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LINK PERFORMANCE FUNCTIONS FOR HIGH OCCUPANCY VEHICLE LANES OF FREEWAYS

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Abstract. High Occupancy Vehicle (HOV) lanes are widely used on freeways and play an important role in network design and management. Likewise, link performance functions serve as an essential tool for transport system analysis. This paper aims to support network analysis by providing a tailored link performance function for HOV lanes contiguous with general motor lanes on freeways. Specifically, real traffic data is used for model calibration and evaluation that was assembled from the Performance Measurement System (PeMS) maintained by the California Department of Transportation. Three alternative models for link performance functions of HOV lanes on freeways are developed, which take traffic performance on both HOV lanes and adjacent sets of general motor lanes into consideration. To calibrate the parameters of the models, linear regression is made through stepwise and enter methods and nonlinear regression is carried out using sequential quadratic programming. Statistical analysis together with an evaluation using real traffic data is conducted to evaluate the validity of the proposed models. Our results show that all the three proposed models for contiguous HOV lanes on freeways are statistically significant and perform better in representing real traffic condition with regards to a traditional link performance function, with one specific nonlinear model best supported.

Keywords: high occupancy vehicle lane, freeway, link performance function, regression model, traffic estimation.

Introduction

A High Occupancy Vehicle (HOV) lane is a motor vehicle lane exclusively allowed for HOV traffic to travel on, including buses, carpools and all other vehicles, which carry two or more persons. HOV lanes are widely used on freeways in some countries, e.g. the US. The application of HOV lanes helps to improve the average travel speed, increase the average number of persons per vehicle, enhance bus operations and reduce delay during peak hours, leading to significant improvement in transportation system reliability and level of service (Krimmer, Venigalla 2006; Daganzo *et al.* 2008; Kwon, Varaiya 2008). For example, according to 'The 2005 Urban Mobility Report', travel time on 19 surveyed lanes declined by 20% on average after HOV lanes had been employed, which obviously relieved the previous severe congestions on these lanes (Schrank, Lomax 2005). For another example, Li *et al.* (2007) recommended that exclusive bus lanes should be transformed into HOV lanes and demonstrated the necessity and feasibility of applying HOV lanes on Yan'an East tunnel and Siping-Zhongshan East Road in Shanghai, China. Besides traffic performance, HOV lanes are effective in reducing vehicle emissions and improving air quality (Boriboonsomsin, Barth 2008; Shewmake 2012; Fontes *et al.* 2014). Thus, in the US, federal policies encourage the construction of HOV lanes and restrict funding for mixed-flow lanes in metropolitan areas that do not meet federal air quality standards such as Los Angeles, San Francisco (FHWA 2016). Although HOV lanes have not been widely used in some countries and regions, the application of HOV lanes would become a tendency mainly for the following two reasons:

- rapid development of Intelligent Transportation System (ITS) is beneficial to further use of HOV lanes by provision of traffic information (Lee *et al.* 2010);
- the residential layouts in densely populated urban areas lead to a large number of travellers with the same origins and destinations in daily trips, which probably creates the prevailing conditions for incentivizing rideshare and use of HOV lanes (Chen, W., Chen, B. 2003).

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In order to promote the deployment of HOV lanes on freeways, the characteristics of traffic streams in HOV lanes should be explored, based on which we can estimate traffic conditions, manage traffic flow and evaluate transport systems. Link performance functions can be used to characterize traffic streams, describing relationships among traffic factors including infrastructure configuration elements (lane, ramp, shoulder, etc.) and traffic stream parameters (volume, speed, etc.). Therefore, this paper focuses on contiguous HOV lanes, where no buffer separates HOV lanes and general motor lanes, allowing HOV traffic to enter and exit a HOV lane along its length. It aims to propose a link performance function for contiguous HOV lanes on freeways, which is the primary contribution of this study.

The remainder of this paper is organized as follows. The Section 1 summarizes the main research outcomes in HOV lanes and link performance functions. In Section 2, data of roadway information and traffic performance are collected and processed, in preparation for model calibration and evaluation. In Section 3, three alternative models for link performance functions of freeway HOV lanes are developed and calibrated. In Section 4, the proposed models are examined and compared through statistical analysis and the evaluation based on real traffic data. Finally, last section summarizes outcomes, makes conclusions and presents future studies.

1. Literature review

So far, most existing studies on traffic performance of HOV facilities have focused on evaluation of HOV lanes. Based on 'before and after study', some researchers explored the effects of adding new HOV lanes and the effects of lane conversion from mixed-flow to HOV lanes and vice versa (Kim 2000; Wang 2011; Shewmake, Jarvis 2014). Typically, Sisiopiku *et al.* (2010) made a detailed alternatives analysis and cost-benefit analysis with the help of Traffic Software Integrated System (TSIS) and Integrated Development Assessment System (IDAS), in order to determine the operational, environmental and economic impacts of HOV lanes on traffic condition in Birmingham, Alabama. Chow *et al.* (2010) researched the influences of HOV lanes with regard to vehicular trip reduction and congestion relief based on hypothetical scenarios. Boriboonsomsin and Barth (2007) explored the vehicle emissions contributed from HOV lane configurations, using a new emissions modelling methodology that integrates a microscopic traffic simulation model (PARAMICS) with a modal emissions model (CMEM). Most of these studies made positive evaluation of HOV lanes (Jou *et al.* 2005). However, some exceptions exist. For example, Dahlgren (1998) pointed out that in certain condition, adding a HOV lane to a three-lane freeway can be less effective than adding a general-purpose lane. Plotz *et al.* (2010) concluded that pure HOV lanes contribute little to relieving congestion without additional managed use and pricing features. Almost all previous studies have been conducted on specific scenarios, which contribute to the substantial divergence in the findings. Thus, a quantifiable methodology that takes into consideration all the relevant factors is needed in order to evaluate the performance of existing HOV lanes and potential additional HOV lanes. Dedicated link performance functions can be used to carry out this analysis, since they integrate the fundamental traffic flow characteristics. Hence, in this study, we introduce a link performance function tailored for HOV lanes. To the best of our knowledge, studies on link performance functions of HOV lanes, especially on freeways, have not been extensively explored before.

Among all the existing link performance functions, three types are the most predominant: Bureau of Public Roads (BPR) function, Transport and Road Research Laboratory (TRRL) function and Highway Capacity Manual (HCM) method (Wang 2010; Highway Capacity Manual 2010). The BPR function is the most influential and widely-used link performance function, which adapts well to freeways, multilane highways as well as arterial streets. The TRRL function is developed based on data from traffic field investigations conducted in England, corresponding well with urban arterial streets. The HCM method can generally forecast speeds more accurately than the BPR model, especially in an oversaturated condition. However, the computations required by the HCM model can hardly be implemented by software due to its high complexity, to the point that in the Highway Capacity Manual (2000, 2010), the use of the BPR function is strongly recommended as an alternative and practical link performance function. For all these three aforementioned link performance functions, the modelling devices are developed for generic motor lanes. Given the general-motor-lane data collected and employed for model calibration, they all assume a default traffic composition, a majority of which are cars. Considering the discrepancy between this assumption and the HOV traffic, none of these three well-developed functions would be a justified match with the operation of HOV lanes.

Besides these three functions, further studies have also proposed some variations on link performance functions. Nie and Zhang (2005) made an attempt to propose a delay function with both high modelling accuracy and First-In-First-Out (FIFO) consistency. Pulugurtha and Pasupuleti (2010) developed the working of methodology to estimate travel time and its variations, congestion score and reliability of each link in the network. Zhang *et al.* (2013) formulated a three-layer neutral network model based on sparse probe vehicle data and estimated the complete travel time for individual vehicle traversing the link. Hans *et al.* (2015) proposed a method of estimating travel times on isolated urban arterials by using Variational Theory (VT). However, most of these functions are established for homogeneous traffic on a general motor lane, which consider uniform traffic comprised of cars in most cases. Since compared to other vehicles, HOVs have distinctive traffic characteristics in terms of speed and flexibility, these functions are not entirely appropriate for HOV lanes (Zhang *et al.* 2013). In addition, several studies considered the impacts of different vehicle classes on delay functions, and proposed separate models for a variety of lanes in terms of different traffic compositions. Thomas *et al.* (2012) proposed the development of several volume delay functions specific to different traffic conditions, but vehicular flow on HOV lanes was not investigated in that study. Lu *et al.* (2016) developed link travel time functions using microscopic traffic-simulation based four-step method for heterogeneous traffic on freeways, wherein high-occupancy vehicles and oversized vehicles were accounted for. However, different from research on HOV lanes, in that work, a variety of vehicles was allowed to travel on freeway lanes, including Low-Occupancy Vehicles (LOVs).

In a transport system, general motor lanes are the most popular lanes, but there exist some special lanes including bicycle lanes, bus exclusive lanes, HOV lanes and emergency lanes. Not only do special lanes play an important role in transportation systems, but they are often seen side-by-side with general motor lanes in a road segment without any barrier between them. Inevitably, traffic flow on general motor lanes will interfere with that on an adjacent special lane and this may cause network inefficiencies. Previously, some research efforts have been focused on the traffic flow interference between general lanes and exclusive bus lanes (Arasan, Vedagiri 2010; Khoo *et al.* 2014), or general lanes and bicycle lanes (Chen *et al.* 2016), but few investigated such situation for HOV lanes. To address the gaps of previous research, this study aims to propose a link performance function dedicated to HOV lanes, and in the meantime, seeks to explore the interaction of general motor lanes and contiguous HOV facilities.

2. Data

In this section, we collect and process the real traffic data, which is the preparatory work for model formulation. The regression models developed in Section 3 will be calibrated and assessed by using the data obtained from this section.

2.1. Data collection

2.1.1. Data collection site

In this study, two categories of data are necessary, *i.e.* roadway information data and traffic performance data. These data are accessible from the online database of Performance Measurement System (PeMS) maintained by California Department of Transportation (Caltrans) (CDoT 2014). In this study, data are collected at seven collection sites on freeways in the state of California shown in Figure and Table 1. The HOV lane at each site is contiguous, which allows HOV traffic to enter and exit the HOV lane along its length. Further, the collected traffic data at these sites are of high quality, which will be stated later.

2.1.2. Roadway information data

Roadway information data collected from the seven sites are summarized in Table 2. All the roadway attributes are used for computing values of the Free Flow Speed (FFS) and capacity in link performance functions.

Note that in this study, the right-side lateral clearance refers to the outer shoulder of the whole roadway containing HOV lanes and general motor lanes, rather than a buffer between HOV lanes and general motor lanes. At all these seven sites, the HOV lane is the median lane and on the left side of the general motor lanes, wherein no buffer separation is employed between them.

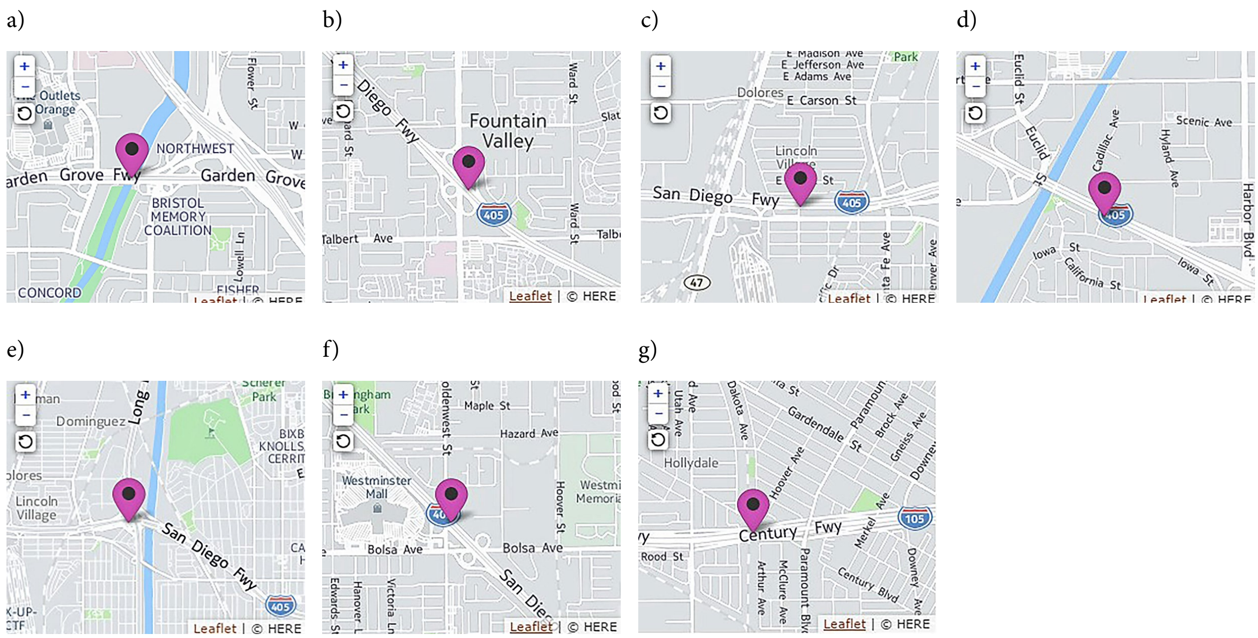

Figure. Data source: a – Site No 1; b – Site No 2; c – Site No 3; d – Site No 4; e – Site No 5; f – Site No 6; g – Site No 7

Table 1. Data collection site

Site No	No of freeway	County	City	Location (CA PM / Abs PM)[a]	Direction (H–L or L–H)[b]	LDS[c]
1	SR22-E	Orange	Orange	9.9 / 11.5	L–H	1202889
2	I405-N	Orange	Fountain Valley	13.74 / 13.5	L–H	1201626
3	I405-S	Los Angeles	Long Beach	8.45 / 32.2	H–L	771884
4	I405-S	Orange	Costa Mesa	12.16 / 11.9	H–L	1209226
5	I405-S	Los Angeles	Long Beach	7.63 / 31.4	H–L	771867
6	I405_N	Orange	Westminster	17.92 / 17.7	L–H	1201844
7	I105-W	Los Angeles	Paramount	14.4 / 14.4	H–L	715816

Notes:
a) CA PM – California jurisdictional post mile; Abs PM – absolute post mile;
b) H–L – from high PM to low PM; L–H – from low PM to high PM;
c) LDS – loop detector station.

Table 2. Roadway information data collected

Site No	Lane width [ft]	Lanes in one direction[a]	Outer shoulder width [ft][b]	Total ramp density [ramps/mile]	Population	Roadway use
1	12.0	3	6	1.3	urbanized	median lane is HOV lane
2	12.4	5	10	1.4		
3	11.2	6	10	1.4		
4	11.6	7	10	0.9		
5	11.2	4	0	1.4		
6	12.4	5	10	1.4		
7	12.7	6	10	1.2		

Notes:
a) including both general motor lanes and HOV lanes;
b) i.e. right-side lateral clearance for the whole roadway.

2.1.3. Traffic performance data

The traffic performance data collected include traffic flow on each general motor lane (i.e. mainline lane) and HOV lane, and speed on each HOV lane for every five-minute interval during the periods when the health of detectors is 100%. The traffic performance data are summarized in Table 3. In this study, every observation corresponds to one five-minute interval, including all the flow and speed data collected during the five minutes on a certain site, which is defined as a data set. Some collected data sets are discarded, because the flow on HOV lane is zero and no effective travel speed is recorded.

As is shown in Table 3, the time periods for different sites are not exactly the same, resulting in the difference in the number of the collected data sets. This is because the situation when the detector condition is 100% good does not necessarily happen concurrently for all the studied sites. Only data collected from the 100% health detector are acceptable regardless of the time periods.

2.2. Data processing

In this research, we seek to develop the link performance function on HOV lanes, which requires the computation of the FFS, capacity and flow rate. These variables are determined in accordance with the methods provided by Highway Capacity Manual (2010).

2.2.1. Free flow speed and capacity

The FFS can be calculated based on roadway information data by the equation:

$$FFS = 75.4 - f_{LW} - f_{LC} - 3.22 \cdot TRD^{0.84}, \quad (1)$$

where: f_{LW}, f_{LC}, TRD are adjustment factors for lane width and right-side lateral clearance of the whole roadway (not a buffer between HOV and general motor lanes), and total ramp density, respectively.

FFS derived from Eq. (1) is rounded to the nearest 5 mi/h (Highway Capacity Manual 2010). For capacity, it can be determined by FFS accordingly (e.g. 70 mi/h corresponding to 2400 pc/h/ln, 65mi/h to 2350 pc/h/ln). For details in methods of how to determine f_{LW}, f_{LC} and capacity, please refer to Highway Capacity Manual (2010). Based on the collected roadway information data in Section 2.1.2, it has been determined that the FFS and capacity are 70 mph and 2400 pc/h/ln for all the seven investigated sites.

2.2.2. Flow rate

For the flow rate, the hourly flow rates under the equivalent base condition are required in order to calculate the flow-to-capacity ratios later, while the collected data are five-minute flow rates under prevailing conditions. Thus, we convert the raw data to the required. Collected five-

Table 3. Summary of traffic performance data collected

Site No	Date (day/month/year)	Time period	Granularity [min]	Number of data sets		
				Collected	Discarded	Effective
1	01/12/2014 – 03/12/2014	from 6:00:00 to 23:59:59	5	648	0	648
	01/03/2015 – 03/03/2015			648	0	648
	01/08/2015 – 03/08/2015			648	6	642
2	01/12/2014 – 03/12/2014			648	0	648
	01/03/2015 – 03/03/2015			648	0	648
	01/08/2015 – 03/08/2015			648	0	648
3	01/12/2014 – 03/12/2014			648	0	648
	01/03/2015 – 03/03/2015			648	0	648
	01/08/2015 – 03/08/2015			648	0	648
4	01/01/2014 – 03/01/2014			648	3	645
	01/02/2014 – 03/02/2014			648	1	647
	01/04/2014 – 03/04/2014			648	1	647
5	02/12/2014			216	0	216
	01/03/2015 – 02/03/2015			432	0	432
	03/08/2015			216	0	216
6	02/12/2014			216	0	216
	01/03/2015 – 02/03/2015			432	0	432
	03/08/2015			216	0	216
7	02/12/2014			216	0	216
	01/03/2015 – 02/03/2015			432	0	432
	03/08/2015			216	0	216
Total				10368	11	10357

Detector condition for all the time periods investigated

Working	Percent [%]	Suspected errors	Percent [%]
Bad	0.00	Line down	0.00
		No data	0.00
		Insufficient data	0.00
		Card off	0.00
Good	100.00	Intermittent	0.00
		Constant	0.00
		Feed unstable	0.00
		Good	100.00

minute flow data should be multiplied by twelve and then, modified by the equation:

$$v_p = \frac{V}{f_{HV} \cdot f_p}, \tag{2}$$

where: v_p, V are flow rates under equivalent base conditions [pc/h/ln] and prevailing conditions [veh/h/ln] respectively; f_p, f_{HV} are adjustment factors for unfamiliar driver populations and presence of heavy vehicles in the traffic stream respectively.

f_p = 1.0, because of urbanized population type according to Table 2 (Roess *et al.* 2010). For f_{HV}, it can be determined by the equation:

$$f_{HV} = \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)}, \tag{3}$$

where: P_T, P_R represent the proportion of buses and the proportion of Recreational Vehicles (RV) in the traffic stream respectively; E_T, E_R are passenger-car equivalents of one bus and one RV respectively.

Default values of these four parameters are P_T = 0.05, P_R = 0, E_T = 1.5, E_R = 1.2, which can be adopted in general cases (Roess *et al.* 2010). However, values of P_T should be different among all the lanes in this study since a HOV lane exists, which appeals to HOV traffic considerably. Thus, it can be assumed that all the HOVs are driven on the HOV lane. In this case, f_{HV} for mainline lanes is 1.0 while f_{HV} for HOV lane is determined by the equation set below:

$$\begin{cases} f_{HV,HOV} = \dfrac{1}{1 + P_{T,HOV} \cdot (E_T - 1)}; \\ P_{T,HOV} = \dfrac{\left(V_H + \sum_{i=1}^{m} V_{M,i}\right) \cdot 0.05}{V_H}, \end{cases} \tag{4}$$

where: V_H, $V_{M,i}$ are flow rates under prevailing conditions [veh/h/ln] on the HOV lane and each mainline lane respectively; m is the number of mainline lanes.

With the help of Eqs (2)–(4), the collected traffic performance data in Section 2.1.3 can be converted to hourly flow rates under the equivalent base condition for both HOV lanes and mainline lanes. Since there are 8629 effective data sets of traffic performance in total, they cannot be listed here.

3. Model formulation and calibration

In this section, we introduce tailored formulations for link performance functions of HOV lanes on freeways and derive three specific models. The three models are calibrated and analysed.

3.1. Model formulation

The traffic flow on a contiguous HOV lane is inevitably influenced by traffic on the general motor lanes (mainline lanes) at the same directed link (Jang *et al.* 2012; Qi *et al.* 2016). Therefore, traffic performance on the adjacent set of mainline lanes should be taken into consideration when the link performance on a HOV lane is analysed. Since the way that the link travel time on the HOV lane and traffic conditions on different types of lanes interact is unknown, we first propose several different models, and then select the significant one(s). By referring to the mathematical pattern of the most commonly used BPR function, the link performance function for HOV lanes on freeways can be formulated as follows.

Model (I):

$$\begin{cases} T = T_0 \cdot \left(1 + a \cdot \left(\dfrac{v_H}{c_H}\right)^{b_1} \cdot \left(\dfrac{v_M}{c_M}\right)^{b_2}\right); \\ S = \dfrac{FFS}{1 + a \cdot X_H^{b_1} \cdot X_M^{b_2}}. \end{cases} \quad (5)$$

In the first formula of Eq. (5), T and T_0 are link travel time at actual speed and FFS respectively; v_H, v_M are equivalent car flow rates [pc/h] on HOV lane and the adjacent set of mainline lanes at the same site respectively; c_H, c_M are a capacity of HOV lane and a total capacity of all the mainline lanes respectively. Since T and T_0 are equal to link length divided by actual speed S and FFS respectively, and $\dfrac{v_H}{c_H}$, $\dfrac{v_M}{c_M}$ are flow-to-capacity ratios denoted as X_H and X_M respectively, the link performance function can be also formulated in terms of speed as the second equation in Model (I). a, b_1 and b_2 are parameters.

Besides Model (I), given a different interaction of traffic on HOV lanes and that on adjacent mainline lanes, the link performance function for HOV lanes on freeways can be alternatively formulated as the following model.

Model (II):

$$\begin{cases} T = T_0 \cdot \left(1 + a_1 \cdot \left(\dfrac{v_H}{c_H}\right)^{b_1} + a_2 \cdot \left(\dfrac{v_M}{c_M}\right)^{b_2}\right); \\ S = \dfrac{FFS}{1 + a_1 \cdot X_H^{b_1} + a_2 \cdot X_M^{b_2}}, \end{cases} \quad (6)$$

where: a_1, a_2, b_1 and b_2 are parameters; the other denotations are the same as Model (I).

Both Model (I) and Model (II) are alternative models for link performance functions of HOV lanes. Since values determined in Section 2 for speed and *FFS* are used for further analysis, Model (I) and Model (II) in terms of speed are studied below.

Given a freeway segment, FFS is fixed according to Eq. (1), so both models have one dependent variable S and two independent variables X_H and X_M. In nature, both models can be approached using Multiple Nonlinear Regression (MNR) models. However, Model (II) cannot be linearized, while Model (I) can be transformed into a Multiple Linear Regression (MLR) model below:

$$\ln\left(\frac{FFS}{S} - 1\right) = \ln a + b_1 \cdot \ln X_H + b_2 \cdot \ln X_M. \quad (7)$$

As Y, X_1, X_2 and A are substituted for $\ln\left(\dfrac{FFS}{S} - 1\right)$, $\ln X_H$, $\ln X_M$ and $\ln a$, respectively, the linearized Model (I), denoted as Model (I)-L, is achieved:

Model (I)-L:

$$Y = A + b_1 \cdot X_1 + b_2 \cdot X_2, \quad (8)$$

where: Y is dependent variable; X_1, X_2 are independent variables; A, b_1, b_2 are parameters. Model (I)-L is derived from Model (I), so they are strictly not different in terms of the interaction between the flow-to-capacity ratios of the HOV lane and the adjacent motor lanes. However, different techniques for linear and nonlinear regression will be applied to calibrate Model (I)-L and Model (I) respectively, with the different formats of explanatory variables integrated. Thus, we define them as two different models in this work. The three proposed Models (I), (II) and (I)-L are formulations for link performance functions of contiguous HOV lanes on freeways, which we need to calibrate in order to demonstrate their validity.

3.2. Model calibration

In this section, all the effective data collected from Sites No 1–4 (7765 data sets in total) in Section 2 are used to calibrate the parameters of the three proposed models separately. Data analysis is carried out using SPSS 19.0 (*Statistical Product and Service Solutions, 19.0 Version*, see Norusis (2011)).

3.2.1. Model (I)-L

For a MLR model, linear correlation between independent variables should be tested before parameter calibration. If the correlation coefficients among independent variables are high, some independent variables should be considered to be removed from the MLR model. In this study, Pearson correlation coefficient, Kendall correlation coefficient and Spearman correlation coefficient are used to test the correlation between two independent variables of Model (I)-L. Results are summarized in Table 4.

Table 4. Correlations between two independent variables in Model (I)-L

			X_1	X_2
Pearson	X_1	Correlation coefficient	1.000	0.901*
		Sig. (2-tailed)		0.000
		N	7765	7765
Kendall	X_1	Correlation coefficient	1.000	0.654*
		Sig. (2-tailed)		0.000
		N	7765	7765
Spearman	X_1	Correlation coefficient	1.000	0.842*
		Sig. (2-tailed)		0.000
		N	7765	7765

Note: * – correlation is significant at the 0.01 level (2-tailed).

Table 4 shows that Pearson, Kendall and Spearman correlation coefficients are equal to 0.901, 0.654, 0.842 respectively, which indicates that independent variables X_1 and X_2 are significantly positive correlated. Moreover, two-tailed significance probability is obviously less than that under significant level (0.01), so non-correlation assumption should be rejected. Thus, it seems better to remove either X_1 and X_2 due to this significant linear correlation. In terms of traffic characteristics, this correlation reveals that traffic performance of mainline lanes has significant effects on that of HOV lanes. It can be illustrated from the following two perspectives:

- when the flow rate on mainline lanes rises, a portion of HOVs on the mainline lanes tend to change lanes and enter the HOV lane, leading to increase in flow-to-capacity ratio on the HOV lane;
- drivers on the HOV lane can be affected by the change in traffic performance on mainline lanes; they might adjust driving situations such as speed, headway, leading to variation in flow-to-capacity ratio on the HOV lane.

Then a linear regression is conducted to calibrate Model (I)-L. The objective of linear regression is to minimize the sum of squared residuals (the most commonly-used loss function), which is called least square method (Montgomery *et al.* 2012). In addition, in terms of model selection, there are several methods (e.g. enter method, stepwise method, forward method, backward method and remove method), among which enter method and stepwise method are the most widely used. Since either b_1 or b_2 may be considered as zero, stepwise method, with potentially removed independent variable, is more suitable. Also, enter method, with all independent variables remaining, is used as a comparison. The inputs of the linear regression are all of the 7765 data sets $\left\{Y_n, X_{1,n}, X_{2,n}\right\}$ $\left(n=1, 2, \ldots, 7765\right)$ for Sites No 1–4, and the outputs are summarized in Table 5.

In Table 5, Model (I)-LS and Model (I)-LE are achieved using stepwise method and enter method respectively. It shows that in stepwise method, variable X_2 is removed from the Model (I)-LS; while in Model (I)-LE both X_1 and X_2 remain. Based on Table 5, the regression results are analysed as follows:

- in *Part (i) Model Summary*, the adjusted R squared are achieved, which indicate the goodness of fit. The value of an adjusted R squared ranges from 0 to 1. The greater the value is, the better the goodness of fit is. Since there are no uniform criteria of evaluating the goodness of fit through the adjusted R squared, it is hard to determine whether the adjusted R squared 0.291 and 0.325 are high enough or not. However, the goodness of fit for Model (I)-LE is better than that for Model (I)-LS in terms of the adjusted R squared;
- in *Part (ii) ANOVA*, according to F-test on both models, the probability that a value in corresponding F-distribution is higher than the F-value for the model is less than 0.01, which indicates that linear correlations described by both models are highly significant.
- in *Part (iii) Coefficients*, parameter calibration results are shown on the column 'Unstandardized coefficients'. Meanwhile, according to t-test on both models, the probabilities are less than that under highly significant level (0.01), which indicates that all the independent variables have highly significant effects for both regression models.
- in terms of the adjusted R squared, F-test and t-test, the effectiveness of regression for both models are generally good. Despite the greater R squared, Model (I)-LE fails to describe the impacts the independent variables have on the dependent variable appropriately. According to Part (iii), the unstandardized and standardized coefficients of X_2 are −0.734 and −0.425 respectively. Also *Part (iv)* reveals that if X_2 remained, the partial correlation between X_2 and Y would be −0.220. These three negative numbers indicate that X_2 and Y are negatively correlated. In other words, Model (I)-LE states that the larger the flow rate on the mainline lanes is, the faster the vehicles on the HOV lane incline to be driven. It contradicts the fact that heavier traffic flow on mainline lanes can interfere with traffic flow on the HOV lane passively, leading to drop in travel speed on the HOV lane. Therefore, Model (I)-LE should not be accepted.

Table 5. Model (I)-L regression analysis summary

(i) Model Summary

Model	R	R squared	Adjusted R squared	Std. error of the estimate
(I)-LS[a]	0.540	0.291	0.291	0.663898
(I)-LE[b]	0.571	0.326	0.325	0.647717

(ii) ANOVA

Model		Sum of squares	*df*	Mean square	*F*	Sig.
(I)-LS	regression	1407.502	1	1407.502	3193.347	0.000
	residual	3421.625	7763	0.441		
	total	4829.127	7764			
(I)-LE	regression	1572.679	2	786.340	1874.303	0.000
	residual	3256.448	7762	0.420		
	total	4829.127	7764			

(iii) Coefficients

Model		Unstandardized coefficients		Standardized coefficients	t	Sig.
		B	Std. error	Beta		
(I)-LS	constant	−1.399	0.014		−98.712	0.000
	X_1	0.515	0.009	0.540	56.510	0.000
(I)-LE	constant	−1.501	0.015		−101.724	0.000
	X_1	0.880	0.020	0.923	43.053	0.000
	X_2	−0.734	0.037	−0.425	−19.842	0.000

(iv) Excluded Variables

Model		Beta in	t	Sig.	Partial correlation
(I)-LS	X_2	−0.425	−19.842	0.000	−0.220

Notes:
a) predictors: (constant), X_1; dependent variable: Y;
b) predictors: (constant), X_1, X_2; dependent variable: Y.

– according to Table 5, independent variable X_2 is removed from the model, which indicates that: (1) there is significant correlation between X_1 and X_2 as mentioned before; (2) the extent to which X_2 influences dependent variable Y is less than that to which X_1 influences Y; (3) the influences which X_1 has on Y contains those which X_2 has on Y to a large degree. Therefore, although only one independent variable X_1 exists in Model (I)-LS, it has indicated the impacts which traffic flow of mainline lanes has on that of the HOV lane.

Based on analysis above, Model (I)-LS derived from stepwise method is acceptable and the parameter values are as follows:

$A=-1.399;$

$a=\exp\left(A\right)=0.247;$

$b_1=0.515,$

$b_2=0.$

Thus, the calibration result of Model (I)-L is achieved: Model (I)-L:

$$\begin{cases} T=T_0\cdot\left(1+0.247\cdot\left(\dfrac{v_H}{c_H}\right)^{0.515}\right); \\ S=\dfrac{FFS}{1+0.247\cdot X_H^{0.515}}. \end{cases} \quad (9)$$

3.2.2. Model (I) and Model (II)

Both Models (I) and (II) are MNR models, so they can be analysed in the same way. For MNR models, Pearson, Kendall and Spearman correlation coefficients are not applicable since they indicate linear correlation among variables. Even if significant linear correlation exists between two independent variables X_H and X_M in Model (I) and Model (II), no variable should be removed without any further evidence because of the nonlinear relationships among the dependent variable and independent variables.

Similarly to linear regression, the objective of nonlinear regression is to minimize the sum of squared residuals (Seber, Wild 2003). Sequential quadratic programming is adopted in this study to perform this analysis. Through sequential quadratic programming, a quadratic program is established at each iteration in order to determine the direction of the optimization. Then, at each iteration, the estimated parameters should be put into the loss function to calculate the loss. The procedure will terminate when the loss function reaches its minimum. The inputs of the nonlinear regression are all of the 7765 data sets $\left\{S_n, FFS_n, X_{H,n}, X_{M,n}\right\}$ $\left(n=1,2,\ldots,7765\right)$ for Sites No 1÷4, and the outputs are summarized in Table 6.

Table 6 shows that after 12 and 18 iterations, the sum of squared residuals for the nonlinear regression Model (I) and Model (II) reach the minimum respectively, with parameter calibration results achieved. Meanwhile, Table 6 presents standard errors and 95% confidence intervals of parameters.

Table 6. Nonlinear regression analysis summaries

Model (I)

Iteration History

Iteration number[a]	Residual sum of squares	Parameter		
		a	b_1	b_2
0.3	727224.998	0.100	0.100	0.100
1.3	571288.595	0.199	0.100	0.100
2.2	482776.300	0.342	0.845	0.357
3.1	434425.709	0.378	1.078	0.042
4.1	372201.244	0.547	1.249	0.042
5.1	359745.684	0.637	1.414	0.042
6.1	348680.826	0.793	1.716	0.042
7.1	345991.611	0.885	1.867	0.042
8.1	345118.326	0.944	1.945	0.042
9.1	344937.800	0.971	1.971	0.042
10.1	344926.850	0.978	1.974	0.042
11.1	344926.776	0.978	1.974	0.042
12.1[b]	344926.775	0.978	1.974	0.042

Parameter Estimates

Parameter	Estimate	Std. error	95% confidence interval	
			Lower bound	Upper bound
a	0.978	0.027	0.925	1.031
b_1	1.974	0.036	1.904	2.044
b_2	0.042	0.067	−0.089	0.173

ANOVA

Source	Sum of squares	*df*	Mean squares
Regression	28748133.825	3	9582711.275
Residual	344926.775	7762	44.438
Uncorrected total	29093060.600	7765	
Corrected total	625523.498	7764	
Adjusted R squared[c]		0.449	

Model (II)

Iteration History

Iteration number	Residual sum of squares	Parameter			
		a_1	a_2	b_1	b_2
0.3	606210.014	0.100	0.100	0.100	0.100
1.6	554511.187	0.100	0.100	0.708	0.401
2.3	451432.219	0.438	0.431	3.958	1.999
3.2	417952.592	0.706	0.193	4.668	0.904
4.2	399900.351	1.090	0.247	5.912	0.841
5.1	399402.889	1.121	0.256	5.964	0.939
6.3	336166.151	0.843	0.087	3.078	0.326
7.5	318950.576	0.836	0.065	2.310	0.307
8.1	314881.095	1.620	0.093	4.107	0.455
9.2	304488.731	1.454	0.116	3.840	0.472
10.2	298743.823	1.333	0.084	3.219	0.255
11.2	294870.988	1.393	0.073	3.302	0.094
12.1	293072.359	1.506	0.079	3.578	0.013
13.1	292662.116	1.536	0.074	3.560	0.013
14.1	292554.556	1.572	0.074	3.592	0.013
15.1	292506.506	1.614	0.075	3.640	0.013
16.1	292505.744	1.620	0.075	3.647	0.013
17.1	292505.740	1.621	0.075	3.648	0.013
18.1[d]	292505.740	1.621	0.075	3.648	0.013

Parameter Estimates

Parameter	Estimate	Std. error	95% confidence interval	
			Lower bound	Upper bound
a_1	1.621	0.056	1.512	1.730
a_2	0.075	0.005	0.066	0.085
b_1	3.648	0.096	3.459	3.837
b_2	0.013	0.051	−0.086	0.112

ANOVA

Source	Sum of squares	*df*	Mean squares
Regression	28800554.860	4	7200138.715
Residual	292505.740	7761	37.689
Uncorrected total	29093060.600	7765	
Corrected total	625523.498	7764	
Adjusted R squared		0.532	

Notes:
a) major iteration number is displayed to the left of the decimal, and minor iteration number is to the right of the decimal;
b) run stopped after 12 iterations; optimal solution is found;
c) adjusted R squared = 1 – (Residual sum of squares) / (Corrected sum of squares);
d) run stopped after 18 iterations; optimal solution is found.

It shows that all the standard errors are below 0.1, and 95% confidence intervals are relatively small. Thus, the degrees of confidence of all the estimated parameters are high, and the nonlinear regression results are acceptable. Additionally, since the adjusted *R* squared of Model (II) is greater than that of Model (I), Model (II) has a better goodness of fit than Model (I). Thus, the calibration results of Model (I) and (II) are as follows:

Model (I):

$$\begin{cases} T = T_0 \cdot \left(1 + 0.978 \cdot \left(\frac{v_H}{c_H}\right)^{1.974} \cdot \left(\frac{v_M}{c_M}\right)^{0.042}\right); \\ S = \frac{FFS}{1 + 0.978 \cdot X_H^{1.974} \cdot X_M^{0.042}}. \end{cases} \quad (10)$$

Model (II):

$$\begin{cases} T = T_0 \cdot \left(1 + 1.621 \cdot \left(\frac{v_H}{c_H}\right)^{3.648} + 0.075 \cdot \left(\frac{v_M}{c_M}\right)^{0.013}\right); \\ S = \frac{FFS}{1 + 1.621 \cdot X_H^{3.648} + 0.075 \cdot X_M^{0.013}}. \end{cases} \quad (11)$$

As a result, the calibrated Models (I)-L, (I) and (II) have been achieved and depicted as functions (9), (10) and (11), respectively. To summarise, these three derived models are all statistically meaningful in terms of the significance of each explanatory variable and the regression relationship that each model expresses. Through statistical analysis, we have compared the calibration results for Model (I)-L obtained from two different methods, and selected the significant one. In addition, the nonlinear Model (I) and Model (II) have been validated and compared via regression techniques including the 95% confidence interval and the adjusted *R* squared. For further assessment of these three models, they will be tested in the traffic performance estimation process in the next section.

4. Model evaluations

Firstly, the three proposed models can be evaluated through statistical analysis, with the help of some indicators including adjusted *R* squared, *t*-value, *F*-value, significance probability, standard error, 95% confidence interval, etc. In terms of these indicators, the effectiveness of regression for all the three models is good in general according to the aforementioned analysis. In other words, all the three models are reliable in theory. Additionally, Model (II) has a better goodness of fit than Model (I) in terms of the adjusted *R* squared. However, it is hard to compare Model (I)-L with Model (I) or (II) based on statistical analysis only, since:

- essential distinctions exist between linear and nonlinear regression. For example, *F*-test is applicable to linear regression only;
- the independent variables of Model (I)-L are different from those of Model (I) and Model (II), i.e., the former one is the logarithms of the flow-to-capacity ratios while the latter is the ratios themselves.

Furthermore, in order to firmly validate the proposed models and make convincing comparison, we carry out another evaluation using the real traffic data, i.e. the estimation error test. Data used in this test are effective data collected for Sites No 5÷7 (2592 data sets in total), which have not been used during the parameter calibration. Based on these real traffic data, three proposed link performance models for HOV lanes are evaluated by comparing their estimation performance with regards to a traditional BPR model. For Sites No 5÷7, the BPR model below is applied (Nie, Zhang 2005; Roess *et al.* 2010):

$$S = \frac{FFS}{1 + 0.32 \cdot X_H^{7.0}}. \quad (12)$$

The above BPR model serves as the baseline, through which the strength of the HOV-specific model will be demonstrated later.

In the evaluation process, inputs are all the 2592 practically measured and processed data sets excluding actual speeds on the HOV lane $\{FFS_n, X_{H,n}, X_{M,n}\}$ $(n = 1, 2, \ldots, 2592)$ from the three investigated sites. Then, the speeds on the HOV lane for the corresponding 2592 five-time intervals are estimated using the three proposed models and the BPR model. Then, the estimated speeds (theoretically calculated values) are compared to the actual speeds (practically measured values) and estimation errors can be achieved with the following equation:

$$\varepsilon = \left|\frac{S_e - S_a}{S_a}\right| \cdot 100\%, \quad (13)$$

where: ε is the estimation error; S_e, S_a are the estimated speed and actual speed respectively.

As a result, for all these three models, the residuals of the dependent variable, i.e. link travel speed, do not have any apparent distribution across any of the explanatory variables. The homoscedasticity indicates the validity of the proposed regression models. Specifically, for the 2592 data sets, the average estimation errors of Model (I)-L, Model (I) and Model (II) are 5.31, 6.76, and 3.80%, respectively. By contrast, the link travel speed calculated by the traditional BPR function bears 18.82% estimation error. Some analysis is made based on the result:

- in terms of estimation errors, all the three proposed models have advantages over BPR model;
- the estimation errors of Model (I)-L and Model (I) are close, since both models originate from the same model.
- Model (II) has obviously less estimation error among all the four models considered;
- the most important characteristics of traffic flow are randomness and uncertainty caused by many factors such as diverse comprehensive qualities of drivers (Matas *et al.* 2012), so travel speeds might

be quite different in the same condition. Thus, no model can make prediction absolutely precisely. By comparison, estimation errors 5.31, 6.76 and 3.80% are relatively low.

Through the real traffic data based evaluation, it reveals that the three proposed models have higher accuracy than the most commonly-used BPR function, and they are all reliable in practice. Moreover, by contrast, Model (II) is superior to the others in terms of estimation errors.

Conclusions and future studies

The primary objective of this study is to provide a tailored link performance function for contiguous HOV lanes of freeways. Real roadway data and traffic performance were collected from PeMS as maintained by Caltrans (CDoT 2014), and processed using 'Highway Capacity Manual' to determine values of the required variables, i.e. FFS, capacity and equivalent car flow rate. Link performance functions for HOV lanes were modelled in consideration of traffic conditions on both HOV lanes and adjacent sets of mainline lanes. Three potential models were examined: one linear model (Model (I)-L) and two nonlinear ones (Model (I) and Model (II)). The parameter calibration was carried on these three models separately via linear and nonlinear regression based techniques. For the linear model, Pearson, Kendall and Spearman correlation coefficients were used to test the linear correlation among independent variables and accordingly, stepwise method as well as enter method was used for regression. For the two nonlinear models, sequential quadratic programming was adopted to conduct the regression. Finally, evaluation was made to demonstrate the validity of the three developed models. Through the statistical analysis, the three proposed models prove statistically meaningful. Further, among the two nonlinear models, Model (II) has a better goodness of fit. Through the evaluation based on real traffic data, it was shown that all the three proposed models have less estimation errors than the popular BPR function in practice. Additionally, nonlinear Model (II) has the best accuracy overall.

In conclusion, all the three proposed models are reliable and significant in both theory and practice, and the nonlinear Model (II) appears best suited. In the proposed Model (II), the variables (i.e. FFS, capacity, flow rate) integrate location-specific elements (lane width, lane number, ramp density, population, volume, etc.) and describe impacts of general motor lane flow on the contiguous HOV lane flow. Thus, Model (II) can serve as a quantifiable methodology in traffic assessment, specifically:

- it can be used for estimating link travel time and speed on a median HOV lane, as shown in Section 4.
- it can be used in traffic assignment. Although typical BPR functions have been adopted in most cases, this manuscript proposes that Model (II) is more suitable to serve as a tailored link performance function for freeway HOV lanes.

Admittedly, future studies are necessary to further improve the proposed framework. First, in this study, the HOV lane is the median lane of the freeway. Alternatively, a HOV lane can also be located between two general motor lanes, so more configuration patterns should be investigated. Additionally, some important factors, such as accidents, terrain and weather, could have impacts on traffic conditions on HOV lanes, which will be explored in future work.

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