TRANSPORT
ISSN 1648-4142 / eISSN 1648-3480
2018 Volume 33(1): 1–11
doi:10.3846/16484142.2014.994227

OPTIMIZING CAPACITY OF SIGNALIZED ROAD NETWORK WITH REVERSIBLE LANES

Jian Wang[1], Wei Deng[2]

[1]*School of Civil Engineering, Purdue University, United States*
[2]*School of Transportation, Southeast University, China*

Submitted 6 January 2014; resubmitted 12 March 2014, 25 August 2014; accepted 26 August 2014; published online 14 January 2015

Abstract. This paper studies the network capacity problem on signalized road network with reversible lanes. A Mixed Network Design Problem (MDNP) is formulated to describe the problem where the upper-level problem is a mixed integer non-linear program designed to maximize the network capacity by optimizing the input parameters (e.g. the signal splits, circles, reassigned number of lanes and O–D demands), while the lower-level problem is the common Deterministic User Equilibrium (DUE) assignment problem formulated to model the drivers' route choices. According to whether one way strategy is permitted in practice, two strategies for implementing reversible roadway are considered. In the first strategy, not all lanes are reversible and the reversible roadways always hold its ability to accommodate the two-way traffic flow. In the second strategy, one-way road is allowed, which means that all the lanes are reversible and could be assigned to one flow direction if the traffic flow in both directions is severally unsymmetrical. Genetic Algorithm (GA) is detailedly presented to solve the bi-level network capacity problem. The application of the proposed method on a numerical example denotes that Strategy 2 can make more use of the physical capacity of key links (signal controlled links), thus, the corresponding network capacity outperforms it is of Strategy 1 considerably.

Keywords: network capacity; genetic algorithm; mixed network design problem; user equilibrium; signalized road network.

Introduction

To accommodate the increasing traffic flow in urban road network, measures, such as network construction (widening links or building new roads), traffic signal coordination control *et al.* are extensively used by the approach people. Traditionally, the problem of finding the optimal decisions in response to the growing demand can be regarded as a Network Design Problem (NDP). NDP is usually formulated as a bi-level problem where the upper-level problem generally represents the investment decision-making of the transport planner to maximize social welfare, while the lower-level problem models the drivers' route choice decisions. Classically, the NDP is considered in three forms, the first form is Continuous Network Design Problem (CNDP), which deals with the continuous capacity expansion of the existing streets, the second form is Discrete Network Design Problem (DNDP), which deals with adding new streets or lanes to the existing streets, and the third one is Mixed Network Design Problem (MNDP), which deals with both discrete and continuous network design variables (Miandoabchi, Farahani 2011). Thanks to its efficiency in predicting and improving traffic congestion, the NDP has been applied to study various transportation problems, such as congestion pricing (Liu *et al.* 2009; Ekström *et al.* 2009; Yang, Zhang 2003), network reliability (Chen *et al.* 2006; Chootinan *et al.* 2005; Shor, Sharifov 2006; Li 2009) and multiclass problem (Wang *et al.* 2013; Yang, Zhang 2002; Daganzo 1983). Besides, recently, the researchers also gave considerable attention in exploring the effectiveness of NDP method in maximizing network capacity under various assumptions.

The study of network capacity is used to find out how much the total Origin–Destination (O–D) demands a network can accommodate, withstand or handle without exceeding a prescribed degree of saturation while taking users' route choice into account (Wong, Yang 1997). Akin to system travel time, reserve capacity is usually taken as an important performance indicator for a road network, thus the corresponding research is quite necessary and meaningful (Ge *et al.* 2003). Yang and Bell

Corresponding author: Jian Wang
E-mail: *wang2084@purdue.edu*

(1998a) firstly studied the network capacity problem with DNDP. They used a small network to demonstrate that adding a new link may reduce the potential capacity of the network but could be avoided if the concept of reserve capacity put forward by Wong and Yang (1997) is introduced. Gao and Song (2002) extended the concept of reserve capacity by assuming that the demand multipliers between each O–D pair could be different, a bi-level program is formulated to optimize the signal parameters and road capacity increasing to maximize the network capacity. Numerical application denotes that the network continuous design is an efficient measure for increasing the network capacity. Ceylan and Bell (2004) proposed a two-stage approach to study the reserve capacity, the first stage was to optimize signal timing to maximize the network performance, while the traffic was reassigned by logit-based Stochastic User Equilibrium (SUE), and the second stage was to find the largest common multiplier at the optimized signal timing provided by the first stage. Chiou (2007) formulated a mathematical program with equilibrium constraints to maximize the reserve capacity of optimal signal settings based on the models in TRAffic Network StudY Tool (TRANSYT) (Vincent *et al.* 1980), a projected gradient approach is proposed to solve the bi-level programming problem. Additionally, through embedding the concept of reserve capacity, Chiou (2008) further proposed a CNDP to study the network capacity on signalized road network with link capacity expansions. This problem is formulated such that the total travel demand is maximized while the total delays are minimized simultaneously. Miandoabchi and Farahani (2011) developed a mixed-integer bi-level optimizing problem to study the reserve capacity of urban road network which aims to find the optimum configuration of street directions and two-way street lane allocations and the street lane addition projects, in a way that the network capacity is maximized. Chen *et al.* (2002) pointed out the link capacity is susceptible, thus practically, it is a random variable instead of constant. They introduced the concept of network capacity reliability to study the probability of a certain level of traffic demand that the road network can accommodate at equilibrium condition. This topic sparked further research and has been explored from various viewpoints, such as network capacity with degradable links problem (Chen *et al.* 2002; Lo, Tung 2003; Chootinan *et al.* 2005) and network capacity flexibility problem (Yang *et al.* 2000; Kasikitwiwat, Chen 2005; Chen, Kasikitwiwat 2011).

While the aforementioned literatures address the network maximization problem under different scenarios, the program they formulated in studying the problem, however, can only be categorized into CNDP (Gao, Song 2002; Chiou 2008; Chen *et al.* 1999, 2002; Chootinan *et al.* 2005) and DNDP (Miandoabchi, Farahani 2011; Yang, Bell 1998a, 1998b). To our knowledge, not yet a research has been conducted on describing network capacity maximization problem as a MNDP. Besides, apart from the regular network capacity maximization measures, such as link capacity expansion, signal optimizing, road building, no attention has been given on studying how to allocate the number of lanes on reversible roadways to maximize the capacity of signalized road network in city area. Since nowadays, the reversible roadway is commonly seen in the city area and is widely operated by the transportation management to accommodate the routine tidal traffic flow or abruptly souring up traffic volume in certain roads, it is thus quite necessary to find ways to make the optimal reversible roadway strategy from system level to better increase the network capacity.

This paper aims to addressing this problem by formulating a MNDP in which three decisions are simultaneously considered:

- determining the optimal signal settings (circles and signal splits);
- resigning the number of lanes for both direction on reversible roadways;
- deciding the O–D demands for each O–D pair.

Besides, according to different applications of reversible roadways in practice, two reversible lane strategies are considered in this paper, i.e., *Strategy 1: one-way street is not allowed*: in this case, no matter how severely asymmetric of traffic volume is between both flow direction, at least one lane is allocated for each flow direction in the reversible roadway; and *Strategy 2: one way street is allowed*: in this case, all lanes are reversible and can be allocated to major flow direction if the traffic flow on the minor flow direction is quite limited. The network capacity problems with the two reversible roadway strategies are modeled as a MNDP, where upper-level problem is a mixed integer non-linear program, which aims to maximize the network capacity by optimizing parameters, such as the signal splits, circles, reassigned number of lanes and O–D demands, while the lower-level problem is the common Deterministic User Equilibrium (DUE) assignment problem, formulated to compute the equilibrium traffic flows for each network design scenario. Genetic Algorithm (GA) is explicitly presented to solve the network capacity problem with reversible lanes.

The remainder of this article is structured as follows: in the Section 1, some basic notations are defined. The network capacity with reversible lane problem is discussed and formulated in Section 2. In Section 3, a GA is specially designed to solve the proposed bi-level problem. Section 4 presents a numerical example to illustrate the general use of the proposed method in studying the network capacity problem. In addition, comparison is conducted between the resulted network capacity with Strategy 1 and it is with Strategy 2, and detailed discussions are also provided regarding the difference of network capacity. The Last Section concludes the paper.

1. Notations

N – $N = \{1, 2, 3 \cdots, P\}$ be a set of P nodes each of which represents a signal-controlled intersection;

A – the set of links in the network, $A = \{ij, \cdots\}$;

$\tilde{A}$ – the set of signal controlled links;

$\bar{A}$ – the set of all reversible roadway;

a – $a = \{ij, ji\}$, a reversible roadway, $a \in \bar{A}$, which includes two links with different flow direction;

ij – a link from node i to node, $ij \in A$;

r – an origin node;

s – a destination node;

W – a set of all O–D pairs, $W = \{rs, \cdots\}$;

m_{ij} – the allocated number of lanes on link ij;

C_n – circle time for signalized intersection n, $n \in N$;

m_a – the total number of lanes on reversible roadway, $a \in \bar{A}$, it is the summary of the number of lanes of both flow direction;

R_{rs} – the set of all routes between O–D pair rs;

S_n – the set of signal phases for signalized intersection n, $n \in N$;

λ_{hn} – the hth signal splits on intersection n, $h \in S_n$, $n \in N$;

$\boldsymbol{\lambda}$ – a vector of all signal splits, $\boldsymbol{\lambda} = \{\lambda_{hn}, h \in S_n, n \in N\}$;

f_k^{rs} – route flow on kth route between O–D pair rs, $k \in R_{rs}$;

c – saturation flow for a single lane, it is assumed to be 1800 veh/h;

$c_{ij}(\lambda)$ – capacity of a single lane on link ij, it is a function of signal splits;

$t_{ij}(v_{ij})$ – the link travel cost function on link ij, $ij \in A$;

q_{rs} – the O–D demand between O–D pair rs;

$\mathbf{q}$ – the vector of all O–D demands on the network;

v_{ij} – the flow on link ij, $ij \in A$;

$\delta_{ij,k}^{rs}$ – link-route indicators, $\delta_{ij,k}^{rs} = 1$ if a is a link on route k, $k \in R_{rs}$, else $\delta_{ij,k}^{rs} = 0$.

2. Network Capacity with Reversible Lane Problem

2.1. Mathematical Programming for Network Capacity with Reversible Lane Problem

Reversible roadway is one of the most popular methods employed by the approach people to release the peak-hour congestion due to its efficiency in accommodating the severely perturbed traffic flows during peak hours. A reversible roadway is one in which the direction of the traffic flow in one or more lanes or shoulders may be reversed to the opposing direction for some period of time (Wu *et al.* 2009). Usually, through allocating the superfluous lanes of minor flow direction to the major flow direction, the reversible roadway can accommodate more traffic flow. However, traditionally, the decision makers just consider making reversible roadway strategy on a single main road. This may in most cases, shift the congestion from one road to anther road. Thus, it is not desirable measured from system level. It is a plain fact that the flows of different links in the road network are interdependent rather than independent, they interrelate with each other and in some sense, could be regarded as a united system. Therefore, it is more reasonable to make a reversible roadway strategy on the ground of the whole network other than just focus on a single road. In the following, we will present the method to maximize the network capacity through optimizing the two-way street lane allocation and signal parameters in a systemic way.

For a reversible way in the road network $G = (N, A)$, no matter how the number of lanes of both flow directions changes, the total number of the lanes for a reversible roadway is always constant, that is

$$m_{ij} + m_{ji} = m_a, \; ij \in a, \; ji \in a, \; a \in \bar{A}. \tag{1}$$

In order to ensure the capacity is applicable to the real situation, queues and delays at network intersections under equilibrium conditions must be acceptable by users, thus the resulting degree of saturation of any lanes of signal controlled links should not exceed a prescribed maximum acceptable value. i.e., capacity constraints are given as follows:

$$v_{ij} \le m_{ij} P_{ij} c_{ij}(\boldsymbol{\lambda}), \; ij \in \tilde{A}, \tag{2}$$

where: P_{ij} is the maximum acceptable degree of saturation on each lane on $ij \in \tilde{A}$; m_{ij} is the number of lanes on link $ij \in \tilde{A}$.

In addition, the green time and circle time at a signal-controlled intersection and the allocated number of lanes should satisfy some linear constraints, given as:

$$C_{\min} \le C_n \le C_{\max}, \; n \in N; \tag{3}$$

$$\lambda_{\min} \le \lambda_{hn} \le \lambda_{\max}, \; h \in S_n, \; n \in N; \tag{4}$$

$$m_{ij} \text{ is a integer and } m_0 \le m_{ij} \le m_a, \; ij \in a, \; a \in \bar{A}, \tag{5}$$

where: $C_{\min}$, $C_{\max}$ is the minimum and maximum circle time respectively; $\lambda_{\min}$, $\lambda_{\max}$ is the low-bound and up-bound of green split respectively; m_0 is the minimum allocated number of lanes for one flow direction.

Generally, with the economic development and continuously growing population in developing countries, the city urbanization process is accelerated, thus it is rational to assume that demands between certain O–D pair in future are no less than a preset demands, namely.

$$q_{rs} \ge q_{rs}^0, \; \forall rs \in W, \tag{6}$$

where: q_{rs}^0 is the preset O–D demand. Due to the start-up lost time and all red phases, lost time is existed in each signal circle. Thus the green split must satisfy the following relationship:

$$\sum_{h \in S_n} \lambda_{hn} + \frac{L_n}{C_n} = 1, \; \forall n \in N, \tag{7}$$

where: L_n is the fixed lost time per circle length.

Based on the above considerations, the problem to find the maximum network capacity on multi-phase signalized road network can be formulated as the following bi-level programming:

$$\max z = \sum_{r,s} q_{rs}$$

$$\text{subject to}\begin{cases} m_{ij}+m_{ji}=m_a,\ ij\in a,\ ji\in a,\ a\in\bar{A}; \\ v_{ij}\le m_{ij}p_{ij}c_{ij}(\lambda),\ ij\in\tilde{A}; \\ \sum_{h\in S_n}\lambda_{hn}+\frac{L_n}{C_n}=1,\ \forall n\in N; \\ C_{\min}\le C_n\le C_{\max},\ n\in N; \\ \lambda_{\min}\le\lambda_{hn}\le\lambda_{\max},\ h\in S_n,\ n\in N; \\ q_{rs}\ge q_{rs}^0,\ \forall rs\in W; \\ m_{ij}\text{ is a integer and } m_0\le m_{ij}\le m_a,\ ij\in a,\ a\in\bar{A}. \end{cases} \tag{8a}$$

Assume the drivers all have perfect knowledge of the traffic and make a route choice to minimize their travel cost, and the signalized intersection are all controlled with two-phase signals. Then the equilibrium v_{ij} in problem (8a) can be obtained by solving the following DUE problem:

$$\min\sum_{ij\in A}\int_0^{v_{ij}}\left(t_{ij}\left(x,C_m,m_{ij},\lambda_{ij}\right)+d_{ij}\left(C_m,\lambda_{ij},x,m_{ij}\right)\right)d(x)$$

$$\text{subject to}\begin{cases} \sum_{k\in R_{rs}} f_k^{rs}=q_{rs},\ \forall rs\in W; \\ \sum_{r,s}\sum_{k\in R_{rs}} f_k^{rs}\delta_{ij,k}^{rs}=v_{ij},\forall ij\in A; \\ f_k^{rs}\ge 0,\forall k\in R_{rs},\forall rs\in W. \end{cases} \tag{8b}$$

Suppose the approach traffic at an intersection obeys a Poisson distribution, then the following formula suggested by Sheffi (1985) can be used to calculate average delay in problem (8b):

$$d_{ij}=\frac{C_n\left(1-\lambda_{hn}\right)^2}{2(1-\rho)}+\frac{\rho/\lambda_{hn}}{2c_{ij}\left(\lambda_{in}-\rho\right)}, \tag{9}$$

where: c_{ij} is the saturation flow on link ij, $c_{ij}=c\cdot m_{ij}$; λ_{hn} is the split of the signal phase in which the vehicles on link ij are allow movement; ρ is the normalized flow $\rho=v_{ij}/c_{ij}$.

2.2. Two Strategies for Reversible Roadway Implementing

To reverse the direction of roadway is perhaps the most common measure applied by decision makers to accommodate the souring up traffic flow on the major flow direction in peak hour. However, in practice, how to reverse the roadway may be different in terms of whether all the lanes are reversible. In most situations, only a few lanes in the middle are reversible and the roadway is still able to hold two-way traffic flow. For example, the Caldecott Tunnel between Oakland, California and Contra Costa County, California, is only switched the direction of middle bore among the three for rush hour traffic, and at least one lane is maintained for minor flow direction. Another example is US Route 78 in Snellville, Georgia, US. It has 6 lanes in total, of which the middle two lanes were reversible (usually occurring during rush hours) with a varying lane always reserved as a center turn lane; hence at least 3 lanes were used for one direction of travel and 2 lanes for the other. However, in some other cases, the roadway may be fully reversible that all lanes for one flow direction can be shifted to its opposite direction if the traffic is significantly asymmetric. In this scenario, the original two-way road will turn to be one-way road during this period. For example, the Victoria Bridge, in Montreal, Quebec, Canada normally allows for two-way traffic, but during rush hours, in order to hold the souring up unbalanced traffic, it only allows one-way traffic: northbound in the morning, and southbound in the afternoon. Interested readers could refer to Wolshon and Lambert (2006) for more practical applications of those two reversible road strategies.

According to whether one-way road is allowed, we consider two different strategies for implementing reversible roadway, i.e, Strategy 1: one-way road is not allowed, in this case, no matter how small the traffic flow in the minor flow direction, at least one lane is allocated for it, namely, the low-bound of number of lanes parameter is 1 (m_0 = 1 in in Equation (5)); and Strategy 2: one-way road is allowed, that is, all lanes on the roadway are reversible and could be allocated to one flow direction if the traffic volume in both flow direction is severely asymmetric, namely, m_0 = 0 in in Equation (5); For simplicity, the following we will use 'network capacity problem with Strategy 1 or 2' to represent the 'network capacity problem with reversible roadways under Strategy 1 or 2'.

3. Solution Approach

3.1. General Description of Solutions for Bi-Level Mathematical Programming

Similar to other forms of bi-level mathematical programming, our proposed reserve capacity problem (8) is also intrinsically non-convex, and hence is very difficult to solve for a globally optimal solution. The bi-level network capacity problem in the previous research is generally solved with gradient-based algorithm, such as sensitivity analysis based method (Wong, Yang 1997; Gao, Song 2002), projected Quasi-Newton method (Chiou 2008) and augmented Lagrangian algorithm (Meng *et al.* 2001). One of the most important characteristics for gradient-based algorithm is that it can produce quickly enough a solution by following the decent direction of the multivariable target function. However, the gradient-based algorithm has a very strong prerequisite that all designed variables (O–D demands, signal splits and so on) must be continuous and the equilibrium solutions (link flows, route travel time) are differentiable with respect to the designed variables. Therefore, this method is not applicable for our proposed bi-level program since the parameter m_{ij} in problem (8a) is discrete instead of continuous.

It can be seen that the designed variables in network capacity problem (8) not only include the discrete variable (allocated number of lane parameter), but also include the continuous variable (O–D demands, signal splits, circles). Thus the network capacity problem is actually a MNDP which can be solved with Simulated Annealing (SA) (Sun *et al.* 2009) and global optimization algorithm based on a cutting constraint method (Luathep 2011). In this paper, we develop a GA method for the proposed network capacity problem (8). GA is a random search technique based on Darwinian evolution that is originally introduced by Holland (1992). It searches the optimal solution in the feasible region through natural principles of selection. GA is best known for its simple form, less restriction and more powerful search for improvement. It has been applied to a wide range of research area, including engineering, sciences, and commerce (Mathew, Sharma 2009). Generally, GA can find the global satisfactory solution if the parameters are well controlled and enough generations are guaranteed. The procedures for implementing GA for solving the network capacity problem (8) are summarized as:

- *Step 1:* Initialization. Define the GA parameters, such as crossover probability, population size, mutation rate and the range of parameters. Randomly generate an initial population coded by real numerical strings.
- *Step 2:* Using well-known Frank-Wolfe method for lower-level UE assignment problem with the potential solutions given by the population. Then evaluate the fitness of each chromosome in the population.
- *Step 3:* Perform GA operators (i.e. reproduction, crossover, and mutation) to create offspring; increase generation counter.
- *Step 4:* Convergence test: check the stopping criterion. If the termination criterion is met, accept the best individual in population as the approximated optimal solution and stop; otherwise, return to *Step* 2.

3.2. Genetic Algorithm for Solving the Proposed MNDP Problem

As is mentioned before, for simplicity, all the intersections in the network are assumed to be controlled with two-phase signals. Therefore, for each intersection $n, n \in N$, if one signal split is obtained, another signal split can be calculated with Equation (7). For lane allocations, similarly, we can obtain the allocated number of lanes of the opposite direction with Equation (1) if the number of lanes of one flow direction is given in advance. Thus, only half of the signal splits parameter and allocated number of lane parameter are required to be coded in the chromosome. In this paper, each designed variable (i.e. signal splits, circles, O–D demands and allocated number of lanes) in the chromosome is coded using a real value representation. Then for each chromosome, it is represented as:

$$\left(\lambda_{11}, \lambda_{11}, \cdots, \lambda_{1P}, C_1, C_2, \cdots, C_P, \left\{q_{rs}, \forall rs \in W\right\}, \left\{m_{ij}, \forall ij \in \bar{A}\right\}\right),$$

where: $\left\{q_{rs}, \forall rs \in W\right\}$ denote all the O–D demands on the road network. $\left\{m_{ij}, \forall ij \in \bar{A}\right\}$ denote the number of lanes for one flow direction on all reversible roadway. It is very clear that each locas in the chromosome represents an input variable.

Selection, the selection operator in GA is to select the superior chromosomes to generate a next generation according to the fitness. Selection operator must be such that the chromosomes with better fitness will have higher probability to be selected and weaker ones will be withered away in probability. In this paper, the roulette wheel selection method is applied in which the probability to choose a particular chromosome equals to the quotient of its fitness and the total fitness of chromosome in the population.

Crossover, crossover is a process passing the good characteristic of parent chromosome to its child chromosome. Since most of the variables in a chromosome are continuous variables (e.g. circles, signal splits and demands), the arithmetic crossover operator which linearly combine two parent chromosome vectors to produce two new offspring is used for crossover operation. The main procedure for arithmetic crossover operator can be denoted as:

$$\begin{cases} X_A^{t+1} = \alpha X_B^t + \left(1-\alpha\right) X_A^t; \\ X_B^{t+1} = \alpha X_A^t + \left(1-\alpha\right) X_B^t, \end{cases} \tag{10}$$

where: X_A^t, X_B^t are two parent chromosome; X_A^{t+1}, X_B^{t+1} is the produced child chromosome; α is a scalar, $\alpha \in \left[0,1\right]$.

The arithmetic crossover operator makes sure that the produced child chromosome at each generalized genome (represent solution variable) will automatically subject to the upper and lower bound constraints, namely, the generated solutions in the chromosome will automatically satisfy the in Equations (3) and (4). However, the arithmetic crossover operator can not guarantee that the produced gene corresponding to allocated number of lanes in each chromosome will be integers since it is specially designed for continuous variables other than discrete variables. Therefore, the rounding-off method is introduced to produce a 'real' values for allocated number of lanes parameter.

Mutation. This operator replaces the value of the chosen gene with a uniform random value selected between the user-specified upper and lower bounds for that genome. Mutation operator is used to maintain genetic diversity from one generation of a population and to reduce the probability of close breeding. Mathematically, it helps to avoid the genetic algorithm to converge to local optimum value. For the mixed integer nonlinear problem (8a), the mutation operator is formulated as:

$$\rho' = \begin{cases} \rho + \left(upbound - \rho\right) delta, & \text{if } \kappa = 1; \\ \rho - \left(\rho - lowbound\right) delta, & \text{if } \kappa = -1, \end{cases} \tag{11}$$

where: ρ is a locas of the chromosome; ρ' is the mutated locas; *upbound*, *lowbound* denote the minimum and maximum value of the variable which the locas ρ represents; κ is a discrete value which belongs to the set $\left\{-1,1\right\}$, it is generated randomly in the mutation process to decide the direction of the mutation, that is, to mutate downward or upward; *delta* is a continuous random variables used to decide the step of the mutation, $delta \in [0,1]$. Similar to the crossover operator, the allocated number of lane parameter obtained by mutation process is not guaranteed to be an integer. Therefore, the rounding-off method is operated again to estimate the allocated number of lanes.

Fitness function: with previous operations, a population is changed in form and characteristics, which represents a new generation. The crossover and mutation operator make sure that all the input variables in each of the produced chromosome will automatically satisfy corresponding bound constraints (in Equations (4–6)). However, since another half of the input variables (e.g. signal splits and allocated number of lanes) are calculated based on the generated chromosome and Equation (7) and (1), it is very likely that the resulted half number of signal splits may violate the low bound constraint of in Equation (4). Besides, the equilibrium link flow solutions with regard to each chromosome may not satisfy the link capacity constraints (in Equation 2). Those chromosomes are thus recognized as infeasible solution for network capacity problem (8) and must be punished in the fitness function in order to reduce the possibilities of producing the next generation. Following those considerations, the fitness function is then formulated as:

$$F = \begin{cases} F_1; \\ F_2, \end{cases} \tag{12}$$

where:

$$F_1 = Val, \text{ if } \min\left(\frac{1-\lambda_{1n}-10}{C_n-\lambda_{\min}} < 0, n \in N\right);$$

$$F_2 = -\sum_r \sum_s q_{rs} + \gamma \max\Big(0, \max\big(v_{ij} - n_{ij} p_{ij} c_{ij}(\boldsymbol{\lambda})\big)\Big),$$

$$ij \in \tilde{A}, \text{ if } \min\left(\frac{1-\lambda_{1n}-10}{C_n-\lambda_{\min}} \geq 0, n \in N\right),$$

where: Val is a constant that is much greater than the total demand.

Equation (12) denotes that once the signal splits violate the lower bound, the corresponding chromosome will be punished with a large value. In addition, the equilibrium link flows exceeding the link capacity will also be punished. By using the fitness function, one can see that the genetic algorithm will accept those feasible chromosomes according to their fitness. It is worthy noting that the coefficient γ in Equation (12) is in charge of the precision of the calculation results, the higher precision is required, the larger coefficient γ is supposed to be set.

4. Numerical Example

In this section, we will demonstrate the applications of the proposed method with an example road network presented in Fig. 1. It has four O–D pairs (5→2, 6→9, 1→10, 10→1), eighteen links and ten nodes, among which the nodes 3, 4, 7, 8 are signalized intersections controlled with two-phase signals. Bureau of Public Roads (BPR) function is used for the link travel time which is monotonic with respect to link flows, denoted as:

$$t_{ij} = t_0\left(1 + 0.15\left(\frac{v_{ij}}{c \cdot m_{ij}}\right)^4\right), \tag{13}$$

where: the free flow link travel time t_0 and initial allocated number of lanes m_{ij} can be found in Fig. 1.

Equation (13) implies that if the allocated number of lanes approach to 0, e.g. $m_{ij} \to 0$, then the link travel time approaches to positive infinite, e.g. $t_a \to +\infty$. In this scenario, according to the rule of DUE, there will be no traffic flows assigned to the routes which using this link if another route with finite travel time is existed.

The links 54, 43, 32, 67, 78 and 89 in the example network are one-way roads, the rest are two-way roads on which the lanes are assumed to be reversible. Our goal is to optimize the allocated lanes on two-way roads to maximize the network capacity. The lower and upper bounds of signal splits are set as 0.1 and 0.9 respectively. The maximum degree of saturation for all signal-controlled links are taken the same value of p = 0.9. The minimum and maximum circles are 30 seconds and 180 seconds respectively. All signalized intersections have the same lost time 10 seconds in each circle. The initial O–D demands are 1000 veh/h between all O–D pairs. The minimum O–D demands q_{rs}^0 in in Equation (6) are assumed to be their initial value. For GA procedure, the parameters are set as following:

– real code GA is used and considered up to three decimal precision;

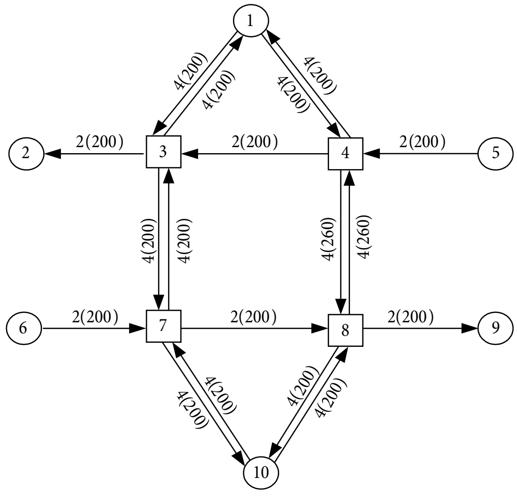

Fig. 1. The example road network (the digits in bracket is value for t_0 [s] and the digits out of bracket is the value for m_{ij})

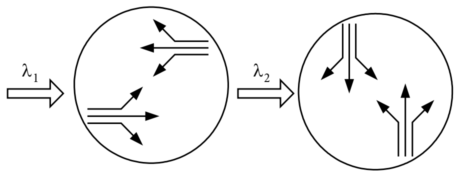

Fig. 2. Signal phase plans for all signalized intersections

– population size 60;
– crossover operator is arithmetic crossover with a probability of 0.8;
– mutation rate, 0.25;
– the *Val* in fitness function (12) is set as 10^8, penalty coefficient are fixed and set as γ = 1000.
– the maximal number of generation is 600.

The software *Matlab* is used to code the GA to solve the bi-level network capacity problem. Figs 3a, b describe the convergence of GA method. It can be seen that GA converges for network capacity problem with both Strategy 1 and Strategy 2 within the maximal generation number, which substantiate its effectiveness in solving the MNDP. Fig. 3c reveals that the GA converges after 250 generation for network capacity problem with Strategy 1, much faster than it is with Strategy 2, which takes more than 530 generations to converge. Besides, it is widely recognized that the difficulty to obtain the equilibrium solutions for DUE problem multiplies when congestion level of the network increases. In this term, the computational cost for network capacity with Strategy 2 is much more expensive than it is for Strategy 1 since the maximum network capacity obtained with Strategy 2 is greater that it is with Strategy 1 as can be seen in Fig. 3c.

Table 1 lists the optimized signal splits, circles and O–D demands for network capacity problem with Strategy 1 and Strategy 2 respectively. It demonstrates that the optimized O–D demands are all increased in comparison with the initial values. For example, the optimized demand for O–D pair 5–2 is 1373.3 veh/h, an increase of 37.3% compared with the original value. Besides, the network capacity, which is the sum of the maximum O–D demands, can also be calculated according to Table 1, which are 12612.7 veh/h and 15498.9 veh/h for Strategy 1 and Strategy 2, respectively. Therefore, we can draw a conclusion that the network capacity with Strategy 2 is dramatically larger than it is with Strategy 1. In the following, we will demonstrate the reasons through deep analysis of the equilibrium results of Strategy 1 and Strategy 2.

Fig. 4 shows the optimized link flows and allocated lanes for Strategy 1. As opposed to the initial state where the lanes on the reversible roadway are symmetrically distributed in both directions (Fig. 1), the optimized allocated lanes on reversible roadway, however, is asymmetry. For example, the allocated number of lanes for link 84 is 2, while the number of lanes in the opposite directions (link 48) is 6. Besides, the traffic flow on each reversible roadway is severally unbalanced in equilibrium state, a direct proof the necessity in reversing the lanes to accommodating the unbalanced traffic demands. Table 2

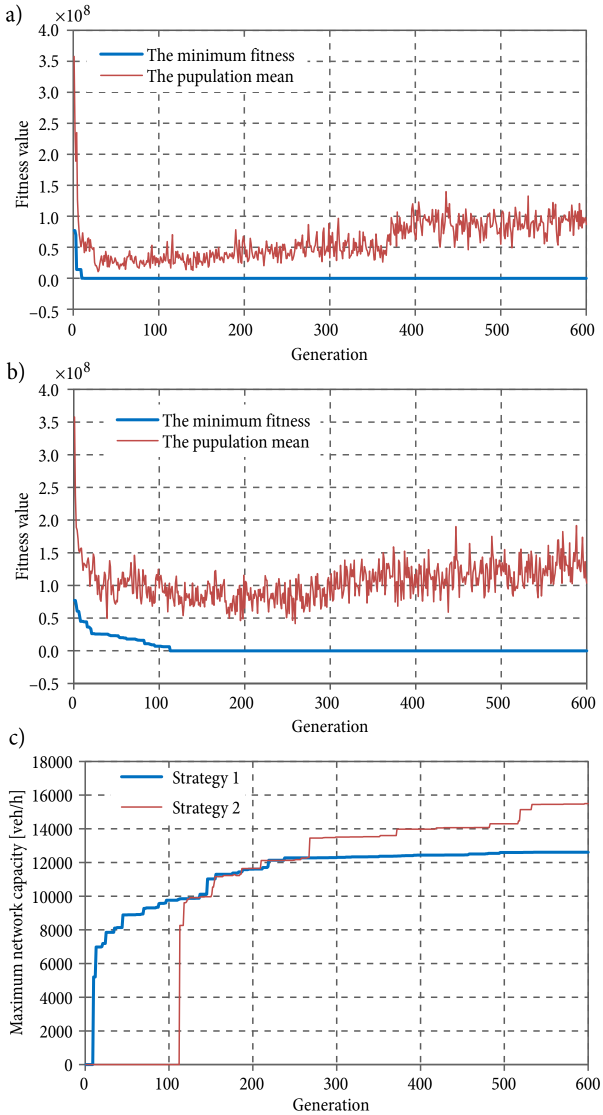

Fig. 3. The solution of network capacity problem with Strategy 1 and Strategy 2: a – the convergence of GA for solving network capacity problem with Strategy 1; b – the convergence of GA for solving network capacity problem with Strategy 2; c – the maximum network capacity with Strategy 1 and Strategy 2 found at each generation

Table 1. The optimum solutions for network capacity problem with Strategy 1 and Strategy 2

Strategies	Signal splits							
	λ_{14}	λ_{24}	λ_{13}	λ_{23}	λ_{17}	λ_{27}	λ_{18}	λ_{28}
Strategy 1	0.424	0.507	0.590	0.354	0.565	0.380	0.508	0.372
Strategy 2	0.393	0.539	0.432	0.487	0.404	0.522	0.399	0.533
Strategies	Circles [s]				O–D demands [pcu/h]			
	C_4	C_3	C_7	C_8	q_{52}	q_{96}	q_{110}	q_{101}
Strategy 1	145.896	178.299	179.496	83.677	1373.3	1644.2	4797.2	4798.0
Strategy 2	149.23	122.88	135.423	147.42	1215.9	1217.1	6783.0	6282.9

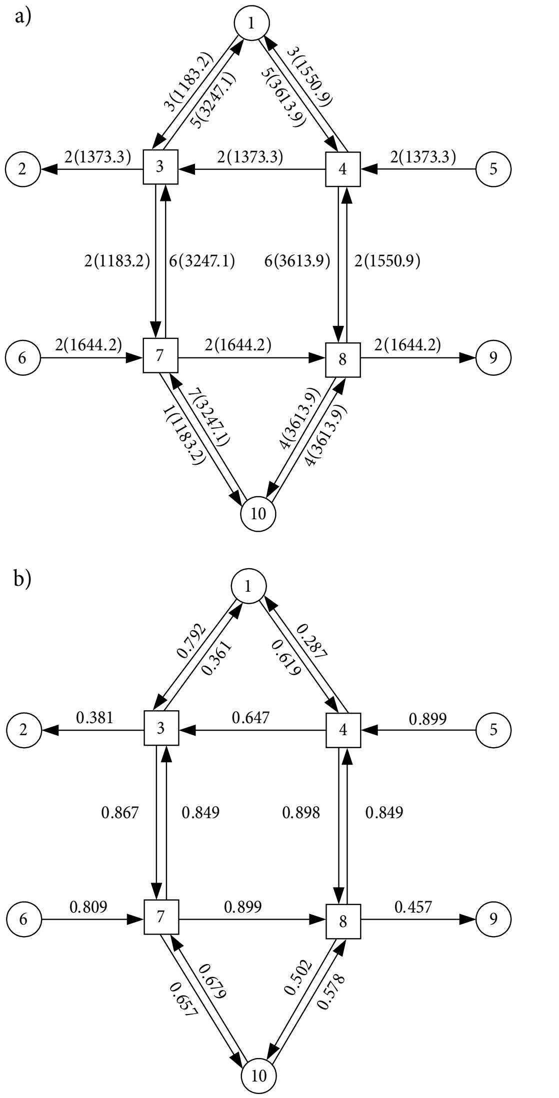

Fig. 4. The solutions for network capacity problem with Strategy 1: a – optimal link flow solutions (in brackets) and number of lanes (out of brackets) for network capacity problem with strategy 1; b – the Volume/Capacity (V/C) of each link at maximum network capacity state of Strategy 1

Table 2. Optimal route flow and route travel time for network capacity problem with Strategy 1

O–D pair	Route [in node order]	Route flow [veh/h]	Route travel time [s]
O–D pair 5→2	5–4–1–3–2	0	943.2
	5–4–3–2	1373.3	728.94
	5–4–8–10–7–3–2	0	1528.8
O–D pair 6→9	6–7–8–9	1644.2	713.73
	6–7–10–8–9	0	913.66
	6–7–3–1–4–8–9	0	1518.4
O–D pair 1→10	1–3–7–10	1183.2	762.69
	1–4–8–10	3613.9	762.69
	1–3–7–8–10	0	998.19
	1–4–3–7–10	0	1009.5
	1–4–3–7–8–10	0	1245.0
O–D pair 10→1	10–8–4–1	1550.9	758.22
	10–7–3–1	3247.1	758.22
	10–7–8–4–1	0	1039
	10–8–4–3–1	0	1018.3
	10–7–8–4–3–1	0	1299.1

Note: the value in dark background is the minimum route travel time between corresponding O–D pair

lists the equilibrium route flows and route travel time of each O–D pair for Strategy 1. It shows that at equilibrium condition, there are diversity feasible routes for each O–D pair, e.g. three feasible routes for O–D pair 5→2 and 6→9 and five feasible routes for O–D pair 1→10 and 10→1. The O–D demands for O–D pair 5→2 and 6→9 are only distributed on one route while the O–D demand is distributed on two least-cost routes, that is, the route 1–3–7–10 and 1–4–8–10 for O–D pair 1→10 and routes 10–8–4–1, 10–7–3–1 for O–D pair 10→1.

Fig. 5a describes the equilibrium link flows and allocated number of lanes for network capacity with Strategy 2. It denotes that when one-way road is allowed, all lanes on the reversible roadway are assigned to one flow direction, making the two-way roads into one-way roads. As a result, the feasible routes between corresponding O–D pairs are significantly reduced in comparison with they are at Strategy 1. For example, there only remain 2 routes for both O–D pair 5→2 and 6→9, less than it is at Strategy 1, which has 3 feasible routes for both O–D pair 5→2 and 6→9. Besides, the O–D demands between 1→10 and 10→1 are only assigned to one feasible route other than two in Strategy 1. This implies that large number of traffic flows on one route in Strategy 1 must be moved to the other route if Strategy 2 is adopted. However, despite that the number of feasible routes of Strategy 2 is less than that of Strategy 1, the network capacity for Strategy 2 increases about 22.9% compared with Strategy 1. This result, to some extent, is beyond the expectation of the road manager. Practically, to avoid the deficiencies associated with Strategy 2, such as confusing road, the mangers generally would not take the one-way strategy to accommodate the maximum increase in travel demand, especially in some arterials. Therefore, even for some extreme situations, such as the seriously asymmetry of the traffic flow on two-way streets, commonly, the road manager are just try to allocate as much lanes as possible for major flow direction other than all the lanes. This strategy however, may result in inefficient use of the link capacity of the minor flow direction. In the following, we will substantiate the claim.

Figs 4b and 5b depicts the Volume/Capacity) (V/C) of each link at maximum network capacity with Strategy 1 and Strategy 2, respectively. Since the reversible roadway in the example network, mainly used to accommodate the traffic demand between O–D pair 1→10 and 10→1, we only analyze the V/C of links used by the routes between O–D pair 1→10 and 10→1. Fig. 4b denotes that although at equilibrium state, the V/C of link 84, 48, 37 and 73, which constitute the maximum cross section of traffic demands between O–D pair 10→1

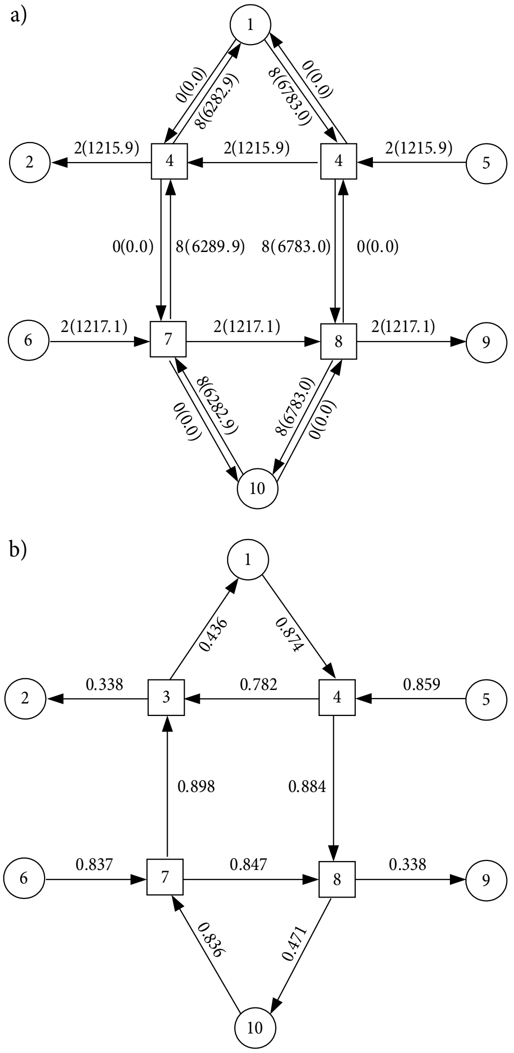

Fig. 5. The solutions for network capacity problem with Strategy 2: a – optimal link flow solution (in brackets) and number of lanes (out of brackets) for network capacity problem with Strategy 2; b – the Volume/Capacity (V/C) at maximum network capacity state of Strategy 2

and 1→10, are very close to the maximum allowable V/C (0.9), the V/C of other signal controlled links, such as links 13, 14, 107 and 108, is far from the maximum allowable value (0.9). This result indicates that Strategy 1 cannot make full use of the capacity of signal-controlled links. When maximum network capacity is arrived for Strategy 2, one can see from Fig. 5b that not only the V/C of links in the maximum cross section (links 73 and 48) approach to the maximal value (0.9), but also the V/C of all other signal controlled links (link 107 and link 14) are close to 0.9 (Fig. 5b). Besides, Table 1 demonstrates that the optimal signal splits at Strategy 2, which control the reversible roadways (e.g. λ_{24}, λ_{23}, λ_{27}, λ_{28}) all outperform they are at Strategy 1, thus more traffic can be accommodated in the main flow direction. Based on the above discussions, we can draw a conclusion that the Strategy 2 can make more use of physical link capacity, thus more network capacity is generated as compared to Strategy 1.

Table 3. Optimal route flow and route travel time for network capacity problem with Strategy 2

O–D pair	Route [in node order]	Route flow [veh/h]	Route travel time [s]
O–D pair 5→2	5–4–1–3–2	–	–
	5–4–3–2	1215.9	722.1976
	5–4–8–10–7–3–2	0	1513.093
O–D pair 6→9	6–7–8–9	1217.1	729.0418
	6–7–10–8–9	–	–
	6–7–3–1–4–8–9	0	1513.3366
O–D pair 1→10	1–3–7–10	–	–
	1–4–8–10	6783.0	794.9546
	1–3–7–8–10	–	–
	1–4–3–7–10	–	–
	1–4–3–7–8–10	–	–
O–D pair 10→1	10–8–4–1	–	–
	10–7–3–1	6282.9	716.5433
	10–7–8–4–1	–	–
	10–8–4–3–1	–	–
	10–7–8–4–3–1	–	–

Note: the value in dark background is the minimum route travel time between corresponding O–D pair.

It is worth noting that the conclusion is well-founded not just for our proposed road network, but is also for all the networks. This is because the feasible search region of parameter m_{ij} for Strategy 2 ($m_{ij} \geq 0$) is larger than it is with Strategy 1 ($m_{ij} \geq 1$). Therefore, the maximum network capacity founded with Strategy 2 is at least no less than it is with Strategy 1. Besides, by changing some two-way streets into one-way streets, the network capacity is not only maximized, thus can hold more traffic volume, but also some users on the network can benefit from the increased speeds and reduced travel times either. Therefore, Strategy 2 is much preferable than Strategy 1. However, it should be noted that although the Strategy 2 improves the network capacity considerably as compared to Strategy 1, the average travel cost is inferior to it is of Strategy 1, with 750.9 s and 752.3 s respectively calculated based on Table 2 and Table 3. This is partly because the Strategy 2 forces large number of derivers to use a further route (the drives used the route 1–3–7–10 in Strategy 1 are forced to use the route 1–4–8–10 in Strategy 2 which is more time taking). Consequently, the reversible roadway strategy-makers should have a system analysis of distinct strategies before put it into practice in order to avoid the possible risks of decreasing the average travel utility.

Conclusions

This paper formulates a MNDP for network capacity maximization on signalized road network with reversible lanes. Based on practical applications, two different reversible roadway design strategies are considered, that is, *Strategy 1: one-way road is forbidden* and *Strategy 2:*

one-way road is allowed. GA is detailedly presented to solve the bi-level network capacity problem. Application of the proposed method in an example road network finds that:

1) The proposed GA can efficiently solve the bi-level network capacity problem with both Strategy 1 and Strategy 2. However, it converges much slower for network capacity problem with Strategy 2 than it is with Strategy 1. This is partly because the resulted network capacity with Strategy 2 is larger than it is with Strategy 1, which makes it more difficult to obtain an equilibrium solution for the low-level DUE problem.
2) Network capacity with Strategy 2 is generally larger than it is with Strategy 1. This is because under Strategy 2, the equilibrium traffic flows can make more use of the physical capacity of signal-controlled links (key links) than it is under Strategy 1. Besides, Strategy 2 allows more the signal splits to be allocated for the main road, thus the physical capacity of main road is increased considerably.
3) Although Strategy 2 improves the network capacity with Strategy 1 effectively, sometimes, the improvement is achieved by sacrificing average travel time. This is because the one way roads generated by Strategy 2 may leave some derivers no choice but to use the more cost routes. Thus, to avoid the possible risks of decreasing the average travel utility, the strategy maker should have a system analysis of distinct strategies before putting it into practice.

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