TRANSPORT
ISSN 1648-4142 / eISSN 1648-3480
2016 Volume 31(4): 416–426
doi:10.3846/16484142.2014.915427

SETTINGS OF SHORT LEFT-TURN LANE AND SIGNAL PHASE SEQUENCE FOR ISOLATED SIGNALIZED INTERSECTIONS

Ronghan Yao

School of Transportation and Logistics, Dalian University of Technology, China

Submitted 27 October 2012; accepted 28 January 2013;
first published online 28 May 2014

Abstract. At signalized intersections, short left-turn lanes are often installed to improve capacity and level of service. However, the blockage or overflow of short left-turn lane often occurs when signal phasing is not coordinated with lane configuration and traffic demands. On the basis of probability theory, the probability of no blockage and overflow of the short left-turn lane is formulated under the three common signal phase plans when the lane next to a short left-turn lane is a through lane. For this case, it is shown that the left-through phasing should be adopted for the study approach when the short left-turn lane is very short and the volume of left-turn vehicles is high enough, and the leading or lagging left-turn phasing should be adopted for the study approach when the short left-turn lane is long enough and the volume of left-turn vehicles is low enough. To optimally allocate the space for each short left-turn lane and the green time for each lane group, a new optimization model is put forward to maximize intersection capacity and guarantee an acceptable level of service for each movement for isolated signalized intersections with short left-turn lanes. The usage of this model is demonstrated by an illustrative example. The results indicate that the intersection capacity can be maximized under the same level of service by integrating the configuration of traffic lanes and the split of signal phases. Finally, the procedure for using the proposed model is given for practical applications.

Keywords: traffic flow; isolated signalized intersections; short left-turn lanes; probability theory; optimization model.

Introduction

To enhance capacity and level of service at signalized intersections, exclusive left-turn lanes are usually added. Because of limited space, such left-turn lanes often exist in the form of short lanes (Akçelik 1998). They are called short left-turn lanes or left-turn bays (Yao 2013). In the past decades, many scholars were devoted to studying the operations of signalized intersections (Dion *et al.* 2004; Ban *et al.* 2011; Xuan *et al.* 2011). These studies do not consider the impact of short lanes on intersection operations because the short lanes are generally regarded as exclusive lanes (Highway Capacity Manual 2000). However, such a short lane has a significant effect on the saturation flow rate for the approach, and further, the capacities of the approach and intersection will be influenced. Recently, some researchers have focused on the short lanes. The relevant achievements can be classified into two aspects: the effect of short lanes on saturation flow rate, capacity or delay, and the determination of the required short-lane length.

Regarding the effect of short lanes on saturation flow rate, capacity or delay, the following progress has been achieved. For the lane group with a short lane, Akçelik (1998) stated that the saturation flow rate can decrease after the queue full discharge time for the short lane. Then, he gave the calculation formula of the saturation flow rate for the average movement. Wu (1999) found that the capacity of shared and short lanes at unsignalized intersections is overestimated by the conventional methods and that is underestimated by the shared lanes' formula, and then gave a more accurate capacity estimation equation based on probability theory. Subsequently, Tian and Wu (2006) considered the probabilistic nature of traffic flow and the effect of queue blockage on the short-lane section, and then proposed a capacity estimation model for a signalized intersection with a short right-turn lane in order to overcome one of the major shortcomings of the current capacity estimation methodologies. Wu (2007) also considered the stochastic nature of traffic flow and the effect of queue

Corresponding author: Ronghan Yao
E-mail: *cyanyrh@dlut.edu.cn*

blockage on the short turn lanes, and then developed a theoretical-empirical model to estimate the total approach capacity for signalized intersections with shared and short lanes. Most recently, Reynolds *et al.* (2010) proposed a computational approach for integrating the effects of short left-turn bays (or pockets) on sustained service rates in a mesoscopic modeling environment. Yin *et al.* (2011) presented theoretical delay models for protected left-turn operations at a pretimed signalized intersection with short left-turn bays during heavy traffic, and compared the given left-turn delay models with the Highway Capacity Manual (2000) delay model.

Regarding the determination of the required short-lane length, Kikuchi *et al.* (1993) first analyzed the required length of the left-turn lane at signalized intersections and presented the recommended lengths for different traffic conditions. To avoid lane overflow and blockage of lane entrance, they also proposed a procedure to determine the length of the Double Left-Turn Lanes (DLTLs) (Kikuchi *et al.* 2004). Then, they further examined the appropriate lengths of turn lanes when a single lane approaches a signalized intersection and is divided into three lanes: left-turn, through, and right-turn (Kikuchi *et al.* 2007). Additionally, Easa and Ali (2005) denoted that the previous guidelines overestimate the available sight distance so that the requirements for intersection elements are underestimated, and then modified the offset between opposing left-turn lanes, left-turn lane length and lane-line width by analyzing the actual available sight distance for left-turn vehicles. Qi *et al.* (2007) asserted that the left-turn lane should be designed to have a length sufficient to store the longest expected queue to prevent lane overflow, and proposed a new method to estimate the storage lengths of left-turn lanes at signalized intersections. Subsequently, they also evaluated the analytical-based traffic models and the simulation-based methods to determine the left-turn lane queue storage length and deceleration length (Qi *et al.* 2012). Yang and Zhou (2011) denoted that the design length of left-turn lanes is appropriate for unsignalized intersections, but it is inapplicable to signalized intersections because the approaches switch between stopped and unstopped by phase. Then, they proposed a new methodology that coordinates the requirement of each component with signal timing for the proper design of left-turn lanes.

The above-mentioned research achievements on short lanes indicate that the short lanes have a significant effect on intersection operations and should be well designed to improve intersection capacity and level of service. The objective of this paper is to investigate the effects of short lane space and signal phase sequence on the probability of no blockage and overflow of short left-turn lane and built a new optimization model to maximize intersection capacity and guarantee an acceptable level of service for each movement.

The rest of this paper is organized as follows:

- *probability of blockage or overflow of short left-turn lane:* formulating the probability of no blockage and overflow of short left-turn lane under the three common signal phase plans, and demonstrating the effects of short lane space and signal phasing on such a probability;
- *mathematical modeling for optimizing short lane space and signal timing:* modeling a new optimization model to maximize intersection capacity under a required maximum control delay for each movement, clarifying the usage of this model by an illustrative example, and presenting a procedure for using the proposed model for practical applications;
- *conclusions:* summarizing the research achievements and conclusions in this paper.

1. Probability of Blockage or Overflow of Short Left-Turn Lane

Without losing generality, assume that the study approach has a single left-turn lane and a single through lane, and consider three signal phase schemes: leading left-turn phasing, lagging left-turn phasing and left-through phasing. As shown in Fig. 1, leading left-turn phasing means that protected left-turn traffic goes through before the opposing through traffic. As shown in Fig. 2, lagging left-turn phasing means that protected left-turn traffic goes through after the opposing through traffic. As shown in Fig. 3, left-through phasing means that protected left-turn traffic and the adjacent through traffic go through during the same phase.

For the leading and lagging left-turn phasing, the signal cycle for one specific approach can be divided into three distinct subperiods, i.e. green phase for the left-turn movement and red phase for the through movement, green phase for the through movement and red phase for the left-turn movement, and red phase for both left-turn and through movements. For the left-through phasing, the signal cycle for one specific approach can be divided into two distinct subperiods, i.e. green phase for both left-turn and through movements, and red phase for both left-turn and through movements.

In addition, we assume that vehicles randomly arrive at an intersection. The random arrival of vehicles can be modeled using Poisson distribution. Before we proceed to formulate the models, we denote: D_1 – length of the study short left-turn lane; h – average queue spacing between adjacent vehicles; $N_1 = \text{fix}\ D_1 / h)$ – the maximum count of queued vehicles on the study short left-turn lane (fix means to round D_1 / h to the nearest integer towards zero); C – signal cycle length; λ_L – average arrival rate of left-turn vehicles; λ_T – average arrival rate of through vehicles; $L(\Delta t)$ = number of arrivals of left-turn vehicles during time interval Δt ; $T(\Delta t)$ – number of arrivals of through vehicles during time interval Δt.

For the leading and lagging left-turn phasing, the effective red time for both left-turn and through movements is: $r_{LT} = C - g_L - g_T$, where: g_L – effective green time for the left-turn movement; g_T – effective green time for the through movement.

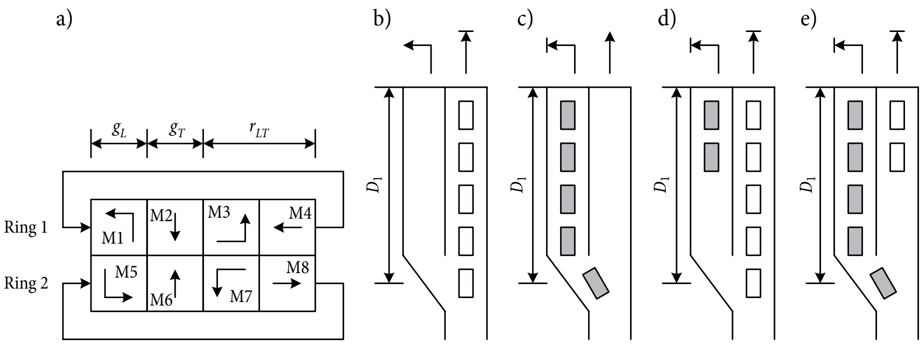


Fig. 1. Blockage and overflow of short left-turn lane under leading left-turn phasing: a – signal phase sequence; b – blockage of short left-turn lane during the green phase for the left-turn movement and red phase for the through movement; c – overflow of short left-turn lane during the green phase for the through movement and red phase for the left-turn movement; d – blockage of short left-turn lane during the red phase for both left-turn and through movements; e – overflow of short left-turn lane during the red phase for both left-turn and through movements

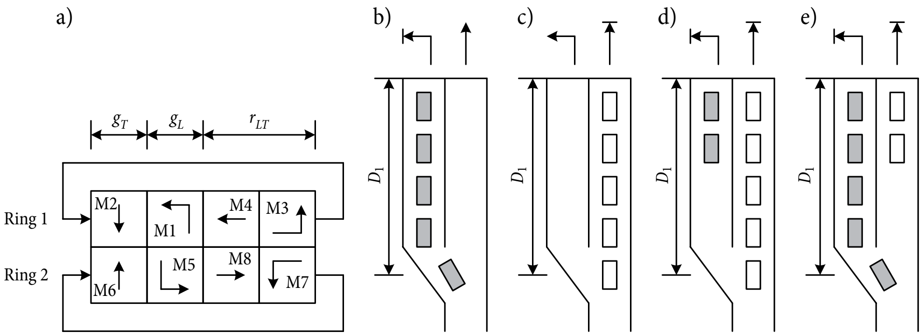


Fig. 2. Blockage and overflow of short left-turn lane under lagging left-turn phasing: a – signal phase sequence; b – overflow of short left-turn lane during the green phase for the through movement and red phase for the left-turn movement; c – blockage of short left-turn lane during the green phase for the left-turn movement and red phase for the through movement; d – blockage of short left-turn lane during the red phase for both left-turn and through movements; e – overflow of short left-turn lane during the red phase for both left-turn and through movements

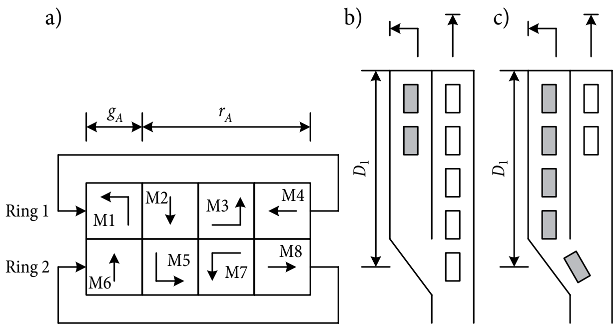


Fig. 3. Blockage and overflow of short left-turn lane under left-through phasing: a – signal phase sequence; b – blockage of short left-turn lane during the red phase for both left-turn and through movements; c – overflow of short left-turn lane during the red phase for both left-turn and through movements

For the left-through phasing, the effective red time for both left-turn and through movements is: $r_A = C - g_A$, where: g_A – effective green time for both left-turn and through movements.

In the following, we will formulate the probability of no blockage and overflow of the short left-turn lane during one cycle for each signal phase plan.

1.1. Leading Left-Turn Phasing

During the green phase for the left-turn movement and red phase for the through movement, the probability of the overflow of the short left-turn lane is $p_o^L = 0$, and the probability of the blockage of the short left-turn lane is:

$$p_b^L = P\left[T\left(g_L\right) \geq N_1\right] = 1 - \sum_{k=0}^{N_1-1} P\left[T\left(g_L\right) = k\right] = 1 - \sum_{k=0}^{N_1-1} \frac{\left(\lambda_T g_L\right)^k e^{-\lambda_T g_L}}{k!}. \quad (1)$$

Thus, the probability of no blockage and overflow of the short left-turn lane during this subperiod is:

$$p_n^L = 1 - p_b^L - p_o^L. \quad (2)$$

During the green phase for the through movement and red phase for the left-turn movement, the probability of the blockage of the short left-turn lane is $p_b^T = 0$, and the probability of the overflow of the short left-turn lane is:

$$p_o^T = P\left[L\left(g_T\right) \geq N_1\right] = 1 - \sum_{k=0}^{N_1-1} P\left[L\left(g_T\right) = k\right] = 1 - \sum_{k=0}^{N_1-1} \frac{\left(\lambda_L g_T\right)^k e^{-\lambda_L g_T}}{k!}. \quad (3)$$

Thus, the probability of no blockage and overflow of the short left-turn lane during this subperiod is:

$$p_n^T = 1 - p_b^T - p_o^T. \tag{4}$$

During the red phase for both left-turn and through movements, the probability of the blockage of the short left-turn lane is:

$$p_b^{LT} = P\left[T\left(r_{LT}\right) \geq N_1\right] P\left[L\left(g_T + r_{LT}\right) < N_1\right] = \left(\sum_{k=N_1}^{\infty} P\left[T\left(r_{LT}\right) = k\right]\right) \times \left(\sum_{k=0}^{N_1-1} P\left[L\left(g_T + r_{LT}\right) = k\right]\right) = \left(1 - \sum_{k=0}^{N_1-1} \frac{\left(\lambda_T r_{LT}\right)^k e^{-\lambda_T r_{LT}}}{k!}\right) \times \left(\sum_{k=0}^{N_1-1} \frac{\left(\lambda_L g_T + \lambda_L r_{LT}\right)^k e^{-\lambda_L\left(g_T + r_{LT}\right)}}{k!}\right) \tag{5}$$

and the probability of the overflow of the short left-turn lane is:

$$p_o^{LT} = P\left[L\left(g_T + r_{LT}\right) \geq N_1\right] P\left[T\left(r_{LT}\right) < N_1\right] = \left(\sum_{k=N_1}^{\infty} P\left[L\left(g_T + r_{LT}\right) = k\right]\right) \times \left(\sum_{k=0}^{N_1-1} P\left[T\left(r_{LT}\right) = k\right]\right) = \left(1 - \sum_{k=0}^{N_1-1} \frac{\left(\lambda_L g_T + \lambda_L r_{LT}\right)^k e^{-\lambda_L\left(g_T + r_{LT}\right)}}{k!}\right) \times \left(\sum_{k=0}^{N_1-1} \frac{\left(\lambda_T r_{LT}\right)^k e^{-\lambda_T r_{LT}}}{k!}\right). \tag{6}$$

Thus, the probability of no blockage and overflow of the short left-turn lane during this subperiod is:

$$p_n^{LT} = 1 - p_b^{LT} - p_o^{LT}. \tag{7}$$

Therefore, the probability of no blockage and overflow of the short left-turn lane during a signal cycle is:

$$p_n^{LTC} = p_n^L \frac{g_L}{C} + p_n^T \frac{g_T}{C} + p_n^{LT} \frac{r_{LT}}{C}. \tag{8}$$

1.2. Lagging Left-Turn Phasing

During the green phase for the through movement and red phase for the left-turn movement, the probability of the blockage of the short left-turn lane is $p_b^T = 0$, and the probability of the overflow of the short left-turn lane is shown in Eq. (3). Thus, the probability of no blockage and overflow of the short left-turn lane during this subperiod is shown in Eq. (4).

During the green phase for the left-turn movement and red phase for the through movement, the probability of the overflow of the short left-turn lane is $p_o^L = 0$, and the probability of the blockage of the short left-turn lane is shown in Eq. (1). Thus, the probability of no blockage and overflow of the short left-turn lane during this subperiod is shown in Eq. (2).

During the red phase for both left-turn and through movements, the probability of the blockage of the short left-turn lane is:

$$p_b^{TL} = P\left[T\left(g_L + r_{LT}\right) \geq N_1\right] P\left[L\left(r_{LT}\right) < N_1\right] = \left(\sum_{k=N_1}^{\infty} P\left[T\left(g_L + r_{LT}\right) = k\right]\right) \times \left(\sum_{k=0}^{N_1-1} P\left[L\left(r_{LT}\right) = k\right]\right) = \left(1 - \sum_{k=0}^{N_1-1} \frac{\left(\lambda_T g_L + \lambda_T r_{LT}\right)^k e^{-\lambda_T\left(g_L + r_{LT}\right)}}{k!}\right) \times \left(\sum_{k=0}^{N_1-1} \frac{\left(\lambda_L r_{LT}\right)^k e^{-\lambda_L r_{LT}}}{k!}\right) \tag{9}$$

and the probability of the overflow of the short left-turn lane is:

$$p_o^{TL} = P\left[L\left(r_{LT}\right) \geq N_1\right] P\left[T\left(g_L + r_{LT}\right) < N_1\right] = \left(\sum_{k=N_1}^{\infty} P\left[L\left(r_{LT}\right) = k\right]\right) \times \left(\sum_{k=0}^{N_1-1} P\left[T\left(g_L + r_{LT}\right) = k\right]\right) = \left(1 - \sum_{k=0}^{N_1-1} \frac{\left(\lambda_L r_{LT}\right)^k e^{-\lambda_L r_{LT}}}{k!}\right) \times \left(\sum_{k=0}^{N_1-1} \frac{\left(\lambda_T g_L + \lambda_T r_{LT}\right)^k e^{-\lambda_T\left(g_L + r_{LT}\right)}}{k!}\right). \tag{10}$$

Thus, the probability of no blockage and overflow of the short left-turn lane during this subperiod is:

$$p_n^{TL} = 1 - p_b^{TL} - p_o^{TL}. \tag{11}$$

Therefore, the probability of no blockage and overflow of the short left-turn lane during a signal cycle is:

$$p_n^{TLC} = p_n^T \frac{g_T}{C} + p_n^L \frac{g_L}{C} + p_n^{TL} \frac{r_{LT}}{C}. \tag{12}$$

1.3. Left-Through Phasing

During the green phase for both left-turn and through movements, the probability of no blockage and overflow of the short left-turn lane is $p_n^G = 1$.

During the red phase for both left-turn and through movements, the probability of the blockage of the short left-turn lane is:

$$p_b^R = P\left[T(r_A) \ge N_1\right] P\left[L(r_A) < N_1\right] = \left(\sum_{k=N_1}^{\infty} P\left[T(r_A) = k\right]\right) \times \left(\sum_{k=0}^{N_1-1} P\left[L(r_A) = k\right]\right) = \left(1 - \sum_{k=0}^{N_1-1} \frac{(\lambda_T r_A)^k e^{-\lambda_T r_A}}{k!}\right) \times \left(\sum_{k=0}^{N_1-1} \frac{(\lambda_L r_A)^k e^{-\lambda_L r_A}}{k!}\right) \quad (13)$$

and the probability of the overflow of the short left-turn lane is:

$$p_o^R = P\left[L(r_A) \ge N_1\right] P\left[T(r_A) < N_1\right] = \left(\sum_{k=N_1}^{\infty} P\left[L(r_A) = k\right]\right) \times \left(\sum_{k=0}^{N_1-1} P\left[T(r_A) = k\right]\right) = \left(1 - \sum_{k=0}^{N_1-1} \frac{(\lambda_L r_A)^k e^{-\lambda_L r_A}}{k!}\right) \times \left(\sum_{k=0}^{N_1-1} \frac{(\lambda_T r_A)^k e^{-\lambda_T r_A}}{k!}\right). \quad (14)$$

Thus, the probability of no blockage and overflow of the short left-turn lane during this subperiod is:

$$p_n^R = 1 - p_b^R - p_o^R. \quad (15)$$

Therefore, the probability of no blockage and overflow of the short left-turn lane during a signal cycle is:

$$p_n^{AC} = p_n^G \frac{g_A}{C} + p_n^R \frac{r_A}{C}. \quad (16)$$

1.4. Comparison of Different Combinations of Short Lane Space and Signal Phase Sequence

Before we will compare the performances of the three above-mentioned signal phase plans, the following criterion is introduced. The higher the probability of no blockage and overflow of the short left-turn lane during a signal cycle under a signal phase plan is, the better the signal phasing is for traffic demands. Otherwise, the signal phasing is worse.

As shown in Fig. 4, the effects of short lane space and signal phase sequence on the probability of no blockage and overflow of the short left-turn lane are illustrated by a numerical example. This example is based on the following assumptions:

– the average queue spacing between adjacent vehicles is 6 m;
– the cycle length is 120 s;
– for the leading and lagging left-turn phasing, the effective green time for the left-turn movement is 20 s, and the effective green time for the through movement is 40 s;
– consider three cases for the left-through phasing, Case 1 is $g_A = g_L$, Case 2 is $g_A = g_T$, and Case 3 is $g_A = (g_L + g_T)/2$;
– consider three levels of short lane space, i.e. 20, 50, and 80 m, representing the short, median and long levels;
– consider three levels of left-turn traffic, i.e. 100, 200, and 300 pcu/h, representing the low, median and high levels;
– the hourly volume of through vehicles ranges from 100 to 700 pcu/h with a 50 pcu/h increment.

As shown, when the short left-turn lane is very short (the short level) and the hourly volume of left-turn traffic is high enough (the median or high level), the left-through phasing is obviously better than the leading and lagging left-turn phasing. When the short left-turn lane is long enough (the median or long level) and the hourly volume of left-turn traffic is low enough (the low or median level), the leading and lagging left-turn phasing is basically better than the left-through phasing, and the leading left-turn phasing is basically better than the lagging left-turn phasing, especially for higher through volumes.

1.5. Discussions and Suggestions

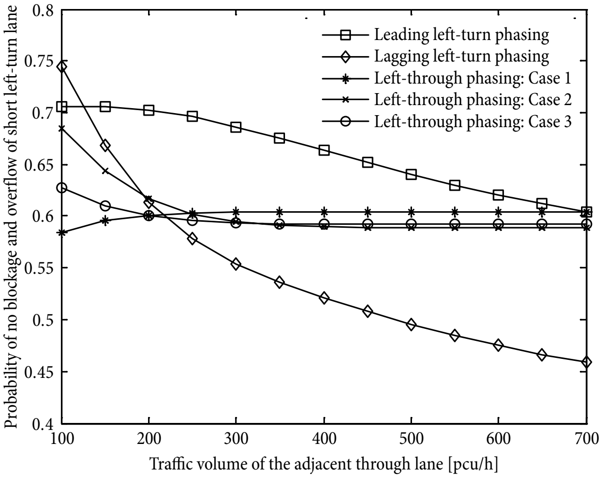
When adopting the left-through phasing, it is suggested that the through lane shown in Fig. 3 should be transformed into a shared left-through lane so that the probability of no blockage and overflow of the short left-turn lane can be increased. Additionally, it is suggested that the lane adjacent to the short left-turn lane should be designed as an exclusive left-turn lane so that the blockage or overflow of the short left-turn lane does not occur when the number of traffic lanes on the study approach is more than two and the hourly volume of left-turn traffic is high enough.

When left turns are permitted or signal phases overlap, the calculation of the probability of no blockage and overflow of the short left-turn lane during a signal cycle will become extremely complex. Therefore, we will develop a new optimization model to analyze the effects of short lane space and signal phase sequence on intersection operations in the next section.

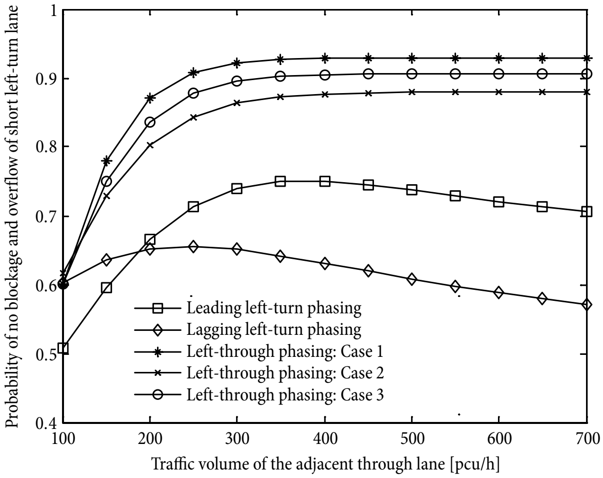
2. Mathematical Modeling for Optimizing Short Lane Space and Signal Timing

2.1. Intersection Capacity and Delay

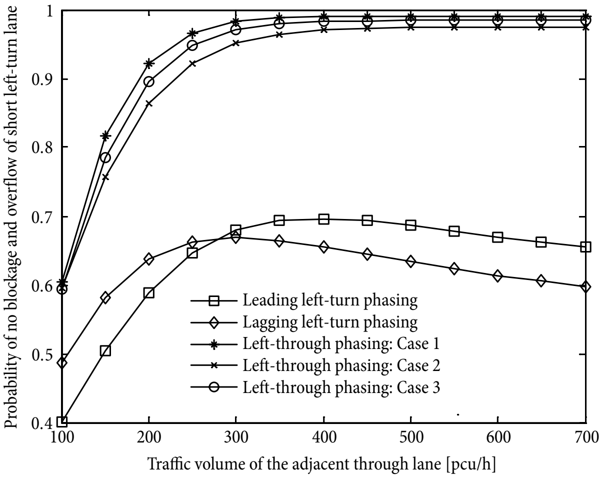
The existing literature shows that capacity and level of service are two significant performance indices to evaluate traffic flow operations at signalized intersections (Highway Capacity Manual 2000; Roess *et al.* 2010). According to Highway Capacity Manual (2000), level of service is a qualitative index and determined by intersection delay.



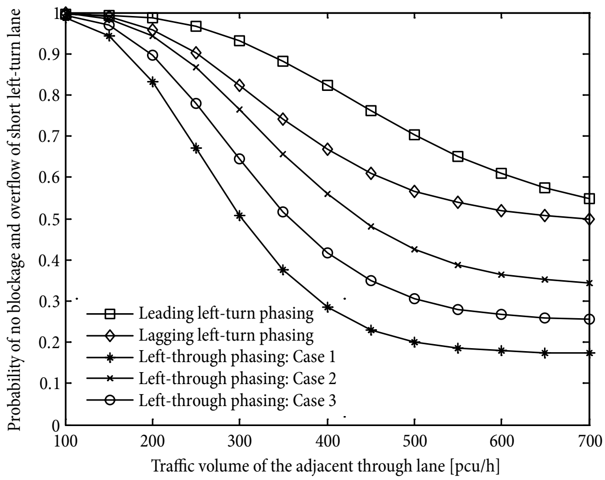
a) $D_1 = 20$ m, $\lambda_L = 100$ pcu/h



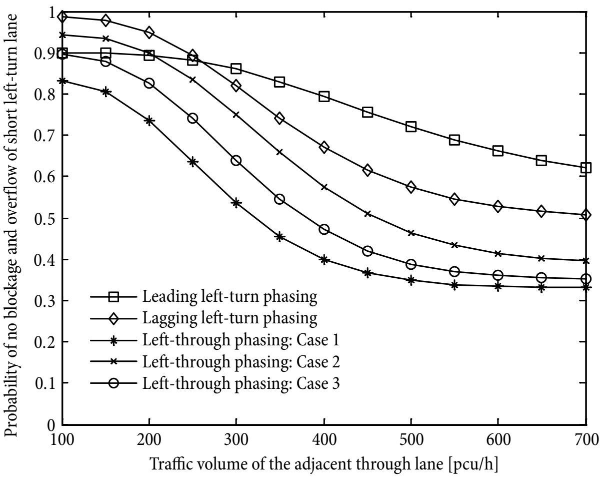
b) $D_1 = 20$ m, $\lambda_L = 200$ pcu/h



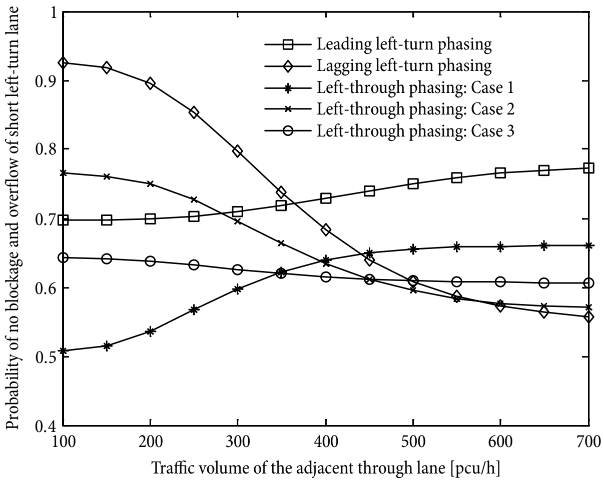
c) $D_1 = 20$ m, $\lambda_L = 300$ pcu/h



d) $D_1 = 50$ m, $\lambda_L = 100$ pcu/h

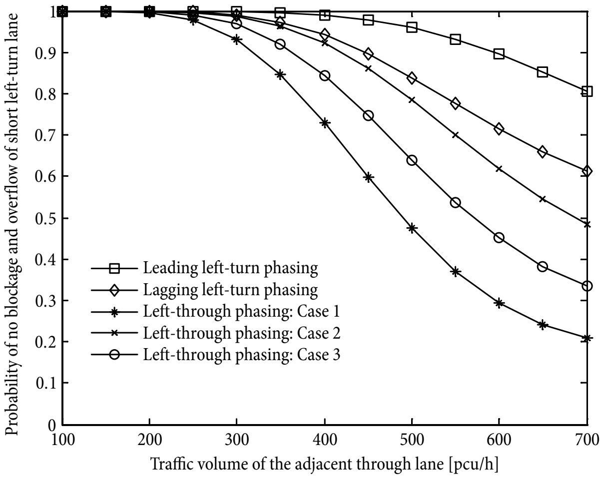


e) $D_1 = 50$ m, $\lambda_L = 200$ pcu/h

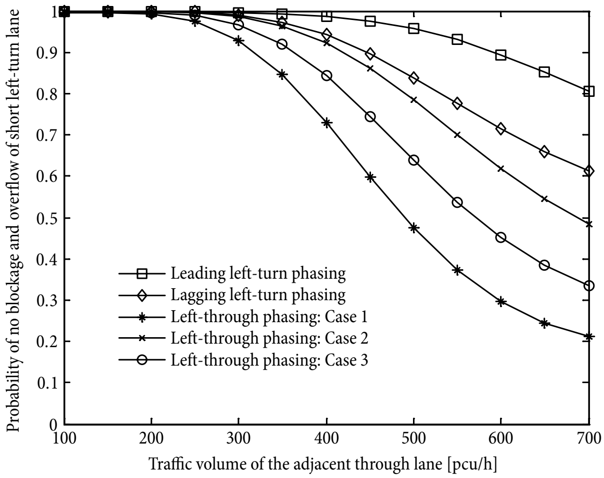


f) $D_1 = 50$ m, $\lambda_L = 300$ pcu/h

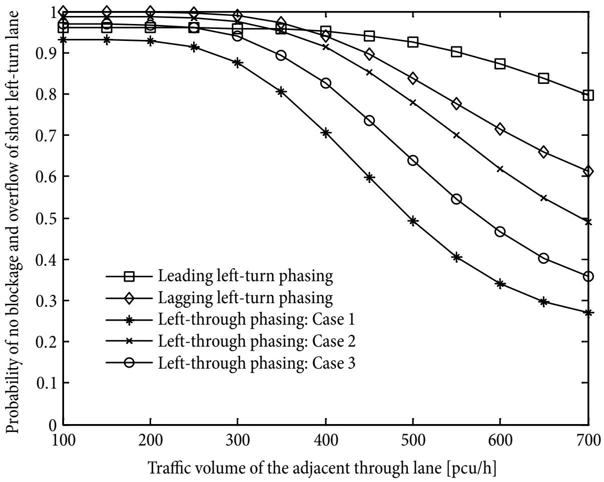
To be continued



g) $D_1 = 80$ m, $\lambda_L = 100$ pcu/h



h) $D_1 = 80$ m, $\lambda_L = 200$ pcu/h



i) $D_1 = 80$ m, $\lambda_L = 300$ pcu/h

Fig. 4. Comparison of the probabilities of no blockage and overflow of the short left-turn lane under different signal phase plans

The previous studies show that the saturation flow rate for the lane group with a short left-turn lane may not be a constant and is related to the effective green time for the lane group and the queue full discharge time for the short left-turn lane in the lane group (Yao 2013). Therefore, the intersection capacity can be calculated as:

$$Q = \sum_{j=1}^{m} Q_j \,, \tag{17}$$

where:

$$Q_j = \begin{cases} \dfrac{1}{C}\left(SF_j \left(\sum_{i=1}^{n} \phi_{ij} g_i \right) + \varphi_j SS_j g_j^c \right), & \sum_{i=1}^{n} \phi_{ij} g_i \ge g_j^c; \\ \dfrac{1}{C}\left(SF_j + \varphi_j SS_j \right) \left(\sum_{i=1}^{n} \phi_{ij} g_i \right), & \sum_{i=1}^{n} \phi_{ij} g_i < g_j^c; \end{cases}$$

$$C = \sum_{i=1}^{n} g_i + n_d l \,; \; g_j^c = D_j t / h \,;$$

Q – intersection capacity [pcu/h]; m – number of lane groups; Q_j – capacity of lane group j [pcu/h]; C – signal cycle length [s]; n – number of signal phases; g_i – effective green time for phase i [s]; n_d – number of discrete signal phases; l – lost time per phase [s]; SF_j – saturation flow rate for the full lanes in lane group j [pcu/h]; ϕ_{ij} – 0–1 variable to identify whether the movement in lane group j has right of way in phase i, if yes, $\phi_{ij} = 1$, otherwise, $\phi_{ij} = 0$; φ_j = 0–1 variable to identify whether a short left-turn lane exists in lane group j, if yes, $\varphi_j = 1$, otherwise, $\varphi_j = 0$; SS_j – saturation flow rate for the short left-turn lane in lane group j [pcu/h]; g_j^c – queue full discharge time for the short left-turn lane in lane group j [s]; D_j – length of the short left-turn lane in lane group j [m]; t – average saturation headway between adjacent vehicles [s]; h = average queue spacing between adjacent vehicles [m].

Also, the intersection delay can be calculated as:

$$d = \frac{\sum_{j=1}^{m} q_j d_j}{\sum_{j=1}^{m} q_j} \,, \tag{18}$$

where:

$$d_j = d_{1j}\left(PF\right) + d_{2j} + d_{3j} \,;$$

$$d_{1j} = \frac{0.5C\left(1-u_j\right)^2}{1-\min\left(1,x_j\right)u_j};$$

$$d_{2j} = 900T\left(\left(x_j-1\right)+\sqrt{\left(x_j-1\right)^2+\frac{8kIx_j}{Q_jT}}\right);$$

$$u_j = \frac{1}{C}\sum_{i=1}^{n}\phi_{ij}g_i;$$

$$x_j = \frac{q_j}{Q_j};$$

d – intersection delay [s/pcu]; q_j – arrival flow rate for lane group j [pcu/h]; d_j – average delay of vehicles passing through lane group j [s/pcu]; d_{1j} – average uniform delay of vehicles passing through lane group j [s/pcu]; d_{2j} – average incremental delay of vehicles passing through lane group j [s/pcu]; d_{3j} – average initial queue delay of vehicles passing through lane group j [s/pcu]; u_j – green ratio for lane group j; x_j – degree of saturation for lane group j; PF – uniform delay progression adjustment factor; T – duration of analysis period [h]; k – incremental delay factor that is dependent on controller settings; I – upstream filtering or metering adjustment factor.

2.2. Consideration of Constraints

To avoid residual queue on short left-turn lane, the effective green time for the lane group with a short left-turn lane should not be less than the queue full discharge time for the short left-turn lane in the lane group, namely:

$$\sum_{i=1}^{n}\phi_{ij}g_i \ge g_j^c,\ \forall\varphi_j = 1. \tag{19}$$

The effective green time for each lane group should not be less than the minimum effective green time, that is:

$$\sum_{i=1}^{n}\phi_{ij}g_i \ge g_{\min}, \tag{20}$$

where: $g_{\min}$ – minimum effective green time for each lane group [s].

The cycle length is the sum of the effective green time for all phases plus the total lost time per signal cycle. It should be between the minimum and maximum cycle lengths, namely:

$$C_{\min} \le \sum_{i=1}^{n}g_i + n_d l \le C_{\max}, \tag{21}$$

where: $C_{\min}$ – minimum cycle length [s]; $C_{\max}$ – maximum cycle length [s].

To guarantee an acceptable level of service for each lane group, the average delay of all vehicles passing through the lane group should not be greater than a specified maximum control delay, that is

$$d_j \le d_{\max}, \tag{22}$$

where: $d_{\max}$ – maximum control delay for each lane group [s].

The effective green time for each phase and the length of each short left-turn lane should all be non-negative, namely:

$$g_i \ge 0,\ D_j \ge 0. \tag{23}$$

2.3. The Optimization Model and Solution Algorithm

According to the above analysis, there should be a maximum intersection capacity under the aforementioned constraints. Therefore, the optimization problem is to maximize the objective function as Eq. (17) under the constraints as Eqs (19), (20), (21), (22), and (23), namely:

$$\min\ -Q = -\sum_{j=1}^{m}\left(SF_j\left(\sum_{i=1}^{n}\phi_{ij}g_i\right)+\varphi_j SS_j D_j t/h\right)\Bigg/\left(\sum_{i=1}^{n}g_i + n_d l\right),$$

$$\text{s. t. } \sum_{i=1}^{n}\phi_{ij}g_i \ge D_j t/h,\ \forall\varphi_j = 1;$$
$$\sum_{i=1}^{n}\phi_{ij}g_i \ge g_{\min};$$
$$C_{\min} \le \sum_{i=1}^{n}g_i + n_d l \le C_{\max};$$
$$d_j \le d_{\max};$$
$$g_i \ge 0,\ D_j \ge 0. \tag{24}$$

Eq. (24) is a single-objective optimization problem which attempts to find a constrained minimum of a function of several variables. The *fmincon* function provided by the language of technical computing MATLAB is one of the tools to solve this kind of problem. Thus, the proposed optimization model can be directly solved using the *fmincon* function.

2.4. Calibration of Parameters in the Optimization Model

On the basis of the relevant literature (Roess *et al.* 2010; Yao 2013), the amber and all-red times (y and ar) are set to be 3 s and 2 s, respectively, and the start-up and clearance lost times per phase (l_s and l_c) are set to be 1.3 s and 2.2 s, respectively. The relationship between the effective and actual green times for a lane group is: $l_s + g_j + l_c = G_j + y + ar$, where: g_j – effective green time for lane group j [s]; G_j – actual green time for lane group j [s].

We assume that the study intersection is controlled by pretimed signals and the traffic arrivals are random. Thus, a value of 0.5 for k, and a value of 1.0 for I and PF are used. Also, we assume that there is no residual queue from a previous period at the start of the analysis period and the duration of the analysis period is 1 h. Therefore, d_{3j} is set to be 0 and T is set to be 1.0.

According to the Guidelines for Traffic Signals (RiLSA) (Road and Transportation Research Association 2003), the minimum effective green time for each lane group is set to be 10 s. According to Highway Capacity Manual (2000), the minimum and maximum cycle lengths are set to be 60 s and 150 s, respectively, and the maximum control delay for each lane group is set to be 55 s.

2.5. An Illustrative Example

The following example is designed to state the settings of short lane space and signal phase sequence at isolated signalized intersections with short left-turn lanes. Fig. 5 shows the channelization scheme and three signal phase plans for a four-leg intersection. Here, right turns are assumed to proceed concurrently with the through traffic. Table 1 lists the saturation flow rates, peak 15-minute flow rates and hourly volumes for all the lane groups at the intersection during a peak-hour period. As shown in Fig. 5a, there is a short left-turn lane, an exclusive left-turn lane, a through lane and a shared through-right lane on each approach. The three signal phase plans are designed on the basis of traffic demands during the peak period. Fig. 5b is the exclusive left-turn phasing, Fig. 5c is the left-through phasing, and Fig. 5d is the overlapping phasing which includes the leading and lagging green phasing in the south-north direction and the exclusive left-turn plus leading green phasing in the east-west direction (Roess *et al.* 2010). In Fig. 5, M1, M2, M3, M4, M5, M6, M7 and M8 are the codes of the lane groups.

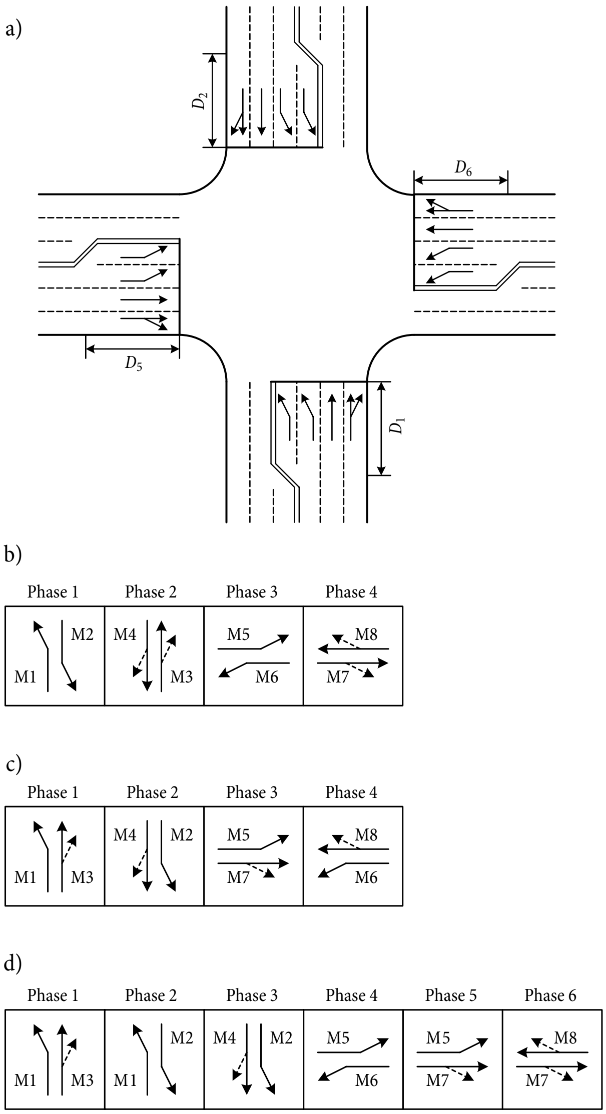


Fig. 5. Channelization scheme and signal phase plans for a four-leg intersection: a – channelization scheme; b – exclusive left-turn phasing; c – left-through phasing; d – overlapping phasing

To consider the fluctuation of traffic stream during the peak-hour period, we adopt the peak 15-minute flow rates to solve the proposed optimization model in order to obtain the optimal combination of lane space and green splits. On the other hand, to evaluate the operational situation of traffic stream during the peak-hour period, we adopt the hourly volumes to measure the performance indices of the channelization and signal phase schemes.

Table 2 shows the optimization outcomes under the three signal phase plans. It is shown that the intersection capacity under the overlapping phasing is the maximum, that under the exclusive left-turn phasing takes the second place, and that under the left-through phasing is the minimum when the level of service for the intersection is controlled by D.

Table 3 shows the design parameters under the three signal phase plans which can be directly used in practice. It is shown that the overlapping phasing can more flexibly design the short left-turn lane and the green split for each lane group than the exclusive left-turn phasing and the left-through phasing.

The above-mentioned outcomes are obtained via a 1-h analysis period. As we all know, multiple analysis periods are usually created for signalized intersections because traffic flow fluctuates within the whole day. Different signal timing plans are adopted for different analysis periods, but the channelization scheme for an intersection is often constant during a longer period, such as several months or years. Therefore, the procedure for using the proposed model is presented for practical applications, as shown in Fig. 6.

Table 1. Saturation flow rates, peak 15-minute flow rates and hourly volumes

Lane group	Saturation flow rate [pcu/h]*		Peak 15-minute flow rate [pcu/h]	Hourly volume [pcu/h]
	Full lanes	Short lanes		
M1	1810	1810	650	600
M2	1810	1810	600	500
M3	3660	–	500	400
M4	3660	–	400	300
M5	1810	1810	550	500
M6	1810	1810	350	300
M7	3660	–	750	700
M8	3660	–	500	400

Note: *These values are assumed on the basis of the research by Akçelik (1998).

Table 2. Optimization outcomes under different signal phase plans

Parameters		Exclusive left-turn phasing	Left-through phasing	Overlapping phasing
Effective green time [s]	Phase 1	31.05	30.07	30.68
	Phase 2	26.53	28.61	4.41
	Phase 3	28.15	32.76	29.25
	Phase 4	33.74	25.54	26.80
	Phase 5	–	–	5.44
	Phase 6	–	–	32.29
Cycle length [s]		133.46	130.98	142.86
Short-lane length [m]	Northbound	93.15	90.22	105.25
	Southbound	93.15	85.83	100.96
	Eastbound	84.44	98.28	96.74
	Westbound	84.44	76.61	80.41
Intersection capacity [pcu/h]		6516.66	6501.87	6566.90
Intersection delay [s/pcu]		49.96	49.14	52.15
Degree of saturation *		0.76	0.76	0.72
Level of service **		D	D	D

Notes: *It is referred to as the degree of saturation for the intersection and the maximum among the degrees of saturation for all the lane groups at the intersection;
**Based on Highway Capacity Manual (2000), the level of service is D when the control delay for an intersection is greater than 35 s and not greater than 55 s.

Table 3. Design parameters under different signal phase plans

Parameters		Exclusive left-turn phasing	Left-through phasing	Overlapping phasing
Cycle length [s] *		133	131	143
Actual green time [s]**	M1	30	29	34
	M2	30	27	32
	M3	25	29	29
	M4	25	27	28
	M5	27	31	31
	M6	27	24	25
	M7	32	31	36
	M8	32	24	31
Short-lane length [m]***	Northbound	96	96	108
	Southbound	96	90	102
	Eastbound	90	102	102
	Westbound	90	78	84

Notes: *The design value of cycle length is obtained from its calculated value by omitting decimal fractions smaller than 0.5 and counting all others, including 0.5, as 1;
**The design value of actual green time is obtained from its calculated value by omitting decimal fractions smaller than 0.5 and counting all others, including 0.5, as 1. The calculated value of actual green time is the effective green time plus the lost time per phase and then minus the summation of the amber and all-red times;
***The design value of short-lane length rounds its calculated value to the nearest integer multiple of the average queue spacing between adjacent vehicles towards infinity.

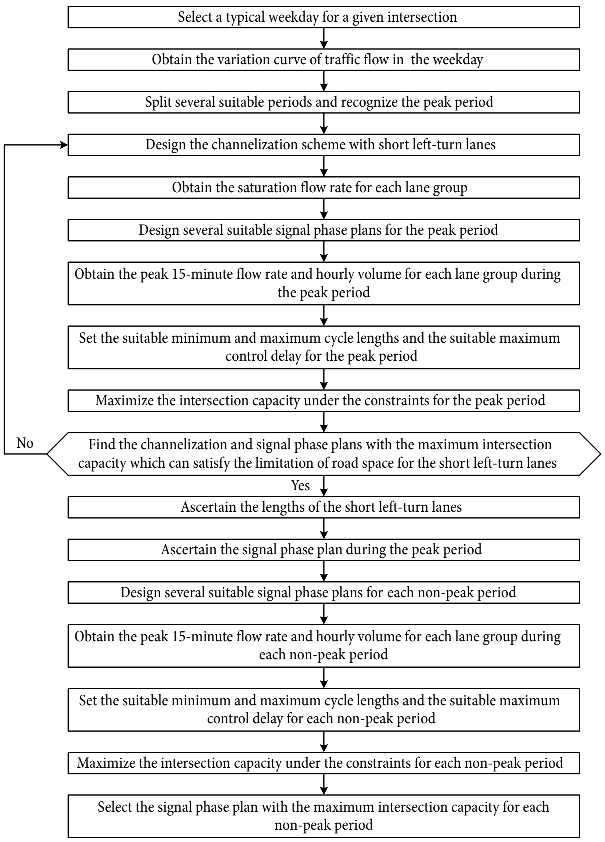


Fig. 6. Procedure for using the proposed model

Conclusions

Short left-turn lanes are often installed to improve capacity and level of service at signalized intersections. However, the blockage or overflow of the short left-turn lane often occurs when the lane next to a short left-turn lane is a through lane. Based on probability theory, the probability of no blockage and overflow of the short left-turn lane is formulated under the three common signal phase plans, i.e. the leading left-turn phasing, the lagging left-turn phasing and the left-through phasing. The numerical example is demonstrated to investigate the effects of short-lane length and left-turn volume on the probability of no blockage and overflow of the short left-turn lane. The numerical results indicate that the left-through phasing should be adopted for the study approach when the short left-turn lane is very short and the volume of left-turn vehicles is median or high, and the leading or lagging left-turn phasing should be adopted for the study approach when the short left-turn lane is long enough and the volume of left-turn vehicles is low enough. When the left-through phasing is adopted for the study approach, it is recommended that the lane adjacent to the short left-turn lane should be designed as a shared left-through lane in order to decrease the probability of the blockage or overflow of the short left-turn lane. When the number of traffic lanes on the study approach is more than two, it is recommended that the lane adjacent to the short left-turn lane should be designed as an exclusive left-turn lane in order to avoid the blockage or overflow of the short left-turn lane.

To optimally allocate the space for each short left-turn lane and the green time for each lane group, a new optimization model is developed to maximize intersection capacity under a specified level of service for each movement for isolated signalized intersections with short left-turn lanes. Then, an illustrative example is given to explore the effects of different channelization and signal phase schemes on intersection operations. According to traffic demands during the peak period, the three reasonable signal phase plans are designed. It is shown that the intersection capacity can be increased by adopting the overlapping phasing under the same level of service. In practice, the optimal combination of short lane space and green splits should be determined by applying the proposed procedure.

Acknowledgements

This research has been funded by the National Natural Science Foundation of China (Grant No. 50808035).

References

Akçelik, R. 1998. *Traffic Signals: Capacity and Timing Analysis*. Australian Road Research Board (ARRB) Transport Research Ltd, Victoria, Australia. 108 p.

Ban, X.; Hao, P.; Sun, Z. 2011. Real time queue length estimation for signalized intersections using travel times from mobile sensors, *Transportation Research Part C: Emerging Technologies* 19(6): 1133–1156. http://dx.doi.org/10.1016/j.trc.2011.01.002

Dion, F.; Rakha, H.; Kang, Y.-S. 2004. Comparison of delay estimates at under-saturated and over-saturated pre-timed signalized intersections, *Transportation Research Part B: Methodological* 38(2): 99–122. http://dx.doi.org/10.1016/S0191-2615(03)00003-1

Easa, S.; Ali, M. 2005. Modified guidelines for left-turn lane geometry at intersections, *Journal of Transportation Engineering* 131(9): 677–688. http://dx.doi.org/10.1061/(ASCE)0733-947X(2005)131:9(677)

Highway Capacity Manual. 2000. Transportation Research Board (TRB), National Research Council. Washington, U.S.A. 1134 p.

Kikuchi, S.; Chakroborty, P.; Vukadinovic, K. 1993. Lengths of left-turn lanes at signalized intersections, *Transportation Research Record* 1385: 162–171.

Kikuchi, S.; Kii, M.; Chakroborty, P. 2004. Lengths of double or dual left-turn lanes, *Transportation Research Record* 1881: 72–78. http://dx.doi.org/10.3141/1881-09

Kikuchi, S.; Kronprasert, N.; Kii, M. 2007. Lengths of turn lanes on intersection approaches: three-branch fork lanes – left-turn, through, and right-turn lanes, *Transportation Research Record* 2023: 92–101. http://dx.doi.org/10.3141/2023-10

Qi, Y.; Guo, L.; Yu, L.; Teng, H. 2012. Estimation of design lengths of left-turn lanes, *Journal of Transportation Engineering* 138(3): 274–283. http://dx.doi.org/10.1061/(ASCE)TE.1943-5436.0000299

Qi, Y.; Yu, L.; Azimi, M.; Guo, L. 2007. Determination of storage lengths of left-turn lanes at signalized intersections, *Transportation Research Record* 2023: 102–111. http://dx.doi.org/10.3141/2023-11

Reynolds, W. L.; Zhou, X.; Rouphail, N. M.; Li, M. 2010. Estimating sustained service rates at signalized intersections with short left-turn pockets: mesoscopic approach, *Transportation Research Record* 2173: 64–71. http://dx.doi.org/10.3141/2173-08

Road and Transportation Research Association. 2003. *Guidelines for Traffic Signals (RiLSA)*. 182 p.

Roess, R. P.; Prassas, E. S.; McShane, W. R. 2010. *Traffic Engineering*. 4th edition, Prentice Hall. 744 p.

Tian, Z. Z.; Wu, N. 2006. Probabilistic model for signalized intersection capacity with a short right-turn lane, *Journal of Transportation Engineering* 132(3): 205–212. http://dx.doi.org/10.1061/(ASCE)0733-947X(2006)132:3(205)

Wu, N. 2007. Total approach capacity at signalized intersections with shared and short lanes: generalized model based on a simulation study, *Transportation Research Record* 2027: 19–26. http://dx.doi.org/10.3141/2027-03

Wu, N. 1999. Capacity of shared-short lanes at unsignalized intersections, *Transportation Research Part A: Policy and Practice* 33(3–4): 255–274. http://dx.doi.org/10.1016/S0965-8564(98)00041-X

Xuan, Y.; Daganzo, C. F.; Cassidy, M. J. 2011. Increasing the capacity of signalized intersections with separate left turn phases, *Transportation Research Part B: Methodological* 45(5): 769–781. http://dx.doi.org/10.1016/j.trb.2011.02.009

Yang, J.; Zhou, H. 2011. Integrating left-turn lane geometric design with signal timing, *Journal of Transportation Engineering* 137(11): 767–774. http://dx.doi.org/10.1061/(ASCE)TE.1943-5436.0000282

Yao, R. 2013. Sensitivity analysis of optimization models for isolated intersections with short left-turn lanes on approaches, *Journal of Advanced Transportation* 47(1): 28–42. http://dx.doi.org/10.1002/atr.1185

Yin, K.; Zhang, Y.; Wang, B. X. 2011. Modeling delay during heavy traffic for signalized intersections with short left-turn bays, *Transportation Research Record* 2257: 103–110. http://dx.doi.org/10.3141/2257-12