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OPTIMIZING LIMITED-STOP BUS SERVICES ALONG A PUBLIC TRANSIT CORRIDOR WITH A DIFFERENTIAL FARE STRUCTURE

Chunyan TANG[1], Ying-En GE[2*#], William H. K. LAM[3]

[1]*College of Transportation Engineering, Dalian Maritime University, China*
[2]*College of Transport and Communications, Shanghai Maritime University, China*
[3]*Dept of Civil and Environmental Engineering, The Hong Kong Polytechnic University, China*

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Abstract. Limited-stop bus services are a highly efficient way to release more potential of the public transit system to meet travel demand, especially under constraints on vehicle fleet size and transportation infrastructure. This work first proposes a visualized fare table for the design of limited-stop bus services along a public transit corridor, along which many lines of public transit carry a heavy load of demand back and forth every working day. Based on this proposed fare table, a set of fare strategies and desired aims of fare policy, a differentiated fare structure is established to improve social equity and increase revenue. The nature of the structure can help travellers understand how to be charged between their origins and destinations (e.g. flat, time-based, stop-based or quality-based pricing) and then plan their trips efficiently. Secondly, a model is formulated to minimize the total social cost in designing a fixed demand limited-stop bus service system with a differentiated fare structure. Thirdly, numerical results are carried out with sensitivity analysis within three scenarios of differentiated fare structures. It is found that a differentiated fare structure has a great effect on passenger path choice behaviour and resulting optimal design of bus services. An attractive feature of this differentiated fare structure is that it could not only enhance the operator's revenue and social equity but also reduce passenger transfers and social cost.

Keywords: public transit, limited-stop bus service, differentiated fare structure, social cost, environmental improvement.

Introduction

Public transit plays a crucial role in meeting the need for individual mobility and environmental improvement. There has been a considerable increase in the number of urban resident trips and in travel distance in the past few decades, which arose from the continuous growth in both car ownership and urban area as well as rise in urban population. As a result of this growth in car trips, traffic congestion and transport pollution issues have turned out to be a global concern. In order to solve these issues, many policy makers, transportation planners, and researchers consider public transit an efficient way out. Transit routes with high demand (e.g., routes passing through a Central Business District (CBD) or resident area of great density) are generally seen as public transit corridors (Leiva *et al.* 2010). Apparently, in practical operations, supply is not efficient to accommodate passenger demand spatially due to the use of common full-length services. The limited-stop service scheme may be a highly efficient way to release more potential of the public transit system to meet needs of these routes, especially under the constraints on vehicle fleet sizes, land use, transportation infrastructure, etc. (Furth, Day 1985). Besides, this scheme is expected to reduce bus operational emissions because of the reduction in the number of "stops and goes", which shall make a great contribution to environmental improvement, in particular air quality. The investigation in the existing literature has shown that the limited-stop service scheme can save a huge cost to the society (Leiva *et al.* 2010). In these research efforts, attention has been paid to consideration of capacity constraints, dwell time, fare collection systems, etc. (Tirachini, Hensher 2011; Tang *et al.* 2016).

The fare structure of public transit has a great impact on traveler choices, e.g. whether to travel, when to travel, how to travel, etc. Therefore, in optimizing the design of limited-stop services, the fare structure is a key factor to capture. To determine it, we need specify or identify the most suitable fare policy, strategies, and enforcement technologies (e.g. types of fare payment and collection tech-

*Corresponding author. E-mail: *yege@shmtu.edu.cn*

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nologies). Fleishman *et al.* (1996) illustrated an interaction among these elements. Depending on specific needs and situations, transit agencies identify a set of goals to offer a guide of fare restructuring and technology development. These goals can be categorized into four aspects: customer-related, financial, managerial, and political. A fare strategy is a fundamental component of the fare structure. Fare strategies can be grouped into two basic categories: flat and differentiated ones. The flat one requires all passengers pay the same fare while the differentiated fare strategy allows passengers to pay a fare dependent on one or more factors, such as length of trip, time of day, quality of service, etc. The existing fare strategies may be further refined to eight categories below (Fleishman *et al.* 1996; Chien, Tsai 2007):

1) *flat fare*: passengers are charged the same amount of fare;
2) *distance-based or zonal fare*: a fare is determined by the distance or amount of zones a trip covers;
3) *time-based fare*: a fare depends on when to start and how long a trip lasts, e.g. whether to travel at peak time or whether a trip is long enough to extend to the peak time;
4) *quality-based fare*: a fare is related to which service a passenger receives, e.g. express, limited-stop, short-turn or local services;
5) *cost-based fare*: a fare is a function of operating costs, e.g. air-conditioning cost, staff wages;
6) *route-based fare*: a fare is associated with which zones a bus goes through, such as CBD, residential zones, work places, or tourist places;
7) *patron-based fare*: a fare depends on types of passengers, such as students, senior citizens, or disabled passengers;
8) *market-* or *consumer-based fare*: a fare is dependent on the frequency of use and willingness to repay, such as passes and discounted tickets.

The advantages and disadvantages of each of the above fare strategies have been discussed widely for years in terms of efficiency, convenience, and equity. Flat fare is recognized to be the simplest and most convenient, though it ignores equity. Alternatively, the differentiated fare can completely display social equity while its implementation requires the use of hi-tech collection systems, e.g. smart bus/strip card (Fleishman *et al.* 1996). There is no doubt that a huge obstacle exists for us to adopt a differentiated fare in implementing public transit services. However, as the new technologies are less and less costly, they have become more applicable for use in collecting bus fare on board. The problem has become how to develop easy-to-understand fare structures. Moreover, there are considerable studies on effects of a flat and a differential fare on revenues. Ling (1998) evaluated effects of flat and differential fare structures on passenger demand, revenues, passenger-km, and consumer surplus. It is found that a differential fare structure may be able to bring out a positive effect on passenger demand and revenues. In the study of maximizing operator's profit by Chien and Tsai (2007), an optimization model is developed for optimizing operating headway and differential time- and zone-based fare structures taking service capacity constraint into account. Results show that differential fare structures may achieve the higher profit, compared with a flat fare. Li *et al.* (2009) examined transit fare structures in consideration of monopoly and oligopoly market regimes with uncertainty. Borndörfer *et al.* (2012) discussed fare effects on passenger travel behaviour when four different objectives are established and used, including the maximization of demand, revenue, profit and social welfare. In this study, distance-based fare structure and single/monthly fare structure are considered. The results from these objectives indicate that distance-based fare structure is preferred by passengers with short travel distance. In addition, Tsai *et al.* (2013) also illustrated such significant impacts of distance-based fare on passengers' travel behaviour by formulating a daily profit maximization problem for an intercity transit system. In the recent study, in order to optimize fare, Zhang *et al.* (2014) proposed a bi-level programming model taking account of the disturbance of unexpected effects and adverse weather conditions. Weather conditions have a great effect on walk times and travel comfort levels. It is found that different fare structures should be used in different kinds of weather conditions. Compared with a flat fare, the existing efforts indicate that the differential fare structure not only proves more potential in solving social equity concern, but also can gain a greater increase in operators' revenues. To the best of our knowledge, most studies on a differential fare structure mainly focus on the design of all-stop services. This paper aims to investigate the effects of a differential fare structure on limited-stop service optimization and users' path choices, and then assist fare decision makers in determining an attractive fare.

One key contribution of this paper is to design a fare structure table. It shifts a differential fare structure from an abstract way into a visualized one. In the existing literature, it is found that one of the most difficult things for delaying the adoption of a differential fare structure is the difficulty for users to understand how to be charged. This may not be as important to passengers as transit operators think it is, especially when new technologies, including smart card and payment systems, help to facilitate the provision of various types of fare structures, but this perception obviously remains. This paper proposes a fare structure table to tackle this difficulty. The fare structure table is similar to a table of Origin–Destination (O–D) pairs, which shows a fare between each pair of all stops. Passengers could get this table by smart phone apps, Internet, and platform messaging, and then plan their trips to minimize expected travel costs. Apparently, public transit operators could get an attractive fare structure table under the regulation of fare policy and various strategies. Thus, this fare structure table not only offers the users a way to understand the fare to pay quickly but also makes the operator able to establish an attractive differential fare structure easily. It is interesting to investigate effects of this scheme on air quality and environmental improvement. To avoid complicating the current discussion on the new type of far structures, we will analyse environmental effects of such fare structures applied to limited-stop bus services in another piece of research work.

Following this introductory section, Section 1 systematically illustrates the representation of a fare structure table and how to formulate a differential fare structure table based on given various fare strategies. Section 2 formulates a model for optimizing limited-stop public transit services with a differential fare structure. We display a series of sensitive analysis by optimizing the proposed model in Section 3. Last section summarizes the findings in this paper and points out further research directions in this arena.

1. Representation of fare structure

1.1. A basic fare structure table

Fare is of considerable importance to bus operations and management as well as to passengers. A low fare may reduce revenue and attract more passengers, whereas a high fare may increase the revenue and reduce demand. Therefore, it is necessary to determine an attractive fare that benefits both users and operators or minimizes the total social cost. Compared with the pervious literature, this paper presents an abstract fare structure, in particular a complex and differential fare structure, in a form of table that shows a new and visualized fare structure for the users and operator of public transit service, like a matrix of O–D pairs, where each origin or destination is a bus stop. This fare table consists of fares between each pair of all stops along a bus corridor, whether these two stops are successive or not. Suppose that there is a bus corridor with n stops, as shown in Figure 1. We define a value of fare u_{ij} for each pair (i, j) of all stops along this corridor. Table 1 gives a basic fare structure for this bus corridor. In a fare structure table for a limited-stop public transit service line, the value of each of those elements with a stop where the bus does not stop is assumed to be $+\infty$.

Figure 1. A sample corridor with n stops

Table 1. A basic fare structure table for the corridor in Figure 1

O \ D	1	2	…	i	…	j	…	$n-1$	n
1	0	u_{12}	…	u_{1i}	…	u_{1j}	…	u_{1n-1}	u_{1n}
2	u_{21}	0	…	u_{2i}	…	u_{2j}	…	u_{2n-1}	u_{2n}
⋮	⋮	⋮	0	…	…	…	…	…	…
i	u_{i1}	u_{i2}	⋮	0	…	u_{ij}	…	u_{in-1}	u_{in}
⋮	⋮	⋮	⋮	⋮	0	…	…	…	…
j	u_{j1}	u_{j2}	⋮	u_{ji}		0	…	u_{jn-1}	u_{jn}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	0	…	…
$n-1$	u_{n-11}	u_{n-12}	⋮	u_{n-1i}	⋮	u_{n-1j}	⋮	0	u_{n-1n}
n	u_{n1}	u_{n2}	⋮	u_{ni}	⋮	u_{nj}	⋮	u_{nn-1}	0

In Table 1, u_{ij} may completely or partly differ from each pair of stops along the corridor in Figure 1. If the value of u_{ij} equals the value of u_{ji}, the resulting fare table is a symmetrical one. If all values of a basic fare table are the same and constant, it means that the bus operator charges all travellers a flat fare, regardless of distance of travel, time of day, quality of service or trip route.

Clearly, if a line does not stop at a stop i, then the value of u_{ij} or u_{ji} is set to $+\infty$ for all $j \neq i$. Therefore, this table can be used for representation of a fare structure for a limited-stop public transit service.

It is noteworthy that the fare structure table of a flat fare strategy has the same constant value for all its elements. If there are multiple stops (denoted by $s_1, s_2, ..., s_k$) between a pair of stops (i, j), the fare for travel between (i, j) is not necessarily the sum of these fares $u_{is_1}, u_{s_1s_2}, u_{s_2s_3}, ..., u_{s_kj}$ but the value u_{ij} that is already given in the Table 1.

1.2. A differential fare structure table

If the elements of a fare structure table vary in accord with the aforementioned strategies, including distance-based, time-based, quality-based, cost-based, route-based, patron-based, and market-based strategies, the table defines a differential fare structure table. The choice of fare strategies depends on purposes of a fare policy. For instance, in order to attract more passengers to use public transit in off-peak periods, the transit service planner may consider varied discounts to lower fare rates during off-peaks so that some users may be led from peak time to off-peak time. If a fare policy aims to increase the operator's revenues and achieve social equity, it is essential to have a comprehensive set of differentiable fare strategies in setting bus fare rates.

The structure of such tables enables travellers intuitively to understand how to be charged between their origins and destinations. For instance, in a stop-based fare table, elements of upper triangular show $u_{ij} < u_{ij+1}$, because the more stops are included within a trip the higher fare is required. New technologies, such as platform messaging, smart phone, and the Internet are available conveniently for showing this table to each public transit service user.

1.3. Discussion: one table for one line?

It is easy to have one table for each line. An advantage for doing so is that the planner or travellers can intuitively understand the fare structure once a line is picked up. The downside is that it occupies a lot of resources to cope with at least as many tables as the number of lines if the fare does not vary due to the difference in time of day.

A different way to handle this is that a fare structure table is specified for all pairs of stops in a network. Based on this, the planners may vary the fare for a specific pair of stops taking into account certain factors, such as time of day, distance between two stops, etc. The fare for a pair of stops along a line can also be a decision variable for the planners to optimize.

2. Model formulation

The objective of this investigation is to minimize the total social cost for a bus corridor, which is defined as a sum of user costs and operator cost. The proposed model is primarily used for the planning and operations of limited-stop public transit services. To establish the objective function of this minimization problem, the following assumptions are made:

1) the characteristics of a bus corridor and a set of feasible limited-stop service lines on it are given, including instance, station location, route zones, stop-spacing and length of it. In this study, a fixed O–D demand matrix is considered along a dedicated corridor (Leiva *et al.* 2010; Tirachini 2007);
2) although passengers randomly arrive at their own origin stops, the rates of their arrivals are assumed to be constant on average. Under the assumption that a passenger always chooses the best stop to transfer, passenger assignment reflects the existence of a set of attractive itinerary segments for each O–D pair that minimizes the expected travel time.

As discussed above, the social cost SC consists of operator cost OC and user costs UC, which can be written as:

$$SC = OC + UC. \tag{1}$$

The operator cost is incurred by vehicle operations and transit lines. Thus, the operator cost OC for a set of lines L can be expressed as (Leiva *et al.* 2010):

$$OC = \sum_{l \in L} \left(R_C \cdot C_l + R_H \cdot H_l \right) \cdot f_l \,, \tag{2}$$

where: L is a set of lines l serving the bus corridor; C_l is the round trip time of line l; H_l is the total length of line l in both directions; R_C is the hourly average operating cost per vehicle; R_H is the transit line operating cost per kilometre per vehicle; f_l is the frequency of line l and measures the number of buses sent out for line l per hour.

The total user cost is the sum of waiting time cost UC_{TW}, in-vehicle time cost UC_{TV}, transfer cost UC_{TT}, and bus fare cost UC_{fare}, i.e.:

$$UC = UC_{TW} + UC_{TV} + UC_{TT} + UC_{fare}. \tag{3}$$

The total waiting time cost UC_{TW} for a user is his or her waiting time multiplied by the value of waiting time, and then the total user waiting time cost can be expressed as:

$$UC_{TW} = \sum_{w \in W} \sum_{s \in S} V_s^w \cdot P_{TW} \cdot TW_s \,, \tag{4}$$

where: V_s^w is passenger flow for a pair w on route segment s; P_{TW} is the value of waiting time savings for a user; TW_s is waiting time on route segment s; S is a set of route segments on the corridor; W is a set of O–D pairs.

The stages of a passenger's journey are called route segments (De Cea, Fernández 1993), each of which is defined as a fictitious link with a start node, an end node and a subset of attractive lines. The waiting time on route segment s, TW_s, depends on bus service frequencies and can be modelled as follows:

$$TW_s = \frac{k}{\sum_{l \in L} f_l^s} \,, \tag{5}$$

where: k is a parameter depending on the distribution of bus arrivals at each stop (when the bus arrival is Poisson distributed, k will be 1).

The in-vehicle travel time that a user experiences on a bus consists of the running time and all the dwell time at each intermediate stop visited by the bus. The in-vehicle time cost can be formulated as:

$$UC_{TV} = \sum_{w \in W} \sum_{s \in S} V_s^w \cdot P_{TV} \cdot TV_s ; \tag{6}$$

$$TV_s = \frac{\sum_{l \in L} RT_l^s \cdot f_l^s + ST^s}{\sum_{l \in L} f_l^s} ; \tag{7}$$

$$ST^s = \sum_{i \in P_s} \max \left(\sum_{k \in S_i^+} \sum_{w \in W} V_k^w \cdot \frac{f_l^k}{\sum_{l \in L} f_l^k} \cdot t_b , \; \sum_{k \in S_i^-} \sum_{w \in W} V_k^w \cdot \frac{f_l^k}{\sum_{l \in L} f_l^k} \cdot t_a \right), \tag{8}$$

where: P_{TV} is the value of in-vehicle time savings for a user; TV_S *is* in-vehicle time on route segment s; RT_l^s is the running time for line l on route segment s; ST^s is dwell time on route segment s; V_k^w is passenger flow on route segment k; t_b is the average boarding time per passenger; t_a is the average alighting time per passenger; S_i^+, S_i^- are two sets of route segments respectively departing and arriving at stop i; P_s is the set of stops on route segment s, except the stop arrived at by route segment s.

Equation (8) is the sum of the bigger one of passenger boarding and alighting times at each stop along segment s because it is assumed here that the processes of boarding and alighting are simultaneous (with different doors for boarding and alighting) and that boarding and alighting flows are independent of each other.

Transfer costs include the cost of time for passengers to walk from one stop to another for transfer and other costs to the passenger due to the transfer but bus fare and waiting time. θ_{trans} is the value of the monetary penalty a user bears due to transfer. T^w is the total passenger flow for O–D pair w. The total transfer cost can be expressed as:

$$UC_{TT} = \sum_{w \in W} \theta_{trans} \cdot \left(\sum_{s \in S} V_s^w - T^w \right). \tag{9}$$

The fares that passengers pay for their whole trips depends on the number of transfers and the bus fare to be paid for each segment of bus journey. It is also assumed that passengers pay full price, i.e. undiscounted bus fares regardless of boarding at an original stop or at a transfer stop. The bus fare cost can be written mathematically as:

$$UC_{fare} = \sum_{w \in W} \sum_{s \in S} U_s \cdot V_s^w; \tag{10}$$

$$U_s = \sum_{s \in S} \sum_{i \in P} \sum_{j \in P} \delta_s^{ij} \cdot U_{ij}, \tag{11}$$

where: U_s is the fare cost on route segment s; U_{ij} is a bus fare between stops i and j, which is determined on the basis of fare strategies and a basic fare u_{ij}; δ_s^{ij} equals 1 if route segment s connects stops i and j, and 0 otherwise.

Thus, the optimization problem of bus services can be formulated as an optimization model as follows:

$$\min_{\{f_l, f_l^s\}} \Bigg(R_C \cdot \Bigg(\sum_{s \in S} \sum_{l \in L} RT_l^s \cdot f_l^s + \sum_{i \in P} \sum_{l \in L} \max \left(\sum_{k \in S_i^+} \sum_{w \in W} V_k^w \cdot \frac{f_l^k}{\sum_{l \in L} f_l^k} t_b, \sum_{k \in S_i^-} \sum_{w \in W} V_k^w \cdot \frac{f_l^k}{\sum_{l \in L} f_l^k} \cdot t_a \right) \Bigg) + R_H \cdot \sum_{l \in L} H_l \cdot f_l + \sum_{w \in W} \sum_{s \in S} V_s^w \cdot P_{TV} \times \frac{\sum_{l \in L} RT_l^s \cdot f_l^s + \sum_{i \in P_s} \max \left(\sum_{k \in S_i^+} \sum_{w \in W} V_k^w \cdot \frac{f_l^k}{\sum_{l \in L} f_l^k} \cdot t_b, \sum_{k \in S_i^-} \sum_{w \in W} V_k^w \cdot \frac{f_l^k}{\sum_{l \in L} f_l^k} \cdot t_a \right)}{\sum_{l \in L} f_l^s} + \left(\sum_{w \in W} \sum_{s \in S} V_s^w \cdot P_{TW} \cdot \left(\frac{1}{\sum_{l \in L} f_l^s} \right) + \sum_{w \in W} \theta_{trans} \cdot \left(\sum_{s \in S} V_s^w - T_w \right) + \sum_{s \in S} \sum_{w \in W} U_s \cdot V_s^w \right) \Bigg) \tag{12}$$

subject to:

$$0 \le f_l^s \le f_l, \text{ integer } \forall l \in L, \forall s \in S; \tag{13}$$

$$\sum_{s \in S_i^+} V_s^w - \sum_{s \in S_i^-} V_s^w = \begin{cases} T_w, & \text{if } i = O; \\ -T_w, & \text{if } i = D; \\ 0, & \text{otherwise,} \end{cases}$$

$$\forall i \in \{1, ..., n\}, \ \forall w \in W; \tag{14}$$

$$q_{l,i} \cdot \tau_q \cdot f_l \ge \sum_{s \in S_i^+} \sum_{w \in W} V_s^w \cdot \frac{f_l^s}{\sum_{l \in L} f_l^s},$$

$$\forall s \in S, \ \forall l \in L, \ \forall i \in P. \tag{15}$$

In this formulation, the objective function consists of four parts, which together defines the total cost to the operator and the passengers. The first part represents vehicle operating costs. The second one is the costs of transit line operations. The third one is in-vehicle time costs to passengers. The fourth one consists of waiting time costs, transfer costs, and fare costs.

In the above optimization model, the constraint set (14) is to ensure passenger flow continuity at each stop, and constraint (15) is set for the desired occupancy on the bus in order to maintain a comfortable average passenger load and eliminate the number of standees (if any). In addition, $q_{l,i}$ is the vehicle capacity of line l departing from stop i, τ_q is a safety margin factor for vehicle capacity and P is a set of all stops on a corridor.

The model formulated above is a nonlinear integral model and we use the branch and bound algorithm coded in LINGO (*https://www.lindo.com/index.php/products/lingo-and-optimization-modeling*) to solve it.

3. Numerical analysis

Same as in Leiva *et al.* (2010), a sample dedicated bus corridor is 6.6 km long in both directions and consists of 10 stops. A set of attractive bus service lines (23 lines) along the bus corridor is shown in Figure 2. The distance between two successive stops is the same and constant, and the running time between two stops is also presented in Figure 2. The O–D demand matrix in Table 2 represents the demand for the duration of one hour between each O–D pair along the corridor. According to the definition of a fare structure table defined in Subsection 1.1, we will consider a flat fare structure as a basic table, of which all elements but those on the diagonal are u_{ij} = \$0.17. The use of such a flat fare structure not only simplifies the process of calculation, but also clearly shows how to formulate a differential fare structure table based on this. Table 3 represents parameter definitions and values.

Figure 3 shows how a trial solution converges to the satisfactory solution.

3.1. Time-based fare structure

Essentially, a time-based fare structure is one dependent on how much time a passenger may need while completing a trip or tour. This implies that travellers in the peak period may pay more than those who make their trips in the off-peak period. Implicitly, this may shift travel demand from the peak time to the off-peak time. Considering time-based fare strategy, a differential fare between stops i and j in Equation (11), U_{ij}, during transit operational time period t, can be formulated as follows:

$$U_{ij}^t = \lambda^t \cdot u_{ij}, \tag{16}$$

where: U_{ij}^t represents a fare between stop i and stop j during transit operation time period t; λ^t is a parameter associated with a basic bus fare u_{ij} during period t.

Passengers sometimes may need to transfer once or more during their trips. Thus, there exists an additional fee associated with transfer. In this section, a transfer discount is provided for passengers involved in transferring.

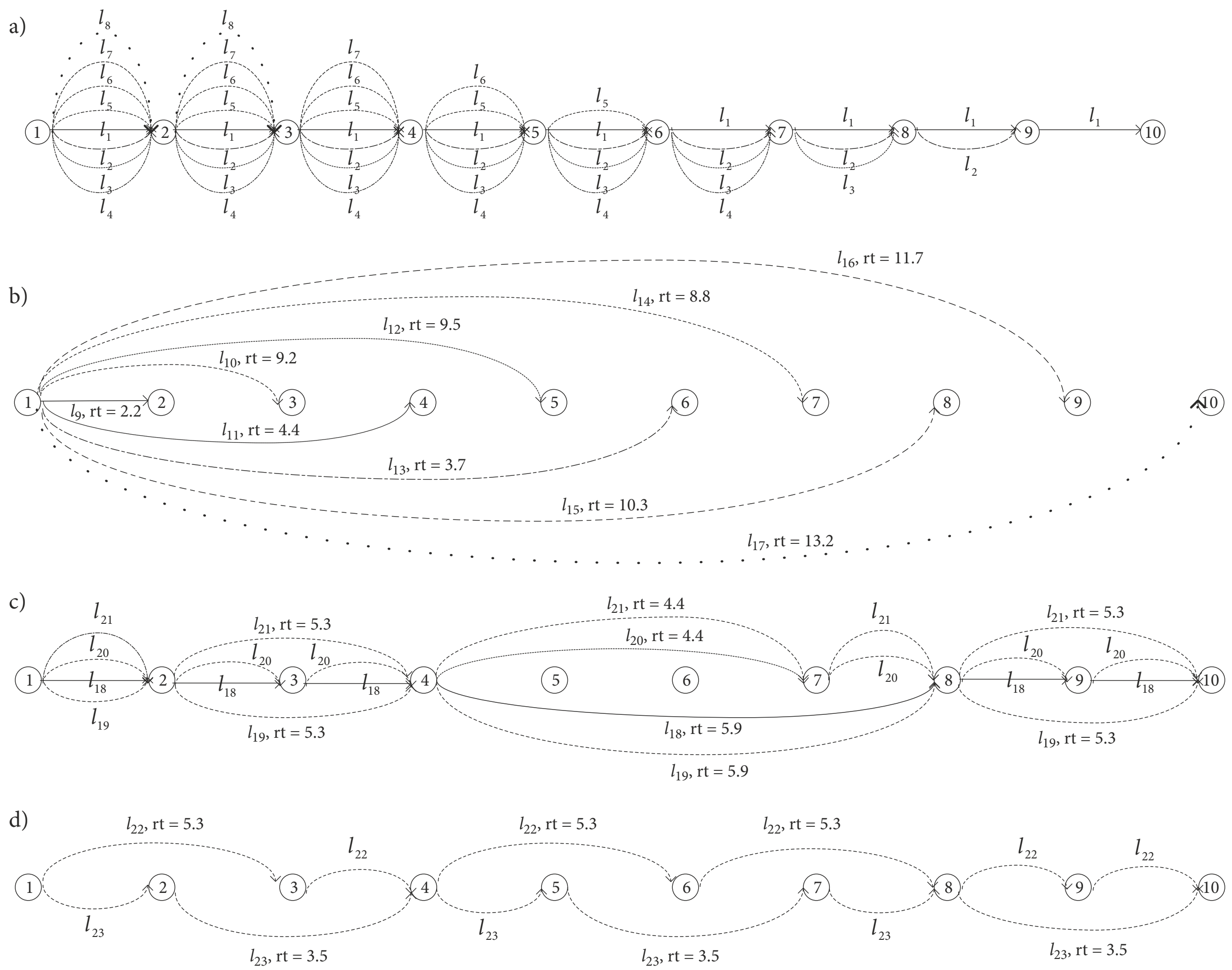

Figure 2. A corridor scenario (source: Leiva *et al.* 2010): a – normal services (short-turn and full-length); b – express services; c – limited-stop services

Table 2. O–D trip demand matrix on the bus corridor (source: Tirachini (2007))

O \ D	1	2	3	4	5	6	7	8	9	10
1	0	600	189	166	63	45	341	606	726	395
2	3620	0	11	9	4	2	20	35	43	23
3	790	38	0	5	2	1	10	18	22	12
4	1585	75	82	0	1	0	3	4	4	3
5	282	13	14	14	0	2	13	24	28	16
6	187	9	9	9	8	0	13	22	27	14
7	263	13	13	13	12	9	0	11	14	8
8	2631	125	136	129	117	86	107	0	36	19
9	336	16	17	17	15	11	14	18	0	67
10	4425	211	229	218	198	144	179	232	200	0

A passenger who transfers need pay the full fare of his or her first route segment and is assumed only to pay transfer fares for transfer route segments. This transfer fare is represented as the full fare of the transferred services multiplied by a transfer discount. Considering a transfer discount parameter π added in the Equation (10), the total fare costs are given as follows:

$$UC_{fare} = \sum_{s \in S^{+}_{i=O}} \sum_{w \in W} U^{t}_{s} \cdot V^{w}_{s} + \sum_{s \in S^{+}_{i \neq O}} \sum_{w \in W} \pi \cdot U^{t}_{s} \cdot V^{w}_{s}, \quad (17)$$

where: $S^{+}_{i=O}$ is the set of route segments departing from the origin stop of a pair w; $S^{+}_{i \neq O}$ is the set of route segments departing from other stops except the origin stop of a pair w; the first part of equation is full fare costs for users, and the second part is transfer discount fare costs.

Table 3. Parameter definitions and values

No	Parameter	Descriptions	Value	Unit
1	q	capacity of bus	80	passengers
2	t_b	boarding time per passenger	2	s/pas
3	t_a	alighting time per passenger	1	s/pas
4	P_{TT}	value of in-vehicle time	1.5	$/h
5	P_{WT}	value of waiting time	3	$/h
6	θ_{trans}	monetary penalty	0.067	$/pas
7	R_C	hourly operating cost	6.185	$/bus h
8	R_H	operating cost of distance	0.375	$/bus km
9	τ_q	safety margin factor of capacity	0.9	

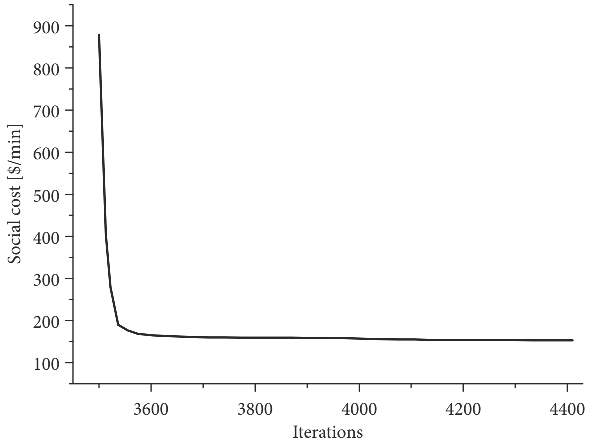

Figure 3. A solution convergence process corresponding to the 2nd column scenario in Table 4

The optimal results on designing limited-stop services are shown in Figure 4, under a time-based differential fare structure. In Figure 4, the fare varies from off-peak time period to peak time period, which is expressed by a time-varying multiplier λ^t of the basic fare, including 0.5, 1, 1.5, and 2. When a bus fare is less than the basic fare, normal services (lines 1…8) play a vital role in bus services. However, as the fare rises, limited-stop services (lines 9…23) become more and more important. When the fare is greater than a certain amount, the optimal set of lines tends to stabilize, since a high fare prevents passengers from transferring in their trips, which is showed at the point without transfer discounts in Figure 5. As a result, passenger travel behaviour remains invariant and thus the service network stays in equilibrium.

As shown in Figure 5, we analyse impacts of time-based fares and transfer fares on passenger transfer behaviour. Three types of transfer discounts are considered, including no discount ($\pi = 1$), 50% discount ($\pi = 0.5$), and 100% discount ($\pi = 0$). There are a great number of passenger transfers at a low fare in Figure 5. As a result, users' in-vehicle times and waiting times may increase because of additional boarding and alighting at bus stops due to transfers.

The number of passenger transfers may gradually decline as a discount of transfer fare decreases, as shown in Figure 5. Passengers will not transfer in their trips when a bus fare is more than $0.17 without transfer discounts. However, with the increasing discount of transfer fare, it may trigger passengers to transfer again, because low transfer costs could offset long waiting time or in-vehicle time costs incurred by direct services. The number of passenger transfers increases up to the biggest if passengers have a 100% discount of transfer fare. In addition, there is a great fluctuation in the number of passenger transfers as transfer fare discounts change, when passengers are charged low fares. It indicates that passengers with low fares may have a higher perception of transfer fare discounts than those passengers with high fares.

Table 4 displays optimal results in detail with consideration of different transfer fare discounts. Total fare costs present a little change, though there is a great increase in the number of transfer passengers when transfer fare discounts increase from 0 to 50%. This indicates that a gain in transfer fare discounts compensates for the transfer fare costs generated by the increased number of passenger transfers. The total fare costs are the smallest at 100% transfer fare discount, because passengers in transferring achieve the greatest savings due to free transfer. The total social cost shows an increased trend as transfer fare discounts rise. This is because extra waiting times and vehicle dwell times generated by the increased passenger transfers result in a great increase in other user costs and operating costs, and these increased costs are not offset by gains in transfer fare discounts.

It presents a great change in service patterns as transfer fare discounts change, as shown in Figure 6. Based on operation distances, those bus lines on the corridor are divided two categories: short-turn services with short-distance operation, and full services with full-length operation. In Figure 6, line numbers from 2 to 16 are short-turn services, while other lines are full-length services. The needs of line services present an increasing trend, especially in short-turn services, when transfer fare discounts increase. This indicates that short-turn services prove more profitable for meeting the increased needs of passenger transfers and minimizing the social cost for users and operator.

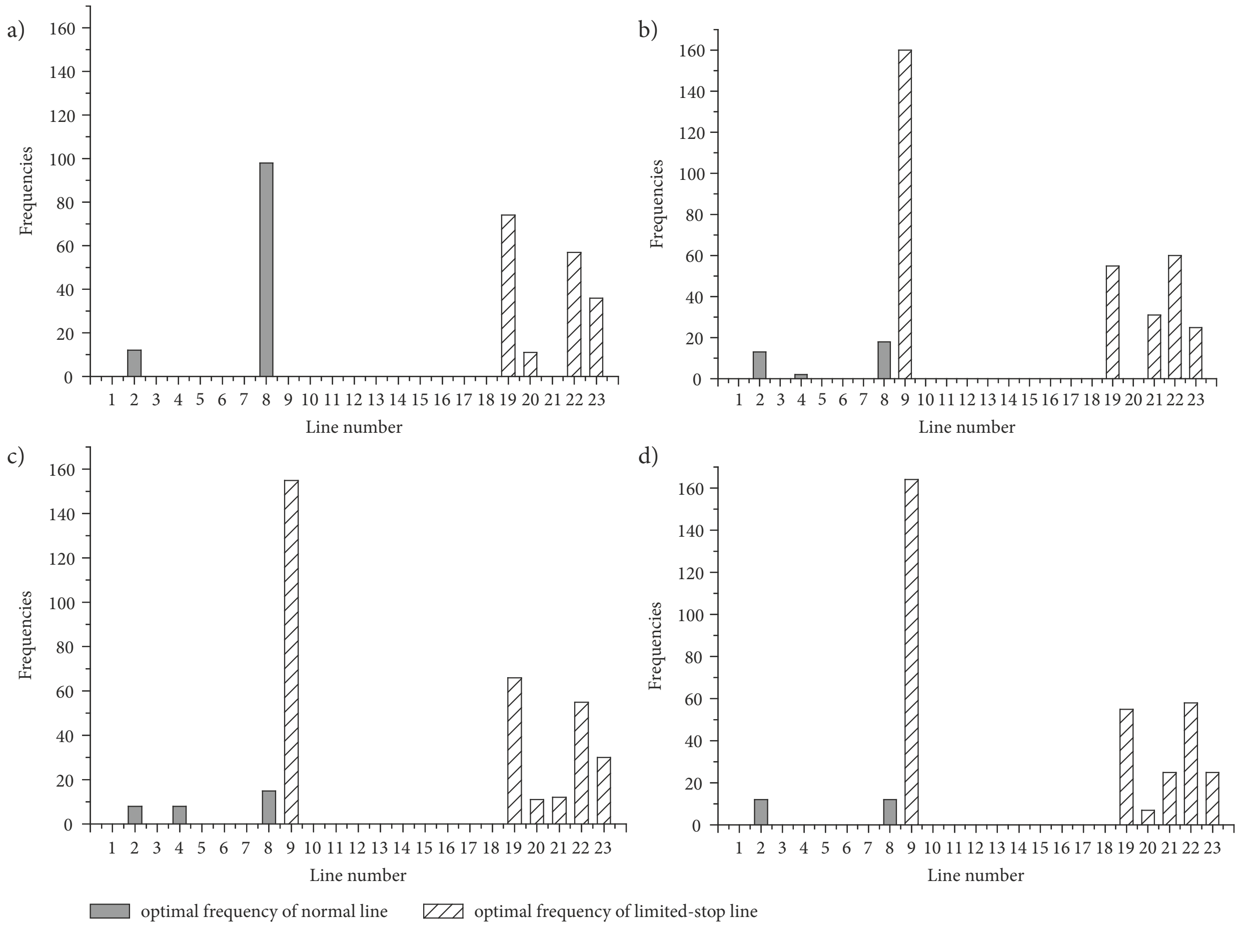

Figure 4. Optimal line frequencies with no transfer discounts under different times of day: a – 0.5 multiplier of a basic bus fare; b – a basic bus fare; c – 1.5 multiplier of a basic bus fare; d – 2 multiplier of a basic bus fare

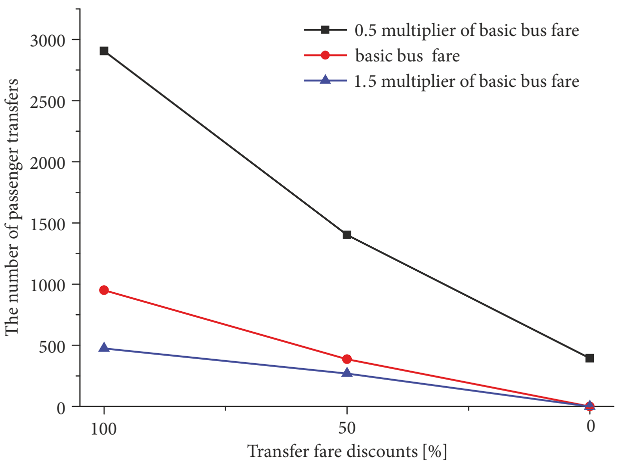

Figure 5. Effects of time-based fares with transfer discounts on the number of transfers

Table 4. The results of different transfer discounts with a 0.5 multiplier of basic bus fare

	Transfer discounts		
	0%	50%	100%
Total social cost [$/min]	153.9496	157.3916	163.6306
Total user cost [$/min]	131.4145	133.0558	137.4549
Fare cost [$/min]	29.0847	29.5104	28.5361
Other user cost [$/min]	102.3298	103.5454	108.9188
The number of transfers [passengers]	395	1403	2906
Total operator cost [$/min]	22.5351	24.3358	26.1756
Running cost [$/min]	20.8597	22.5807	24.3196
Dwell time cost [$/min]	1.6754	1.7551	1.8560

3.2. Stop-based fare structure

A distance-based or zonal fare structure is used widely in most cities. A few cities also adopt a stop-based fare structure, such as Dalian in China. In this study, a stop-based fare structure is constructed on the basis of characteristics of the bus corridor and a given basic fare structure. In Equation (18), passengers only need to pay a fixed fare if their travel stops are no more than n stops. They will be charged an additional fee when traveling more than n stops. This additional fee is determined based on the number of stops on passenger trips. When a stop-based fare strategy is adopted, a differential fare between stops i and j in Equation (11), U_{ij}, during transit operational time period t, is expressed as:

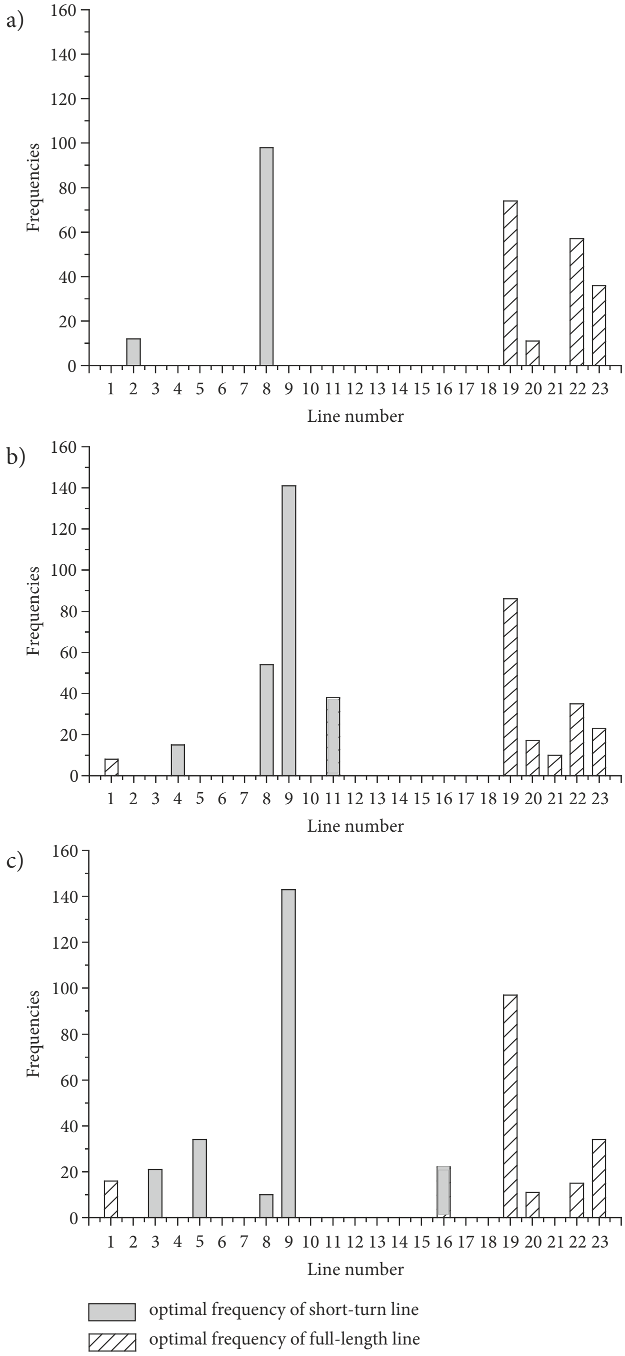

Figure 6. Frequencies of optimal lines with three transfer fare discount scenarios: a – 0% discount for transfer; b – 50% discount for transfer; c – 100% discount for transfer

$$U_{ij}^{t} = \begin{cases} \lambda^{t} \cdot u_{ij}, & \text{if } \left|j-i\right| \le n; \\ \lambda^{t} \cdot u_{ij} + \left|j-i\right| \cdot \beta^{ij} \cdot \lambda^{t} \cdot u_{ij}, & \text{if } \left|j-i\right| > n, \end{cases} \quad (18)$$

where: U_{ij}^{t} represents a fare between stop i and stop j during operational time period t; β^{ij} is a parameter of an additional distance fee; i is a boarding stop; j is an alighting stop; λ^{t} is 0.5 and indicated a relationship between a basic fare and a fare during operation time period t; n is the maximum number of stops, which passengers may visit without additional distance fees (n = 4).

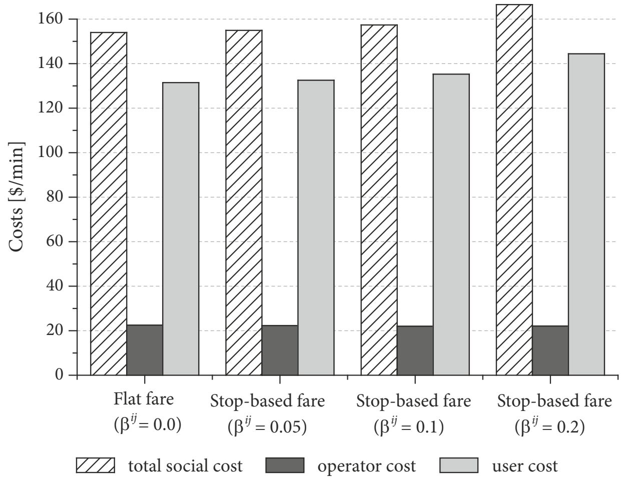

Figure 7. Optimal results of a flat fare and stop-based fares

Using stop-based fares generates greater social costs and user costs, compared with the use of a flat fare, as shown in Figure 7. But there is a little change in operator costs, and even smaller than that generated by using a flat fare. This indicates that using stop-based fares can benefit operators to some extent. In addition, with the increase of the value of additional distance fee parameter β^{ij}, the number of lines using limited services keeps stable, though the whole service patterns have a great change in Figure 8. This change mainly takes place among lines with the use of short turn services. The changes in service patterns stem from passenger travel behaviour change. There is a great change in passenger transfer behaviour when the value of the additional fee parameter β^{ij} increases, as shown in Figure 9. The number of passenger transfers reduces first down to the minimum, and then increases. This indicates that service patterns integrated with a suitable stop-based fare can efficiently reduce the number of passenger transfers.

Figure 10 shows specific changes of passenger transfers as the value of β^{ij} increases. It is found that transfer O–D pairs are completely different in different scenarios. As shown in Figure 10a, without additional fees involved, only an O–D pair with a long travel distance from stop 1 to stop 10 has a transfer. The travel route and transfer stop of this O–D pair w (1, 10) are displayed in Figure 11. While the value of β^{ij} increases to 0.05, passengers with a long travel distance might not transfer, e.g. those between O–D pair w (1, 10). There is a high fare on the route segment connecting stop 1 and stop 9, due to an additional fee required. In order to reduce fare costs, O–D pair w (1, 10) will give up transferring at stop 9. This indicates that additional fees are unprofitable for transfer O–D pairs with long travel distances. Additional fees may induce a few other O–D pairs with short travel distance to transfer, as shown in Figure 10b, though the total number of passenger transfers presents a reduced trend. Interestingly, when the value of the additional fee parameter β^{ij} increases up to more than 0.05, passengers with long distance transfer again (see Figure 10c and Figure 10d), and the number of passenger transfers gradually increases, as indicated in Figure 9.

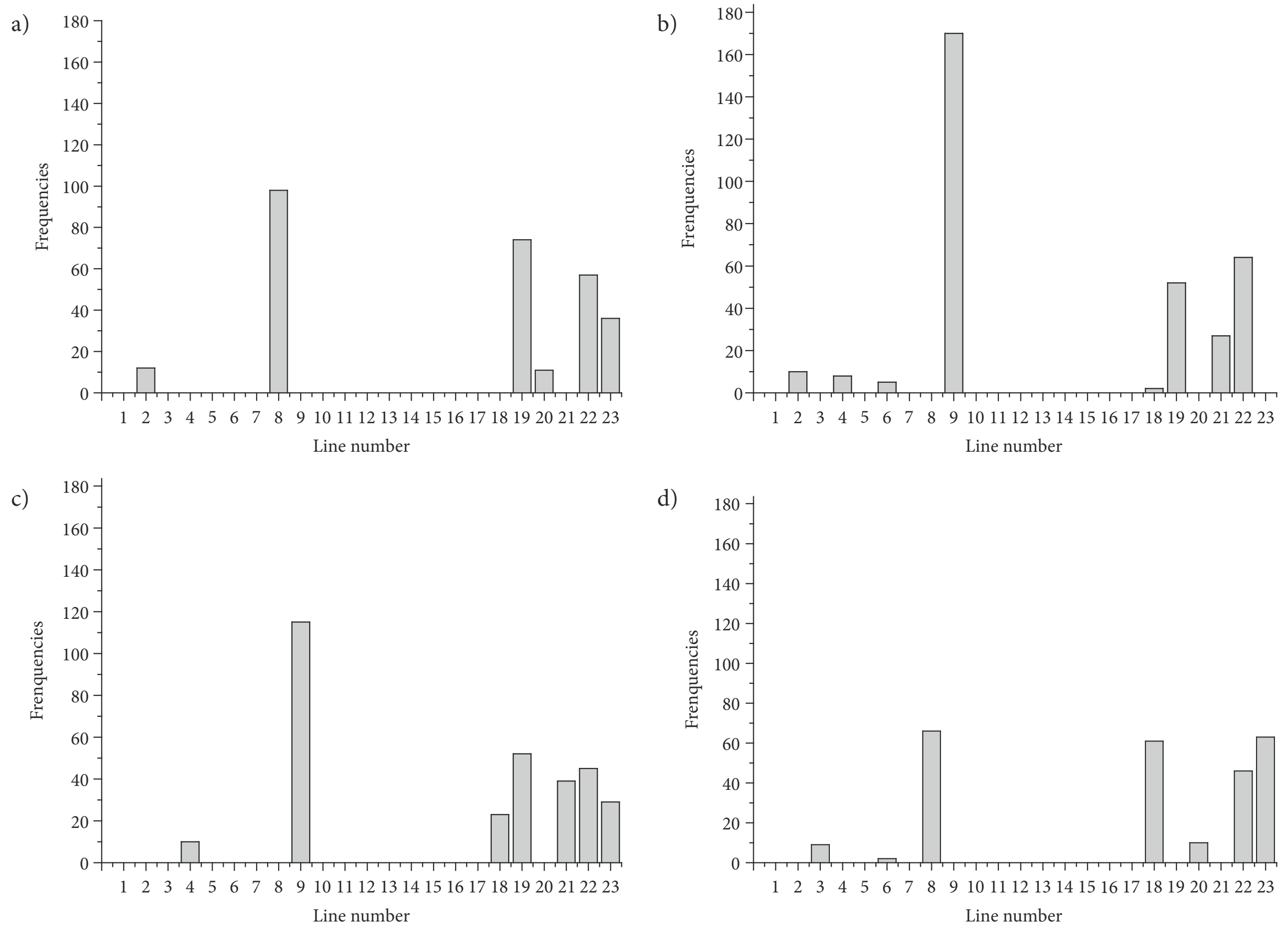

Figure 8. Frequencies of optimal lines for different values of β^{ij}: a – β^{ij} = 0; b – β^{ij} = 0.05; c – β^{ij} = 0.1; d – β^{ij} = 0.2

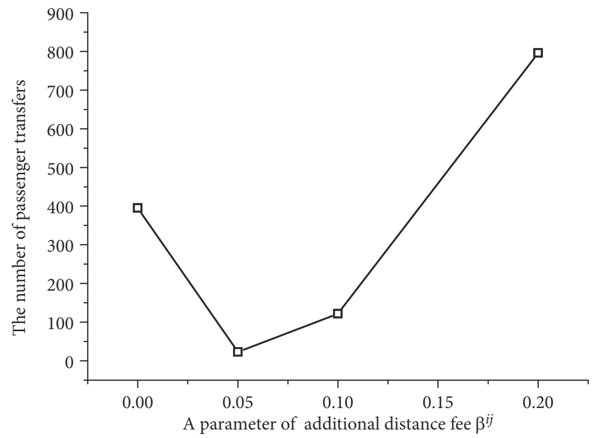

Figure 9. The number of passenger transfers for different values of β^{ij}

This stems from the fact that additional fees for passengers with long trips to pay for are more than transfer fares, so that passengers tend to transfer in their trips. Accordingly, in public transport operations, more extra services, such as infrastructure and staff services, may be provided for the increased transfer passengers, though this is not explicitly noted in the objective function. A beneficial stop-based fare determined by fare makers should present a great trade-off between passenger transfers and service efficiency.

3.3. Quality-based fare structure

In this paper, different feasible line services are used in the corridor, including normal, express, and limited-stop services. Compared with normal services, express and limited-stop services have shorter travel times because of visiting less stops. In order to present social equity, different fares should be offered for these lines based on their provided service levels. It is reasonable that passengers pay more for using lines with greater service levels indicated by the non-trivial savings of travel times. According to a given basic fare structure, the quality-based fare function may be formulated as followes:

$$U_l^t = \lambda^t \cdot u_{ij}^l + N^l \cdot \theta^l \cdot \lambda^t \cdot u_{ij}^l, \tag{19}$$

where: U_l^t is a bus fare of line l at period t; λ^t is set to 0.5 and indicates a relationship between a basic fare and a fare at time period t; u_{ij}^l is a basic bus fare associated with line l connecting origin stop i and destination stop j; θ^l is a parameter of additional fee related to line l for saving travel times; N^l is the number of stops that line l skips at period t. The more stops line l skips, the greater fare line l has.

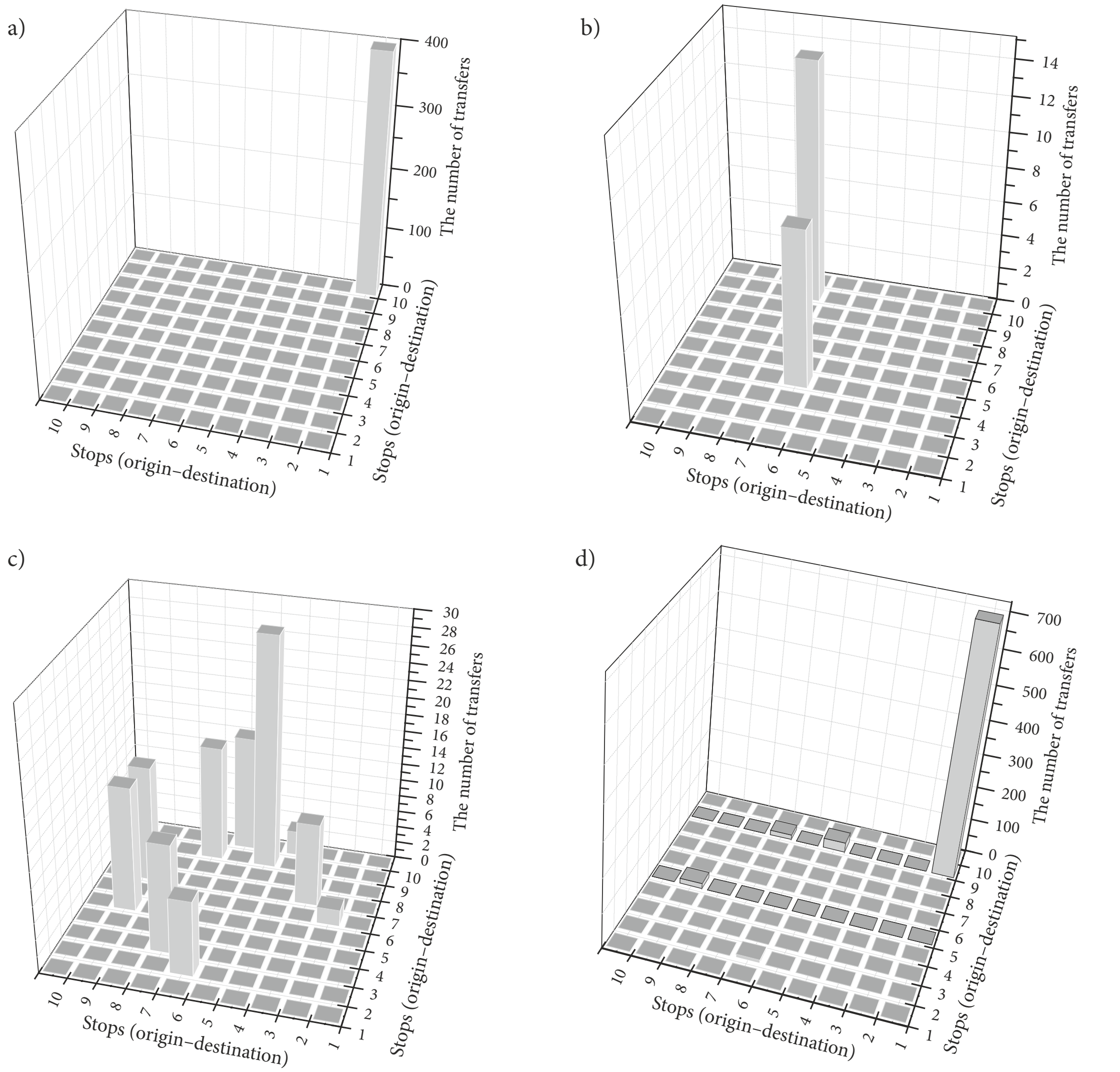

Figure 10. A change of O–D pairs transfer with the increment of β^{ij}: a – β^{ij} = 0; b – β^{ij} = 0.05; c – β^{ij} = 0.1; d – β^{ij} = 0.2

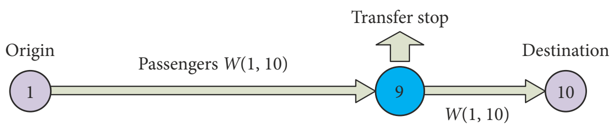

Figure 11. Transfer route of O–D pair (1, 10) under β^{ij} = 0 (corresponding to Figure 10a)

According to Equation (19), the quality-based fare on segment s in Equation (11), U_s, during operational time period t, is expressed as:

$$U_s^t = \frac{\sum_{l \in L} U_l^t \cdot f_l^s}{\sum_{l \in L} f_l^s}, \tag{20}$$

where: U_s^t represents a fare on segment s during transit operational period t.

Table 5 displays the characteristics of feasible lines.

Figure 12 shows the optimal results with five scenarios respectively corresponding to θ^l = 0.1, 0.2, 0.3, 0.4, 0.5. In this study, the parameter of additional service fee θ^l has the same value for all lines in the same scenario. For instance, θ^l equals 0.1, regardless of express, limited-stop, or normal lines. The bus fares of lines with no skipped stops are fixed. We divide all lines into two types: normal services that do not skip stops from original to destination, and skipped services. As shown in Figure 12, lines 1…9 are normal services while the other lines 10…23 are skipped services. When the value of additional fee θ^l increases a little, some passengers use bus lines with skipped services, such as lines 18, 19, 20, 22 and 23 in Figure 12a. However, with the increase in the additional fee, the number of passengers using skipped services decreases. When the value of θ^l is more than 0.4, only normal services are provided for passengers. Due to high fares, there are no passengers to use skipped services, and thus skipped services are abandoned, as shown in Figure 12d and Figure 12e. It indicates that normal services prove more profitable than skipped services, when a high additional service fee is required. In addition, the more stops a line skips, the easier it is given up by passengers. Since such line charges passengers more, so that passengers might give it up when they face a trade-off between bus fare and other costs (e.g. waiting time cost, in-vehicle time cost, transfer cost, etc.).

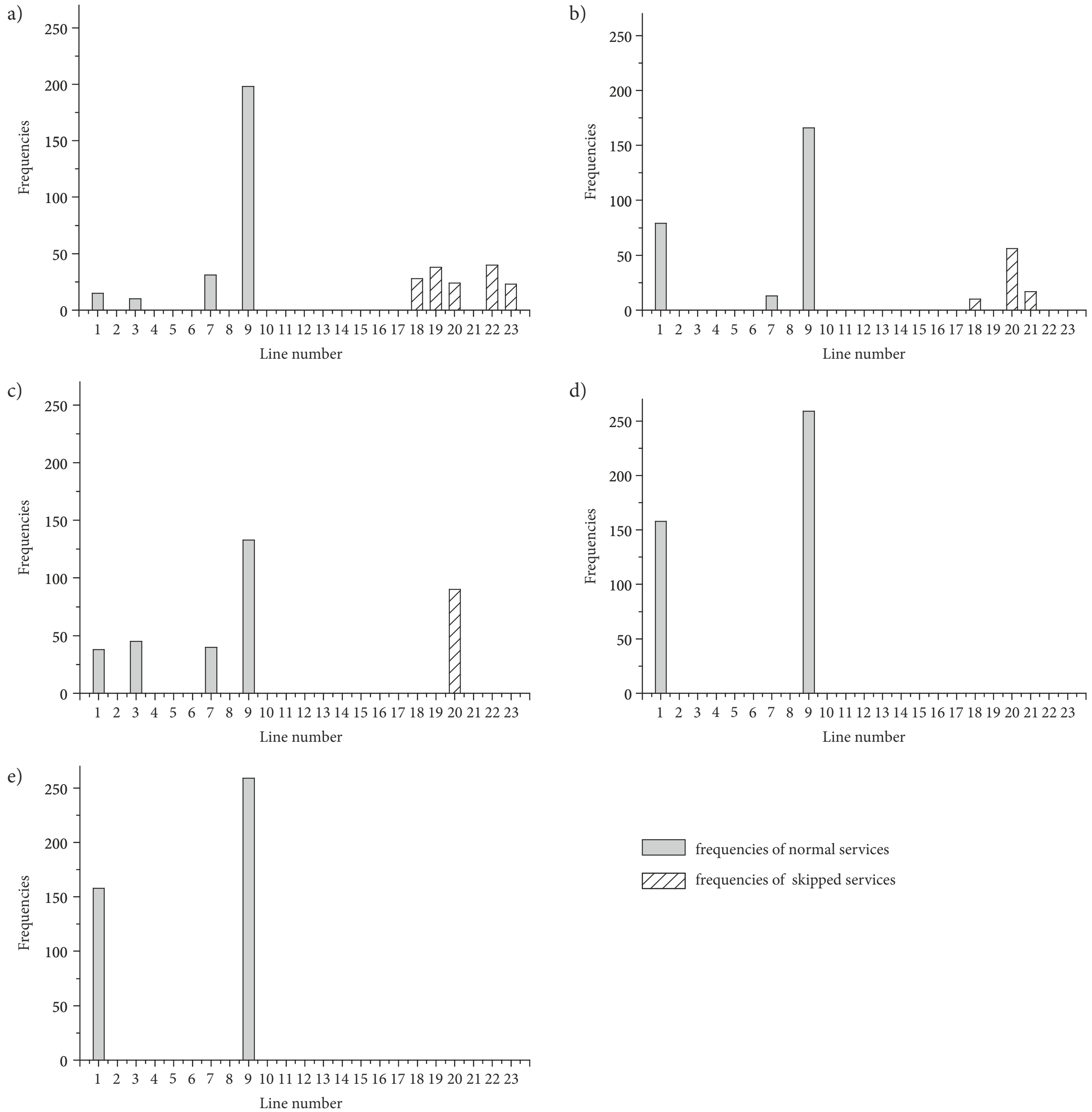

Figure 12. Optimal line frequencies for different values of θ^l: a – $\theta^l = 0.1$; b – $\theta^l = 0.2$; c – $\theta^l = 0.3$; d – $\theta^l = 0.4$; e – $\theta^l = 0.5$

For instance, line 19 that skips ten stops, and lines 22–23 with six stops to skip, respectively, are removed first, as shown in Figure 12b. In Figure 12c, there is only line 20 with four skipped stops left with a higher value of θ^l. Therefore, it presents that fare decision makers should construct an attractive quality-based fare under practical operation conditions, in case all skipped services are abandoned due to a high quality-based fare.

As the value of the additional service fee parameter θ^l increases from 0 to 0.2, the number of passenger transfers gradually reduces, as shown in Figure 13. It presents that a little increase in the additional service fee can restrain passengers from transferring in their trips. When the value of parameter θ^l is between 0.2 and 0.4, the number of passenger transfers rapidly increases up to the greatest. This is because, as indicated in Figure 12a–c, passengers may give up some skipped services, and thus turn to transfer in order to minimize their travel costs with the increasing value of parameter θ^l. In addition, it is found that the number of passenger transfers becomes a constant, when the value of θ^l is more than 0.4. This demonstrates that the passenger travel behaviour may not change if the value of an additional service fee increases beyond a great threshold.

Table 5. Characteristics of feasible lines

Transit line	Number of skipped stops	Number of visiting stops	Length	Running time (excluding dwell times)
1	0	20	6.6	39.6
2	0	18	5.87	35.2
3	0	16	5.13	30.8
4	0	14	4.4	26.4
5	0	12	3.67	22
6	0	10	2.93	17.6
7	0	8	2.2	13.2
8	0	6	1.47	8.8
9	0	4	0.73	4.4
10	2	4	1.47	5.8
11	4	4	2.2	8.8
12	6	4	2.93	11.8
13	8	4	3.67	14.6
14	10	4	4.4	17.6
15	12	4	5.13	20.6
16	14	4	5.87	23.4
17	16	4	6.6	26.4
18	4	14	6.6	33.8
19	10	10	6.6	30.2
20	4	16	6.6	35.2
21	8	12	6.6	31.6
22	6	14	6.6	34.2
23	6	14	6.6	34.2

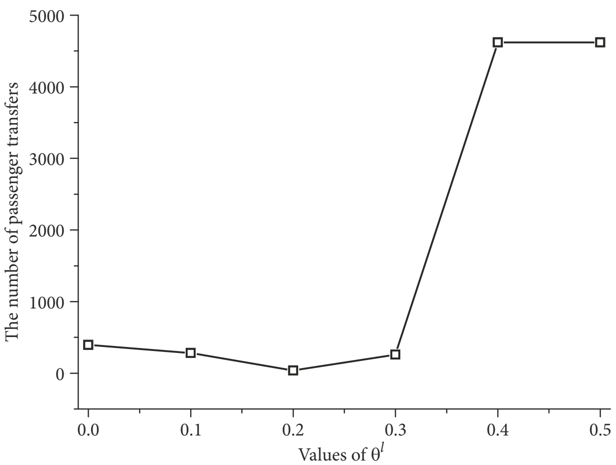

Figure 13. The number of passenger transfers for different values of θ^l

Concluding remarks

Fare plays a vital role in traveller choice decisions on the use of public transit. An attractive set of fare rates can not only improve the operator's revenue and social equity, but also better match supply and demand on the public transit service market. A great deal of the literature has contributed to the identification of advantages and disadvantages of a flat or a differential fare strategy. Apparently, a differential fare strategy shows a great potential for achieving social equity and revenue growth. Moreover, with the latest available technologies (e.g. smart bus card, automated collection system), a differential fare strategy has become feasible for the public transit service market. In this paper, we transform a hidden and complex fare into a visualized and easy–to-understand fare table or matrix so that we can obtain varied differential fare tables on the basis of a given fare policy, a set of fare strategies, and a set of desired objectives with the support of available latest technologies. In the optimization of limited-stop public transit services, it is found that a differential fare structure has a great effect on passenger transfer behaviour and the option of bus service patterns.

In a time-based fare structure, the optimization of bus services indicates that, in order to minimize the total social cost, more limited-stop services shall be provided to improve the level of service when a high bus fare is charged during the peak time. In considering different discounts of transfer fare, it is found that a gap in the numbers of passenger transfers with high and low fares decreases as transfer fare discounts reduce. Passengers may not be willing to transfer in their trips if there is a high fare and no reasonable discount for transferring. Thus, during the peak time, the operator should provide more limited-stop services if they want to improve the revenue by charging a high fare. Moreover, it will not result in a great change in passenger demand.

The stop-based fare structure is also investigated in this paper. The passenger path choice behaviour is very sensitive to an additional fee due to more stops. This presents a great change in service patterns and the number of passenger transfers. This change mainly takes place among lines using short turn services. The number of passenger transfers first declines, and then increases with the growth of an additional fee. This is because passengers with long travel distance try to have no or less transfers in their trips when an additional fee increases. However, when this fee increases and is more than a threshold so that the incremental fare is greater than a transfer fare, the passengers with long travel distance will tend to take a transfer. This offers transport planners and decision makers a way to influence passenger transfer behaviour and improve the level of bus services.

In this paper, a set of feasible bus services is taken as an input in the optimization problem for a public transit corridor. Based on the qualities of bus services, a differential fare structure is constructed. With the increasing value of an additional service fee generated by travel time savings, the number of passenger transfers falls down and the use of skipped services presents a reduced trend. When this value increases beyond a threshold, all skipped services are abandoned due to high fares. Meanwhile, the number of passenger transfers increases up to the maximum. It demonstrates that a suitable quality-based fare helps to reduce the number of passenger transfers.

In the modern multi-modal urban transportation system, the contribution of applying this differential fare structure to limited-stop bus services mainly results from two aspects. One is that it may drive more travellers to leave their cars and turn to public transit services because of a higher level of the transit service. The other is that the reduction in the number of "stops and goes" significantly decreases transport emissions while vehicles are accelerating. A quantitative analysis of this in a multi-modal system is an ongoing piece of our research work.

Further research may extend the proposed technique to model limited-stop bus services with elastic demand and provision of information on bus arrivals. Bus fares are usually so low in many cities that travellers may not be really sensitive to the change in bus fares in short term, but a change in the bus fare may have a long term effect. By contrast, travellers are more sensitive to the change in journey times (either bus dwell or running time or service access time) because of the increasing value of time savings in particular in the urban area. Generally, the demand for public transit services is a function of fare, varied trip times and their reliability, as well as other factors affecting service levels. Limited-stop services most definitely offer a way to reduce journey times and improve service reliability. In addition, as mobile and intelligent devices that can help travellers check when a target bus arrives have become more and more common, personalizing bus services is growing to be a feasible way to drive more and more travellers to leave their cars at home and take public transit services. A differential fare structure shall be part of the scheme of public transit service personalization. Therefore, it is timely to carry out research on these issues.

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