

## STUDY ON DYNAMIC INFLUENCE OF PASSENGER FLOW ON INTELLIGENT BUS TRAVEL SERVICE MODEL

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**Abstract.** To improve the service quality and convenience of bus travel services, this paper proposes the Intelligent Bus Travel Service Model (IBTSM). The IBTSM makes it possible to provide a travel strategy considering every aspect of bus travel, specifically, delay in the peak period arising from limited carrying-capacities of buses. A three-step approach was executed toward implementing the IBTSM. First, the bus travel-time was predicted using Long Short-Term Memory (LSTM). Next, the crowding level in the bus was evaluated using a fuzzy expert system, based on which a reasonable start-off time was planned, and the delay caused by large passenger flow was circumvented. The *k*-Nearest Neighbours (*k*-NN) algorithm was used to provide input data of passenger flow. In this study, the correlation between passenger flow variation and bus services was investigated to extend the provisions of the travel strategy to include start-off time scheduling and target bus selection, rather than only bus running-time estimation. The proposed model was evaluated using a bus in China as a case study, and its reliability and positive impact on promoting both the quality of bus services and development of intelligent travel were demonstrated.

**Keywords:** intelligent bus travel service, urban travel, advanced public transportation system, passenger flow, travel-time prediction, fuzzy expert system inference.

### Introduction

With increased urbanization in China, the urban bus has become a very important mean of transportation. Its advantages include low charges, wide coverage, and large capacity. However, some shortcomings of the urban bus, such as unstable travel-time and uncertainty of time interval between buses, are significant bottlenecks. The conception of the Intelligent Transportation System (ITS) (Gerland 1993; Chen 2017) is a good solution for these problems. As a subset of the ITS, the Advanced Public Transportation System (APTS) deserves further examination, because of the prevalence of bus travel (Kumar *et al.* 2018). The APTS provides bus passengers with useful information such as travel-time, vehicle positioning, transfer information, and route planning. However, it is necessary to develop an approach to utilizing this information to make the urban bus more attractive and convenient for bus passengers. In contemporary China, it is not only the traffic situation that can affect bus travel, the passenger flow is also a key factor (Mazloumi *et al.* 2010). Although it is rather common for passengers to get stranded in the peak periods, scant study has focused on reducing time

uncertainty to improve travel experiences. Therefore, it is necessary to improve the comprehensiveness and effectiveness of the ITS and the APTS.

This study establishes an integrated model named Intelligent Bus Travel Service Model (IBTSM) to make the entire bus trip a smooth one for passengers, especially those who travel at peak period. The IBTSM aims to enhance bus travel experience by accurate bus travel-time estimation incorporating the evaluation of the dynamic variation in passenger flow. Although the traffic prediction and passenger flow received much attention, the integrated systems have hardly been applied for improving bus travel service. It means that the topic of traffic prediction and passenger flow are always discussed individually. While, the travel service information provided by the IBTSM takes both the bus travel-time and variation of passenger flow into account. In relation to the travel-time, the accuracy of the travel service information can be enhanced by a more suitable approach; and on the basis of passenger flow, this paper proposes a new method that improves APTS integrity for bus passengers'

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start-off time decision. Most passengers highly desire to arrive their destination before a specific time. However, the delay in peak period, arising from the limited load capacity, disturbs the plans to different extent (Dimitrov *et al.* 2017). The IBTSM, being capable of travel strategy formulation, can fulfil this requirement. The service is intended to ensure that passengers arrive on time and save them unnecessary waiting time. The content of a travel strategy includes start-off time, route planning, target bus selection, and Estimated Time of Arrival (ETA), making it possible for the passengers to receive these above-mentioned instructions in advance. Therefore, the IBTSM devotes to improve the bus passengers' travel experiences, provides visualized bus travel-planning, and motivates further development of the ITS.

The rest of this paper is organized as follows: Section 1 reviews previous studies on bus travel service. Section 2 describes the characteristics of the IBTSM. In Section 3, the IBTSM framework is established. Section 4 demonstrates the reliability and universality of the IBTSM. The conclusions are drawn in the final section.

## 1. Literature review

One of the most popular applications of the ITS is in the bus system. As the basis of the ITS, the provision of travel-time is an important issue (Chen 2017; Yu *et al.* 2006; Yang *et al.* 2016). Because of the overall dynamics of the transportation system, there is often a disparity between the actual bus travel-time and the scheduled time (Hua *et al.* 2018). Bus travel-time prediction has attracted extensive scholarly attention. Early related studies were always implemented through statistical-based methods. Patnaik *et al.* (2004) establish a multivariate regression model for estimating the bus arrival time at all downstream stops using the automatic passenger counting system. Sun *et al.* (2007) developed a historical average algorithm to determine the bus arrival time. They also applied time-series analysis to predict travel-time based on the correlation between time-series and travel-time among historical data. Williams and Hoel (2003) implemented a seasonal AutoRegressive Integrated Moving Average (ARIMA) model aimed at predicting short-term traffic based on the internal relationship taken from historical data. However, although the above-mentioned models are easy to calibrate, establish, and understand, their prediction accuracy is relatively low (Chang *et al.* 2010).

Based on computer techniques, some complex algorithms, such as Kalman filtering (Cathey, Dailey 2003; Shalaby, Farhan 2004), Support Vector Machine (SVM) (Yu *et al.* 2006, 2010; Huang, Ran 2003), and Artificial Neural Network (ANN) (Chen *et al.* 2004; Wang *et al.* 2014; Huang, Ran 2003; Yu *et al.* 2011), have been developed and applied to the area of travel-time prediction. Among all, the ANN has proven to be one of the most widely used and efficient prediction algorithm, because of its good prediction accuracy and strong capacity to deal with complex non-linear problems (Patnaik *et al.* 2004;

Lin *et al.* 2013); however, some internal parameters and transfer functions in the ANN model are determined empirically. The easily implemented and commonly used multi-layer perceptron-type ANN was selected to conduct travel-time prediction based on the time-of-day, day-of-week, and weather conditions. The results generally demonstrated better indication of the bus-arrival time between two adjacent time points, compared with the timetable (Chen *et al.* 2004). A radial basis function neural network model was established by Wang *et al.* (2014) to learn the historical bus travel data that was considered the basis of the subsequent online adjustment. This approach achieved better predicting performance, compared to the linear regression and Back-Propagation (BP) neural networks. A more sophisticated ANN approach that considers passenger flow was developed by Amita *et al.* (2016); this approach outperforms the traditional method. Xu *et al.* (2019) presented an ANN-based travel-time prediction method that incorporates spatial-temporal relevancy to infer the travel-time distribution; they achieve relatively high prediction accuracy with low expenditure. To improve the prediction accuracy, some revised algorithms, such as the Bayesian inference theory (Van Hinsbergen *et al.* 2009) and particle swarm optimization (Ji *et al.* 2016), are combined with the ANN. These combined algorithms achieve vastly better prediction performance than the separate ones. However, although these methods considerably improve the accuracy of the original models, they do not overcome the fundamental drawback of various types of the ANN algorithm that assumes that the sample data is independent (Zhou *et al.* 2017). As a result, the prediction accuracy was decreased as the data-to-data relationship was neglected.

Generally, previous studies on bus travel-time focused solely on bus driving. Moreover, a majority of them were described from the perspective of technological progress. However, receiving accurate bus arrival information is also important to passengers to reduce their anxieties and waiting time at the bus stop (Yu *et al.* 2011). One of the most critical problems is the delay caused by a large number of passengers. Previous studies on bus passenger flow mainly focused on providing a reasonable schedule (Zhao *et al.* 2011; Zhang *et al.* 2017), and improving the efficiency of the public transportation service (Liu *et al.* 2017; Bai *et al.* 2017) from the perspective of bus companies. Gradually, some scholars began to consider the influence of huge crowds on traffic and passenger delay. Nagatani (2001) explores the interaction between buses and passengers, and stresses that scant attention has been paid the necessity of devising measures to minimize passenger delay, a matter of great significance to the public. Dimitrov *et al.* (2017) conclude that the load does not impact average passenger waiting time at bus stops when the public transport passenger load was not close to bus capacity. Ding and Xu (2016) constructed a forecasting method for passenger flow based on the theory of angle expenses to obtain the waiting-passenger distribution. The results indicate that delay to the passenger would disrupt travel

plans, and lead to changes in transportation modes. Although bus scheduling can solve the problem of passenger delay, no effective method has been proposed considering the passengers. Therefore, this study aims to explore the problem of passenger delay from the perspective of bus passengers. This is achieved by guiding passengers on planning their start-off time and itinerary via the IBTSM to minimize the difference between the actual arrival time and the expected arrival time as much as possible.

## 2. Model description

### 2.1. Research scope

This study covers the entire process of traveling by bus. This process is divided into four parts: “walking to the bus stop”, “waiting for the bus”, “bus driving”, and “walking to the terminal”. The process is illustrated in Figure 1. In this figure,  $Bus_i$  is the target bus recommended by the IBTSM for the passengers.  $Bus_{i-1}$  is the bus immediately before  $Bus_i$ . Similarly,  $Bus_{i-2}$  is the bus preceding  $Bus_{i-1}$ . “Bus driving” in this study refers to the bus running-time from  $Stop_m$  to  $Stop_n$ , in which,  $Stop_m$  represents a bus station located between the origin stop and the terminal stop of a bus line. Therefore,  $Stop_m$  is defined as the passenger’s departure bus stop, while  $Stop_n$  is the passenger’s arrival stop. The goal of the IBTSM is for the passengers to board the target bus, and not be stranded at the station. The earlier they arrive at the station, the higher the possibility of boarding the target bus in an orderly manner. “Departure” refers to the departure of buses, and “start-off” refers to the moment the passengers set out. With the objective of reducing the uncertainties of bus travel, the characteristics of the four components are systematically analysed. “Walking to bus stop” and “walking to terminal” refer to the process of walking to the bus stop and walking to the destination, respectively. To simplify the model, it is assumed that the speed of walking to the bus stop and walking to the terminal is constant at 1.2 m/s. Therefore, the walking time  $t_w$  was assumed to be prone to few uncertainties and be easily measured by Equation (1):

$$t_w = \frac{S}{\bar{v}}, \tag{1}$$

where:  $\bar{v}$  represents the constant speed;  $S$  is the walking distance between the start-off position and  $Stop_m$ .

Compared with the two walking processes, bus travel period is characterized by great uncertainty, because the transportation system, with people involved and its time variability, is a complex system. It is subject to timing, multiple influencing factors, and non-linearity (Marfia, Roccetti 2011; Chan *et al.* 2012). Therefore, a bus travel prediction approach was applied to explore its underlying rule.

With increasing attention on APTS construction, on-board Global Positioning System (GPS) positioning facility for urban buses has become a standard configuration in many cities in China. Although this significantly eliminates waiting time uncertainty, large passenger flow during peak period increases the degree of congestion, and might even result in the passenger failing to get on the approaching bus. Thus, with the aim of providing a practical travel strategy, the dynamic passenger flow was taken into consideration as well. The framework of the IBTSM, according to the scope of the study, is presented in the next section.

### 2.2. The IBTSM description

The goal of the IBTSM is to enable bus passengers to get on the target bus. There are two key problems to be solved. The first is identifying the target bus. The second is determining how the passenger can board the target bus successfully. The target bus can be identified by the bus travel-time prediction. However, the second problem depends on the dynamic influence of the passenger flow. Because it is hard to change the bus schedule to minimize the delay at a bus stop, the desired effect can be achieved by guiding the passengers’ behaviour. Therefore, the framework of the IBTSM is designed to solve these two key problems. The IBTSM is a data-driven model. The function of the IBTSM is realized by analysing large amounts of historical data. The function realization process of the entire model decomposes the bus travel process, and runs it step-by-step according to the goals of each stage. Generally, the boarding time is initially deduced through the ETA. Next, the start-off time is determined based on the influence of dynamic passenger flow. Then, the simplified walking time is calculated. Ultimately, the acquired information is published, and travellers are provided with travelling strategies.

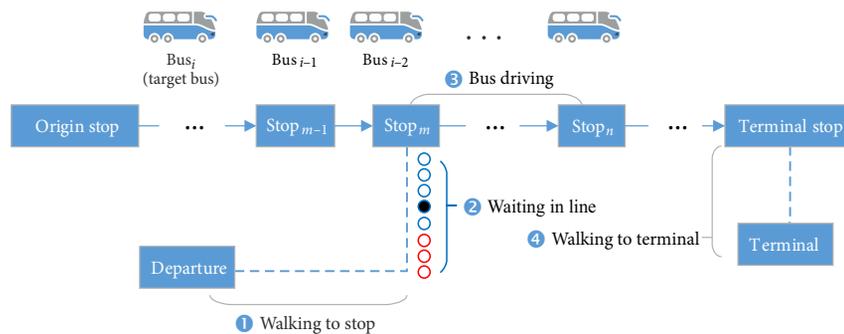


Figure 1. Overall process of bus travel

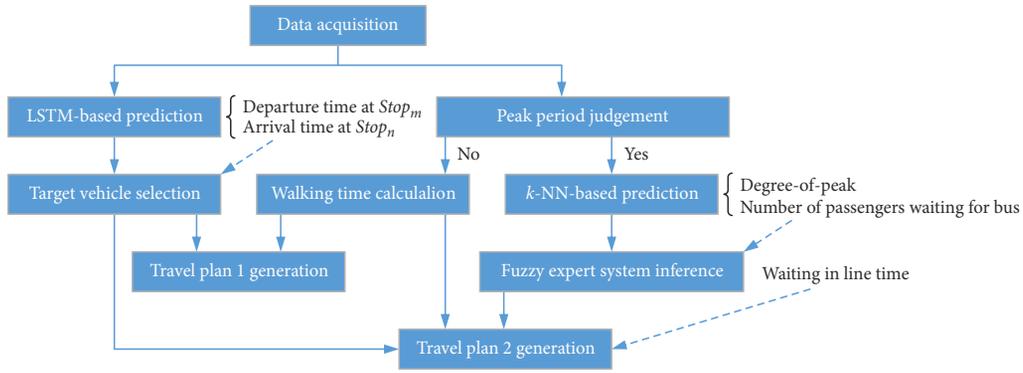


Figure 2. Flow diagram of the IBTSM

Figure 2 illustrates the detailed implementation process of the IBTSM. First, historical bus running data are collected. Second, Long Short-Term Memory (LSTM)-based travel-time prediction is implemented to determine the target bus. This step solves the first problem and generates Travel Plan 1 in the context where there is no dynamic influence of passenger flow. However, the peak period presents more complications. In this situation, the  $k$ -Nearest Neighbours ( $k$ -NN) algorithm is applied to predict the passenger flow. Based on the  $k$ -NN prediction results, the fuzzy expert system infers a reasonable start-off decision strategy. Thus, Travel Plan 2 is generated, and the second problem is thereby solved. Finally, the travel plans are presented to the passengers.

The critical uncertain problems in the overall travelling process are considered in the IBTSM. Considering the limitation of the LSTM model, the IBTSM, with the goal of devising convenient bus travel for passengers, tackles the uncertainty caused by dynamic passenger flow. Therefore, the fuzzy expert system, through proper user instruction, eliminates the negative impact of dynamic passenger flow to the greatest possible extent. Because the input of the fuzzy expert system relates to situations in the near future, the  $k$ -NN, a stable prediction approach, is employed for providing the predicted passenger flow data.

### 2.3. The LSTM

The transportation system is subject to uncertainty, timing, and multi-factorial characteristics. Thus, traffic prediction is a thorny issue. Travel-time prediction is at the basis of providing travel services. Therefore, the establishment of a time-variant, non-linear, and multi-input prediction model is the key to ensuring accurate and reliable traffic prediction.

The ANN has powerful information-processing capability that has been amply demonstrated in many different areas. In this paper, the LSTM, a special Recurrent Neural Network (RNN), is applied for travel-time prediction. The LSTM enables to solve the prediction problem in the near future, which greatly satisfies the research objective of travel-time prediction. Depending on its capability of

dealing with the time-series and non-linearity and reflecting the changing status or degree of things over time, the LSTM can overcome the disadvantages of the RNN gradient diffusion and gradient explosion by replacing the RNN's hidden layer with a memory module (Gers *et al.* 2000). It has been applied for short-term traffic forecast for road traffic control (Zhao *et al.* 2017). Similarly, Petersen *et al.* (2019) propose an LSTM-based bus travel-time prediction model that can detect irregular peaks in bus travel-time and provide satisfactory prediction. The realization of the LSTM's "long-term memory" relies on the transformation of memory cells, and most importantly, the control of the "cell state".

As shown in Figure 3, controlling the cell state is a three-step operation involving the forgetting gate, input gate, and output gate. The forgetting gate produces a number  $f_t$  (between 0 and 1) to control the forgetting-level of the previous unit state, according to the previous output  $h_{t-1}$  and current input  $x_t$ . The input gate generates a candidate vector  $\bar{C}_t$  to control the inclusion of new information in updating the cell state  $C_t$ . The output gate generates a number  $o_t$  (between 0 and 1) to control the degree of participation through the *sigmoid* function in the current cell state. Based on the LSTM-based travel prediction, the IBTSM can capture the short-term bus running rules. Thus, it provides the possibility of determining the passenger's travel plan in relation to a specific bus. This study defines this specific bus as the target bus. Bus arrival is a discrete process; hence, it is necessary for passengers to transform the timespan of their travel plan according to an accurate vehicle selection.

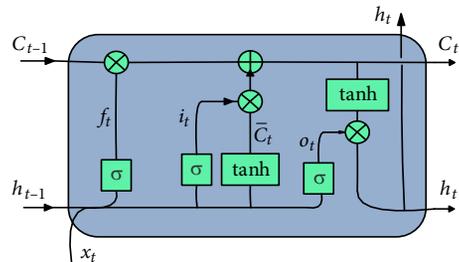


Figure 3. Unit structure of the LSTM

## 2.4. *k*-NN algorithm

However, bus passenger flow conforms to a certain rule of change with time. Accurate passenger flow information in the near future can be obtained through systematic data analysis. Because the non-parametric method has proven to be effective in many studies (Lin *et al.* 2013), a widely used non-parametric regression method, the *k*-NN algorithm (Hu *et al.* 2008), is adopted to predict the passenger flow. The estimated passenger flow is used as an input variable in the fuzzy expert system for deciding the start-off time.

The execution process of the *k*-NN algorithm requires a training dataset (including both features and labels) and prediction dataset (including features only). In the training dataset, the closest *k* samples to the features of the prediction data are determined. These *k* samples are composed of several labels that represent different characteristics waiting for distinguishing. Therefore, the prediction data can be labelled based on majority of the label category in these *k* samples. “The nearest neighbour” is measured according to the Euclidean distance, as shown in Equation (2):

$$d_i = \sqrt{(x_i^1 - \hat{x}^1)^2 + (x_i^2 - \hat{x}^2)^2 + (x_i^3 - \hat{x}^3)^2 + \dots + (x_i^n - \hat{x}^n)^2}, \quad (2)$$

where:  $d_i$  represents the distance between the predicted data and the *i*th training data;  $x_i^1$  is the first feature of the *i*th training data;  $\hat{x}^1$  represents the first feature of the predicted data; *n* is the total number of features of the dataset.

## 2.5. Fuzzy expert system

The fuzzy expert system is formed by combining the expert system and fuzzy logic. The expert system has the capacity of intelligent information-processing, and can make imprecise reasoning using symbolic reasoning engines (Azadeh *et al.* 2008). The fuzzy logic provides an inference mechanism for the expert system. In the fuzzy expert system, the entire process includes four sub-processes: fuzzification, inference, composition, and defuzzification (Hornik, Ruf 1997). Fuzzy inference is the core of the fuzzy expert system. It is a process of logical reasoning for uncertain and fuzzy problems using knowledge base and reasoning rules. The fuzzy reasoning completes the non-linear mapping from the input space to the output space that is denoted as follows: “if *a* is  $\tilde{X}$ , then *b* is  $\tilde{Y}$ ”. The fuzzy expert system demonstrates its superiority in the study of complex systems, and finally realizes the goal of enabling the computer make decisions according to human’s will (Azadeh *et al.* 2008).

Based on the current situation of bus scheduling, the fuzzy expert system helps determine the behavioural decision of passengers (how long ahead of time to start-off). It should meet the following assumption that the passengers wait in line after arriving at  $Stop_m$  to ensure orderly boarding.

## 3. Modelling

The IBTSM is composed of multi-stage processes as shown in Section 2. Detailed information on every step in each stage is illustrated in Figure 4 and explained in the following subsections.

### 3.1. The LSTM-based travel-time prediction

In this study, historical bus running data were acquired using GPS positioning, and hourly weather data issued by the meteorological bureau were collected. Meanwhile, bus passenger flow data for two weeks were obtained using a field survey. The bus of No 976 in Shanghai was chosen as a case study. Some basic information of bus No 976 is as follows. The entire length of the route is approximately 14 km and there are 25 stops along it. The operating time is from 5:30 to 22:30 every day. On-board GPS positioning is installed on each bus. The Lingzhao road, Shangnan road station, was chosen as  $Stop_m$ , and the Longyang road metro station was defined as  $Stop_n$ . The total length of the route is 12.7 km.

In a two week timespan, 1332 effective operation data from  $Stop_m$  to  $Stop_n$  was obtained, including departure and arrival time. Considering the influence of external factors on bus travel, a total of 10 variables were identified as the input variables of the LSTM. These 10 variables can be divided into two categories according to their properties. Three of them were labelled time-related factors, including month, week, and day. Variables in this category did not only reflect specific days; they also reflected some underlying information, such as different traffic laws and characteristics on different working days and legal holiday information. Another category was weather-related factors, including dry-bulb temperature, dew-point temperature, humidity, air pressure, visibility, wind speed, and precipitation intensity. They were all obtained from the hourly meteorological data issued by the meteorological station. As indicated in a previous study (Chen *et al.* 2004), not considering the weather condition may result in inaccurate prediction results. With the participation of hourly temperature, precipitation and sky condition, they obtained the desirable traffic prediction result. In addition, as shown in the correlation analysis, the traffic flow is proved influenced by weather information including temperature, humidity, visibility, wind speed and gust, dew-point, cloud layer height, and general weather conditions. Different weather conditions also cause the changes of free flow speed, capacity and trip-maker decision (Akin *et al.* 2011). For example, the wet road surface is reflected through index of air humidity and the various air temperature may lead to different driver behaviours. The factors in this category impact travel-time prediction to varying extents. Therefore, the decision was made to retain all as the input variables. All the input variables in the LSTM are primary data that have not been pre-processed.

The departure time and arrival time are determined as two output variables. Similar to the travel-time, these two output variables can reflect the time-series characteristic, and provide possibilities for target bus selection.

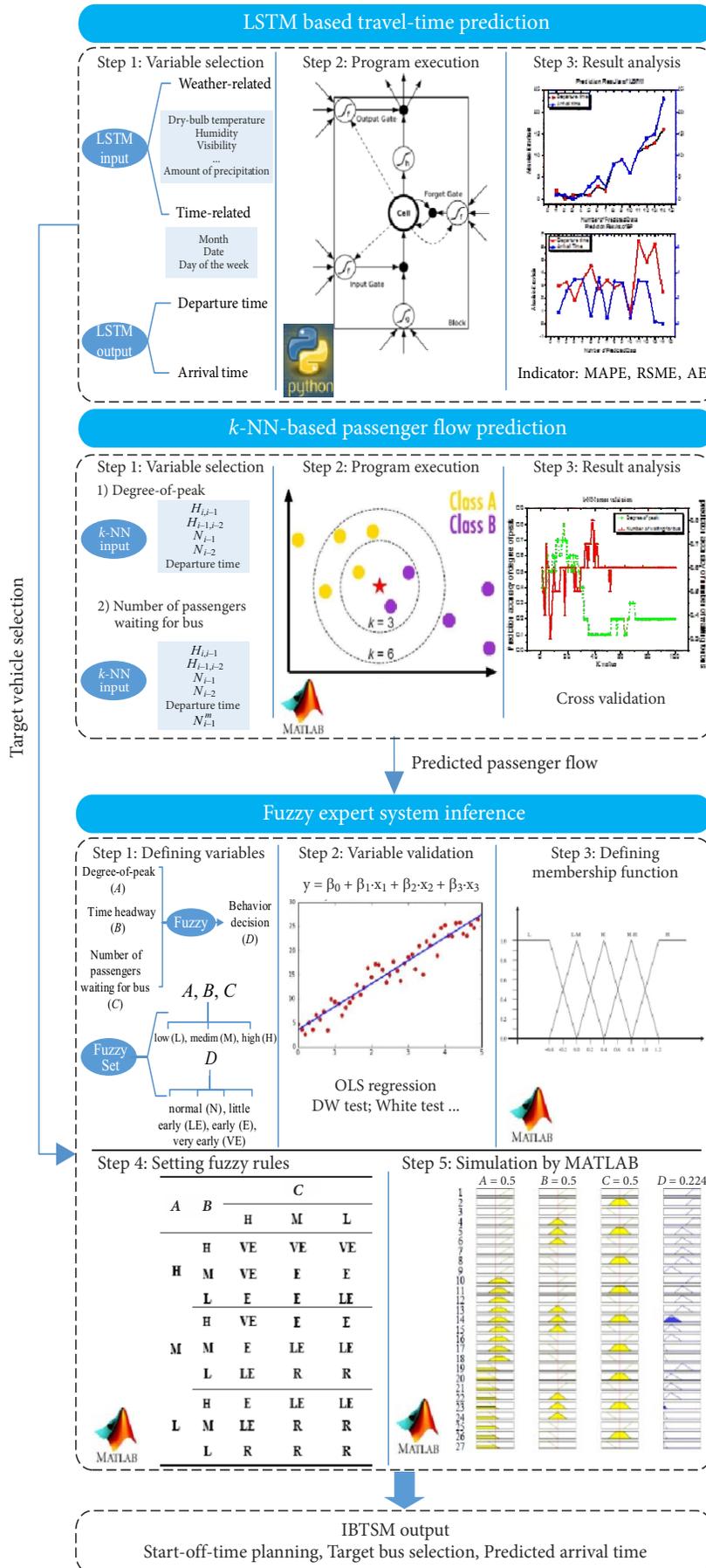


Figure 4. Implementation process of the IBTSM

Thus, 10 input features and two prediction objects were incorporated into the initial dataset. Details on the variables and their units are given in Table 1. Following Table 1, the initial dataset was converted into the time-series dataset, which is adapted to supervised learning. Of this time-series dataset, 99% of the data (1318) were used for the training set, and the last 1% (14), as the test set. By means of trial and error, the hidden layer was determined using six neurons. The batch size was 128, and the maximum number of iterations was 1000. The optimizer of Adam’s algorithm was employed and the LSTM program was executed in Python (<https://www.python.org>).

Table 1. Input and output data information of the LSTM

Variable	Data field	Unit
Dry-bulb temperature	8...29	°C
Dew-point temperature	-6...19	°C
Humidity	12...94	%
Air pressure	1002...1023	hPa
Visibility	2...10	km
Wind speed	3.6...39.6	km/h
Precipitation intensity	0...19	mm/h
Month	4	-
Week	3...4	-
Day	Monday...Friday	-
Departure time	5:32...22:36	-
Arrival time	6:03...23:13	-

### 3.2. Fuzzy expert system inference

#### 3.2.1. Variables definition

Identifying the proper influential factors is an important task for establishing the fuzzy expert system. Considering the correlation of the bus running rules and the passenger flow variation, the degree-of-peak *A*, time headway *B*, and number of people waiting at *Stop<sub>m</sub>* when *Bus<sub>i</sub>* comes *C* are regarded as three influential factors of behavioural decision *D*. The variable *A* does not refer to traffic flow but number of passengers. Variable *B* is the key indicator of behavioural decision, because the number of passengers waiting at the station gradually increases with the extension in time headway. In fact, it reflects the relative number of passengers, because time headway extension happens with the process of passengers’ accumulation. Variable *C* is specifically referenced, because a long queue may contribute to a crowd of delayed passengers. Variable *D* is the start-off strategy via catching the target bus provided by the LSTM. Therefore, the discussion of the behavioural decision exists as an independent stage in this study.

To convert this practical issue into a fuzzy set problem, *A*, *B*, and *C* are each divided into three affairs in their own fuzzy sets to represent the degree, tagged High (H), Medium (M), Low (L). Likewise, *D* is composed of four affairs in the entire fuzzy sets: Very Early (VE), Early (E), Little Early (LE), and Regular (R).

#### 3.2.2. Variable validation

To verify that *A*, *B* and *C* have an impact on *D*, the Ordinary Least Square (OLS) is used to conduct multivariate regression analysis. In this regression analysis, *A*, *B*, and *C* are respectively represented by numerical variables,  $x_1$ ,  $x_2$ ,  $x_3$ .  $x_1$  is the total number of people riding in the bus and waiting at the stop when a bus arrives *Stop<sub>m</sub>*.  $x_2$  is the crisp value of time headway, and  $x_3$  is the number of people waiting at *Stop<sub>m</sub>* when the bus arrives *Stop<sub>m</sub>*. *D* is regarded as the explanation of variable *y*. *y* is a relative value in the regression equation, not the crisp value of time. When  $y > 1$ , passengers are to prepare for start-off in advance. On the contrary, when  $y < 1$ , the bus is not fully-loaded, thus eliminating the need to start-off early. Hence,  $y = 1$  is defined as the condition of the bus being fully-loaded without stranded passengers. *y* is determined by Equation (3). According to Equation (3), the fully-loaded state refers to the existence of delayed passengers when the bus leaves *Stop<sub>m</sub>*. When there are 85 people aboard the bus, it is defined as the state of fully-loaded. At that time, it is considered no passenger delay based on statistical average data. The linear regression equation is shown in Equation (4).

$$y = \frac{\text{number of people boarding} + \text{number of passenger delaying}}{\text{number of defined fully-loaded}}; \tag{3}$$

$$y = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \beta_3 \cdot x_3. \tag{4}$$

By collecting the field data of the bus arrival at *Stop<sub>m</sub>*, the bus arrival time and passenger number data from 7:00 to 9:00 on work days over the course of two weeks were obtained. The regression-equation-fitting explanatory variables and explained variables were obtained by running the STATA software (<https://www.stata.com>). Then, the overall reliability of the equation and the accuracy of each parameter were tested. The regression result is shown in Table 2, where the judgment coefficient of  $R^2$  equals 0.8814, for 232 effective statistical samples. The explanatory variables –  $x_1$ ,  $x_2$ ,  $x_3$ , are statistically significant at the 1, 5, and 10% levels, respectively, which shows good

Table 2. Regression results checklist

OLS form	$y = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \beta_3 \cdot x_3$
$\beta_0$	-0.393 (0.119)
$\beta_1$	0.0123** (0.000)
$\beta_2$	0.091* (0.037)
$\beta_3$	0.0027** (0.003)
<i>n</i>	232
<i>F</i> (3, 228)	564.93
DW test	1.82
$R^2$	0.8814
Mean VIF	1.38
Probability > $\chi^2$	0.2679

Note: \*\*, \* mean 1 and 5% are statistically significant.

explaining results to  $y$ . Furthermore, the regression equation meets the requirements of the  $F$ -test. The White test, Durbin–Watson (DW) test, and Variance Inflation Factor (VIF) test indicated that there was no heteroscedasticity, sequence autocorrelation, and multicollinearity in the regression analysis. From the perspective of realistic significance, the increase of  $x_1, x_2, x_3$  contributes to the rising of  $y$ . The result coincides with the physical truth. Therefore, the reasonability of the variable setting of the fuzzy expert system is established.

### 3.2.3. Defining the membership function

According to the statistical data of the samples in the established regression equation, the ranges of the crisp value of  $A, B,$  and  $C$  are defined, and the corresponding fuzzy description, which is clarified in the Section 3.2.1, is shown in Table 3. Specifically, the normalization is based on the linear interpolation. For behavioural decision,  $D$  the earlier the bus arrives at  $Stop_m$ , the higher probability of the passenger boarding the target bus in an orderly fashion. Supposing that  $H_{i,i-1}$  is the time headway of  $Bus_i$  and  $Bus_{i-1}$ ,  $N_{i,i-1}$  is the number of passengers coming to  $Stop_m$  between  $Bus_{i-1}$  departing and  $Bus_i$  arriving,  $L_{i,i-1}$  of  $N_{i,i-1}$  delay after  $Bus_i$  departing  $Stop_m$ , the crisp value of the start-off time  $T$  in advance can be calculated via Equation (5). Considering the phenomenon of delay may happen with  $Bus_{i-1}, Bus_{i-2}, \dots$  the modified  $T'$  is determined by Equation (6). Thus, the detailed fuzzy classification information of  $D$ , which is pre-defined in the Section 3.2.1, is shown in Table 4.

$$T = \frac{L_{i,i-1}}{N_{i,i-1}} \cdot H_{i,i-1}; \quad (5)$$

Table 3. Fuzzy classification of variables  $A, B, C$

Variable	Fuzzy description	Range of crisp value	Unit	Range after normalization
A	H	[80, 100]	Person	[0.6, 1]
	M	[70, 90]		[0.4, 0.8]
	L	[50, 80]		[0, 0.6]
B	H	[2, 4]	Hundred second	[0.5, 1]
	M	[1, 3]		[0.25, 0.75]
	L	[0, 2]		[0, 0.5]
C	H	[15, 30]	Person	[0.5, 1]
	M	[10, 20]		[0.25, 0.75]
	L	[0, 15]		[0, 0.5]

Table 4. Fuzzy classification of  $D$

Fuzzy description	Range of crisp value [hundred second]	Range of normalization
VE	[6.9, 11.5]	[0.6, 1]
E	[3.45, 9.20]	[0.3, 0.8]
LE	[0, 5.75]	[0, 0.5]
R	[0, 1.15]	[0, 0.1]

$$T' = \frac{L_{i,i-1}}{N_{i,i-1}} \cdot H_{i,i-1} + \frac{L_{i-1,i-2}}{N_{i-1,i-2}} \cdot H_{i-1,i-2} + \frac{L_{i-2,i-3}}{N_{i-2,i-3}} \cdot H_{i-2,i-3} + \dots = \sum_{\lambda=0}^{i-1} \frac{L_{i-\lambda,i-\lambda-1}}{N_{i-\lambda,i-\lambda-1}} \cdot H_{i-\lambda,i-\lambda-1}. \quad (6)$$

According to the fuzzy classification of each variable, the triangle function and trapezoid function are jointly used to construct the membership functions of  $A, B, C,$  and  $D$ , as shown in Figure 5. For improving the integrity of the system, the coverage degree of the membership function  $\varepsilon$  is as close as possible to 0.5.

### 3.2.4. Setting fuzzy rules

The establishment of the fuzzy rule base is the core of the fuzzy expert system. It is also the basis for fuzzy inference. The establishment of the fuzzy rule base mainly depends on extracting some useful information from the statistical samples. The variables in the original multivariate regression equation vary in the different units. Therefore, the degree of influence on the explained variable cannot be measured. The Standardized Regression Coefficients (SRCs) are used for providing a quantitative measure of the variable sensitivity. It is an intuitive indicator of variable sensitivity. The method of calculating the SRCs is shown in Equation (7), where  $s_i$  is the estimated standard deviation of  $x_i$ , and  $s_y$  is the estimated standard deviation of  $y$ . Equation (8) is the standardized regression equation. Same variation degree of  $x_1$  causes  $y$  to vary widely:

$$U_{SRC}(x_i, y) = \frac{\beta_i \cdot s_i}{s_y}; \quad (7)$$

$$y = \beta_0 + 0.89 \cdot x_1 + 0.53 \cdot x_2 + 0.08 \cdot x_3. \quad (8)$$

By combining the preset fuzzy sets and variable sensitivity, 27 fuzzy rules are established. The fuzzy rule base is shown in Table 5.

## 3.3. $k$ -NN-based passenger flow prediction

The  $k$ -NN is used to provide input data for the fuzzy expert system, because the passenger flow-related variables also need to be known ahead. Whether the  $k$ -NN shows good performance determines the availability of the entire model. There are two objects in the  $k$ -NN prediction, variables  $A$  and  $C$ . Considering the characteristic of time-series, five variables, including  $H_{i,i-1}$ , number of people riding on  $Bus_{i-1}$ ,  $H_{i-1,i-2}$ , number of people riding on  $Bus_{i-2}$  and departure time of  $Bus_i$ , are determined as the  $k$ -NN inputs for  $A$ . According to the measured range of the crisp value – [50, 100], five intervals are demarcated equally to reflect different peak periods and are defined as labels. In the prediction of  $C$ , the variable number of people getting on  $Bus_{i-1}$  at  $Stop_m$  is regarded as the 6th characteristic variable. Meanwhile,  $C$  is also divided into five groups represented by five labels covering the entire range of the crisp value – [0, 30].

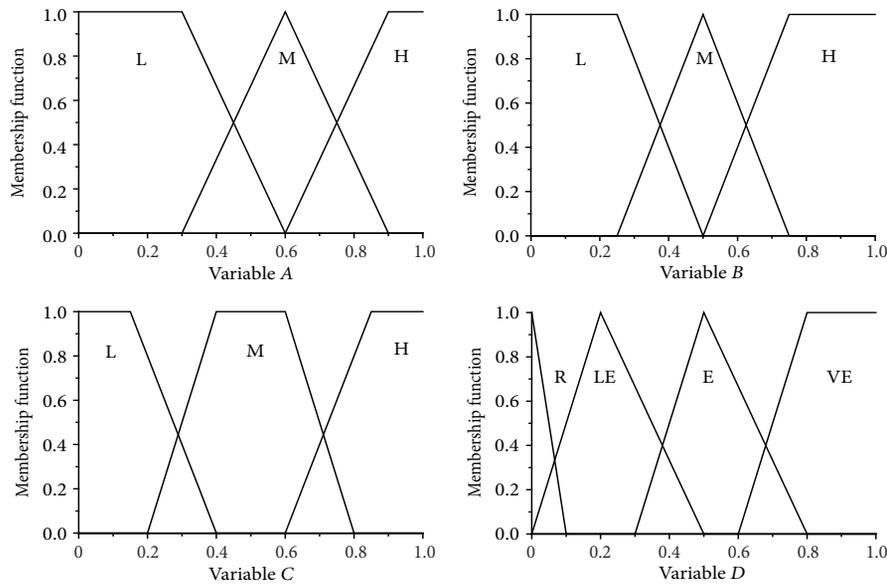


Figure 5. Membership function of each variable

Table 5. Fuzzy rule base

A	B	C		
		H	M	L
H	H	VE	VE	VE
	M	VE	E	E
	L	E	E	LE
M	H	VE	E	E
	M	E	LE	LE
	L	LE	R	R
L	H	E	LE	LE
	M	LE	R	R
	L	R	R	R

### 4. Result and discussion

The reliability of the IBTSM mainly depends on the accuracy of the outcomes in each phase. Accurate outcomes ensure that the error is propagated within the allowable range. In this section, the results of three stages (the LSTM-based travel-time prediction, the *k*-NN-based passenger flow prediction, and the fuzzy expert system inference) and the effectiveness of the IBTSM are discussed.

#### 4.1. The LSTM

After the calculation of the iteration, the absolute error between the LSTM-based simulation results and the actual value are shown in Figure 6; it shows the deviation between the predicted bus departure time, arrival time, and the actual data. The prediction error exhibits a gradually increasing trend with the passage of time, which reflects the recursion characteristic of the LSTM. Consequently, the first four groups in the test set demonstrate good prediction result, and the absolute error of the predicted departure time and arrival time does not exceed 2.5 m,

indicating that the prediction results are effective for the coming hour. To prove the superiority of the LSTM method, one frequently used prediction method, the BP neural network, is selected for comparison in this study. As shown in Figure 7, the results of the BP neural network prediction are relatively unstable. Although part of the prediction data shows a large deviation, some exhibit good performance. It is difficult to determine which is accurate or imprecise in advance.

From another point of view, the actual bus travel-time can also be considered as a key indicator of the effectiveness of the LSTM approach. The actual bus travel-time can be calculated according to the departure time and the arrival time. To measure the accuracy and stability of the prediction result, the Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RSME) are used to evaluate the prediction effect of the LSTM and the BP neural network. The MAPE is used to determine the prediction accuracy, while the RSME is used to measure the discrete degree of the sample and stability of the prediction. The calculation of these two indicators is represented in Equations (9) and (10). Table 6 shows the bus travel-time prediction performance of the data in the test set. The MAPE of the LSTM is 3.56% and is smaller than the 5.16% generated by the BP neural network. The RSME of the LSTM is 2.35, while the RSME of the BP neural network is 3.09. These all indicate that the LSTM is much more stable than the BP neural network, and its prediction error is smaller.

$$MAPE = \frac{1}{N} \cdot \sum_{i=1}^N \left| \frac{t_i - \hat{t}_i}{t_i} \right| \cdot 100\%; \tag{9}$$

$$RSME = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^N (t_i - \hat{t}_i)^2} \tag{10}$$

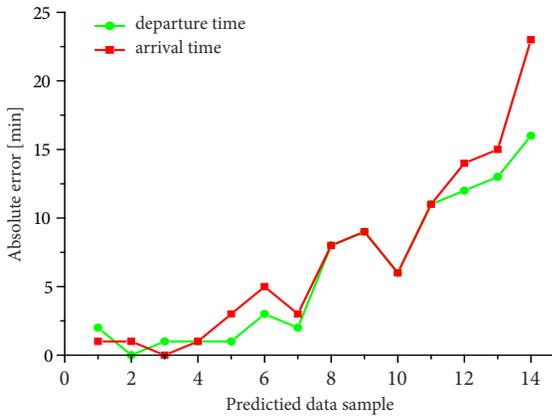


Figure 6. Absolute error of LSTM prediction

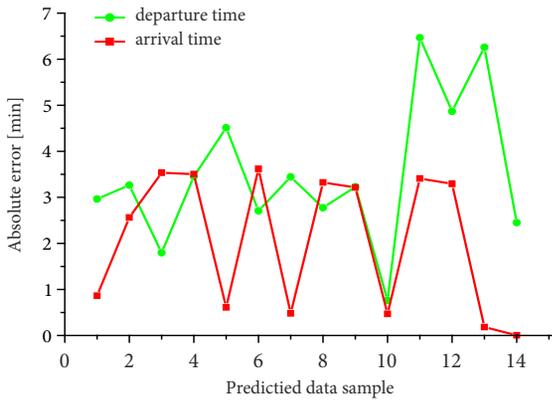


Figure 7. Absolute error of the BP neural network prediction

Table 6. Comparison of the travel-time prediction results

Indicator	LSTM	BP neural network
MAPE [%]	3.56	5.16
RSME	2.35	3.09

Nevertheless, only the prediction result of the near future is required for providing the decision basis for passengers. Because of the unique recursive characteristics of the LSTM, the prediction error of the LSTM will gradually increase with the increase in the number of prediction data. Consequently, the error accumulation relating to the near future has been controlled to a certain extent, and a relatively ideal result can be obtained. However, the BP neural network assumes every piece of data is completely independent. In this way, the LSTM has stronger ability to mine the underlying data link than the BP neural network. The experiment result showed above buttresses this point. Therefore, the LSTM is more suitable for travel-time prediction and selection of target buses.

#### 4.2. Fuzzy expert system inference

Based on the defined membership functions and fuzzy rules, the simulation of the fuzzy expert system is implemented using the fuzzy toolbox in MATLAB (<https://www.mathworks.com/products/fuzzy-logic.html>). Com-

puter arithmetic can significantly improve the efficiency of calculation, while avoiding a mass of matrix iteration, and forming a visual interface. The default Mamdani algorithm is deployed for fuzzy inference, and the centroid method is used in the defuzzification process. Figure 8 shows the surface of fuzzy inference among *ABD*, *BCD*, and *ACD*. Figure 9 shows the interface of the fuzzy rules in the fuzzy toolbox.

Using pre-set fuzzy rules in the MATLAB, this study verifies the validation of the fuzzy expert system. The calculated value  $T'$  is compared with the simulated one, and the absolute error is selected as the evaluation index. Table 7 presents 13 sets of random data following normalization processing. The mean absolute error is 1.9 m, similar to that of the LSTM prediction, and the standard deviation is a mere 0.94. Therefore, the fuzzy expert system can be considered to satisfy the basic requirement of the IBTSM with considerable stability.

Overall, due to the complexity of the transportation system and the dynamic variation in passenger flow, it is difficult to accurately evaluate the behavioural decision of the passengers. Through the validation of the variable, it is apparent that the multivariate regression equation possesses a goodness of fit. However, this study did not choose linear regression as the basis of behavioural decision, because of the following points. Firstly, *A*, *B* and *C* do not rigorously comply with the linear relation with *D*, while the least square method is only a linear-fitting process. The fuzzy expert system is a non-linear fitting process that can better reflect the reality based on fuzzy rules. In addition, the value of the behavioural decision obtained by linear regression is not stable, which means that it is larger or smaller than the real value indeterminately. In the process of fuzzification and defuzzification, a conservative algorithm is adopted to leave room for adjusting the behavioural decision and responding to emergencies or system inaccuracy. Furthermore, the fuzzy expert system is closer to the human thought process and accords with human cognition. The fuzzy expert system furnishes passengers with information on general start-off time, and possesses great capacity for generalization and adaptation.

#### 4.3. The *k*-NN result

Based on the variable preset, the *k*-NN program is executed in MATLAB. The *k* value, as a critical parameter, should be determined in advance. This process is usually based on operating experience. As a result, trial and error is deployed for determining the *k* value. To avoid accidental fluctuations in the forecast accuracy, cross-validation is implemented with the *k* value varying from 1 to 100. All 232 statistical samples used to establish the regression equation are included in the *k*-NN model building. Of the 232, 222 are utilized as the training data, and the remaining 10 samples compose the test set. The result of the *k*-NN with varying *k* values is shown in Figure 10. The blue dash-dot line represents the accuracy rate of predic-

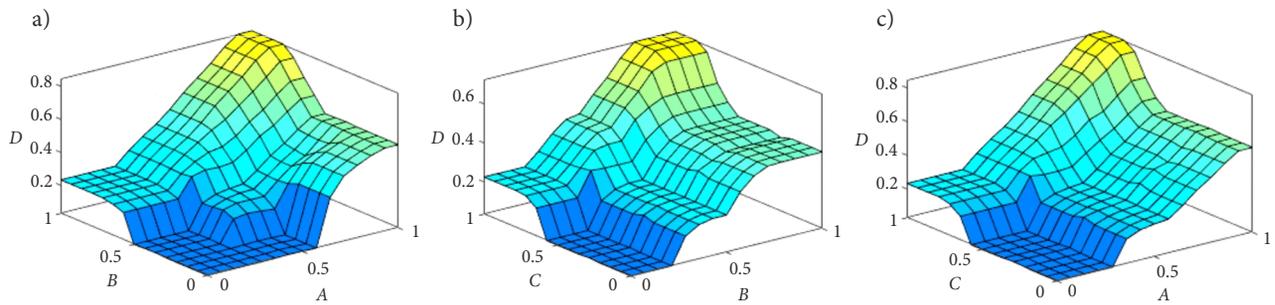


Figure 8. Surface of fuzzy inference: a – inference of ABD; b – inference of BCD; c – inference of ACD

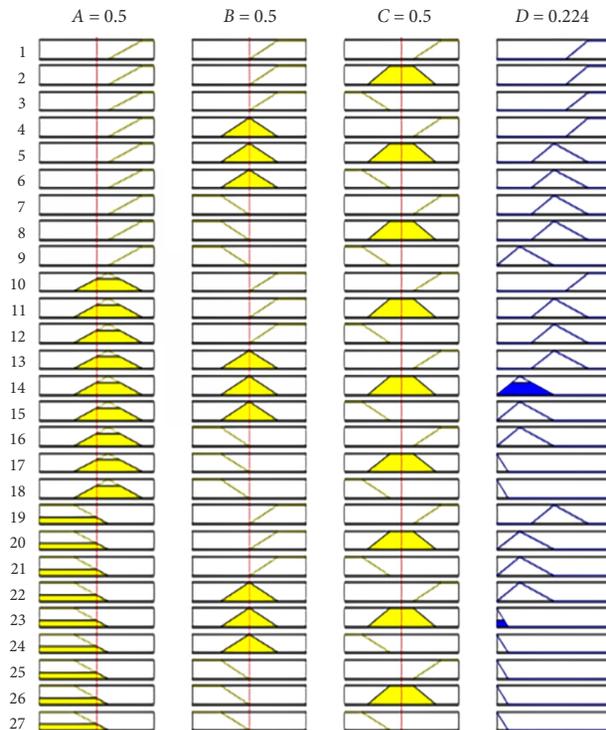


Figure 9. Fuzzy rules in fuzzy toolbox

tion using variable A. The red line represents the accuracy rate of prediction using variable C. When  $k = 18$ , the prediction accuracy of A is as high as 80%. When  $k = 38$  or  $k = 39$ , the accuracy rate of C is 80%. The cross-validation proves that the  $k$ -NN can yield reliable prediction performance within a limited sample size.

#### 4.4. Evaluation of the IBTSM

The IBTSM guides passengers to make reasonable decisions based on the influence of the dynamic passenger flow. The IBTSM is proven to be capable of completing expected tasks. The most important thing is that the IBTSM can effectively incorporate the influence of dynamic passenger flow. In practice, bus arrival time can be determined by incorporating the LSTM prediction into real-time GPS positioning. Similarly, through broader data collection, the IBTSM can capture the passenger flow information more accurately, thereby improving the accuracy of the model.

Table 7. Verification of the fuzzy expert system result

Number	A	B	C	Calculated value [min]	Simulated result [min]	Absolute error [min]
1	0.68	0.02	0.70	4.2	6.4	2.2
2	0.73	0.38	0.63	5.0	7.6	2.6
3	0.53	0.59	0.23	7.8	6.7	1.1
4	0.70	0.50	0.47	9.0	6.7	2.3
5	0.92	0.42	0.80	10.8	14.0	3.2
6	0.75	0.61	0.53	12.8	9.9	2.9
7	0.52	0.58	0.37	6.0	6.5	0.5
8	0.70	0.31	0.73	7.2	8.3	1.1
9	0.78	0.58	0.63	10.3	9.6	0.7
10	0.73	0.91	0.43	14.4	12.6	1.8
11	0.38	0.44	0.10	2.7	3.7	1.0
12	0.42	0.01	0.50	2.7	0.7	2.0
13	0.75	0.92	0.57	9.5	13.0	3.5

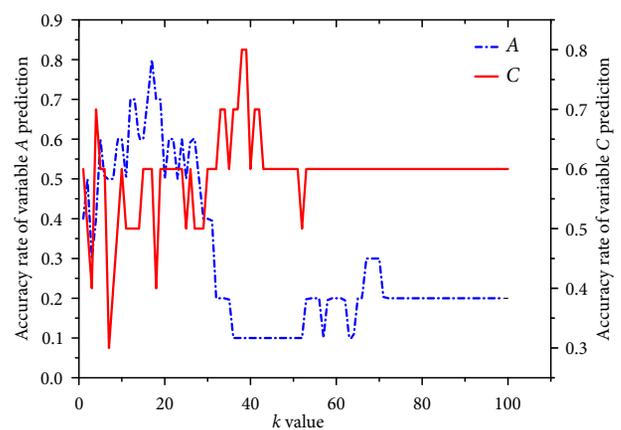


Figure 10. Prediction accuracy of  $k$ -NN with varying  $k$  value

## Conclusions

To improve bus service quality, this study aimed to improve the comprehensiveness and effectiveness of the APTS through the IBTSM. The IBTSM integrates these improvements and promotes them to form a systematic process. In the IBTSM, problems associated with bus travel services are explored from unique perspectives to ensure the maximum convenience of passengers. At the same

time, the IBTSM considers overall process of bus travel, including walking, bus driving, and waiting for bus. The consideration of the integral process greatly reduces the deviation in the result caused by uncertainty and makes it easier to determine the start-off time. An in-depth study is conducted on bus travel-time prediction using the LSTM. Not only does it do a good job of forecasting, it is also the basis for the selection of the target bus, which optimizes the effectiveness of the APTS. Meanwhile, a module of passenger strategy exploration is incorporated into the IBTSM to ensure for convenience of passengers during the peak period. In this module, the fuzzy expert system inference is applied to guide the start-off time planning for passengers to board the target bus. This module is regarded as the supplement of the APTS. Generally, the IBTSM, through the analysis of the overall process of bus travel, furnishes passengers with an intelligent bus travel plan, thus enabling them to arrive on time. The intelligent bus travel plan is delivered to the passengers in a timely manner through a mobile application.

Studies investigating passenger flow variation are required to critically to guide passengers through the bus travel experience. It is the basis of furnishing passengers with the start-off time scheduling and target bus recommendation all day long. As a result-oriented model, the IBTSM is capable of satisfying the requirement of arriving at the specified time in different scenarios. The IBTSM eliminates the need for bus passengers to worry about delay or long waiting time, because all the possible situations are considered in advance. Compared with previous studies on bus travel-time prediction, this study gives priority to the provision of travel service, rather than merely exploring traffic laws. Therefore, the IBTSM greatly improves passengers' bus travel experience, and reduces the stress caused by daily travelling. This will also boost bus travel services, as the IBTSM makes bus travel an attractive approach. Moreover, the IBTSM plays a certain role in promoting the development of intelligent travel and smart life.

### Author contributions

*Sha Liu* and *Xiang Li* conceived the study and were responsible for the design and development of the data analysis.

*Chuanni He* and *Xiang Li* were responsible for data collection and analysis.

*Sha Liu* and *Xiang Li* were responsible for data interpretation.

*Xiang Li* wrote the first draft of the paper.

### Disclosure statement

All the authors have no conflict of interest.

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