory after the rebalancing operation.

$$NewStaInv_{zib} \geq StaInv_{zib} + \sum_k \sum_j FIZ_{zjikb} - \sum_k \sum_j FIZ_{zijkb} + StaPre_{zib} - StaSS_{zib}, \quad (38)$$
$$\forall z, b, \forall i \in I_1;$$
$$NewStaInv_{zib} \geq StaInv_{zib} + \sum_k \sum_j FIZ_{zjikb} - \sum_k FOZ_{pokb} + StaPre_{zib} - StaSS_{zib}, \quad (39)$$
$$\forall z, b, \forall i \in I_2, o \in o_3.$$

3. Numerical examples

In order to show the capability of the proposed mathematical model, 2 numerical examples are presented. The main difference between the 2 numerical examples is the number of BSS zones in NE 1 and NE 2 are 3 and 4 zones, respectively (except central headquarters and maintenance center). In the following, numerical examples are modeled and solved, and the results are presented. Then, the sensitivity analysis is performed on one of the examples. As pointed before, zone 0 is assigned to the central headquarter and zone $z + 1$ to the maintenance center. The central headquarter is responsible for controlling the zones and coordinating between zones. It contains 2 types of bicycles to meet the zone station's needs, 6 trucks with different capacities and employing cost to rebalancing operation and a mobile truck to collect the bicycles that require maintenance service from these zones. Tables 2–11 show the parameter values of 2 numerical examples.

Table 2. Dimension of the 2 numerical examples

Title	Amounts in	
	NE 1	NE 2
Number of zones z	3	4
Number of stations in each zone i	4	4
Bicycles' types b	2	2
Number of trucks k	6	6
Bicycles' transportation cost within the zones α	50	50
Bicycles' transportation cost between the zones β	100	100
Surplus bicycles holding γ	5000	5000

Table 3. Data related to the trucks for both numerical examples

Truck's type	Capacity based on bicycles separation		Constant applied cost $TruImp_k$	Allowed distance to be driven $TruDis_k$
	Type 1	Type 2		
Truck 1	40	40	200000	30000
Truck 2	50	50	150000	15000
Truck 3	60	60	30000	40000
Truck 4	50	50	200000	25000
Truck 5	40	30	200000	25000
Truck 6	50	50	220000	30000

Table 4. The distance between stations for both numerical examples

Zone	Beginning i	Destination j			
		0	1	2	3
Zone 1	0	0	1442	2600	2010
	1	1442	0	1612	1562
	2	2600	1612	0	894
	3	2010	1562	894	0
Zone 2	0	0	1649	1000	1342
	1	1649	0	1281	1887
	2	1000	1281	0	632
	3	1342	1887	632	0
Zone 3	0	0	1523	1000	1131
	1	1523	0	721	632
	2	1000	721	0	825
	3	1131	632	825	0
Zone 4 (for NE 2)	0	0	1780	1303	2154
	1	1780	0	718	1082
	2	1303	718	0	1659
	3	2154	1082	1659	0

Table 5. The distance (cost of traveling) between zones for NE 1

Beginning o	Destination p				
	0	1	2	3	4
0	0	1342	2121	2846	1736
1	1342	0	3421	3314	2954
2	2121	3421	0	2683	1741
3	2846	3314	2683	0	1617
4	1736	2954	1741	1617	0

Table 6. The distance (cost of traveling) between zones for NE 2

Beginning o	Destination p					
	0	1	2	3	4	5
0	0	1342	2121	2846	2163	1736
1	1342	0	3421	3314	2683	2954
2	2121	3421	0	2683	3314	1741
3	2846	3314	2683	0	4966	1617
4	2163	2683	3314	4966	0	3607
5	1736	2954	1741	1617	3607	0

Table 7. Number of the defective bicycles within stations for both numerical examples

Zone	Station's number	Bicycle type 1 b_1	Bicycle type 2 b_2
Zone 1	0	–1	0
	1	–1	–2
	2	0	0
	3	–1	–2
Zone 2	0	0	–1
	1	0	0
	2	–1	–2
	3	0	–1
Zone 3	0	0	–1
	1	–2	0
	2	–1	–1
	3	0	0
Zone 4 (for NE 2)	0	0	0
	1	–1	–1
	2	–1	0
	3	0	–1

Table 8. Current inventory of each bicycle types in stations for both numerical examples

Zone	Station's number	Bicycle type 1 b_1	Bicycle type 2 b_2
Zone 1	0	5	5
	1	5	10
	2	5	11
	3	5	5
Zone 2	0	5	6
	1	6	7
	2	7	5
	3	5	5

End of Table 8

Zone	Station's number	Bicycle type 1 b_1	Bicycle type 2 b_2
Zone 3	0	5	5
	1	7	5
	2	5	7
	3	5	10
Zone 4 (for NE 2)	0	5	5
	1	5	6
	2	5	5
	3	8	8

Table 9. Capacity of stations for each bicycle type for both numerical examples

Zone	Station's number	Bicycle type 1 b_1	Bicycle type 2 b_2
Zone 1	0	20	20
	1	20	20
	2	20	20
	3	20	20
Zone 2	0	20	20
	1	20	20
	2	20	20
	3	20	20
Zone 3	0	20	20
	1	20	20
	2	20	20
	3	20	20
Zone 4 (for NE 2)	0	20	20
	1	20	20
	2	20	20
	3	20	20

Table 10. Safety stock for bicycle types in stations for both numerical examples

Zone	Station's number	Bicycle type 1 b_1	Bicycle type 2 b_2
Zone 1	0	0	0
	1	5	5
	2	5	5
	3	5	5
Zone 2	0	0	0
	1	5	5
	2	5	5
	3	5	5
Zone 3	0	0	0
	1	5	5
	2	5	5
	3	5	5
Zone 4 (for NE 2)	0	0	0
	1	5	5
	2	5	5
	3	5	5

Table 11. Demand prediction of bicycle types in stations for both numerical examples

Zone	Station's number	Bicycle type 1 b_1	Bicycle type 2 b_2
Zone 1	0	0	0
	1	–1	–6
	2	–2	–1
	3	–10	–4
Zone 2	0	0	0
	1	–3	2
	2	–2	–2
	3	–5	–5
Zone 3	0	0	0
	1	–5	–6
	2	–7	–7
	3	0	0
Zone 4 (for NE 2)	0	0	0
	1	–1	1
	2	–6	–6
	3	0	0

Due to the transmitted information from the zones such as station's capacity, safety stock, station's inventory, a number of predicted demands, and a number of bicycles that require maintenance service, the central headquarter makes the best possible decision through solving the mathematical modeling regarding the single-objective function. For each of the zones, stations are allocated to station 0 to deliver/receive bicycles to support the customers' needs. It should be noted that station 0 is considered a central depot. Station 0 of each zone is responsible for coordinating with the central headquarter by transmitting the stations' information and maintaining the defective bicycles to send them to the center.

In order to test the capability of the proposed model, some numerical examples in various dimensions were generated and solved in GAMS software with a Personal Computer *CPU Core i5 and 5G RAM*. These examples with the information about the value of the main variables and their run time were presented in Table 12. As the results show, the run times are exponentially increased when the dimension is gradually added. Also, Figure 2 shows the trend of solving run time by increasing the variable's value.

As the results show, for larger dimensions, the problem could not be solved through GAMS software. Therefore, this model cannot be solved in real-world BSSs through exact methods absolutely. In this paper, the 2 presented numerical examples could not be solved as exact methods with conventional software in a reasonable time. Dimensions of the problem in terms of complexity for NE 1 are provided in Table 13. Hence, the numerical examples were formulated in the GAMS software environment and then loaded it NEOS Server for Optimization (*https://neos-server.org/neos*) and extracted the optimal solution. Unfortunately, the detailed data for a run time was not

Table 12. The run time report for solving different examples with GAMS software

Numerical example	Zone z	Bicycles type b	Station i	Truck k	Run time [sec]
1	2	2	3	2	9.5
2	2	2	3	3	87.5
3	2	2	3	5	425.6
4	2	2	4	6	1247.5
5	3	2	3	2	3789.5
6	3	2	3	4	10253.3
7 (NE 1)	3	2	4	6	not solved
8 (NE 2)	4	2	4	6	not solved

Table 13. Model statistics for the numerical example 1

Statistics parameter	Quantity
Blocks of equations	42
Single equations	3229
Blocks of variables	16
Single variables	2429
Non-zero elements	26101
Non-linear non-zero elements	4236
Discrete variables	2428

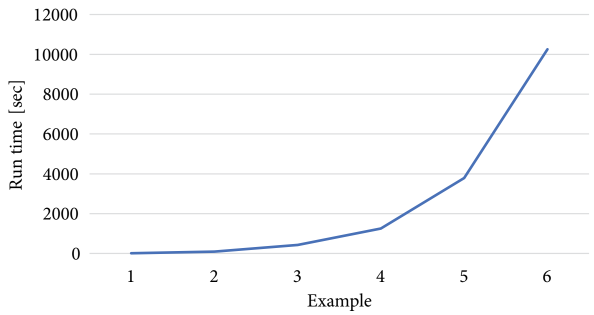

Figure 2. The trend of solving run time with increasing the variable's value

available. So, Figures 3 and 4 illustrate the results of solving the 2 numerical examples.

In order to provide more understanding, a part of the results of solving numerical examples are interpreted in detail. As an instance, the rebalancing operation in zone 3 of the numerical example 1 is described. The central headquarter selects truck 2 to rebalance operation in the zone based on the transmitted information. The truck is selected based on some features like capacity and employing cost. The truck does the rebalancing operation applying 14 bicycles, type 1 and 17 of type 2, to send them to the stations. Firstly, truck number 2 is sent to station 0, and a route is conducted based on the result to stations 2, 1 and 3, respectively, and again goes back to station 0. Each bicycles types are placed in the truck, and they are unloaded and loaded separately. Truck number 2 contains several bicycles types 1 and 2. Also, defective bicycle types 1 and

2 that forms a 4-tuple (14, 17, 0, 0), and this truck visits station 2. According to the parameters, truck 2 delivers 7 bicycles of type 1 and picks up one defective bicycle of type 2, which requires maintenance service. Then, it visits station 1 with 4-tuple (7, 12, 1, 1). 3 bicycles of type 1 and 6 bicycles of type 2 are delivered to this station, and 2 defective bicycles of type 2 that require maintenance service are taken. After rebalancing station 1, the truck visits station 3 with the 4-tuple (4, 6, 3, 1). In this station and similar to the rebalancing operation done in other stations,

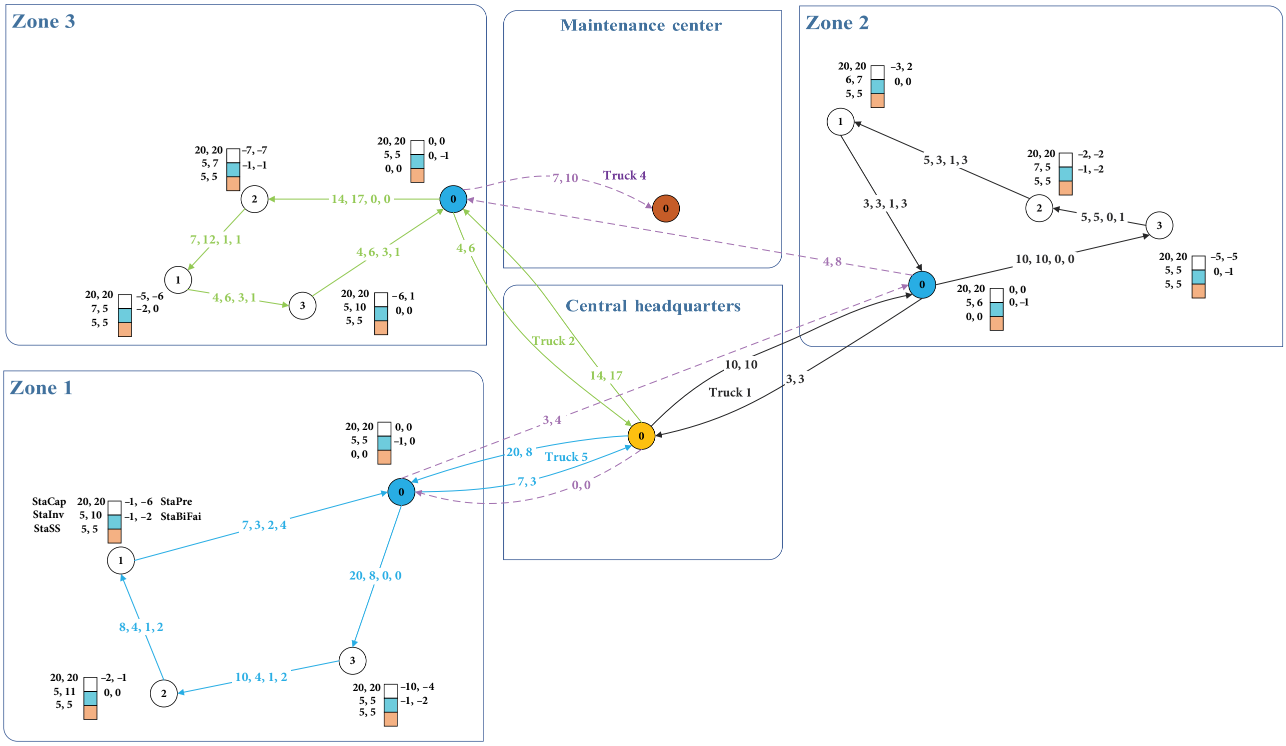

Figure 3. Result of solving numerical example 1

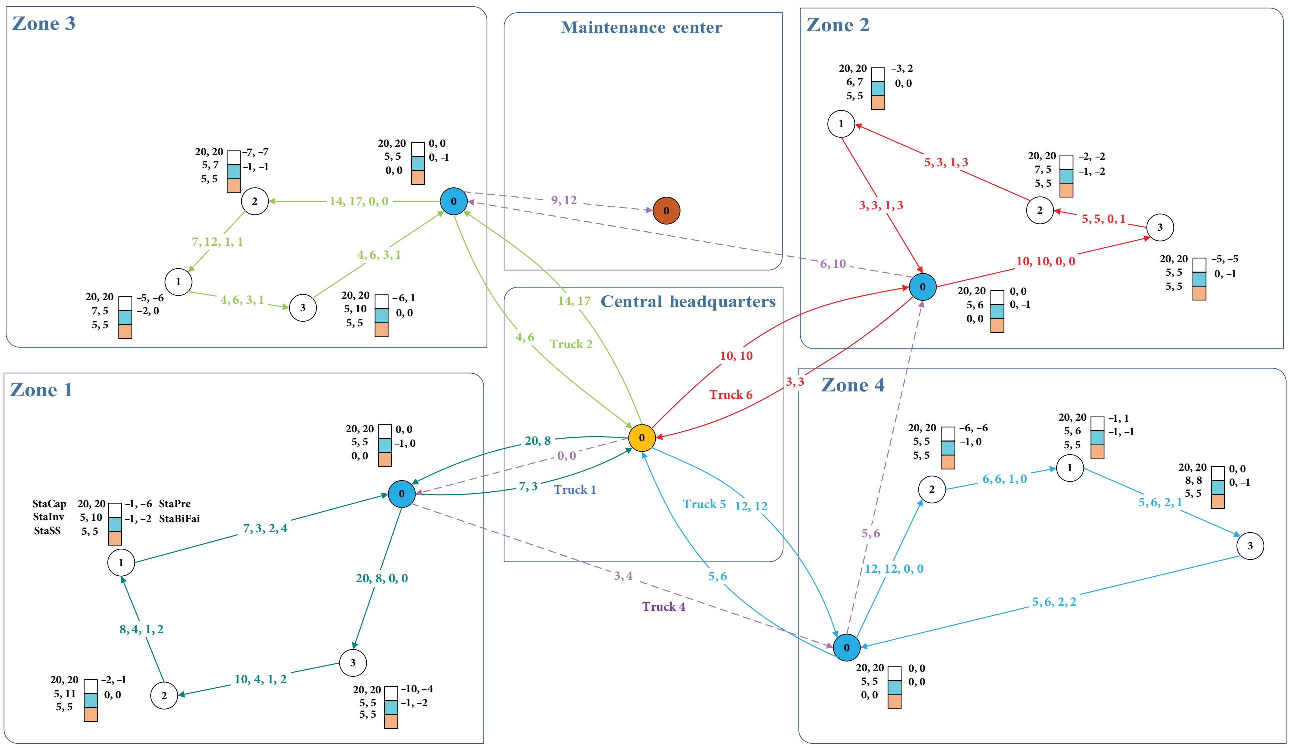

Figure 4. Result of solving numerical example 2

none of the bicycle types are delivered to the station. This station has no bicycle that requires maintenance service. Finally, after visiting station 3, the truck visits station 0 with the 4-tuple (4, 6, 3, 1). The rebalancing operation is done at the track goes back to the headquarters with the ordered pair (4, 6). It is worth mentioning that these values are calculated considering station capacity, current inventory, safety stock, number of the demands, number of defective bicycles, and the surplus bicycles' maintenance cost to reduce the costs.

This is not the end of the rebalancing operation. When the bicycles require maintenance service, they are all collected in station 0 of each zone (zones' depots). The mobile truck is sent from the headquarters to collect the defective bicycles from zones' depots and send them to the maintenance center based on the optimized routing. In this example, truck 4 is employed and based on the routing results of the solving model. Truck 4 is initially sent to zone 1, and it takes 3 bicycles of type 1 and 4 of type 2. Therefore, the truck leaves zone 1 and visits zone 2 by pair of (3, 4). In this zone, one bicycle of type 1 and 4 bicycles of type 2 is also collected in the following, with the pair of (4, 8). Then, the truck visits zone 3 to load defective bicycles, leaves this zone, and goes to the end of rout, visiting the maintenance center by pair (7, 10).

4. Sensitivity analysis

The purpose of sensitivity analysis is to consider the direct effect of parameters on the stability of the mathematical model. In this section, the impact of changes in truck capacity and the cost of holding surplus bicycles at stations on the model results are analyzed. It should be noted that this sensitivity analysis is based on the 2nd numerical example with the characteristics of 4 zones, 4 bicycle stations in each zone, 2 types of bicycles, and 6 types of trucks.

4.1. Investigating the effect of truck's capacity reduction

One of the essential and effective parameters in the rebalancing network of BSS is the capacity of trucks $TCap_{kb}$. The mathematical model intelligently selects the appropriate truck according to the number of demands according to the required capacity and the cost of using them. In order to prove the claim regarding the intelligent selection of trucks for rebalancing operation, the sensitivity analysis of 50% reduction in the capacity of trucks compared to the results of the 2nd numerical example is discussed. The model is solved again in this situation, and new results are illustrated in Figure 5.

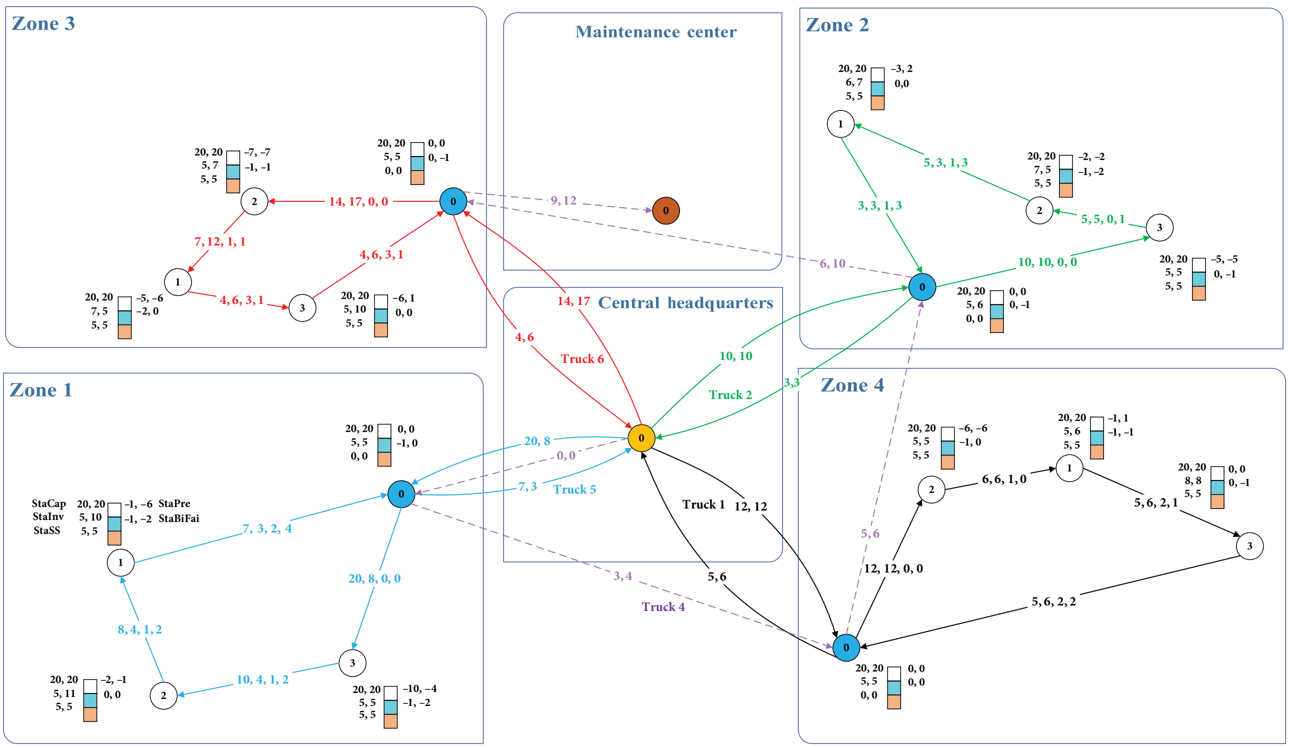

Figure 5. Result of solving by 50% reduction in capacity of trucks

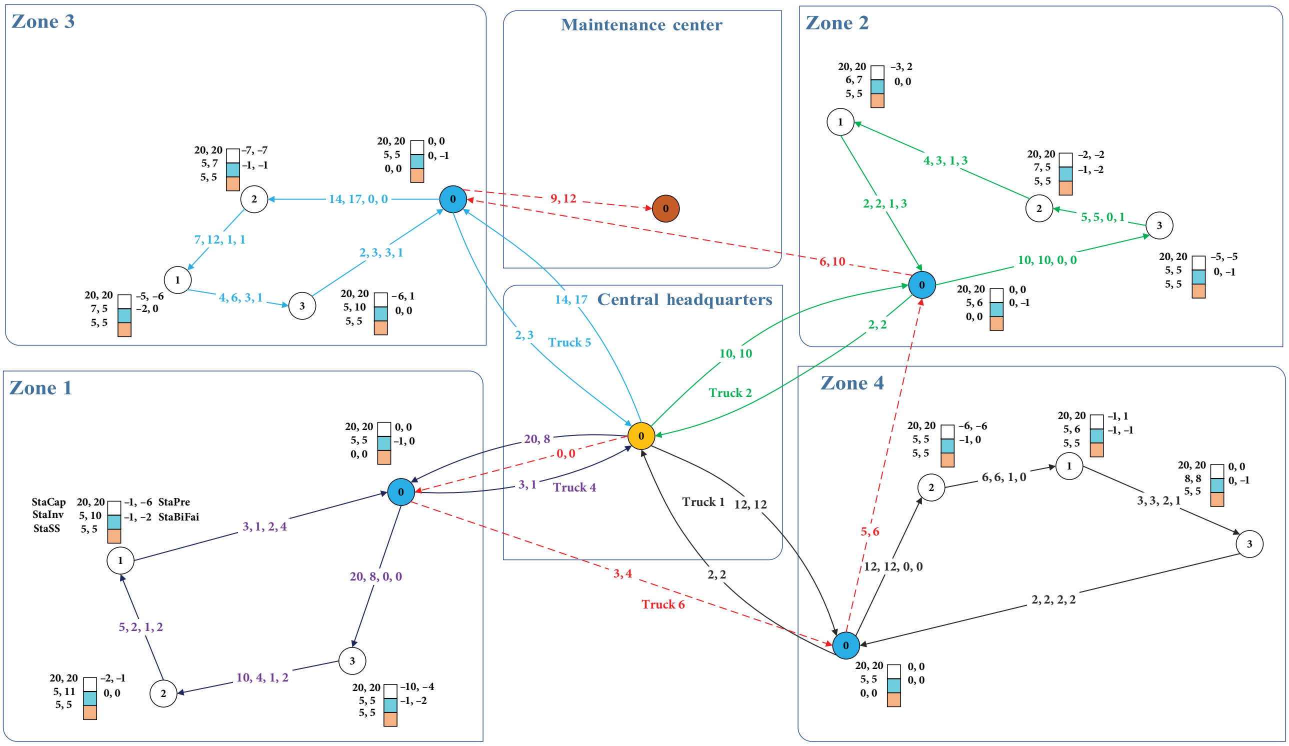

Figure 6. Result of solving without considering holding cost of surplus bicycles

As a result of solving the model in case of a 50% reduction in the capacity of trucks, the selection of trucks for rebalancing operation was changed. Trucks 5, 2, 6 and 1 were selected to rebalance zones 1 to 4, respectively, based on the new capacities, while previously, trucks 1, 6, 2 and 5 were selected, respectively. On the other hand, truck 4 was chosen again for collecting defective bicycles.

4.2. Investigating the effect of holding cost of surplus bicycles

One of the essential and influential parameters of the mathematical model in choosing the optimal solution is the parameter of the holding cost of surplus bicycles in stations γ. For example, when the cost of holding a surplus bicycle is considered zero, the BSS is allowed to store more bicycles at stations. To prove this claim, the sensitivity to this parameter with value of parameter γ equal to zero is done. The new results are compared with solving the basic numerical example. The model is solved again, and new results are illustrated in Figure 6.

Solving the problem in the new situation shows that fewer bicycles have been collected from the zones than solving the basic one. In other words, there are more bicycles left at the stations because there is no cost to keep the extra bicycles at the stations. In the new solution, pair of (9, 8) of bicycle types 1 and 2 are returned to the headquarters, while in the basic numerical example solution pair of (19, 18) bicycle types.

Conclusions

In this research, rebalancing the inventory of the multi-zone BSS with the mobile station and applying maintenance constraints in a static status and being more compatible with the real world, and being implementable have been designed. This problem is considered a multi-product problem containing different types of bicycles, and stations' demand for each bicycles types on stations is assumed to be definite. The instruction of loading/unloading bicycles by a fleet of heterogeneous trucks from different types and unique specifications is determined. Some zones are assigned as headquarters in the model, and some zones are considered workshop centers. The central headquarter is responsible for controlling and coordinating between the zones. This center includes different types of bicycles to meet the needs if there is a shortage in the zones, trucks with different capacities allocated to do the rebalancing operation, mobile stations to collect bicycles from the zones' depots when they require maintenance service. In fact, based on the information that is transmitted from the zones and contains the safety stock, the current inventory of each station, station's capacity, and the number of bicycles that need the maintenance service, the central headquarters makes the best decision to decline the costs of rebalancing operation and transportation routing based on the objective function of the problem. Some of these decisions include the selection of the trucks' numbers and types, which can be proper to carry the bicycles in order to reduce the costs, choosing the sequence of visiting zones stations in order to do rebalancing operation and maintenance, decrease the costs, optimize routing, collect and send the bicycles that require the maintenance service to workshop stations and make a decision in order to apply mobile station in the case that the demand level increase significantly.

Comparing the results of this study with similar research, it is concluded that limited research studies simultaneously considered different types of bicycles and heterogeneous transportation systems to rebalancing operations. Also, it should be noted that none of these researches have done the rebalancing operation applying the multi-zone rebalanced level, considering the maintenance attitude using the mobile station and to increase the customer satisfaction level and optimize this system.

The research has some critical assumptions that could be considered as a limitation, such as:

- the research model was modeled in a static state;
- the number of vehicles in the transport fleet and the number of bicycles are sufficiently available;
- the mathematical model was planned for a single-period model;
- in this paper, in order to solve the mathematical model, an exact algorithm was used, while for large-scale dimension, meta-heuristic algorithms should be used.

There are some suggestions for future development, such as:

- the research model can be examined in dynamic mode;
- regarding the shortage of bicycles in the zones, 2 types of policies based on permissible shortages or unauthorized shortages can be considered by considering some penalties;
- the research model can be developed in a multi-period and multi-objective;
- demand from one type of bicycle can be covered with another type considering some penalties;
- in order to solve the problem in larger dimensions, meta-heuristic algorithms can be used.

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