



## OVERHEAD CRANE ANTI-SWING SYSTEM BASED ON THE PONTRYAGIN'S MAXIMUM PRINCIPLE

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Submitted 21 February 2014; resubmitted 15 October 2014; accepted 7 November 2014

**Abstract.** Overhead cranes are widely used at industrial enterprises for transportation of materials and products. They are successfully adaptable to technological processes used at an enterprise and their exploitation is inexpensive; in addition, the price of cranes is relatively low. However, extension of requirements set for technological processes results in shortened time for transportation and stiffened requirements for accuracy of cargo delivery and cargo safety. In the attempts to satisfy the latter requirements, particular attention is paid to swings of the cargo-holding rope. There is a number of factors that cause increased requirements for the control system of the crane drive. The slewing movement affects the total system of the crane and aggravates the crane movement control. In modern overhead cranes, the abilities and qualification of an operator (who is assisted by a certain anti-swing system) predetermine the cargo swings and the accuracy of its positioning. The said circumstance latterly caused a particular attention to computerisation of overhead crane control. However, a nonlinearity of the mechanical system of a crane and complicated control of swings often cause undesirable swings, in particular in the beginning and the end of cargo transporting process, thus reducing the efficiency of usual crane control systems. In addition, it should be taken into account that the parameters of a crane, as a controlled mechanical system, depend on the cargo and the conditions of its transportation. Consequently, a development of an effective cargo swing reduction system is a currently topical engineering problem.

**Keywords:** overhead crane; anti-swing system; Pontryagin's principle; dynamics; stability.

### Introduction

This problem is known in the case of controlling the working movements of a crane, when a payload is moving with a flat motion or a motion compounded of a few flat movements (Auernig 1986; Lee *et al.* 1997; Tanaka, Kouno 1998; Alli, Singh 1999). It is usually realised in the control system of the dislocation of the cargo attachment point. The speed of the cargo attachment point is changed ensuring that the movement of the cargo consists of three (or at least two) motion phases: the first phase – acceleration of the payload to achieve defined velocity; the second phase – with constant velocity of the cargo; and the third phase – decreasing the velocity to zero. In the case there is no second phase, the movement control is called the bang-bang type.

If the initial conditions are not zeroed and/or disturbances appear when a movement is being realised, then closed control systems are used (Ridout 1989; Fliess *et al.* 1991). In the case of controlling the slewing motion in mobile cranes, the system controlling the angular

velocity of the jib can be applied. Then on input of the system, the jib angular acceleration profile is assumed (Parker *et al.* 1995).

A method of controlling the mobile crane slewing motion is presented in the paper. The method allows us to transfer the cargo from one point to a selected second (target) point, with simultaneous minimizing of the swings when the working motion is finished. Control is achieved in a close-loop control system of cargo movement by controlling the transfer of the cargo attachment point (Kłosiński 2005).

A nonlinear controller is proposed for the trolley crane systems using the Lyapunov method and a modified version of sliding-surface control is then utilised to achieve cart position control (Vikramaditya, Rajamani 2000). However, the sway angle dynamics has not been considered for stability analysis. A simple proportional derivative controller is designed to asymptotically regulate the overhead crane system to the desired position with natural damping of sway oscillation (Fang



et al. 2001). In (Burg et al. 1996), the crane system is regulated by using the variable transformation method. The passivity property of mechanical system is utilised to regulate the overhead crane system (Karkoub, Zribi 2002). Authors of the paper (Liu et al. 2005) propose an adaptive sliding mode control method with fuzzy tuning of slope of sliding surface for a 2-dimension overhead crane. In the paper (Yang, J. H., Yang, K. S. 2007), a non-linear control scheme incorporating parameter adaptive mechanism is devised to ensure the overall stability of a closed-loop system. By applying the designed controller, the position error will be driven to zero while the sway angle is rapidly damped to achieve swing stabilisation. Stability proof of the overall system is given in terms of the Lyapunov concept.

The paper (Uchiyama et al. 2013) presents a simple model of rotary crane dynamics that includes only significant centrifugal and Coriolis force terms. This simple model allows analytical solutions of differential equations of the model to be derived and allows residual vibration suppression without sensing it, using only horizontal boom motion, can be generated by solving only algebraic equations numerically.

In the paper (Smoczek, Szpytko 2013), evolutionary-based algorithm for fuzzy logic-based data-driven predictive model of Time Between Failures (TBF) and adaptive crane control system design is proposed. The heuristic searching strategy combining the arithmetical crossover, uniform and non-uniform mutation and deletion/insertion mutation is developed for optimizing the rules base and tuning the triangular-shaped membership functions to increase the efficiency and accuracy of a fuzzy rule-based system. The evolutionary algorithm was employed to design a fuzzy predictive model based on the historical data of operational states monitored between the failures of the laboratory scaled overhead traveling crane electronic equipment. The fuzzy predictive model of TBF was implemented in the supervisory system created for supporting decision-making process through forecasting upcoming failure and delivering the user-defined maintenance strategies.

## 1. Mathematical Model

In the present research, a model of an overhead crane developed upon using the program package MATLAB/Simulink® (<http://se.mathworks.com/products/simulink>) is used. The research works (Kłosiński 2005) based on the said model showed that considerable cargo swinging appears on its transportation by an overhead crane. The swinging causes worsening of the equipment performance because usually reduction of the cargo swinging amplitude down to the permissible level in any available way is required. The present paper discusses an opportunity of the control optimisation for increasing the accuracy, shortening the time of cargo delivery and reducing the related energy consumption.

To reduce the cargo swinging, the following measures may be applied:

- special laws related to cargo moving velocity

changes on its acceleration and braking (Tanaka, Kouno 1998);

- special filters of the signal that pre-sets the velocity of movement of the cargo attachment point (Slivinskas et al. 2010);
- special adaptive controls for the crane's drive (Yang, J. H., Yang, K. S. 2007) according to the coordinates;
- equipment that in certain moments applies to a force to the cargo and the direction of such a force is opposite to the current deviation of the cargo. A stabilizing force may be also applied to the cargo attachment point by using special control of the crane's engines. It is interesting to compare both ways of control. Such equipment may be realised by applying the Pontryagin's Maximum Principle (Pontryagin et al. 1969). Examples of realisation of control equipment based on the Maximum Principle are known in the industry (Slivinskas et al. 2010).

The research uses the model of an overhead crane developed using the program package MATLAB/Simulink®. The research works based on the said model showed that considerable cargo swinging appears on its transportation by an overhead crane. The dynamic model of an overhead crane is presented in Fig. 1. The design structure of an overhead crane consists of three parts: a cargo lifting mechanism, a trolley and a bridge.

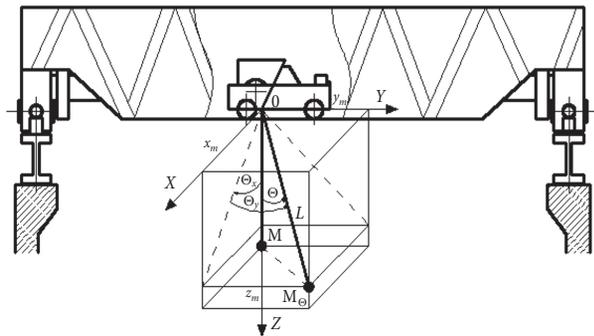


Fig. 1. The dynamic model of a crane

The dislocation of the mass  $M_{\Theta}$  may be described by the following coordinates:

$$x_m = L \sin \Theta_x \cos \Theta_y; \quad (1)$$

$$y_m = L \sin \Theta_y, \quad (2)$$

where:  $L$  – the length of the rope:

$$L = L_0 + \delta_L, \quad (3)$$

where:  $L_0$  – the length of the rope without a cargo predetermined by a position of the rope drum;  $\delta_L$  – the elongation of the rope caused by a cargo.

The angle of the vertical deflection of the rope:

$$\Theta = \frac{\sqrt{x_m^2 + y_m^2}}{L} \quad (4)$$

and vertical movement of the cargo caused by swinging:

$$z_m = L(1 - \cos\Theta_x \cos\Theta_y). \quad (5)$$

The expressions (1) to (5) were used for describing kinetic and potential energy of the mechanical system of the lifting mechanism and its dissipative function in the Lagrange equations of the second kind. Experience of AB ‘Vilnius kranai’ (<http://www.vk.lt>) was evaluated and creating a mathematical model the deviation of rope from the vertical did not exceed 5 degrees. After the relevant processing in accordance with the methods of computerised simulation described in (Augustaitis *et al.* 2011), the expressions of the principal coordinates of the system were obtained that usually are expressed as follow:

$$\ddot{\Theta}_x + \frac{H_2}{ML_0^2} \dot{\Theta}_x + \frac{g}{L_0} \Theta_x = \frac{\ddot{X}_0}{L_0}; \quad (6)$$

$$\ddot{\Theta}_y + \frac{H_2}{ML_0^2} \dot{\Theta}_y + \frac{g}{L_0} \Theta_y = \frac{\ddot{Y}_0}{L_0}; \quad (7)$$

$$M\ddot{\delta}_L + H_1\dot{\delta}_L + C_1\delta_L = Mg \cos\Theta - M\ddot{L}_0; \quad (8)$$

$$\ddot{X}_0(M + M_x) + H_x\dot{X}_0 = F_x + M\ddot{L}_0\Theta_x; \quad (9)$$

$$\ddot{Y}_0(M + M_y) + H_y\dot{Y}_0 = F_y + M\ddot{L}_0\Theta_y, \quad (10)$$

or in the form of a transfer function of the model of the mechanical part (Fig. 2):

$$\Theta_x = \frac{L_0 M \ddot{X}_0 + f_x}{L_0^2 M p^2 + H_2 p + L_0 M g};$$

$$\Theta_y = \frac{L_0 M \ddot{Y}_0 + f_y}{L_0^2 M p^2 + H_2 p + L_0 M g};$$

$$\delta_L = \frac{M g \cos\Theta}{M p^2 + H_1 p + C_1};$$

$$X_0 = \frac{F_x + L_0 M \ddot{\Theta}_x}{(M + M_x) p^2 + H_x p};$$

$$Y_0 = \frac{F_y + L_0 M \ddot{\Theta}_y}{(M + M_y) p^2 + H_y p}, \quad (11)$$

where:  $p = \frac{d}{dt}$  – operator of differentiation.

Modelling of the mechanical part (as well as in the expressions 6 to 11) was accomplished using the following values of the constant parameters: standard gravity  $g = 9.81 \text{ m/s}^2$ ; mass of the cargo  $M = 1000 \text{ kg}$ ; mass of the trolley  $M_x = 5000 \text{ kg}$ ; mass of the trolley and the crane  $M_y = 10000 \text{ kg}$ ; damping in the rope attachment point  $H_2 = 1 \text{ Nms/rad}$ ; damping in the rope  $H_1 = 1000 \text{ Ns/m}$ ; initial tension stiffness of the rope  $C_1 = 1716000 \text{ N/m}$ ; damping along the axis  $X_0$  in the trolley  $H_x = 1000 \text{ Ns/m}$ ; damping along the axis  $Y_0$  in the crane  $H_y = 2000 \text{ Ns/m}$  (the data were obtained with the help of tests carried out at AB ‘Vilnius kranai’). The non-linearity of the rope elongation dependence from load is evaluated in the mathematical model.

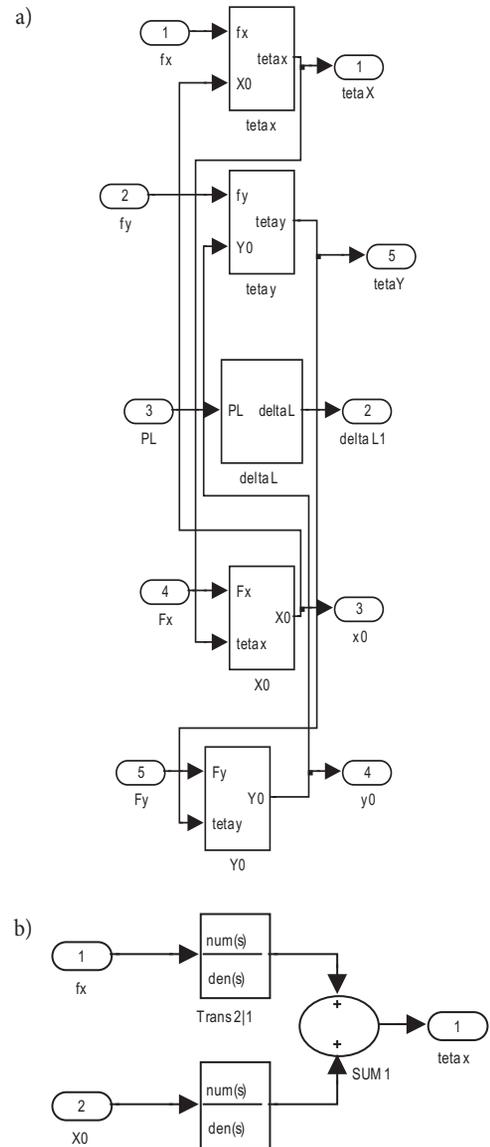


Fig. 2. The model of the mechanical part (a); the model of the subsystem where the angle  $\Theta_x$  is taken into account (b)

For a convenience, in description of operation of the crane’s mechanisms, two additional coordinates (expressed through the principal coordinates of the dynamic system) and their derivatives were introduced:

$$X_L = L_0 \Theta_x; \quad (12)$$

$$\dot{X}_L = L_0 \dot{\Theta}_x; \quad (13)$$

$$X_M = X_0 + X_L; \quad (14)$$

$$\dot{X}_M = \dot{X}_0 + \dot{X}_L. \quad (15)$$

The obtained model describes the mechanical part of the crane and the changes of the dislocation of the cargo caused by changes of the point of rope attachment in two directions  $X$  and  $Y$  and by swinging of the cargo on the rope at angles  $\Theta_x$  and  $\Theta_y$ . To the entries  $F_x$  and  $F_y$  of the model, the forces generated by the engines of the trolley and the crane are applied and the said forces

predetermine the movement of the attachment point. The engines are controlled by controllers.

The algorithm of the controllers may be based on the Pontryagin's Maximum Principle. Pontryagin *et al.* (1969) discusses examples of application of the said Principle. If the trolley moves at a low dissipation and in absence of a restoring force that cannot exceed the value predetermined by the power of the engine, the optimum phase trajectory consists of two parabolas. In the first segment (the acceleration), the control parameter  $u \equiv 1$  and the phase trajectory is described by one parabola of the family:

$$x = \frac{1}{2}(dx)^2 + s. \tag{16}$$

In the second segment (deceleration), the control parameter  $u \equiv -1$ , i.e. on reversing of the engine, the phase point transfers to the parabola that crosses the zero point of coordinates:

$$x = -\frac{1}{2}(dx)^2 + s, \tag{17}$$

where:  $s$  – arbitrary constant. The phase trajectory in absence of a restoring force and dissipation is presented in Fig. 3.

The mechanical system of the trolley (9) accepted in the model differs from the one discussed in (Pontryagin

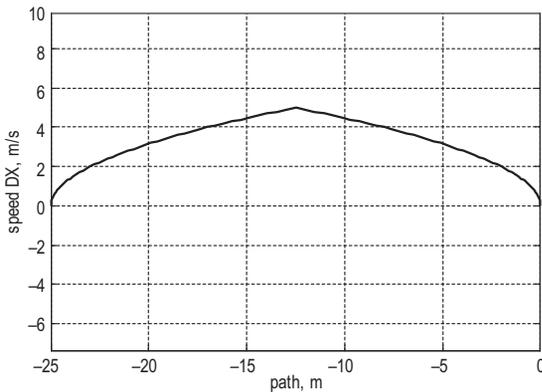


Fig. 3. The optimum phase trajectory in absence of a restoring force and dissipation

*et al.* 1969), by considerable dissipation and existence of a member that takes into account the variable force of response applied to the case of the trolley from the rope. For exploring the mechanical system of the trolley, a special model presented below (K1) was developed.

The model operates as follows: after starting of movement, force from the engine of the trolley is applied to the entry  $F_x$  of the model and movement of the trolley along the axis  $X_0$  toward the zero point of coordinates starts. In the segments of acceleration and deceleration, the phase trajectory is described by the expressions (16) and (17) upon inconsiderable dissipative forces. If dissipation is considerable, the curves of acceleration and deceleration differ from parabolic shape.

Because of this, in the control subsystem (Fig. 4b), the value of the velocity when reversing of the force generated by the engine takes place was chosen in experimental way upon striving to ensure it to be close to zero on passing the origin of coordinates and switching-off the engine.

The cycle of movement of the trolley with a cargo is shown in Fig. 5.

As it may be seen from Fig. 5d, the phase trajectory of free vibrations of the cargo in respect of the attachment point is a set of spiral curves. It is shown in (Pontryagin *et al.* 1969) that the optimum phase trajectory (including a case of pendulum) may be formed of semicircles that cross the zero point of coordinates. Such a trajectory is presented in Fig. 6. If the initial phase point is located below the shown trajectory, the phase point under the impact of the control parameter  $u \equiv 1$  gets onto the arc marked as a top down triangle, then the value of the control becomes  $u \equiv -1$  and deceleration starts and continues until the phase point achieves the zero point of coordinates. If the initial phase point is located over the shown trajectory, the phase point under the impact of the control parameter  $u \equiv -1$  gets onto the arc marked as a top up triangle, then the value of the control becomes  $u \equiv 1$  and acceleration starts and continues until the phase point achieves the zero point of coordinates.

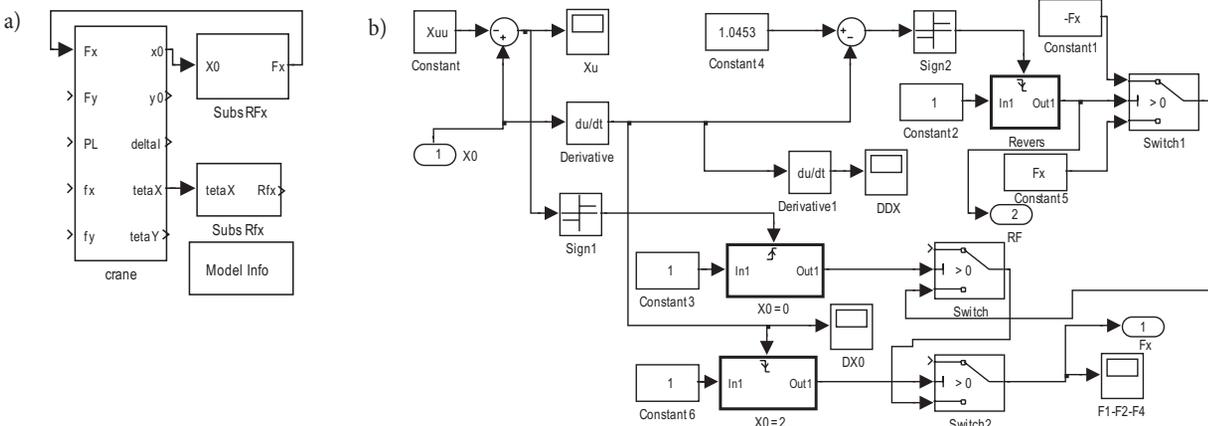


Fig. 4. Model K1 and control subsystem

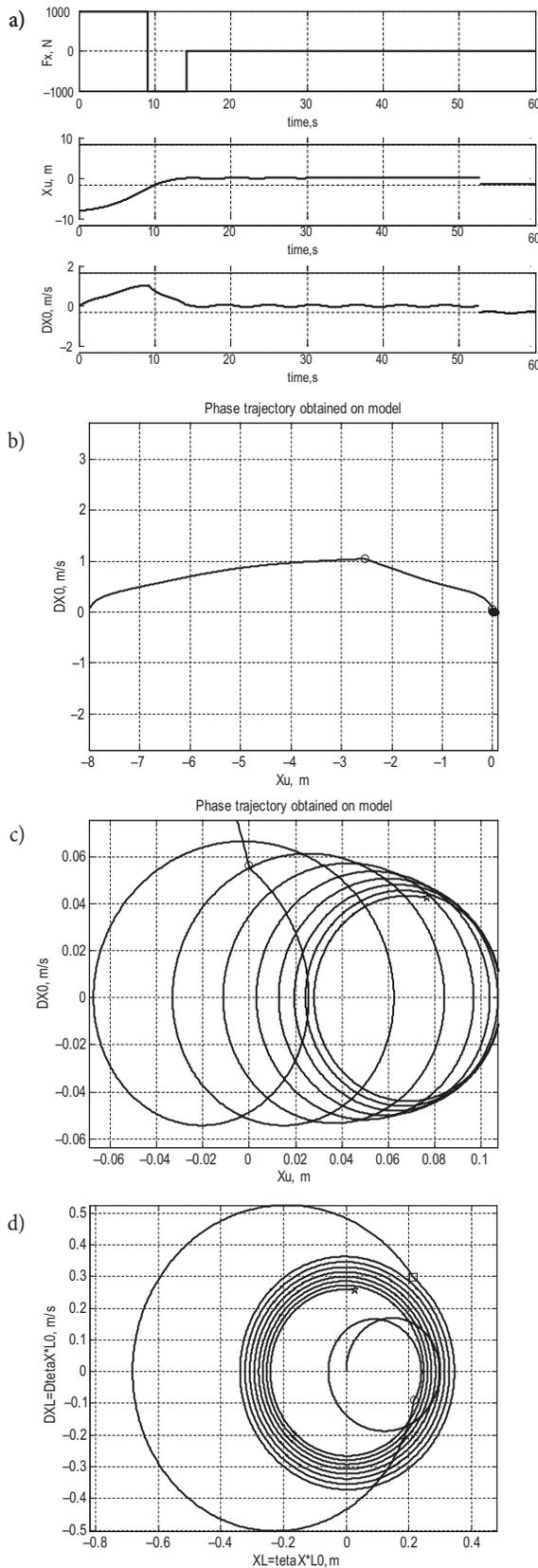


Fig. 5. The cycle of movement of the trolley with a cargo: a – a change of the force generated by the engine, dislocation and velocity; b – the phase trajectory of the trolley; c – the end of the phase trajectory; d – the phase trajectory of the cargo in respect of the attachment point (a circle means the start of reversing; a square – the moment of switching-off the engine; the asterisk – the end of the phase trajectory)

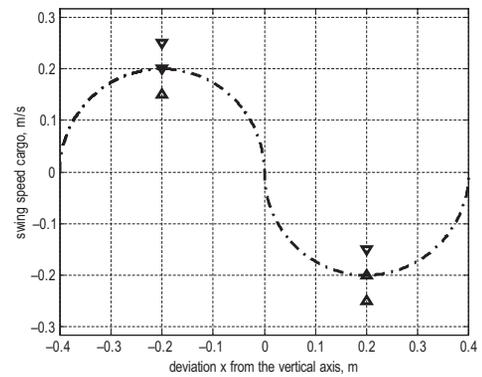


Fig. 6. Optimum phase trajectory when a restoring force presented in the mechanical system

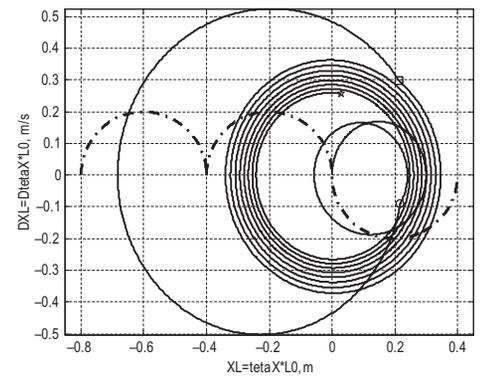


Fig. 7. Phase trajectory of the cargo in respect of the attachment point on the model and switchover phase trajectories

In our case, for the left segment of positive velocities with a centre in the point  $(-R, 0)$ , where  $R$  is the radius of the arc and speed of cargo swing:

$$dx = \sqrt{2xR - x^2}, \tag{18}$$

for the right segment of negative velocities with a centre in the point  $(R, 0)$ :

$$dx = -\sqrt{2xR - x^2}. \tag{19}$$

As it may be seen from Fig. 7, the phase trajectory of the cargo obtained on the base of the model and the switchover phase trajectories intercross in several points; the coordinates of three of them are provided in Table. As it is shown in (Pontryagin *et al.* 1969), when a cargo is over the switchover line for the first and subsequent times, the value of the control parameter becomes opposite. For examining the efficiency of the expressed provision, a model of the cargo controlling system (K2) was developed. Its Simulink<sup>®</sup>-diagram is presented in Fig. 8 b.

Model K2 operates as follows: before starting a movement, a force from the engine of the trolley is applied to the entry  $F_x$  of the model and movement of the trolley along the axis  $X_0$  (controlled by the block  $RF_x$ ) starts. After a reverse of the engine, the signal  $RF_x$  from the block  $RF_x$  activates the cargo control system  $Rfx$  that determines the dislocation of the cargo on the phase trajectory (using blocks Point 1 to Point 3) and provides instructions to switchover control  $fx$ . The obtained results are shown in Figs 9–10.

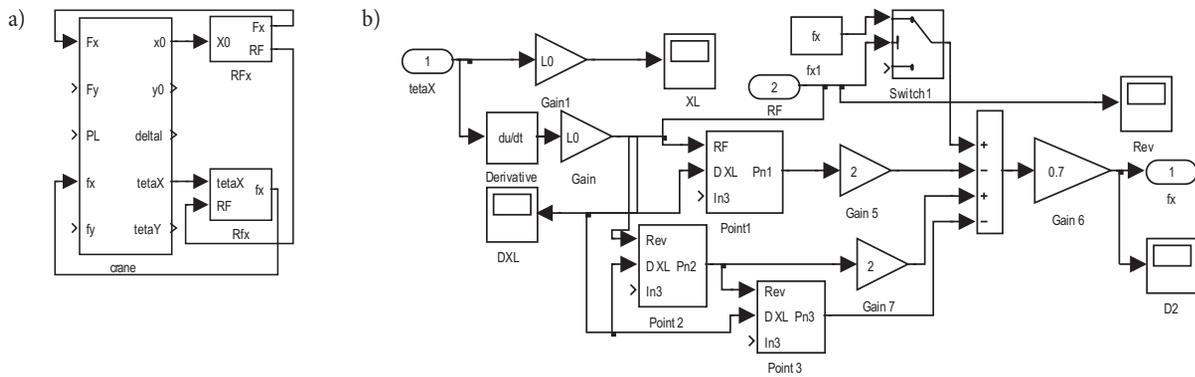


Fig. 8. Model K2 and the cargo controlling system

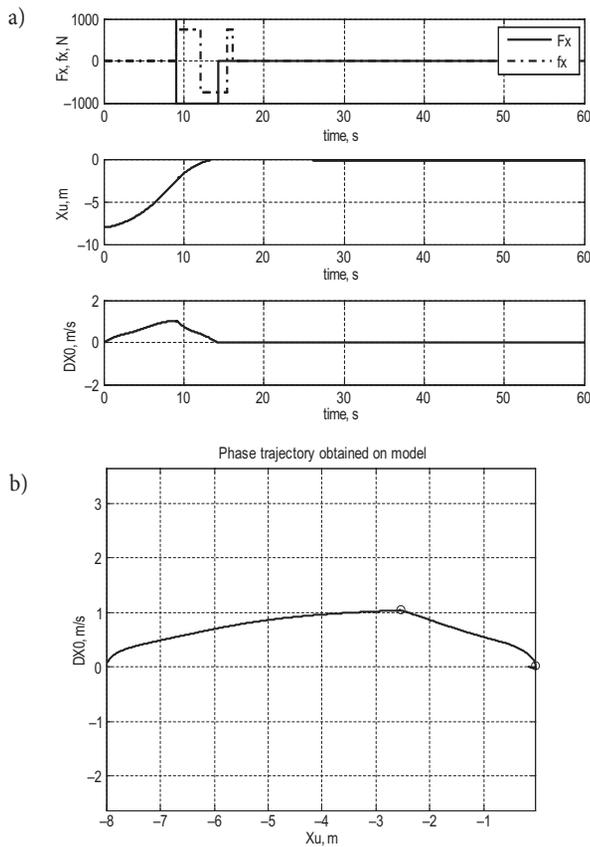


Fig. 9. The cycle of movement of the trolley with a cargo: a – a change of the force generated by the engine, dislocation and velocity; b – the phase trajectory of the trolley (a circle means the start of reversing; a square – the moment of switching-off the engine; the asterisk – the end of the phase trajectory)

As it may be seen from the provided data, three switching actions result in intensive reduction of the amplitude of the cargo swinging from about 0.5 m to about 0.0005 m = 0.5 mm (the amplitude of damped vibrations). By the way, it was found that the amplitude of such residual vibrations increases considerably if the moments of the control switching actions are not chosen precisely.

In a more convenient method for cargo swing reduction, the control parameters in the both channels are summarised and then are transferred to the drive of the trolley.

Table. The factual coordinates of the control switching points (Fig. 10)

Switching point	Point 1	Point 2	Point 3
Time [s]	12.0900	15.3600	16.1000
Speed DXL [m/s]	0.1718	-0.0598	$-1.4965 \cdot 10^{-4}$
Deviation XL [m]	-0.5054	0.0230	$-5.5820 \cdot 10^{-4}$

## Conclusions

1. The research of the model showed that application of the Pontryagin's Maximum Principle enables to reduce considerably the amplitude of cargo swings and to accelerate the process of cargo stabilisation.
2. If the dissipation forces in the lifting mechanism are small and the cargo is moved using a trolley-pushing mechanism, the cargo swings during one minute. By reversing the engine of the said mechanism, the time of swinging can be reduced to 10 s.
3. When the dissipation forces are considerable, swings are reduced by a system of measuring the deviation of the rope from the vertical position and the speed of such a deviation. Thus, the deviation of the factual phase trajectory from the optimum one that conforms to the Pontryagin's Maximum Principle is assessed. The use of signals from the said system for controlling the engine of the trolley-pushing mechanism ensures effective damping of cargo swings.
4. On minimising the time of cargo swings over the cargo delivery point, the engine of the trolley-pushing mechanism is reversed for several times. The number of reverses and the moment of the first reverse are interdependent. Performed investigations shown three switching actions that result in intensive reduction of the amplitude of cargo swinging from approximately 0.5 m to approximately 0.0005 m = 0.5 mm (the amplitude of damped vibrations). The efficiency of swinging reduction closely depends on the moment of reversing start. Determination of successful reversing start needs additional investigation.
5. In reality, the number of reverses should not exceed three, because a larger number of them causes a reduction in the intervals between them; in addition, the deviation of the cargo becomes less than the required accuracy of delivery.

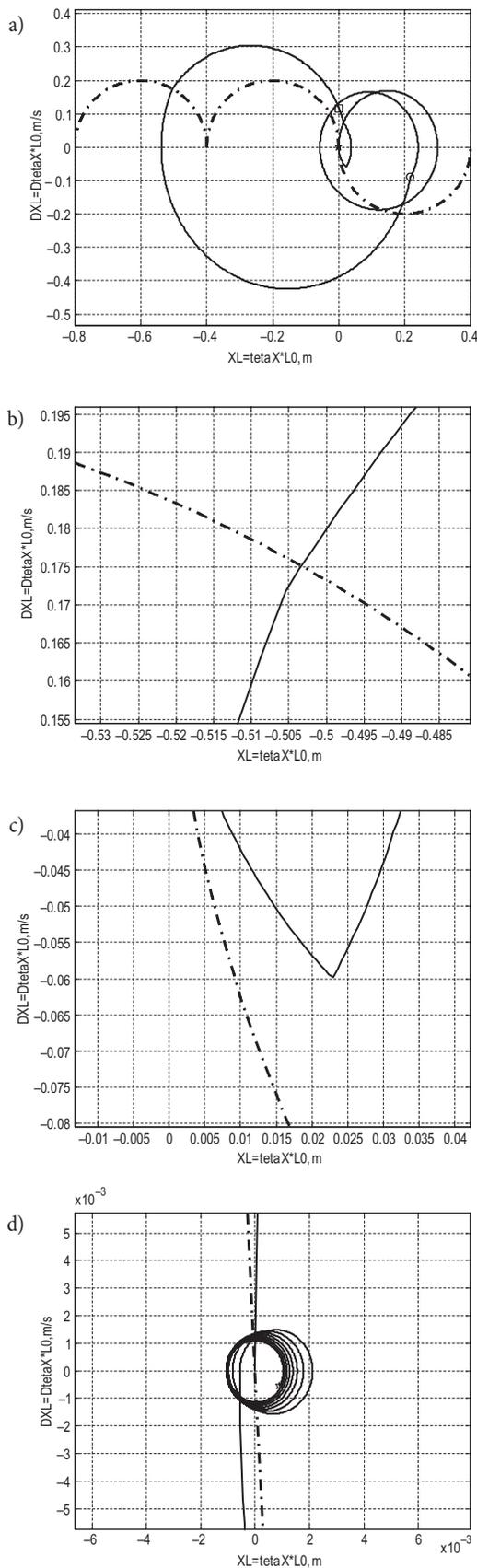


Fig. 10. The phase trajectory of the cargo in respect of the point of attachment on the model and the phase trajectories of changeover (a) 1, 2, 3 – the surrounding of the switching points (closed up); b – Point 1; c – Point 2; d – Point 3

6. If the dissipative properties of the crane's bridge are assessed, an analogous control scheme may be also applied for the engine of the crane's bridge-pushing mechanism, when the cargo is transported in the direction perpendicular to the bridge.
7. The theoretical results will be used for the development of crane control. This requires experimental tests carried out by AB 'Vilniaus kranai'.

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