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EVALUATE PUBLIC-PRIVATE-PARTNERSHIP'S ADVANCEMENT USING DOUBLE HIERARCHY HESITANT FUZZY LINGUISTIC PROMETHEE WITH SUBJECTIVE AND OBJECTIVE INFORMATION FROM STAKEHOLDER PERSPECTIVE

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Abstract. Public-Private-Partnership (PPP) as an efficient mode to provide public services through the government and social capital's cooperation has been in China for more than 30 years. In this paper, we propose an approach to evaluate PPP's advancement in different areas based on the subjective and objective information fusion. At first, we establish an index system from the perspective of the stakeholder. Then, considering that double hierarchy hesitant fuzzy linguistic term set (DHHFLTS) that has two hierarchies of linguistic term sets can describe the subjective linguistic information more accurately, it is applied in the paper to depict the subjective information. By applying the entropy of the DHHFLTS, a programming model is proposed to derive the attribute weight through combining subjective evaluation with objective data. In addition, we develop the double hierarchy hesitant fuzzy linguistic PROMETHEE combining the subjective and objective information (DHHFL-PROMETHEE-S&O) method. At last, we illustrate the index system and the method with the PPP's advancement evaluation problem, and we can find the best choice based on the ranking result. Meanwhile, we also find that the objective information and the subjective information are complementary in the evaluation process.

Keywords: Public-Private-Partnership, double hierarchy hesitant fuzzy linguistic term set, PROMETHEE, subjective and objective information fusion.

JEL Classification: C60, D81.

Introduction

Public-Private-Partnership (PPP), which can be dated back to 1980s, has become popular in China recently. Many different agencies defined PPP in different ways: The World Bank defined it as a long-term contract between social capital and the government for providing public assets or services, in which the social capital bear risk and management responsibility,

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and the remuneration is linked to the performance of public projects (The Word Bank Group, 2017). British Ministry of Finance defined PPP as long-term contracts where the social capital designs, builds, finances and operates an infrastructure project. (British HM Treasury, 2017). China Ministry of Finance holds the view that the PPP is a kind of long-term cooperation relationship between social capital and the government in infrastructure and public services. The common model is that social capitals are responsible for the infrastructure's design, construction, operation and maintenance, and gain benefits from payments of users or the government. Government departments are responsible for public services' pricing and quality supervision (Ministry of Finance of the People's Republic of China, 2014). From all above, we can see that the PPP is a kind of agreement or cooperation relationship between social capital and government to provide public services.

In some way, the PPP has many positive effects on the development of China. From the government's point of view, it reduces the risk of the government. Moreover, it brings more sources of funding and less cost for the government, which relieves the government's financial stress. It is an efficient tool for the government to develop and manage public infrastructure and services. From the social capital's point of view, there are more opportunities and more scopes for investment. From the society's point of view, the efficiency of the public services increases, which is conducive to social and economic development (D. W. Brinkerhoff & J. M. Brinkerhoff, 2011; Cruz & Marque, 2014; Kwak, Chih, & Ibbs, 2009; L. Y. Tang, Q. E. Tang, & Cheng, 2010; Cui, Liu, Hope, & Wang, 2018).

In May 2015, Premier Keqiang Li proposed to promote the PPP model in the State Council regular meeting. Since then, there has been a PPP construction boom in China. In May 2017, the total number of the PPP projects reaches 12286 and the total amount of investment increases to 1453.56 billion yuan (Ministry of Finance of the People's Republic of China, 2017). To provide a relatively open and transparent PPP environment, Chinese Financial Ministry and Chinese Development and Reform Commission established respective web sites to present the basic information of the PPP projects and released reports quarterly. The contents of the reports are quite basic and less targeted, which just show the objective data but no assessment or rankings of different cities. Besides, evaluating the PPP's advancement in different areas is meaningful for social capital to make a proper decision and help the government know about the level of the PPP's promotion in China. Therefore, the aim of this paper, evaluating the PPP's advancement in different regions, is meaningful.

From the literature review in Subsection 2.2, we can find that the researchers first established the related index system, and then made the evaluations based on the index system. That is to say, we can take the indexes as the criteria or the evaluating standards. Hence, in this paper, we start from the point of multi–criteria to evaluate the advancement of PPP. As a classical decision-making method, PROMETHEE was originally proposed by Mareschal, Brans, and Vincke (1984). After that, the method has been extended into various forms, and many researchers combined the method with some other methods (Sennaroglu & Celebi, 2018; Jayant & Sharma, 2018). In this paper, we focus on the PROMETHEE II (completing ranking) approach. Its essence is to compare each pair of alternatives which cannot be compared directly at first by using preference functions, and form a preference relation between them. Then, it needs to rank alternatives based on the positive outranking flow, negative outranking flow and net flow (Brans & Mareschal, 2005). Compared with other decision-making methods, it can process information with a reasonable degree of accuracy (W. X. Li & B. Y. Li, 2010). There are six classical functions based on different principles to transform the deviations between different alternatives into preferences, which are clear and almost meet all the situations (Brans & Mareschal, 2005). To make it more practical, V. Podviezko and A. Podviezko (2010) proposed two functions: Multistage preference function and the eighth C-shape preference function. In addition, one of the principles of the PROMETHEE method is to make the parameters contain some economic significations, which help us fix them easily and improve its stability (Brans Vincke, & Mareschal, 1986). Hence, in this paper, we use it to assess the PPP's advancement.

The evaluation information that we can obtain contains the subjective information from the experts and the objective information from official websites. On the one hand, because of the complexity of the real environment, it is difficult for us to pick out which alternative is better based on objective data only. Experts' evaluation which combines their experience is a good supplement. On the other hand, if there are some tiny differences among objective data, the subjective information usually considers them as the same degree because of the fuzziness of the linguistic expressions. For example, there are two alternatives A and B, and the costs are 1000 dollars and 998 dollars, respectively. If we use linguistic terms to describe the attribute "cost", then we use the same linguistic term, "high". While, in this situation, objective data are more sensitive to distinguish the difference values among different alternatives, and it can support the subjective information to tell the differences among the alternatives. The objective information and the subjective information are complementary with each other, Hence, the evaluations for the PPP projects in this paper are carried out from two aspects, i.e., the subjective evaluation and the objective evaluation.

The subjective evaluation is commonly used in practical situations, such as the expert systems (Sioshansi, 1983) and the recommendation systems (Yano, Sueyoshi, Shinohara, & Kato, 2003) etc. Linguistic terms (Finch, 2000) have been widely applied in these situations and people are more comfortable providing their information in linguistic terms (Yager, 2016), because they are convenient to use and consistent with people's expression habits. However, since the knowledge background, the cognitions and experience of the experts are different, then all the evaluation information that they provided is important and none of them can be ignored. Moreover, when the experts want to express more detailed information such as "a little good" and "extremely bad", the traditional linguistic terms cannot describe it well. In order to describe the linguistic information more accurately, Gou, Liao, Xu, and Herrera (2017a) proposed the DHHFLTS which has two hierarchies of linguistic term sets. The first hierarchy is used to describe the basic properties of objectives or alternatives through adjectives, such as "good", "bad", "medium", etc. The second hierarchy is applied to express the degrees of the linguistic terms in the first hierarchy through adverbs, such as "little", "extremely". For the objective evaluation, some properties of objects can only be expressed by crisp numbers, such as quantity, density, etc.

In order to obtain the useful decision results, the subjective information and the objective information need to be integrated. One of the most important tasks for fusing the subjective information and the objective information is to minimize the deviation between these different kinds of information. Xu and Chen (2007) converted the data matrix into different forms, and they established a model to minimize the deviations between different preference relations for deriving weights. Ma, Fan, and Huang (1999) derived the subjective weights and the objective weights firstly. Then, they established a programming model to calculate the values of the coefficients to fuse the subjective and objective weights. Palevicius, Podviezko, Sivilevičius, and Prentkovskis (2018) proposed a combined COIN (compensating influences) method of obtaining weights, in which the researchers integrated the weights derived from data and the weights from experts. As a basic tool to describe the uncertainty of objects (Zadeh, 1968), the measure of entropy is important (Yager, 1995) and it has been used in information theory (Shannon & Weaver, 1949), probability theory (Herniter, 1973), physics (Herniter, 1973), etc. In this paper, to reduce uncertain parameters and the uncertainty of the whole decision-making matrix, we define the double hierarchy hesitant fuzzy linguistic (DHHFL) entropy and establish a programming model according to the minimum entropy principle.

In this paper, to ensure the pertinence of the evaluation system, we establish the assessment index system from the stakeholder perspective. Then, we finish a questionnaire survey to elaborate the rationality of the index system. In the evaluation process, we combine the subjective and objective information to calculate the attribute weights and develop the DHHFL-PROMETHEE- S&O method to rank alternatives.

The contributions of this paper are shown as follows:

1) An assessment index system is developed to evaluate the PPP's advancement in different regions. The PPP has been developing in China for more than 30 years, and the idea about evaluating the level of the PPP's advancement is significant for the government and social capital to master the development process of the PPP in our country, but it has been barely discussed before.
2) A programming model is proposed to integrate the subjective and objective information. The concept of DHHFL entropy is given and applied to represent the uncertainty of the decision–making problem. Aiming to minimize the entropy of the subjective evaluation and the objective information, we design a programming model to obtain the attribute weights, which is helpful to reduce the uncertainty.
3) The DHHFL-PROMETHEE-S&O method is developed. In this paper, we use the DHHFLTS to evaluate alternatives and attributes. Its double hierarchy structure can transmit more information and depict information more accurately. The PROMETHEE method applied in this paper can make full use of information. It combines the subjective and objective information in the attribute weights deriving process and the alternatives ranking process.

The rest of this paper is organized as follows: In Section 1, we review some basic concepts related to the DHHFLTS and the related previous researches. Section 2 establishes the assessment index system. Then, we combine the subjective information and the objective information to construct a programming model to derive the attribute weights in Section 3. In Section 4, we develop the DHHFL-PROMETHEE-S&O method, and we apply the method in assessing the level of the PPP's advancement in Deyang, Yibin, Xi'an, and Hanzhong in

Section 5. Moreover, we analyze the sensitivity of the approach and compare the new approach with the double hierarchy hesitant fuzzy linguistic MULTIMOORA (DHHFL–MULTIMOORA) method. Finally, the paper ends with some conclusions, research limitations and future research directions.

1. Preliminaries

1.1. Basic concepts related to the DHHFLTS

Gou et al. (2017a) proposed a novel concept named DHHFLTS which has two hierarchies of linguistic term sets. The first hierarchy is used to describe the basic properties of objectives or alternatives through adjectives, such as "good", "bad", "medium", etc. The second hierarchy is applied to express the degree of the linguistic terms in the first hierarchy through adverbs, such as "little", "extremely", etc. The first and second hierarchies are expressed by two linguistic term sets with additive linguistic evaluation scales: $S=\left\{s_t \,\middle|\, t=-\tau,\ldots,-1,0,1,\ldots,\tau\right\}$ and $O=\left\{o_k \,\middle|\, k=-\delta,\ldots,-1,0,1,\ldots,\delta\right\}$ (τ and δ are positive integers and denote the linguistic scales), respectively.

The mathematical form of the DHHFLTS is: $H_{S_O}=\left\{<x_i, h_{S_O}\left(x_i\right)> \,\middle|\, x_i \in X\right\}$, where x_i is fixed, and $h_{S_O}\left(x_i\right)$ is the set of some values in S and O, denoted as $h_{S_O}\left(x_i\right)=\{{}^l s_{t_{O_k}}\left(x_i\right) \,|\, o_k \in O,$ $s_t\left(x_i\right)\in S,\ t=-\tau,\ldots,-1,0,1,\ldots,\tau, k=-\delta,\ldots,-1,0,1,\ldots,\delta, l=1,\ldots,L\}$ with L being the number of linguistic terms in $h_{S_O}\left(x_i\right)$. $h_{S_O}\left(x_i\right)$ is called Double Hierarchy Hesitant Fuzzy Linguistic Element (DHHFLE) and $s_{t_{O_k}}$ is called Double Hierarchy Linguistic Term (DHLT).

The DHLTs can be transformed into fuzzy numbers using Eq. (1) and Eq. (2) (Gou et al., 2017a) as follows:

$$f\left(s_{t_{O_k}}\right)=\begin{cases}\dfrac{1}{\tau}\times\dfrac{k+\delta}{2\delta}+\dfrac{\tau+t-1}{2\tau}=\dfrac{k+\left(\tau+t\right)\delta}{2\delta\tau}, & \text{if } -\tau+1\le t\le\tau-1\\ \dfrac{1}{2\tau}\times\dfrac{k+\delta}{\delta}+\dfrac{\tau+t-1}{2\tau}=\dfrac{k+\left(\tau+t\right)\delta}{2\delta\tau}, & \text{if } t=\tau\\ \dfrac{1}{2\tau}\times\dfrac{k}{\delta}=\dfrac{k}{2\delta\tau}, & \text{if } t=-\tau\end{cases}; \tag{1}$$

$$f^{-1}\left(\gamma\right)=\begin{cases}s_{[2\tau\gamma-\tau]<o_{\delta\times\left(2\tau\gamma-\tau-[2\tau\gamma-\tau]\right)}>} \text{ or } s_{[2\tau\gamma-\tau]+1<o_{\delta\times\left(\left(2\tau\gamma-\tau-[2\tau\gamma-\tau]\right)-1\right)}>}, & \text{if } 1-\tau\le 2\tau\gamma-\tau\le\tau-1\\ s_{\tau-1<o_{\delta\times\left(2\tau\gamma-\tau-[2\tau\gamma-\tau]\right)}>} \text{ or } s_{\tau<o_{\delta\times\left(2\tau\gamma-\tau-[2\tau\gamma-\tau]\right)-\varsigma}>}, & \text{if } \tau-1\le 2\tau\gamma-\tau\le\tau\\ s_{-\tau<o_{\delta\times\left(2\tau\gamma-\tau-[2\tau\gamma-\tau]\right)}>} \text{ or } s_{1-\tau<o_{\delta\times\left(\left(2\tau\gamma-\tau-[2\tau\gamma-\tau]\right)-1\right)}>}, & \text{if } -\tau\le 2\tau\gamma-\tau\le 1-\tau\end{cases}, \tag{2}$$

where k is the subscript of the second hierarchy linguistic term of the DHLT, t is the subscript of the first hierarchy linguistic term of the DHLT. (τ and δ are the linguistic scales of the first and second hierarchy linguistic terms of the DHLT respectively. The transformation functions are all linear and the range of fuzzy numbers corresponding to the DHLTs is 0–1, which means that the best assessment corresponding to 1 and the worst assessment corresponding to 0. But in fact, even if the linguistic assessment information is best, which can be described as linguistic term "perfect" or some similar words, there are some distances from the ideally perfect situation that can satisfy everyone, the same to the extremely worst assessment information that no one can tolerate. Hence, in this paper, we apply the 0.1–0.9 scale (Zhu, Xu, Zhang, & Hong, 2015) to express the evaluation information. The formulas are shown as follows:

$$f\left(s_{t_{O_k}}\right)=\begin{cases}\dfrac{2}{5\tau}\times t+\dfrac{1}{2}+\dfrac{2k}{5\delta\tau}, & \text{if } -\tau+1\le t\le \tau-1\\ \dfrac{2}{5\tau}\times t+\dfrac{1}{2}+\dfrac{2k}{5\delta\tau}, & \text{if } t=\tau\\ \dfrac{1}{10}+\dfrac{2k}{5\delta\tau}, & \text{if } t=-\tau\end{cases}; \tag{3}$$

$$f^{-1}(\gamma)=\begin{cases} s_{\left[\frac{5\tau\gamma}{2}-\frac{5\tau}{4}\right]<o_{\delta\times\left(\frac{5\tau\gamma}{2}-\frac{5\tau}{4}-\left[\frac{5\tau\gamma}{2}-\frac{5\tau}{4}\right]\right)}>} \text{ or } s_{\left[\frac{5\tau\gamma}{2}-\frac{5\tau}{4}\right]+1<o_{\delta\times\left(\left(\frac{5\tau\gamma}{2}-\frac{5\tau}{4}-\left[\frac{5\tau\gamma}{2}-\frac{5\tau}{4}\right]\right)-1\right)}>}, & \text{if } 1-\tau\le 2\tau\gamma-\tau\le\tau-1\\ s_{\tau-1<o_{\delta\times\left(\left[\frac{5\tau\gamma}{2}-\frac{5\tau}{4}\right]\right)}>} \text{ or } s_{\tau<o_{\delta\times\left(-\left[\frac{5\tau\gamma}{2}-\frac{5\tau}{4}\right]\right)}>}, & \text{if } \tau-1\le 2\tau\gamma-\tau\le\tau\\ s_{-\tau<o_{\delta\times\left(\frac{5\tau\gamma}{2}-\frac{5\tau}{4}-\left[\frac{5\tau\gamma}{2}-\frac{5\tau}{4}\right]\right)}>} \text{ or } s_{1-\tau<o_{\delta\times\left(\left(\frac{5\tau\gamma}{2}-\frac{5\tau}{4}-\left[\frac{5\tau\gamma}{2}-\frac{5\tau}{4}\right]\right)-1\right)}>}, & \text{if } -\tau\le 2\tau\gamma-\tau\le 1-\tau\end{cases}. \tag{4}$$

Moreover, for the DHHFLEs h_{S_O}, $h_{S_{O1}}$ and $h_{S_{O2}}$, the basic operations are:

1) Addition: $h_{S_{O1}} \oplus h_{S_{O2}} = f^{-1}\left(\bigcup_{\eta_1\in f\left(h_{S_{O1}}\right),\eta_2\in f\left(h_{S_{O2}}\right)}\{\eta_1+\eta_2-\eta_1\eta_2\}\right)$;

2) Multiplication: $\lambda h_{S_O} = f^{-1}\left(\bigcup_{\eta\in f\left(h_{S_O}\right)}\left\{1-(1-\eta)^{\lambda}\right\}\right)$;

3) Power: $\left(h_{S_O}\right)^{\lambda} = f^{-1}\left(\bigcup_{\eta\in f\left(h_{S_O}\right)}\{\eta^{\lambda}\}\right)$;

4) Expected value: $\overline{h}_{S_O} = \frac{1}{L}\sum_{l=1}^{L} f\left({}^{l}S_{t_{O_k}}\right)$,

where f is the function that transforms DHLTs into fuzzy numbers, f^{-1} is the function that transforms fuzzy numbers into DHLTs and L is the number of linguistic terms in h_{S_O}, $h_{S_{O1}}$ and $h_{S_{O2}}$ are two different DHHFLEs.

Gou, Liao, Xu, and Herrera (2017b) proposed the Hamming distance to measure the difference between two DHHFLEs, the formula is shown as follows:

$$d_{hd}\left(h_{S_{O_1}},h_{S_{O_2}}\right)=\frac{1}{L}\sum_{\substack{l=1\\ \eta_1\in f\left(h_{S_{O_1}}\right),\eta_2\in f\left(h_{S_{O_2}}\right)}}^{L}\left|\left(\eta_1^l-\eta_2^l\right)\right|, \quad l=1,2,\ldots,L, \tag{5}$$

where f is the function that transforms DHLTs into fuzzy numbers; η_1^l and η_2^l are the l_{th}

linguistic terms of $h_{S_{O_1}}$ and $h_{S_{O_2}}$, respectively. If the two DHHFLEs have different numbers of DHLTs, we can add the mean values between the maximum and minimum values in the short one until the length of the two DHHFLEs is the same.

1.2. Literature review

In this subsection, we recall the previous literature from two aspects: 1) topic, 2) subjective and objective information fusion, which are shown in Figure 1.

To evaluate the PPP's advancement, a reasonable assessment method is required. By reviewing relevant literature, we find that there are less relevant researches which are related to PPP risk assessment and investment assessment. In addition, the main approaches to fuse the subjective information and the objective information can be divided into two categories: "One line" and "Two lines". "Two lines" denotes that the final decision is made according to two ranking results. Its general form is to calculate the subjective and objective attribute weights based on the experts' subjective assessments and the objective data, respectively, and then derive the subjective ranking result and the objective ranking result, and make decisions according to these two kinds of ranking results. "One line" denotes that making final decision bases on one ranking result. It contains two categories: 1) "deduction-summary";

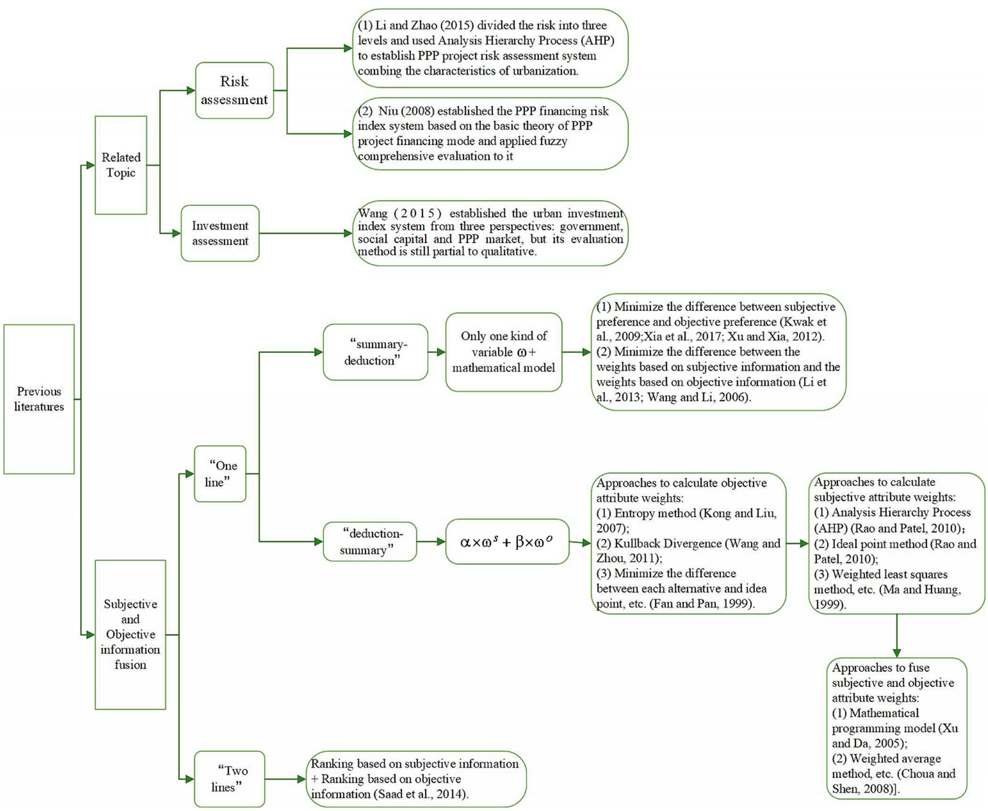


Figure 1. Literature review

2) "summary-deduction". The main distinction between these two categories is whether there exist the subjective weights and the objective weights respectively.

In this paper, we first establish the index system based on the stakeholder theory (Freeman & Phillips, 2010), which is seldom used to establish PPP index system. Besides, we present the result of the questionnaire survey to illustrate the rationality of the index system, while, in the previous literature, there is no step about it. In addition, many evaluation processes in the previous work just used the subjective information, whereas, we integrate both subjective and objective information to assess PPP's advancement. In addition, we define the concept of DHHFL entropy to denote the difference between the subjective information and the objective information, which can measure the confusion degree of information well. Besides, we integrate the subjective information and the objective information in the decision-making step, which is different from that in the previous work, meanwhile, it can use the two kinds of information more comprehensively.

2. Assessment index system

The cooperation and proper management among different stakeholders are crucial for the success of the PPP project (L. Li, Z. F. Li, Jiang, Wu, & Cheng, 2018; Amadi & Carrillo, 2018). Similarly, a successful PPP project should satisfy the requirements of the stakeholders (Liang & Jia, 2018). Hence, in this paper, the essential aim for the assessment index system is to provide a reasonable framework for evaluating the PPP's advancement and reflect the requirements of the main stakeholders i.e., the government's and social capital's requirements. Hence, in this section, we firstly analyze the interest demands of the government and social capital. Then, we detail the assessment index system. Let $C=\{c_1,c_2,\cdots,c_1,\cdots,c_m\}$ denote the set of criteria in the index system.

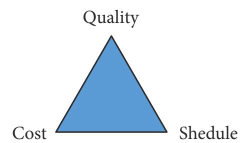
2.1. The interest demands of the government and social capital

Considering that the government and social capital are the main users of the assessment index system, we start from the view of stakeholder theory (Freeman & Phillips, 2010). There are some researches about the PPP's stakeholders. Most of them applied the stakeholder theory into the distribution of benefit and risk, and proposed the interest demands of the government and social capital in different perspectives. Chen (2008) and Yu (2103) started from the perspective of the roles of the government and social capital in the PPP. Jia (2015) and Cheng (2014) started from the perspectives of the project's success and the PPP stakeholders' satisfaction.

On the one hand, the government wants to solve the problem of shortage of the government's funds, increase local finance through PPP projects, meet the public needs for public services and improve social benefits. In addition, it is expected that cooperating with the appropriate social capitals and other stakeholders can consciously abide by laws and regulations to ensure the smooth progress of the project. On the other hand, social capitals pay more attention to the stable return and more investment opportunities. They hope to maintain harmonious cooperation relationship with the government and get supports from the government, too (Chen, 2008; Cheng, 2014; Jia, 2015; Yu, 2013).

Then, we draw a conclusion on the government's and social capital's interest demands for the PPP and the index's expectations that are the summaries of the interest demands contents in Table 1.

Table 1. The interest demands and the index's expectations of the government and social capital

<table>
<tr><th></th><th>Interest demands</th><th>Index's expectations</th></tr>
<tr><td rowspan="4">The Government</td><td>Solve the problem of shortage of the government's funds, increase local finance, reflect the performance of the government, improve the government's governance capacity, and promote local economic development</td><td>The participation situation of social capital</td></tr>
<tr><td>Meet the public needs of public services and improve social benefits</td><td>The performance reflection of the government</td></tr>
<tr><td>Introduce and select the appropriate social capitals to invest in infrastructure construction</td><td>The presentation of public service achievements</td></tr>
<tr><td>Make other stakeholders consciously abide by laws and regulations to ensure the smooth progress of the project</td><td rowspan="2"></td></tr>
<tr><td rowspan="4">Social capital</td><td>Ensure a stable return and the investment to be quickly recovered</td></tr>
<tr><td>Expand the scope of investment, get more investment opportunities and benefits</td><td>Profitability</td></tr>
<tr><td>Be protected by the government through providing preferential policies and improving laws and regulations</td><td>The Government's support</td></tr>
<tr><td>Maintain harmonious relationship with the government</td><td>The Government's capacity</td></tr>
</table>

2.2. The assessment index system

2.2.1. The participation ratio of social capitals (c_1)

There are four kinds of social capitals in the PPP project, i.e., stated-owned enterprise, local stated-owned enterprise, private enterprise and foreign company. In our country, one of the most remarkable problems in the PPP industry is of low entry ratio for private enterprises (Jiao, 2017). Hence, the proportion of the private enterprises in all social capitals is meaningful to reflect the participation situation of private enterprises.

2.2.2. The ratio of the demonstrative projects (c_2)

The identification of the demonstrative project makes sense to the developing and implementing of PPP projects. It ensures the quality of PPP project, and gives full play to the demonstration effects (Ministry of Finance of the People's Republic of China, 2015b). In our country, the Chinese Ministry of Finance PPP Center publishes the lists of the demonstrative projects periodically, based on which, we can know about the PPP's development level and the basic situation of the PPP projects' quality. In addition, it also can reflect the performance of the government on the development of PPP and the achievements on public service. Hence, we put the ratio of the demonstrative projects in the all PPP projects in a region as an index in the assessment index system.

2.2.3. The total number of the PPP projects (c_3), the number of the PPP project types (c_4) and the total investment of the PPP projects (c_5)

The total number of the PPP projects straightly reflects the advancement of the PPP in a region, and the number of the PPP project types mainly depicts the diversity of the PPP project in a region. Chinese Ministry of Finance PPP Center has issued a document about 19 categories of the PPP industries (Ministry of Finance of the People's Republic of China, 2017).

The total investment of the PPP projects is essential to both government and social capital, which reflects the basic situation of the investment level in a region roughly. All above the three indexes are the basic reflection of the PPP's development situation in a region.

2.2.4. The ratio of projects in the implementation period (c_6) and the average time from the identification period to implementation period (c_7)

Schedule is one of the most important factors in project management (Babu & Suresh, 1996). For a PPP project, the whole lifecycle contains identification period, preparation period, purchasing period, implementation period and transition period (Ministry of Finance of the People's Republic of China, 2014). Considering that only when the project really goes into an implementation period, it begins to generate revenue, we mainly focus on the implementation period. The ratio of projects in the implementation period is used to reflect the proportion of the projects which finish purchasing.

The average time from the identification period to the implementation period for the projects depicts the speed of the development of PPP project and reflect the schedule situation.

2.2.5. The average ratio of the return (c_8)

The ratio of return for the project can straightly reflect the profitability of projects. And each project would confirm their expected ratio of return in the preparation period, which we can find on the website. Referring to the information in the PPP project database platform, the average ratio of the return index is used to describe the basic profitability situation of the PPP projects in a region.

2.2.6. The ratio of financial aid from the government (c_9)

The Chinese Ministry of Finance has issued a document about the "award-winning" policy (Ministry of Finance of the People's Republic of China, 2015a). On one hand, it reflects the degree of subsidy on the PPP projects. On the other hand, for social capital, they would like to get strong support from the government (Chen, 2008; Cheng, 2014; Jia, 2005; Yu, 2013). Hence, the ratio of financial aid from the government is applicable to present the degree of the government's support and its capacity.

2.3. A questionnaire survey on rationality of the assessment index system

The questionnaire survey was conducted in a salon to let the experts assess the rationality of the assessment index system. There are 17 experts who have completed the questionnaires, including 11 males and 6 females; 2 from government departments, 5 from social capital, 3 from banks and 7 from fund companies; 3 for 3 years working experience, 14 for more than 4 years. They scored the index system from five aspects: practicality, comprehensiveness, comparability, systematicness and pertinence corresponding to the scores 1–5. The higher the scores, the better assessments to the index system under the five aspects. The results are shown below:

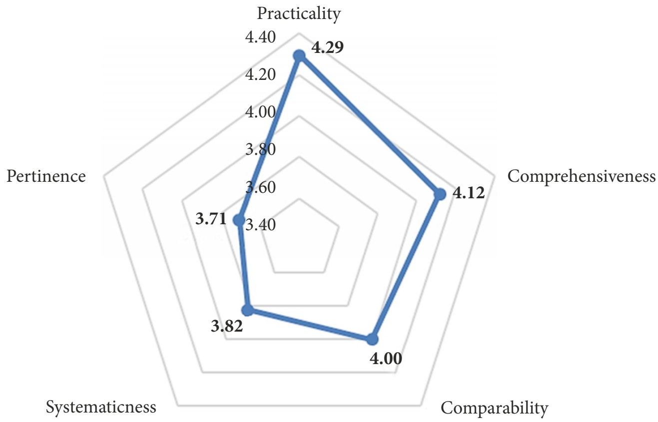


Figure 2. Evaluation results of the assessment index system

As shown in Figure 2, we can see that the system is practical, comprehensive, comparable, systematic and targeted. The average score of each term is more than 3.5, which means that the scores are over average, and practicability gets the highest average score, which indicates that the assessment index system has been recognized. It can be used in the assessment process.

3. Determination of attribute weights

Attribute weights determining is one of the most important things in multi-attribute group decision making (MAGDM) (Liang, Wei, & Cheng, 2016). It has a meaningful influence on the decision-making process, especially in the process of calculating comprehensive assessment values which affects alternatives' ranking straightly. Since both the experts' knowledge backgrounds and objective data are valuable in the evaluation processes, we should combine the subjective information from the experts and the objective information from databases together.

In this section, we first define the concept of the DHHFL entropy, then we put forward a programming model based on the minimum entropy principle to derive the attribute weights. The main approach includes processing subjective information, processing objective information, and constructing a programming model.

3.1. The DHHFL entropy

The concept of entropy was first applied in physical sciences to describe the uncertainty, disorder, or randomness of a probability system (Herniter, 1973). In the 1940s, Shannon and Weaver (1949) proposed the concept of information entropy to describe the uncertainty degree of information. Since then, a lot of researchers have extended the concept of entropy to different areas, such as fuzzy set (Fan & Ma, 2002; Guo & Xin, 2006; Liu, 1992), hesitant fuzzy set (Xu & Xia, 2012) intuitionistic fuzzy set (Xia & Xu, 2012), etc.

In this subsection, we define the concept of the DHHFL entropy to minimize the uncertainty of the DHHFL information.

Definition 1. Let the DHHFLEs A_1 and $\{0.3\}$, whose first hierarchy and second hierarchy base on the linguistic evaluation scales $\pi(A_3, A_4)$ $p_j(A_2, A_4)$ and $O = \{o_k | k = -\delta, \ldots, -1, 0, 1, \ldots, \delta\}$. $E\left(h_{S_O}(x_i)\right)$ is named DHHFL entropy, if it satisfies the following properties:

1) $E\left(h_{S_O}(x_1)\right) = 0$, if and only if $f\left(h_{S_O}(x_1)\right) = 0$ or $f\left(h_{S_O}(x_1)\right) = 1$;
2) $E(h_{S_O}(x_1)) = 1$, if and only if $f\left({}^l h_{S_O}(x_1)\right) + f\left({}^{L\text{-}l+1} h_{S_O}(x_1)\right) = 1$, for $l = 1, 2, \cdots, L$;
3) if $f\left({}^l h_{S_O}(x_1)\right) \le f\left({}^l h_{S_O}(x_2)\right)$ for $f\left({}^l h_{S_O}(x_1)\right) + f\left({}^{L\text{-}l+1} h_{S_O}(x_1)\right) \le 1$ or $f\left({}^l h_{S_O}(x_1)\right) \ge f\left({}^l h_{S_O}(x_2)\right)$ for $f\left({}^l h_{S_O}(x_1)\right) + f\left({}^{L\text{-}l+1} h_{S_O}(x_1)\right) \ge 1$, then $E\left(h_{S_O}(x_1)\right) \le E\left(h_{S_O}(x_2)\right)$, for $l = 1, 2, \cdots, L$;
4) $E\left(h_{S_O}(x_1)\right) = E\left(h_{S_O}(x_1)^c\right)$, where $h_{S_O}(x_1)^c$ is the complement set of $h_{S_O}(x_1)$.

Motivated by the Hesitant fuzzy set entropy formulas (Xu & Xia, 2012), we propose some DHHFL entropy formulas as follows:

$$E_1\left(h_{S_O}\right) = \frac{1}{L\left(\sqrt{2}-1\right)} \sum_{l=1}^{L} \left(\sin \frac{\pi\left(f\left({}^l h_{S_O}\right) + f\left({}^{L-l+1} h_{S_O}\right)\right)}{4} + \sin \frac{\pi\left(2 - f\left({}^l h_{S_O}\right) - f\left({}^{L-l+1} h_{S_O}\right)\right)}{4} - 1 \right); \tag{6}$$

$$E_2\left(h_{S_O}\right) = \frac{1}{L\left(\sqrt{2}-1\right)} \sum_{l=1}^{L} \left(\cos \frac{\pi\left(f\left({}^l h_{S_O}\right) + f\left({}^{L-l+1} h_{S_O}\right)\right)}{4} + \cos \frac{\pi\left(2 - f\left({}^l h_{S_O}\right) - f\left({}^{L-l+1} h_{S_O}\right)\right)}{4} - 1 \right); \tag{7}$$

$$E_3\left(h_{S_O}\right) = -\frac{1}{L \ln 2} \sum_{l=1}^{L} \left(\frac{f\left({}^l h_{S_O}\right) + f\left({}^{L-l+1} h_{S_O}\right)}{2} \ln \frac{f\left({}^l h_{S_O}\right) + f\left({}^{L-l+1} h_{S_O}\right)}{2} + \frac{2 - f\left({}^l h_{S_O}\right) - f\left({}^{L-l+1} h_{S_O}\right)}{2} \ln \frac{2 - f\left({}^l h_{S_O}\right) - f\left({}^{L-l+1} h_{S_O}\right)}{2} \right); \tag{8}$$

$$E_4\left(h_{S_O}\right) = \frac{1}{L\left(2^{(1-s)^t} - 1\right)} \sum_{l=1}^{L} \left(\left(\frac{f\left({}^l h_{S_O}\right) + f\left({}^{L-l+1} h_{S_O}\right)}{2} \right)^s + \left(1 - \frac{f\left({}^l h_{S_O}\right) + f\left({}^{L-l+1} h_{S_O}\right)}{2} \right)^s - 1 \right),$$

$$t \ne 0,\ s \ne 1,\ s > 0. \tag{9}$$

where $l = 1, 2, \ldots L$.

3.2. The approach to determine attribute weights

For a MAGDM problem with double hierarchy linguistic information, there are m alternatives in the finite set $A=\{A_1,A_2,...,A_m\}$, n attributes in the finite set $C=\{C_1,C_2,...,C_n\}$ and q experts in the finite set $E=\{E_1,E_2,...,E_q\}$. The attribute weights are denoted by the vector $\omega^*=\left(\omega_1^*,\omega_2^*,...,\omega_n^*\right)$, where $0\le\omega_j^*\le 1$, and $\sum_{j=1}^{n}\omega_j^*=1$, and the weights of different experts are denoted by the vector $\lambda=\left(\lambda_1,\lambda_2,...,\lambda_q\right)$, where $0\le\lambda_k\le 1$, and $\sum_{j=1}^{n}\lambda_k=1$. The attribute weights are completely unknown in this paper, and the decision matrix is shown as follows:

$$H_{S_O}^k=\left(h_{S_{Oij}}^k\right)_{m\times n}=\begin{pmatrix} h_{S_{O11}}^k & h_{S_{O12}}^k & \cdots & h_{S_{O1n}}^k \\ h_{S_{O21}}^k & h_{S_{O22}}^k & \cdots & h_{S_{O2n}}^k \\ \vdots & \vdots & \vdots & \vdots \\ h_{S_{Om1}}^k & h_{S_{Om2}}^k & \cdots & h_{S_{Omn}}^k \end{pmatrix},\quad k=1,2,...,q; i=1,2,...,m; j=1,2,...,n,$$

where $h_{S_{Oij}}^k$ denotes the assessment values of the alternative A_i over the attribute c_j given by k_{th} expert. All the assessment values are described by the DHFLTs.

Then, we aggregate different DHHFL decision matrices by the weighted sum operator. The formula is shown as follows:

$$H_{S_O}=\left(h_{S_{Oij}}\right)_{m\times n}=\mathop{\oplus}_{k=1}^{q}\lambda_k H_{S_O}^k,\ i=1,2,...m;\ n=1,2,...n;\ k=1,2,...,q. \tag{10}$$

The objective data matrix can be obtained according to actual situations and objective data, which can be expressed as:

$$B=\left(b_{ij}\right)_{m\times n}\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix},\quad i=1,2,...,m;\ j=1,2,...,n,$$

where b_{ij} denotes the real number of the alternative A_i over the attribute c_j. All of them can be measured or calculated by real data.

When we deal with the subjective information, we transform the linguistic terms into the fuzzy numbers with the 0–1 scale. For the objective data, sometimes they may be in different forms or scales, which should be normalized in order to make the calculation process based on subjective and objective information consistent. Hence, the values of objective data matrix should be normalized. The normalization formula is shown as follows:

$$b'_{ij}=\frac{b_{ij}}{\sum_{i=1}^{m}b_{ij}},\ i=1,2,...,m;\ j=1,2,...,n. \tag{11}$$

Cao and Wu (2009) pointed out that the sole optimized function and the lack of judgement information of the experts on attribute weights may cause the irrationality in the programming model, so possible range of the decision variable ω_j should be $\left[0.05,0.5\right]$. By applying the minimum entropy principle, the programming model (M1) combines the subjective and objective matrices to process the subjective and objective information and obtain the attribute weights.

$$\textbf{(M–1)}\ \mathrm{Min}E_1 = -\sum_{i=1}^{m}\sum_{j=1}^{n}\omega_j E\left(f\left(h_{S_{O_{ij}}}\right)\right) =$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\omega_j \frac{1}{L\ln 2}\sum_{l=1}^{L}\left(\frac{f\left({}^{l}h_{S_{O_{ij}}}\right)+f\left({}^{L-l+1}h_{S_{O_{ij}}}\right)}{2}\ln\frac{f\left({}^{l}h_{S_{O_{ij}}}\right)+f\left({}^{L-l+1}h_{S_{O_{ij}}}\right)}{2}+\right.$$

$$\left.\frac{2-f\left({}^{l}h_{S_{O_{ij}}}\right)-f\left({}^{L-l+1}h_{S_{O_{ij}}}\right)}{2}\ln\frac{2-f\left({}^{l}h_{S_{O_{ij}}}\right)-f\left({}^{L-l+1}h_{S_{O_{ij}}}\right)}{2}\right).$$

$$\mathrm{Min}E_2 = -\sum_{i=1}^{m}\sum_{j=1}^{n}E\left(\omega_j b_{ij}\right) = -\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1}{\ln mn}\omega_j b_{ij}\ln\omega_j b_{ij}$$

$$s.t.\ C_\omega,$$

$$\sum_{j=1}^{n}\omega_j = 1,$$

$$0.05 \le \omega_j \le 0.5,$$

$$i = 1,2,\ldots,m;\ j = 1,2,\ldots,n,$$

where ω_j ($j = 1,2,\ldots,n$) denotes the attribute weight, C_ω denotes some constraints for some attributes' weights. To make the model easier, we transform the multi–objective programming model into the single–objective programming model, shown as follows:

$$\textbf{(M–2)}\ \mathrm{Min}E = \mathrm{Min}E_1 + \mathrm{Min}E_2 =$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\omega_j E\left(f\left(h_{S_{O_{ij}}}\right)\right)+\left(-\sum_{i=1}^{m}\sum_{j=1}^{n}E\left(\omega_j b_{ij}\right)\right)=$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\omega_j \frac{1}{L\ln 2}\sum_{l=1}^{L}\left(\frac{f\left({}^{l}h_{S_{O_{ij}}}\right)+f\left({}^{L-l+1}h_{S_{O_{ij}}}\right)}{2}\times\ln\frac{f\left({}^{l}h_{S_{O_{ij}}}\right)+f\left({}^{L-l+1}h_{S_{O_{ij}}}\right)}{2}+\right.$$

$$\left.\frac{2-f\left({}^{l}h_{S_{O_{ij}}}\right)-f\left({}^{L-l+1}h_{S_{O_{ij}}}\right)}{2}\times\ln\frac{2-f\left({}^{l}h_{S_{O_{ij}}}\right)-f\left({}^{L-l+1}h_{S_{O_{ij}}}\right)}{2}\right)$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1}{\ln mn}\omega_j b_{ij}\ln\omega_j b_{ij}$$

$$st.C_\omega,$$

$$\sum_{j=1}^{n}\omega_j = 1,$$

$$0.05 \le \omega_j \le 0.5,$$

$$i = 1,2,\ldots,m; j = 1,2,\ldots,n.$$

where ω_j ($j = 1,2,\ldots,n$) denote the attributes' weights, C_ω denotes some constraints for some attributes' weights.

4. The DHHFL-PROMETHEE-S&O approach

In this section, the DHHFL-PROMETHEE-S&O approach will be discussed in detail. Although there are lots of researchers discussing the combination of the subjective and objective information in the process of MADM (Choua & Shen, 2008; Fan & Pan, 1999; Kong & Liu, 2007; B. Li, Ren, Wang, Wei, & J. Li, 2013; Ma & Huang, 1999; Rao & Patel, 2010; Saad, Ahmad, Abu, & Jusoh, 2014; Wang & Zhou, 2011; Wang & Li, 2006; Xia, Zhang, & Badr, 2017; Xu & Da, 2005; Xu & Chen, 2007; Palevicius et al., 2018), most of them just combined the two kinds of information in the weight deriving process rather than in the decision-making process. However, the decision-making process is an important part in MADM, which finally determines the reasonableness of the results. What's more, the PROMETHEE is a popular decision-making method in MADM, and it makes use of binary relations on a set of potential actions to develop a preference relation among different alternatives (Zhang, Kluck, & Achari, 2009). By using a variety of preference functions, it can dig the experts' preferences to the alternatives based on the evaluation information. Then, it derives the outranking flows through some calculations of the preferences.

In this approach, to derive the net outranking flow, we are required not only to derive the "differences" between two different alternatives over different attributes, but also to transform the "differences" into "preferences". Hence, for the subjective information, the paper uses the DHLTs to describe the experts' assessments, which can present the experts' subjective judgements more accurately. We measure the deviation between two different alternatives over different attributes by Hamming distance. Then, we transform the difference into the preference based on Eq. (14). For the objective information, all the attribute values are crisp numbers. Hence, this paper uses the difference between two different alternatives over different attributes to describe the deviations among them. Then, we transform the difference into the preference through Eq. (15).

After the process of preference deriving based on Eq. (14) and Eq. (15), we fuse the subjective and objective preferences by giving them different importance indices, and then generate a new comprehensive preference which considers both the subjective and objective information. The paper would calculate the subsequent correlation indexes based on the comprehensive preference.

Step 1: Identify attributes

Identifying attributes is the fundamental part for MAGDM. The different attributes should reflect the crucial factors which influence the alternatives' development and embody their traits. We detail the assessment index system in Section 2.

Step 2: Generate the experts' assessment matrices and objective data

The experts' assessment matrices are based on the evaluation of each expert using the DHLTs, and the objective information is expressed as crisp numbers. The concrete details are mentioned in Subsection 3.2, and we use the vector $\lambda=\left(\lambda_1,\lambda_2,...,\lambda_q\right)$ to denote the experts' weights.

Step 3: Determine attribute weights

We combine the subjective and objective information to determine the attribute weights. The specific method to derive the attribute weights is detailed in Section 3, and we use the vector $\omega=\left(\omega_1,\omega_2,...,\omega_q\right)$ to denote the experts' weights.

Step 4: Determine the deviation between each pair of alternatives over different attributes

To derive the preferences, we should calculate the deviation between each pair of alternatives over different attributes. Hence, in this step, we give the different formulas to calculate the deviations.

1) For the subjective decision-making matrix:

We can calculate the deviation between each pair of alternatives over different attributes as follows:

$$d_j^s\left(A_i,A_t\right)=\begin{cases} d_{hdj}^s\left(A_i,A_t\right)=d_{hdj}^s\left(h_{S_{Oij}},h_{S_{Otj}}\right)=\dfrac{1}{L}\sum\limits_{\substack{l=1\\ \eta_1\in f\left(h_{S_{O_{ij}}}\right),\eta_2\in f\left(h_{S_{O_{ij}}}\right)}}^{L}\left|\left(\eta_{ij}^l-\eta_{tj}^l\right)\right| & \text{if } h_{S_{Oij}}\geq h_{S_{Otj}} \\ -d_{hdj}^s\left(A_i,A_t\right)=-d_{hdj}^s\left(h_{S_{Oij}},h_{S_{Otj}}\right)=-\dfrac{1}{L}\sum\limits_{\substack{l=1\\ \eta_1\in f\left(h_{S_{O_{ij}}}\right),\eta_2\in f\left(h_{S_{O_{ij}}}\right)}}^{L}\left|\left(\eta_{ij}^l-\eta_{tj}^l\right)\right| & \text{if } h_{S_{Oij}}< h_{S_{Otj}} \end{cases},$$

$$i=1,2,...m;\, j=1,2,...n;\, l=1,2,...L \tag{12}$$

where f is the function to transform DHLTs into fuzzy numbers; η_{ij}^l and η_{tj}^l are the l_{th} linguistic terms of $h_{S_{O_{ij}}}$ and $h_{S_{O_{tj}}}$, respectively. If the two DHHFLEs have different numbers of DHLTs, then we can add the mean values between the upper and lower values in the short one until the length of the two DHHFLEs is the same.

2) For the objective normalized matrix:

Considering that the data included in the objective normalized matrix are all crisp numbers, we use the difference among real numbers to express the distances between each two alternatives over different attributes.

Hence, we can determine the deviation between each pair of alternatives over different attributes as follows:

$$d_j^o\left(A_i,A_t\right)=\begin{cases} b'_{ij}-b'_{tj} & \text{if } c_j \text{ is benefite attribute} \\ -\left(b'_{ij}-b'_{tj}\right) & \text{if } c_j \text{ is cost attribute} \end{cases},\quad i=1,2,...,m;\; j=1,2,...,n. \tag{13}$$

Step 5: Convert the deviation into the preference

For the PROMETHEE method, there are six kinds of preference functions. Among them, the linear preference function can be adapted in most situations and be suitable for quantitative attributes quite well (Halouani, Chabchoub, & Martel, 2009; Shih, Chang, & Cheng, 2016). Hence, we choose the linear preference function to transfer the deviation into the preference.

1) For the subjective preferences:

$$P_j^s\left(A_i,A_t\right)=f\left(d_j^s\right)=\begin{cases}0 & \text{if} \quad d_j^s \le q_j^s \\ \dfrac{d_j^s-q_j^s}{p_j^s-q_j^s} & \text{if} \quad q_j^s < d_j^s \le p_j^s, \\ 1 & \text{if} \quad d_j^s > p_j^s\end{cases} \quad i=1,2,\ldots,m;\ j=1,2,\ldots,n; \tag{14}$$

2) For the objective preferences:

$$P_j^o\left(A_i,A_t\right)=f\left(d_j^o\right)=\begin{cases}0 & \text{if} \quad d_j^o \le q_j^o \\ \dfrac{d_j^o-q_j^o}{p_j^o-q_j^o} & \text{if} \quad q_j^o < d_j^o \le p_j^o, \\ 1 & \text{if} \quad d_j^o > p_j^o\end{cases} \quad i=1,2,\ldots,m;\ j=1,2,\ldots,n; \tag{15}$$

3) Fuse the two kinds of preferences:

$$P_j\left(A_i,A_t\right)=\alpha P_j^s\left(A_i,A_t\right)+\beta P_j^o\left(A_i,A_t\right), \quad i=1,2,\ldots,m;\ j=1,2,\ldots,n, \tag{16}$$

where q_j^s and q_j^o are the indifference thresholds under the subjective and objective information respectively; p_j^s and p_j^o are the strict thresholds under the subjective and objective information, respectively. Generally, the values of them are confirmed by the experts (V. Podvezko & A. Podviezko, 2010). α and β are the importance indexes for the subjective preferences and the objective preferences which are decided by the decision makers, and $\alpha + \beta = 1$. If $\alpha = 1$ and $\beta = 0$, then the decision makers do not consider the objective preferences. If $\alpha = 0$ and $\beta = 1$, then the decision makers do not consider the subjective preferences.

Step 6: Calculate the overall comprehensive preference index between each pair of alternatives

We calculate the overall comprehensive preference between each pair of alternatives by Eq. (24):

$$\pi\left(A_i,A_t\right)=\sum_{j=1}^{n}\omega_j^* P_j\left(A_i,A_t\right), \ i=1,2,\ldots,t,\ldots,m;\ j=1,2,\ldots,n. \tag{17}$$

Step 7: Calculate the positive outranking flow and the negative outranking flow

1) Positive outranking flow:

$$\phi^+\left(A_i\right)=\frac{1}{m-1}\sum_{A_t\in A}\pi\left(A_i,A_t\right), \quad i=1,2,\ldots,t,\ldots,m; \tag{18}$$

2) Negative outranking flow:

$$\phi^-\left(A_i\right)=\frac{1}{m-1}\sum_{A_t\in A}\pi\left(A_t,A_i\right), \quad i=1,2,\ldots,t,\ldots,m, \tag{19}$$

where A is the alternatives set $A=\left\{A_1,A_2,A_3\ldots A_t\ldots A_m\right\}$.

Step 8: Derive the net flow

The net flow can be calculated by:

$$\phi\left(A_i\right)=\phi^+\left(A_i\right)\text{-}\phi^-\left(A_i\right), \quad i=1,2,\ldots,t,\ldots,m. \tag{20}$$

Step 9: Derive the ranking result

We can rank the alternatives $A_i\left(i=1,2,\ldots,m\right)$ in descending order of the net flow $\phi\left(A_i\right)$. The larger the net flow, the better the alternative.

5. Case study

In this section, we illustrate the DHHFL-PROMETHEE-S&O approach with an example. We introduce the background first. Then, we implement the algorithm step by step. Finally, there is a discussion about the sensitivity of the approach and comparison with the DHHFL-MULTIMOORA method.

5.1. Evaluation of the PPP's advancement

The PPP has been very popular in China recently. Each city is promoting the PPP vigorously. Moreover, knowing about the basic situation of the PPP's advancement is meaningful for the government and social capital. Considering about geographical relations and the completeness of data, we choose four cities, Deyang (A_1), Yibin (A_2), Xi'an (A_3) and Hanzhong (A_4) as evaluation alternatives, and use the DHHFL-PROMETHEE-S&O method to evaluate the PPP's advancement in these four cities to help the government and social capital know about the outcomes of the PPP's promotion and decide which city is worthy to invest in.

Step 1: Identify attributes

The assessment index system has been established in Section 2.

Step 2: Generate the experts' assessment matrices and objective data

Firstly, there are three experts evaluating the four cities under the assessment index system. Their evaluation outcomes are shown as follows:

The weight for each expert is denoted by the vector $\lambda=\left(0.3,0.4,0.3\right)$ (in this paper, since the experts' weights are not the point we focus on, then we just give the experts' weights beforehand). The first hierarchy and second hierarchy linguistic terms set are shown below:

$S=\left\{s_{-2}=\text{very bad}, s_{-1}=\text{bad}, s_0=\text{medium}, s_1=\text{good}, s_2=\text{very good}\right\}$;

If $s_t \geq s_0$, then $O=\left\{o_{-2}=\text{far from}, o_{-1}=\text{little}, o_0=\text{just right}, o_1=\text{much}, o_2=\text{entirely}\right\}$;

If $s_t < s_0$, then $O=\left\{o_{-2}=\text{etirely}, o_{-1}=\text{much}, o_0=\text{just right}, o_1=\text{little}, o_2=\text{far from}\right\}$.

The evaluation information of the first expert:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\!\!\begin{array}{c} \begin{array}{ccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \end{array} \\ \begin{bmatrix} \{s_{-1_{o_1}}\} & \{s_0\} & \{s_{-1_{o_{-2}}}\} & \{s_0, s_{1_{o_1}}\} & \{s_{-1}, s_{-1_{o_1}}\} & \{s_{-1_{o_{-1}}}\} & \{s_{-1_{o_{-1}}}, s_{-1}\} & \{s_{1_{o_1}}, s_2\} & \{s_{-1_{o_{-1}}}\} \\ \{s_1, s_{2_{o_{-1}}}\} & \{s_{-1_{o_{-2}}}\} & \{s_2\} & \{s_2\} & \{s_0, s_{1_{o_1}}\} & \{s_{-1}\} & \{s_{1_{o_1}}\} & \{s_0\} & \{s_{-1_{o_{-1}}}, s_{-1}\} \\ \{s_{-1_{o_{-1}}}\} & \{s_2\} & \{s_{-1_{o_1}}\} & \{s_{-1_{o_{-1}}}\} & \{s_0\} & \{s_0, s_{1_{o_1}}\} & \{s_{-2}\} & \{s_{-1_{o_1}}\} & \{s_{-2}\} \\ \{s_{-1}\} & \{s_{1_{o_1}}, s_2\} & \{s_{-1_{o_{-1}}}\} & \{s_0\} & \{s_{-1_{o_{-2}}}\} & \{s_{1_{o_{-1}}}\} & \{s_0, s_{1_{o_1}}\} & \{s_0, s_{1_{o_{-1}}}\} & \{s_{-2}\} \end{bmatrix} \end{array}.$$

The evaluation information of the second expert:

$$
\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \end{array} \\
\left[\begin{array}{ccccccccc}
\{s_0\} & \{s_{-1_{o_1}}, s_0\} & \{s_{-2}\} & \{s_{-2}\} & \{s_0\} & \{s_{-2_{o_1}}\} & \{s_{-1_{o_1}}, s_0\} & \{s_{1_{o_1}}\} & \{s_{-1_{o_{-1}}}\} \\
\{s_{1_{o_1}}\} & \{s_{-1_{o_1}}, s_0\} & \{s_{1_{o_1}}, s_2\} & \{s_{1_{o_1}}, s_2\} & \{s_{1_{o_1}}\} & \{s_0\} & \{s_{1_{o_1}}\} & \{s_0\} & \{s_{-1_{o_{-1}}}\} \\
\{s_0\} & \{s_2\} & \{s_0\} & \{s_{-1_{o_1}}, s_0\} & \{s_{1_{o_1}}\} & \{s_0, s_{1_{o_1}}\} & \{s_{-1_{o_{-1}}}, s_{-1_{o_1}}\} & \{s_{-1_{o_{-1}}}\} & \{s_{-2}\} \\
\{s_{-1_{o_1}}\} & \{s_2\} & \{s_0\} & \{s_{-1_{o_{-1}}}, s_{-1_{o_1}}\} & \{s_{-1_{o_{-1}}}, s_0\} & \{s_{-1_{o_{-1}}}, s_0\} & \{s_0\} & \{s_{1_{o_1}}\} & \{s_{-2}\}
\end{array}\right]
\end{array}.
$$

The evaluation information of the third expert:

$$
\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \end{array} \\
\left[\begin{array}{ccccccccc}
\{s_0, s_1\} & \{s_0, s_1\} & \{s_{-2}, s_{-1_{o_{-1}}}\} & \{s_{-2}, s_{-1_{o_{-1}}}\} & \{s_0\} & \{s_{-2}, s_{-1_{o_{-1}}}\} & \{s_{-1_{o_1}}, s_0\} & \{s_{1_{o_1}}\} & \{s_{-1_{o_{-1}}}\} \\
\{s_{1_{o_{-1}}}, s_2\} & \{s_{-1_{o_1}}, s_0\} & \{s_2\} & \{s_2\} & \{s_0\} & \{s_0\} & \{s_0, s_{1_{o_{-1}}}\} & \{s_0\} & \{s_{-1_{o_{-1}}}, s_{-1}\} \\
\{s_0, s_1\} & \{s_2\} & \{s_0\} & \{s_0\} & \{s_{0_{o_1}}, s_{1_{o_1}}\} & \{s_0, s_{1_{o_{-1}}}\} & \{s_{-2}, s_{-1_{o_{-1}}}\} & \{s_{-1_{o_{-1}}}\} & \{s_{-2}\} \\
\{s_0\} & \{s_2\} & \{s_{-2_{o_1}}, s_0\} & \{s_0\} & \{s_{-1_{o_1}}, s_0\} & \{s_0\} & \{s_0\} & \{s_0\} & \{s_{-2}\}
\end{array}\right]
\end{array}.
$$

Then, we collect projects' information on the official website of Chinese Ministry of Finance as follows:

$$
\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \end{array} \\
\left[\begin{array}{ccccccccc}
37.50\% & 37.50\% & 8 & 5 & 1979047 & 3.7\% & 11.67 & 6.85\% & 0.11\% \\
48.15\% & 14.52\% & 62 & 13 & 4025228 & 19.42\% & 9.5 & 6.37\% & 0.22\% \\
36.40\% & 57.14\% & 21 & 6 & 8740678 & 25\% & 14.7 & 5.54\% & 0.03\% \\
20\% & 52.94\% & 17 & 7 & 1785348 & 12.5\% & 9.6 & 6.41\% & 0.04\%
\end{array}\right]
\end{array},
$$

where the relevant data come from the website of the PPP center of the Chinese Financial Ministry. The measurement unit of c_5 is ten thousand yuan, the measurement unit of c_7 is months, the other attributes are percentage or the numbers, so they do not have special measurement unit and their measurement units are 1.

Step 3: Determine attribute weights

To make the calculating process simpler, we transform DHHFLTs into fuzzy numbers by Eq. (3) and Eq. (4). The results are shown as follows:

The information from the first expert:

$$
\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \end{array} \\
\left[\begin{array}{ccccccccc}
\{0.4\} & \{0.5\} & \{0.1\} & \{0.5,0.8\} & \{0.3,0.4\} & \{0.2\} & \{0.2,0.3\} & \{0.8,0.9\} & \{0.2\} \\
\{0.7,0.8\} & \{0.1\} & \{0.9\} & \{0.9\} & \{0.5,0.8\} & \{0.3\} & \{0.8\} & \{0.5\} & \{0.2,0.3\} \\
\{0.2\} & \{0.9\} & \{0.4\} & \{0.2\} & \{0.5\} & \{0.5,0.8\} & \{0.1\} & \{0.4\} & \{0.1\} \\
\{0.3\} & \{0.8,0.9\} & \{0.2\} & \{0.5\} & \{0.1\} & \{0.6\} & \{0.5,0.8\} & \{0.5,0.6\} & \{0.1\}
\end{array}\right]
\end{array}.
$$

The information from the second expert:

$$
\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \end{array} \\
\left[\begin{array}{ccccccccc}
\{0.5\} & \{0.4,0.5\} & \{0.1\} & \{0.1\} & \{0.5\} & \{0.2\} & \{0.4,0.5\} & \{0.8\} & \{0.2\} \\
\{0.8\} & \{0.4,0.5\} & \{0.8,0.9\} & \{0.8,0.9\} & \{0.8\} & \{0.5\} & \{0.8\} & \{0.5\} & \{0.2\} \\
\{0.5\} & \{0.9\} & \{0.5\} & \{0.4,0.5\} & \{0.8\} & \{0.5,0.8\} & \{0.2,0.4\} & \{0.2\} & \{0.1\} \\
\{0.4\} & \{0.9\} & \{0.5\} & \{0.2,0.4\} & \{0.2,0.5\} & \{0.2,0.5\} & \{0.5\} & \{0.8\} & \{0.1\}
\end{array}\right]
\end{array}.
$$

The information from the third expert:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{array}{c} \begin{array}{ccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \end{array} \\ \begin{bmatrix} \{0.5,0.7\} & \{0.5,0.7\} & \{0.1,0.2\} & \{0.1,0.2\} & \{0.5\} & \{0.1,0.2\} & \{0.4,0.5\} & \{0.8\} & \{0.3\} \\ \{0.6,0.9\} & \{0.4,0.5\} & \{0.9\} & \{0.9\} & \{0.5\} & \{0.5\} & \{0.5,0.6\} & \{0.5\} & \{0.3\} \\ \{0.5,0.8\} & \{0.9\} & \{0.5\} & \{0.5\} & \{0.6,0.8\} & \{0.5,0.6\} & \{0.1,0.2\} & \{0.2\} & \{0.1\} \\ \{0.5\} & \{0.9\} & \{0.2,0.5\} & \{0.5\} & \{0.4,0.5\} & \{0.5\} & \{0.5\} & \{0.5\} & \{0.1\} \end{bmatrix} \end{array}.$$

According to Eq. (10), we fuse the outcomes from three experts into one subjective decision–making matrix, which is shown below:

$$\begin{array}{c|ccccccccc} & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \\ \hline A_1 & \{0.472,0.55\} & \{0.462,0.5,0.539,0.571\} & \{0.1,0.131\} & \{0.254,0.272,0.426,0.446\} & \{0.447,0.472\} & \{0.171,0.2\} & \{0.346,0.371,0.381,0.392,0.405,0.416,0.424,0.447\} & \{0.8,0.838\} & \{0.23\} \\ A_2 & \{0.722,0.817\} & \{0.322,0.359,0.369,0.403\} & \{0.868,0.9\} & \{0.868,0.9\} & \{0.653,0.737\} & \{0.447\} & \{0.737,0.754\} & \{0.5\} & \{0.217,0.247\} \\ A_3 & \{0.424,0.563\} & \{0.9\} & \{0.472\} & \{0.472\} & \{0.676,0.737\} & \{0.5,0.532,0.62,0.644,0.654,0.676,0.737,0.754\} & \{0.14,0.17,0.237,0.264\} & \{0.266\} & \{0.1\} \\ A_4 & \{0.405\} & \{0.877,0.9\} & \{0.337,0.424\} & \{0.337,0.424\} & \{0.239,0.28,0.369,0.403\} & \{0.497,0.532\} & \{0.5,0.62\} & \{0.654,0.676\} & \{0.1\} \end{array}.$$

Then, we use Eq. (11) to normalize the objective data, shown as follows:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{array}{c} \begin{array}{ccccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \end{array} \\ \begin{bmatrix} 0.264 & 0.231 & 0.074 & 0.161 & 0.12 & 0.061 & 0.257 & 0.272 & 0.275 \\ 0.339 & 0.09 & 0.574 & 0.419 & 0.244 & 0.32 & 0.209 & 0.253 & 0.55 \\ 0.256 & 0.352 & 0.194 & 0.194 & 0.529 & 0.412 & 0.323 & 0.22 & 0.075 \\ 0.141 & 0.327 & 0.157 & 0.157 & 0.108 & 0.206 & 9.6 & 0.255 & 0.1 \end{bmatrix} \end{array}.$$

After processing the subjective and objective information, we can calculate the attribute weights according to (M2). Consider that the government pays more attention to the cost, quality and schedule of the PPP projects, and social capital focus on the profitability of the PPP projects, we let the weights of c_2, c_5, c_6, and c_7 be not less than 0.1, the weight of c_8 be not less than 0.2.

$$\mathrm{Min}E = -\sum_{i=1}^{4}\sum_{j=1}^{9}\omega_j^* \frac{1}{L\ln 2}\sum_{l=1}^{L}\left(\frac{f\left(h_{S_{O_{ij}}}{}^{l}\right)+f\left(h_{S_{O_{ij}}}{}^{L-l+1}\right)}{2}\times\ln\frac{f\left(h_{S_{O_{ij}}}{}^{l}\right)+f\left(h_{S_{O_{ij}}}{}^{L-l+1}\right)}{2} + \right.$$

$$\left. \frac{2-f\left(h_{S_{O_{ij}}}{}^{l}\right)-f\left(h_{S_{O_{ij}}}{}^{L-l+1}\right)}{2}\times\ln\frac{2-f\left(h_{S_{O_{ij}}}{}^{l}\right)-f\left(h_{S_{O_{ij}}}{}^{L-l+1}\right)}{2} \right)$$

$$-\sum_{i=1}^{4}\sum_{j=1}^{9}\frac{1}{\ln 36}\omega_j^* b'_{ij}\ln\omega_j^* b'_{ij}$$

$$st.\begin{cases} \sum_{j=1}^{n}\omega_j^* = 1; 0.05 \le \omega_j^* \le 0.5; \\ 0.1 \le \omega_2^*; 0.1 \le \omega_5^*; 0.1 \le \omega_6^*; \\ 0.1 \le \omega_7^*; 0.2 \le \omega_8^*; \\ i = 1,2,3,4;\ j = 1,2,3\ldots 9 \end{cases}$$

We can derive the attribute weights: $\omega_1^* = 0.05$, $\omega_2^* = 0.3$, $\omega_3^* = 0.05$, $\omega_4^* = 0.05$, $\omega_5^* = 0.1$, $\omega_6^* = 0.1$, $\omega_7^* = 0.1$, $\omega_8^* = 0.2$, $\omega_9^* = 0.05$.

Step 4: Determine the deviation between each pair of alternatives over different attributes.

1) For the subjective decision-making matrix:

According to Eq. (12), we calculate the subjective deviation between each pair of alternatives over each attribute. The results are shown in Table 2.

Table 2. Subjective deviations among alternatives over each attribute

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
$d_j^s(A_1,A_2)$	–0.259	0.155	–0.769	–0.535	–0.236	–0.262	–0.348	0.319	–0.015
$d_j^s(A_1,A_3)$	0.031	–0.382	–0.357	–0.123	–0.247	–0.424	0.195	0.553	0.13
$d_j^s(A_1,A_4)$	0.106	–0.371	–0.265	–0.065	0.137	–0.329	–0.162	0.154	0.13
$d_j^s(A_2,A_1)$	0.259	–0.155	0.769	0.535	0.236	0.262	0.348	–0.319	0.015
$d_j^s(A_2,A_3)$	0.276	–0.537	0.412	0.412	–0.012	–0.193	0.543	0.234	0.132
$d_j^s(A_2,A_4)$	0.365	–0.525	0.504	0.504	0.372	–0.068	0.186	–0.165	0.132
$d_j^s(A_3,A_1)$	–0.031	0.382	0.357	0.123	0.247	0.454	–0.195	–0.553	–0.13
$d_j^s(A_3,A_2)$	–0.276	0.537	–0.412	–0.412	0.012	0.193	–0.543	–0.234	–0.132
$d_j^s(A_3,A_4)$	0.089	0.012	0.092	0.092	0.384	0.125	–0.357	–0.399	0
$d_j^s(A_4,A_1)$	–0.106	0.371	0.265	0.065	–0.137	0.329	0.162	–0.154	–0.13
$d_j^s(A_4,A_2)$	–0.365	0.525	–0.504	–0.504	–0.372	0.068	–0.186	0.165	–0.132
$d_j^s(A_4,A_3)$	–0.089	–0.012	–0.092	–0.092	–0.384	–0.125	0.357	0.399	0
q_j^s	0	0	0	0	0	0	0	0	0
p_j^s	0.25	0.5	0.35	0.5	0.35	0.4	0.4	0.4	0.13

2) For the objective normalized matrix:

According to Eq. (13), we calculate the objective deviation between each pair of alternatives over each attribute and in this case "The average time for project to be executed (month) c_7" is the cost attribute. The results are shown in Table 3.

Table 3. Objective deviations among alternatives over each attribute

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
$d_j^O(A_1,A_2)$	–0.075	0.1418	–0.5	–0.258	–0.124	–0.259	–0.048	0.019	–0.275
$d_j^O(A_1,A_3)$	0.008	–0.121	–0.12	–0.032	–0.409	–0.351	0.067	0.052	0.2
$d_j^O(A_1,A_4)$	0.123	0.095	–0.083	–0.065	0.012	–0.145	–0.046	0.018	0.175
$d_j^O(A_2,A_1)$	0.075	–0.142	0.5	0.258	0.124	0.259	0.048	–0.019	0.275
$d_j^O(A_2,A_3)$	0.083	–0.263	0.38	0.226	–0.285	–0.092	0.114	0.033	0.475

End of Table 3

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
$d_j^O(A_2, A_4)$	0.198	–0.237	0.417	0.194	0.136	0.114	0.002	–0.002	0.45
$d_j^O(A_3, A_1)$	–0.008	0.121	0.12	0.032	0.409	0.351	–0.067	–0.052	–0.2
$d_j^O(A_3, A_2)$	–0.083	0.263	–0.38	–0.226	0.285	0.092	–0.114	–0.033	–0.475
$d_j^O(A_3, A_4)$	0.116	0.026	0.037	–0.323	0.421	0.206	–0.112	–0.035	–0.025
$d_j^O(A_4, A_1)$	–0.123	0.095	0.083	0.065	–0.012	0.145	0.046	–0.018	–0.175
$d_j^O(A_4, A_2)$	–0.198	0.024	–0.417	–0.194	–0.136	–0.114	–0.002	0.002	–0.45
$d_j^O(A_4, A_3)$	–0.116	–0.026	–0.037	0.032	–0.421	–0.206	0.112	0.035	0.025
q_j^o	0	0	0	0	0	0	0	0	0
p_j^o	0.1	0.12	0.4	0.2	0.35	0.25	0.12	0.1	0.5

where q_j^s and q_j^o are the indifference thresholds under the subjective and objective information respectively. If the deviation between two alternatives over some attributes is bigger than the value of the indifference threshold, then there will be preference between the two alternatives. Hence, we usually let it be zero, which means that if there is any tiny difference between two cities, then there would exist preference between them. p_j^s and p_j^o are the strict thresholds under the subjective and objective information, respectively. If the deviation between two alternatives over some attributes is bigger than the value of the strict threshold, then the preference between the two alternatives is 1. In other words, the smaller the strict threshold, the more sensitive for forming the strict preference between two cities (Brans & Mareschal, 2005).

Step 5: Convert the deviations into the preferences

After the subjective deviation and the thresholds have been determined, we can derive the comprehensive preference relation Eq. (14) – Eq. (16), where $\alpha = \beta = 0.5$ (where α and β are the importance indexes for the subjective preferences and the objective preferences which are decided by the decision makers, and $\alpha + \beta = 1$). The following Table 4 shows the comprehensive preferences relation among alternatives over every attribute.

Table 4. Comprehensive preference relation among alternatives over each attribute

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
$p_j(A_1, A_2)$	0	0.655	0	0	0	0	0	0.494	0
$p_j(A_1, A_3)$	0.102	0	0	0	0	0	0.523	0.76	0.7
$p_j(A_1, A_4)$	0.712	0.396	0	0	0.213	0	0	0.283	0.675

End of Table 4

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
$p_j(A_2,A_1)$	0.875	0	1	1	0.514	0.828	0.635	0	0.333
$p_j(A_2,A_3)$	0.915	0	0.975	0.912	0	0	0.975	0.399	0.975
$p_j(A_2,A_4)$	1	0	1	0.985	0.695	0.228	0.241	0	0.95
$p_j(A_3,A_1)$	0	0.882	0.65	0.203	0.853	1	0	0	0
$p_j(A_3,A_2)$	0	1	0	0	0.424	0.426	0	0	0
$p_j(A_3,A_4)$	0.678	0.121	0.178	0.092	1	0.569	0	0	0
$p_j(A_4,A_1)$	0	0.767	0.483	0.228	0	0.702	0.394	0	0
$p_j(A_4,A_2)$	0	0.6	0	0	0	0.085	0	0.217	0
$p_j(A_4,A_3)$	0	0	0	0.08	0	0	0.913	0.674	0.025

Step 6: Calculate the overall comprehensive preference index between each pair of alternatives

We use Eq. (17) to calculate the comprehensive preference index: $\pi(A_1,A_2)=0.295$, $\pi(A_1,A_3)=0.244, \pi(A_1,A_4)=0.266, \pi(A_2,A_1)=0.358, \pi(A_2,A_3)=0.366$, $\pi(A_2,A_4)=0.313$, $\pi(A_3,A_1)=0.493, \pi(A_3,A_2)=0.385, \pi(A_3,A_4)=0.214$, $\pi(A_4,A_1)=0.375, \pi(A_4,A_2)=0.232$, $\pi(A_4,A_3)=0.231$.

Step 7: Calculate the positive outranking flow and the negative outranking flow

After calculating the comprehensive preference indexes for each pair of alternatives, we can derive the positive outranking flow and the negative outranking flow of each alternative. The results are:

$\phi^+(A_1)=0.269$, $\phi^+(A_2)=0.346$, $\phi^+(A_3)=0.373$, $\phi^+(A_4)=0.28$, $\phi^-(A_1)=0.459$, $\phi^-(A_2)=0.304$, $\phi^-(A_3)=0.536$, $\phi^-(A_4)=0.517$.

Step 8: Drive the net flow

Based on the positive outranking flow and the negative outranking flow, we can calculate the net flow according to Eq. (20). The results are: $\phi(A_1)=-0.19$, $\phi(A_2)=0.042$, $\phi(A_3)=-0.163$, $\phi(A_4)=-0.237$.

Step 9: Derive the ranking result

According to the results, we get the ranking: $A_2 \succ A_3 \succ A_1 \succ A_4$. We can find that the city Yibin is the best one. In other words, for the government, the advancement in Yibin is the best one, and for social capitals, Yibin is worthy to invest based on the basic situation of the PPP project's advancement.

5.2 Discussions

In this subsection, we analyze the change of the ranking results, when the uncertain parameters α and β change. Then, we compare the DHHFL-PROMETHEE-S&O method with the DHHFL-MULTIMOORA method.

5.2.1 Sensitivity analysis

In this method, there are two parameters α and β, which are not certain. Different values of α and β represent the degrees of the evaluators' preferences to the subjective and objective information. Hence, the values of these two parameters may affect the ranking result to some extent. When we give α and β different values between 0 to 1, the positive outranking flow, the negative outranking flow, the net flow and ranking results can be seen in Table 5.

Table 5. Sensitivity analysis

		A_1	A_2	A_3	A_4	Ranking results
$\alpha = 0$	ϕ^+	0.289	0.323	0.414	0.199	$A_2 \succ A_3 \succ A_1 \succ A_4$
$\beta = 1$	ϕ^-	0.49	0.273	0.445	0.496	
ϕ		−0.201	0.05	−0.031	−0.297	
$\alpha = 0.1$	ϕ^+	0.285	0.327	0.406	0.215	$A_2 \succ A_3 \succ A_1 \succ A_4$
$\beta = 0.9$	ϕ^-	0.484	0.28	0.463	0.5	
ϕ		−0.199	0.047	−0.057	−0.285	
$\alpha = 0.2$	ϕ^+	0.281	0.332	0.398	0.231	$A_2 \succ A_3 \succ A_1 \succ A_4$
$\beta = 0.8$	ϕ^-	0.478	0.286	0.481	0.505	
ϕ		−0.197	0.046	−0.083	−0.274	
$\alpha = 0.3$	ϕ^+	0.277	0.336	0.389	0.247	$A_2 \succ A_3 \succ A_1 \succ A_4$
$\beta = 0.7$	ϕ^-	0.471	0.292	0.499	0.509	
ϕ		−0.194	0.045	−0.11	−0.261	
$\alpha = 0.4$	ϕ^+	0.273	0.341	0.381	0.263	$A_2 \succ A_3 \succ A_1 \succ A_4$
$\beta = 0.6$	ϕ^-	0.465	0.298	0.518	0.513	
ϕ		−0.192	0.043	−0.137	−0.25	
$\alpha = 0.5$	ϕ^+	0.269	0.346	0.373	0.28	$A_2 \succ A_3 \succ A_1 \succ A_4$
$\beta = 0.5$	ϕ^-	0.459	0.304	0.536	0.517	
ϕ		−0.19	0.042	−0.163	−0.237	
$\alpha = 0.6$	ϕ^+	0.264	0.35	0.364	0.295	$A_2 \succ A_1 \succ A_3 \succ A_4$
$\beta = 0.4$	ϕ^-	0.452	0.31	0.554	0.521	
ϕ		−0.188	0.04	−0.189	−0.226	
$\alpha = 0.7$	ϕ^+	0.26	0.355	0.356	0.311	$A_2 \succ A_1 \succ A_4 \succ A_3$
$\beta = 0.3$	ϕ^-	0.446	0.316	0.572	0.525	
ϕ		−0.186	0.039	−0.216	−0.214	

End of Table 5

		A_1	A_2	A_3	A_4	Ranking results
$\alpha = 0.8$	ϕ^+	0.256	0.36	0.348	0.327	$A_2 \succ A_1 \succ A_4 \succ A_3$
$\beta = 0.2$	ϕ^-	0.439	0.322	0.59	0.529	
ϕ		–0.183	0.038	–0.242	–0.202	
$\alpha = 0.9$	ϕ^+	0.252	0.364	0.339	0.343	$A_2 \succ A_1 \succ A_4 \succ A_3$
$\beta = 0.1$	ϕ^-	0.433	0.329	0.608	0.533	
ϕ		–0.181	0.035	–0.269	–0.19	
$\alpha = 1$	ϕ^+	0.248	0.369	0.331	0.359	$A_2 \succ A_4 \succ A_1 \succ A_3$
$\beta = 0$	ϕ^-	0.426	0.335	0.626	0.537	
ϕ		–0.178	0.034	–0.295	–0.178	

We can easily find that even though the values of α and β are changing, A_2 is still the best. However, with the values of α becoming larger and larger, the values of β become smaller and smaller, which means that the subjective information becomes more important, the net flow of A_2 is getting smaller.

Meanwhile, we can find that the ranking position of A_3 is getting more and more backward. When $\alpha = 0$ and $\beta = 1$, which means that we just use the objective information to evaluate the four cities, A_3 ranks second. When $\alpha = 0.6$ and $\beta = 0.4$, the ranking of A_3 becomes third. When $\alpha = 0.6$, $\beta = 0.4$, the ranking of A_3 starts becoming fourth.

Figures 3a–3c present the change trends of four alternatives' positive outranking flow, negative outranking flow and net flow when the value of α changes. It shows us the reason of the change in the ranking results as the value of α changes.

For the alternative A_1, as α changes from 0 to 1, the positive outranking flow and the negative outranking flow decrease, but the trend of the former is smaller than the latter. So, the trend of the net flow increases; For the alternative A_2, as α changes from 0 to 1, the positive outranking flow and the negative outranking flow increase, but the increasing trend of the former is smaller than the latter. So, the trend of the net flow decreases; For the alternative A_3, as α changes from 0 to 1, the positive outranking flow decreases, and the negative outranking flow increases. So, the trend of the net flow presents a dramatic decrease; For the alternative A_4, as α changes from 0 to 1, the positive outranking flow and the negative outranking flow are increasing, but the increasing trend of the former is bigger than the latter. So, there is an increase trend of the net flow.

From the above sensitive analysis, we can see that as the weight of subjective information becomes larger, the ranking position of the alternative A_3 goes back, and the ranking positions of the alternative A_4 and the alternative A_1 go forward. Figure 3 can present the reasons from the point of view of the difference between the subjective preference and the objective preference.

Figure 4 mainly presents the objective preferences and the subjective preferences for different alternatives over different attributes. For the alternative A_3, we can see that the value of $SPj\left(A3, At\right)$ is smaller than the value of $OPj\left(A3, At\right)$ over the attributes c_1, c_2, c_5, c_6, and the value of $SPj\left(At, A3\right)$ is larger that the value of $OPj\left(At, A3\right)$ over the attributes c_8 and c_9.

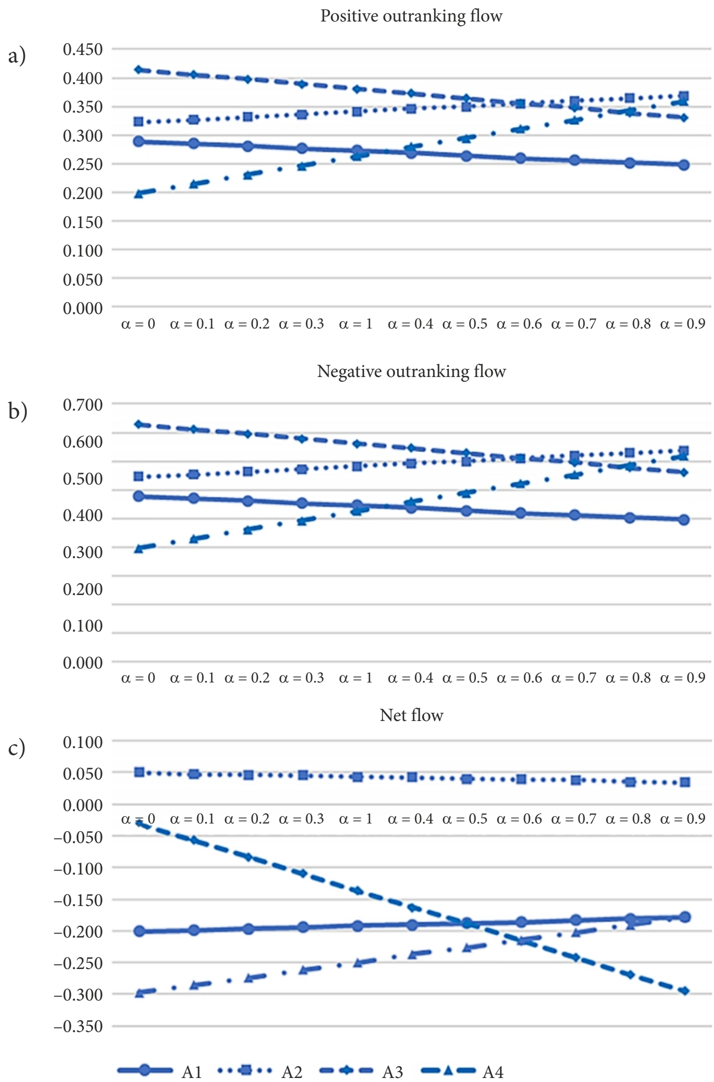


Figure. 3. Sensitivity of positive outranking flow, negative outranking flow and net flow

In fact, bacause of the complexity of the environment, it is difficult for us to make out which is better based on objective data only. It needs the experts' evaluation which combines their experiences. If we just consider the objective information, then the ranking of the alternative A_3 should be superior to the alternative A_4 and the alternative A_1, but when we consider the subjective inforation, it reduces the effects of the objective information. The subjective information from experts can supplement with their experiences for the objective information.

The same to the alternative A_4 and the alternative A_1, there are some differences between the subjective preferences and the objective preferences. The values of $SPj\left(Ai,At\right)$ are bigger than the values of $OPj\left(Ai,At\right)$ and the values of $SPj\left(At,Ai\right)$ are smaller than $OPj\left(At,Ai\right)$.

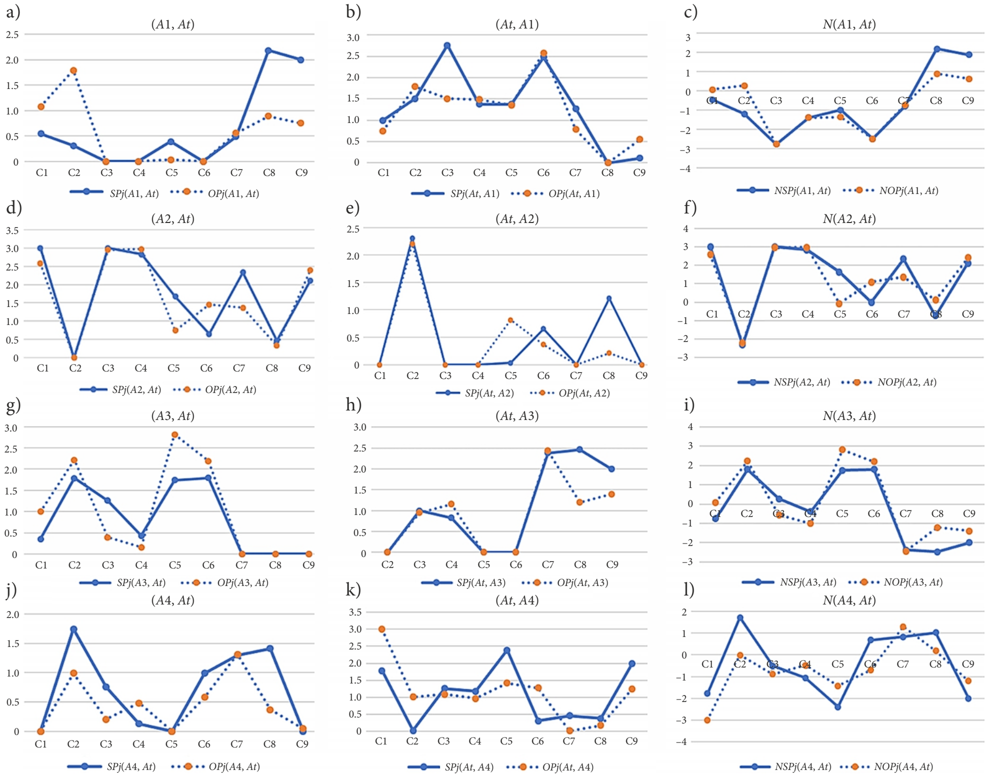


Figure 4. The comparision between the subjective preference and the objective preference

where $SPj(Ai, At)$ denotes the degrees that the alternative A_i is superior to the other alternatives over the j_{th} attribute based on the subjective information. $SPj(At, Ai)$ denotes the degree that the alternative A_i is inferior to the other alternatives over the j_{th} attribute based on the subjective information. $OPj(Ai, At)$ denotes the degree that the alternative A_i is superior to the other alternatives over the j_{th} attribute based on the objective information. $OPj(At, Ai)$ denotes the degree that the alternative A_i is inferior to the other alternatives over the j_{th} attribute based on the objective information. $NSPj(Ai, At)$ denotes the net degree that the alternative A_i is superior to the other alternatives over the j_{th} attribute based on the subjective information, and $NSPj(Ai, At) = SPj(Ai, At) - SPj(At, Ai)$. $NOPj(Ai, At)$ denotes the net degree that the alternative A_i $(i = 1,2,3,4)$ is superior to the other alternatives over the j_{th} attribute, and $NOPj(Ai, At) = OPj(Ai, At) - OPj(At, Ai)$.

Hence, the values of $NSPj(Ai, At)$ are larger than the values of $NOPj(Ai, At)$. Hence, as the weight of the subjective information becomes larger, the ranking positions of the alternative A_4 and the alternative A_1 goes forward.

Figure 5 presents the values of the net flows for the four alternatives. We can find that the ranking is $A_2 \succ A_3 \succ A_1 \succ A_4$ when we only consider the objective information, and the ranking is $A_2 \succ A_4 \succ A_1 \succ A_3$ when we only consider the subjective information. Taking the alternative A_3 as an example, the net flow of the alternative A_3 is better than A_1 and A_4 if we just consider the objective information. But, even if A_3 presents better preformance on the

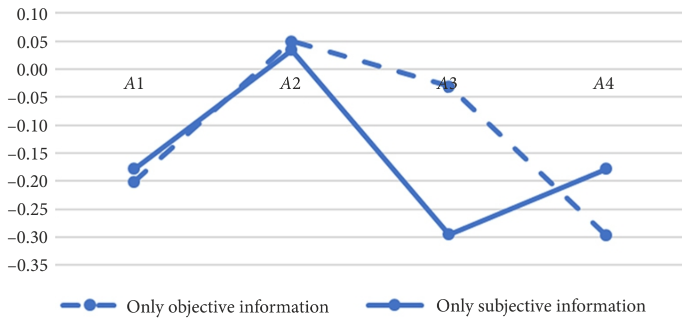


Figure 5. The comparision on the net flow

objective inormation, in fact, when the experts evaluate the preformance of A_3 combining their experiences, A_3 is not better than A_1 and A_4. Hence, when the importance degree of the subjective information increases, the values of the net flow gets closer to the values that consider the subjective information only. As Table 5 shows, the ranking positions of the four alternatives change during the process of combining the subjective and objective information.

From the sensitivity analysis, we can see that the method proposed in this paper is robust relatively, and the subjective information and the objective information are complementary. The subjective information depicted by the DHHFLTS can intergrate the experts' experiences to the evaluation process. And the objective information can help the subjective information which is mainly presented in fuzzy form tell the differences among different alternatives better. Hence, the combination of the subjective information and the objective information can use the information more efficiently.

5.2.2 Comparative analysis

The traditional decision-making approaches usually consider the subjective information only. So, we make some comparisons between the DHHFL-PROMETHEE-S&O and the DHHFL-MULTIMOORA method proposed by Gou et al. (2017a) before.

The DHHFL-MULTIMOORA method mainly considers three aspects, i.e., the DHHFL ratio system (DHHFLRS), the DHHFL reference point (DHHFLRP) and the DHHFL full multiplicative form (DHHFLMF). The essential steps and results are shown as follows:

1. DHHFLRS

Firstly, we transform each DHHFLE into the normalized forms based on the expected values. The formula is shown as follows:

$$h'_{S_{O_{ij}}} = \bar{h}_{S_{O_{ij}}} \Big/ \sum_{i=1}^{n} \bar{h}_{S_{O_{ij}}}, \quad i = 1,2,\ldots m; j = 1,2,\ldots n. \tag{21}$$

Then, we calculate the summarizing ratio Φ_i, shown as:

$$\Phi_i = \sum_{j=1}^{\theta} h'_{S_{O_{ij}}} - \sum_{j=\theta+1}^{n} h'_{S_{O_{ij}}}, \quad i = 1,2,\ldots m; j = 1,2,\ldots n, \tag{22}$$

where θ denotes the number of the benefit attributes, $n-\theta$ denotes the number of the cost attributes. Φ_i denotes the best performance value of the i-th alternative. The larger the value of Φ_i, the better the attribute. In this case, we use the same linguistic scale to describe the situation whether it is good or bad, so there are no cost attributes in the linguistic evaluation.

The results of this aspect are: $\Phi_i(A_1)=1.893$, $\Phi_i(A_2)=2.923$, $\Phi_i(A_3)=2.102$, and $\Phi_i(A_4)=2.083$.

2. DHHFLRP

Firstly, we determine the maximal objective reference points $\mathrm{M}_j(j=1,2,\ldots,n)$. It is determined by Eq. (23):

$$\mathrm{M}_j=\begin{cases}\max\limits_i\left\{h_{S_{O_{ij}}}\right\}, \text{if the attribute is benefit attribute}\\ \min\limits_i\left\{h_{S_{O_{ij}}}\right\}, \text{if the attribute is cost attribute}\end{cases}, \; i=1,2,...,m;\; j=1,2,...,n. \tag{23}$$

Then we calculate the distances between each DHHFLE and M_j using the Euclidean distance, as Eq. (24) shows. Finally, we rank the alternatives according to the Min–Max metric, shown as follows:

$$\min_i\left(\max_j\left\{D\left(h_{S_{O_{ij}}}, \mathrm{M}_j\right)\right\}\right). \tag{24}$$

The results are shown in Table 6.

Table 6. The results of the DHHFLRP

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	$\max\limits_j\left\{D\left(h_{S_{O_{ij}}}, \mathrm{M}_j\right)\right\}$
A_1	0.259	0.384	0.769	0.54	0.248	0.461	0.349	0	0.015	0.769
A_2	0	0.538	0	0	0.016	0.21	0	0.32	0	0.538
A_3	0.278	0	0.412	0.412	0	0	0.545	0.553	0.133	0.553
A_4	0.368	0.016	0.504	0.504	0.387	0.147	0.193	0.154	0.133	0.504

3. DHHFLMF

For the DHHFLMF, U_i presents the overall utility of each alternative. The calculation method is shown as follows:

$$U_i=\frac{B_i}{C_i}, i=1,2,\ldots,m, \tag{25}$$

where B_i denotes the results of $\prod_{j=1}^{\theta}\overline{h}_{S_{O_{ij}}}$ for the i_{th} alternative, C_i denotes the results of $\prod_{j=\theta+1}^{n}\overline{h}_{S_{O_{ij}}}$ for the i_{th} alternative. The larger the value of U_i, the better the alternative. The results of the DHHFLMF are shown in Table 7.

At last, we rank the alternatives according to these three aspects. The results are shown in Table 8.

Table 7. The results of the DHHFLMF

	B_i	C_i	U_i
A_1	6.79×10^{-5}	/	6.79×10^{-5}
A_2	5.87×10^{-3}	/	5.87×10^{-3}
A_3	2.41×10^{-4}	/	2.41×10^{-4}
A_4	3.22×10^{-4}	/	3.22×10^{-4}

Table 8. The ranking results of the DHHFL–MULTIMOORA method

	DHHFLRS	DHHFLRP	DHHFLMF
Rankings	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_4 \succ A_2 \succ A_3 \succ A_1$	$A_2 \succ A_4 \succ A_3 \succ A_1$

The three ranking results based on the DHHFL- MULTIMOORA method are not the same. In this case, we can choose the best alternative A_2, which is the same with the result of the DHHFL-PROMETHEE-S&O method. Hence, we can deem that the DHHFL-PROMETHEE-S&O method is credible.

There are three rankings, and they are not the same, so it is not easy to ascertain the ranking results among different alternatives. However, in the DHHFL-PROMETHEE-S&O approach, there is just one ranking result, which shows the assessment results clearly.

What's more, in the DHHFL-MULTIMOORA method, it does not consider the attribute weights, which cannot classify the different importance degrees among different attributes. However, in fact, the attribute weights cannot be ignored in the decision–making process. In the DHHFL- PROMETHEE-S&O method, we derive the attribute weights by combining the subjective and objective information, which is better than the DHHFL-MULTIMOORA method.

From all the discussions above, we can find that the method in this paper uses the information more comprehensively by combining the subjective information and the objective information. In addition, the objective information and the subjective information are complementary. The subjective information can be used to evaluate the advancement of PPP by integrating the experience of experts, meanwhile, sometimes the objective information can be more sensitive to help the subjective information underline the differences among them. Besides, when we give the different values for the parameters α and β, the best alternative does not change, from which we can see that the method is robust relatively. Through the comparison between the DHHFL-MULTIMOORA method that proposed by Gou et al. (2017a), we can see that the method proposed in this paper is more convenient for us to find the best alternative by one ranking result and we consider the attribute weight. However, there are still some weaknesses of the method. Firstly, the values of the parameters α and β that would affect the final ranking to some extent are not confirmed, which adds uncertain parameters into the method. Then, the method needs both the subjective information and the objective information, which is more complicated than the methods that only consider one kind of information.

Conclusions

During recent years, the Chinese government pays more and more attention on the development and advancement of the PPP in all parts of the country. Meanwhile, social capitals become increasingly active to take part in it. The reasonable evaluation of the PPP's advancement can provide real-time information for the government and help social capitals make a proper decision about investment.

In this paper, we first design an assessment index system to help the government evaluate the PPP's advancement in China. Then, for the purpose of evaluating completely, we combine the subjective evaluation and the objective information and propose a programming model with the DHHFL information to derive the attribute weights. Since the DHHFLTS can describe linguistic information more precisely, we develop the DHHFL-PROMETHEE-S&O method to process the subjective and objective information and get the ranking of alternatives. Then, we apply this method to evaluate the advancement of the PPP in Deyang, Yibin, Xi'an and Hanzhong, and verify the rationality and efficiency of the evaluation method by sensitivity analysis and comparison analysis respectively. We find that the method is robust relatively, and the subjective information can be well complemented with the objective information. From the assessment results, we can find that the PPP's advancement in Yibin is the best both in the subjective assessments and the objective assessments. But for Xi'an and Hanzhong, the ranking results are changed when we give the subjective information and the objective information different importance weights. Although the performance of Xi'an is better than Hanzhong in some attributes based on the objective information, but when the experts combine their experiences, they may think that the performance of Xi'an is not better than Hanzhong. Hence, we need to consider both the objective data and the subjective information to decide which is better.

In this paper, we combine the subjective information and the objective information in the evaluation process to make the decision–making method more reasonable. While, there are still some research limitations. From the application perspective, since the research topic is new and there are less relevant references, then the index system is not perfect. In addition, because of the lack of the objective data, the number of the city in the case study is limited. From the theory perspective, the approach proposed in this paper is more complicated than some other approaches. Besides, the weights of the subjective information and the objective information are not confirmed and there is no mature method to confirm them, which improve the uncertainty of the result.

In the future, we can try to establish the index system from more detailed perspectives, such as the perspective of the investor, the perspective of the government, etc., to make the whole process more reasonable and practical. Then, considering the lack of the data, we may find some new methods to cope with evaluation problems in the incomplete information environment, which is closer to the practice. To perfect the whole approach, we may try to employ the statistic theory to confirm the values of the parameters α and β, which is more reasonable and scientific. For the way to combine the subjective information and the objective information, we may use consensus theory to adjust them and derive the ranking results by making them "reach consensus".

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References

Amadi, C., & Carrillo, P. (2018). Stakeholder management in PPP projects: external stakeholders' perspective. *Built Environment Project and Asset Management, 8*(4), 403-414. https://doi.org/10.1108/BEPAM-02-2018-0048

Babu, A. J. G., & Suresh, N. (1996). Project management with time, cost, and quality considerations. *European Journal of Operational Research, 88*(2), 320-327. https://doi.org/10.1016/0377-2217(94)00202-9

Brans, J. P., & Mareschal, B. (2005). *PROMETHEE methods.* New York: Springer. https://doi.org/10.1007/0-387-23081-5_5

Brans, J. P., Vincke, P., & Mareschal, B. (1986). How to select and how to rank projects: The PROMETHEE method. *European Journal of Operational Research, 24*(2), 228-238. https://doi.org/10.1016/0377-2217(86)90044-5

Brinkerhoff, D. W., & Brinkerhoff, J. M. (2011). Public-private partnerships: Perspectives on purposes, publicness, and good governance. *Public Administration and Development, 31*(1), 2-14. https://doi.org/10.1002/pad.584

British HM Treasury. (2017). *Collection Public Private Partnership*. Retrieved from https://www.gov.uk/government/collections/Public-Private-Partnershipps

Cao, Q. W., & Wu, J. (2009, September). Research on hybrid multiple attribute decision-making problems based on subjective weight and objective weight integrated. In *International Conference on Management and Service Science*. Wuhan, China: IEEE Press. https://doi.org/10.1109/ICMSS.2009.5301130

Chen, F. (2008). *Study on interest coordination and distribution to stakeholders in PPP project*. Chongqing University.

Cheng, X. (2014). Research on target system of PPP project based on stakeholder satisfaction. *Project Management Technology, 12*(11), 72-76.

Choua, S. Y., & Shen, C. Y. (2008). A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes. *European Journal of Operational Research, 189*(1), 132-145. https://doi.org/10.1016/j.ejor.2007.05.006

Cruz, C. O., & Marque, R. C. (2014). Theoretical Considerations on Quantitative PPP Viability Analysis. *Journal of Management in Engineering, 30*(1), 122-126. https://doi.org/10.1061/(ASCE)ME.1943-5479.0000184

Cui, C. Y., Liu, Y., Hope, A., & Wang, J. P. (2018). Review of studies on the public-private partnerships (PPP) for infrastructure projects. *International Journal of Project Management, 36*(5), 773-794. https://doi.org/10.1016/j.ijproman.2018.03.004

Fan, J. L., & Ma, Y. L. (2002). Some new fuzzy entropy formulas. *Fuzzy Sets* & *Systems, 128*(2), 277-284. https://doi.org/10.1016/S0165-0114(01)00127-0

Fan, Z. P., & Pan, D. H. (1999). A subjective and objective synthetic method for multiple attribute decision making. *Systems Engineering, 13*(5), 397-404.

Finch, B. G. (2000). *Linguistic Terms and Concepts*. UK: Macmillan Education. https://doi.org/10.1007/978-1-349-27748-3

Freeman, R. E., & Phillips, R. A. (2010). *Stakeholder Theory*. UK: Cambridge University Press. https://doi.org/10.1017/CBO9780511815768

Gou, X. J., Liao, H. C., Xu, Z. S., & Herrera, F. (2017a). Double hierarchy hesitant fuzzy linguistic term set and MULTIMOORA method. *Information Fusion, 38*(C), 22-34. https://doi.org/10.1016/j.inffus.2017.02.008

Gou, X. J., Liao, H. C., Xu, Z. S., & Herrera, F. (2017b). *Some Distance and Similarity Measures of Double Hierarchy Hesitant Fuzzy Linguistic Term Sets* (Working Paper). School of Business, Sichuan University.

Guo, X. Z., & Xin, X. L. (2006). Some new generalized entropy formulas of fuzzy sets. *Journal of Northwest University, 36*(4), 529-532.

Halouani, N., Chabchoub, H., & Martel, J. M. (2009). PROMETHEE-MD-2T method for project selection. *European Journal of Operational Research, 195*(3), 841-849. https://doi.org/10.1016/j.ejor.2007.11.016

Herniter, J. D. (1973). An entropy model of brand purchase behavior. *Journal of Marketing Research, 10*(4), 361-375. https://doi.org/10.2307/3149383

Jayant, A., & Sahrma, J. (2018). A comprehensive literature review of MCDM techniques ELECTRE, PROMETHEE, VIKOR and TOPSIS applications in business competitive environment. *International Journal of Current Research, 10*(2), 65461-65477.

Jia, C. Y. (2015). *Research on the Success Criteria of PPP Project*. China: Chongqing University Press.

Jiao, X. P. (2017). *Director of the PPP center of the ministry of finance: the ministry of finance has insisted on the reform of PPP*. Retrieved from http://www.cpppc.org/zh/pppyw/4740.jhtml

Kong, F. H., & Liu, Y. (2007). A new fuzzy MADM algorithm based on subjective and objective integrated weights. *International Journal of Information and Systems Sciences, 1*(3), 1-6. https://doi.org/10.1109/ICSSSM.2007.4280142

Kwak, Y. H., Chih, Y. Y., & Ibbs, C. W. (2009). Towards a comprehensive understanding of Public Private Partnerships for infrastructure development. *California Management Review, 51*(2), 51-78. https://doi.org/10.2307/41166480

Li, B., Ren, J. L., Wang, C. Z., Wei, C. Y., & Li, J. (2013, July). A multi-attribute decision method combining subjective and objective information. In *International Conference on Management Science and Engineering*. Heilongjiang, China: IEEE Press. https://doi.org/10.1109/ICMSE.2013.6586364

Li, L., Li, Z. F., Jiang, L., Wu, G. D., & Cheng, D. J. (2018). Enhanced cooperation among stakeholders in PPP mega-infrastructure projects: A China study. *Sustainability, 10*(8), 2791. https://doi.org/10.3390/su10082791

Li, W. X., & Li, B. Y. (2010). An extension of the PROMETHEE II method based on generalized fuzzy numbers. *Expert Systems with Applications, 37*(7), 5314-5319. https://doi.org/10.1016/j.eswa.2010.01.004

Li, Y., & Zhao, L. (2015). An empirical study on building risk evaluation system of PPP project under the background of new urbanization – A case study of Shanghai Xinzhuang CCHP project. *Reform of Economic System, 5*, 17-23.

Liang, X., Wei, C. P., & Cheng, X. Y. (2016, November). Entropy measures for extended hesitant fuzzy linguistic term sets and their applications, In *International Symposium on Knowledge and Systems Sciences.* Kobe, Japan: Springer. https://doi.org/10.1007/978-981-10-2857-1_13

Liang, Y. H., & Jia, H. J. (2018). Key success indicators for PPP projects: evidence from Hong Kong. *Advances in Civil Engineering, 2018*(6), 1-10. https://doi.org/10.1155/2018/9576496

Liu, X. (1992). Entropy, distance measure and similarity measure of fuzzy sets and their relations. *Fuzzy Sets & Systems, 52*(3), 305-318. https://doi.org/10.1016/0165-0114(92)90239-Z

Ma, J., Fan, Z. P., & Huang, L. H. (1999). A subjective and objective integrated approach to determine attribute weights. *European Journal of Operational Research, 112*(2), 397-404. https://doi.org/10.1016/S0377-2217(98)00141-6

Mareschal, B., Brans, J. P., & Vincke, P. (1984). PROMETHEE: a new family of outranking methods in multicriteria analysis. In J. P. Brans (Ed.), *Operational Research '84* (pp. 477-490). Amsterdam, North-Holland: Elsevier.

Ministry of Finance of the People's Republic of China. (2014). *Notification of operation guidelines on the operation of the Public Private Partnership mode*. Retrieved from http://www.cpppc.org/zh/bmwj/2669.jhtml

Ministry of Finance of the People's Republic of China. (2015a). *Notice of the Ministry of Finance on the implementation the complement policy of PPP project*. Retrieved from http://www.cpppc.org/zh/bmwj/2643.jhtml/

Ministry of Finance of the People's Republic of China. (2017). *National PPP Integrated Information Platform Project Library*. Retrieved from http://www.cpppc.org:8086/pppcentral/map/toPPPMap.do

Ministry of Finance of the People's Republic of China. (2015b). *Notice of the ministry of finance on the implementation of the demonstration PPP project*. Retrieved from http://www.cpppc.org/zh/gzdt/1905.jhtml

Niu, X. F. (2008). *PPP risk indicator system and fuzzy comprehensive evaluation*. Chongqing University Press.

Palevicius, V., Podviezko, A., Sivilevičius, H., & Prentkovskis, O. (2018). Decision-aiding evaluation of public infrastructure for electric vehicles in cities and resorts of Lithuania. *Sustainability, 10*(4), 904. https://doi.org/10.3390/su10040904

Podvezko, V., & Podviezko, A. (2010). Dependence of multi-criteria evaluation result on choice of preference functions and their parameters. *Technological and Economic Development of Economy, 16*(1), 143-158. https://doi.org/10.3846/tede.2010.09

Rao, R. V., & Patel, B. K. (2010). A subjective and objective integrated multiple attribute decision making method for material selection. *Materials & Design, 31*(10), 4738-4747. https://doi.org/10.1016/j.matdes.2010.05.014

Saad, R., Ahmad, M. Z., Abu, M. S., & Jusoh, M. S. (2014). Hamming distance method with subjective and objective weights for personnel selection. *The Scientific World Journal, 2014*. Article ID 280146. https://doi.org/10.1155/2014/280146

Sennaroglu, B., & Celebi, G. V. (2018). A military airport location selection by AHP integrated PROMETHEE and VIKOR methods. *Transportation Research Part D Transport & Environment*, *59*, 160-173. https://doi.org/10.1016/j.trd.2017.12.022

Shannon, C. E., & Weaver, W. (1949*). Mathematical theory of communication*. US: University of Illinois.

Shih, H. S., Chang, Y. T., & Cheng, C. P. (2016). A generalized PROMETHEE III with risk preferences on losses and gains. *International Journal of Information & Management Sciences, 27*(2), 117-127.

Sioshansi, F. P. (1983). Subjective evaluation using expert judgment: An application. *IEEE Transactions on Systems Man & Cybernetics, 13*(3), 391-397. https://doi.org/10.1109/TSMC.1983.6313170

Tang, L. Y., Tang, Q. E., & Cheng, W. L. (2010). A review of studies on Public-Private Partnership projects in the construction industry. *International Journal of Project Management, 28*(7), 683-694. https://doi.org/10.1016/j.ijproman.2009.11.009

The Word Bank Group. (2017). *PPP reference guide*. Retrieved from https://pppknowledgelab.org/guide/sections/1-introduction/

Wang, H. L. (2015). Preliminary research on urban PPP investment rating index. *Sub National Fiscal Research*, *8*, 9-14.

Wang, J., & Zhou, H. W. (2011). Subjective and objective weight combined decision-making method based on Kullback divergence. *Journal of University of Jinan, 25*(4), 414-417.

Wang, Z. X., & Li, Q. X. (2006). An approach to integrate the final weights based on the subjective and objective weights. *Communication on Applied Mathematics and Computation, 20*(1), 87-92.

Xia, M. M., & Xu, Z. S. (2012). Entropy/cross entropy-based group decision making under intuitionistic fuzzy environment. *Information Fusion, 13*(1), 31-47. https://doi.org/10.1016/j.inffus.2010.12.001

Xia, M. M., Zhang, R. T., & Badr, Y. (2017). Choquet-based multi-criteria decision making with objective and subjective information. *Journal of Intelligent & Fuzzy Systems, 30*(2), 773-781. https://doi.org/10.3233/IFS-151798

Xu, Y. J., & Da, Q. L. (2005). An objective and subjective synthetic approach to determine weights based on ideal-solution for multi-attribute decision making. *Industrial Engineering and Management*, *10*(4), 45-47.

Xu, Z. S., & Chen, J. (2007, December). A subjective and objective integrated method for MAGDM problems with multiple types of exact preference formats. In *International Conference on Intelligent Data Engineering and Automated Learning*. Birmingham, UK: Springer. https://doi.org/10.1007/978-3-540-77226-2_16

Xu, Z. S., & Xia, M. M. (2012). Hesitant fuzzy entropy and cross-entropy and their use in multiattribute decision-making. *International Journal of Intelligent Systems, 27*(9), 799-822. https://doi.org/10.1002/int.21548

Yager, R. R. (1995). Measures of entropy and fuzziness related to aggregation operators. *Information Sciences, 82*(3-4), 147-166. https://doi.org/10.1016/0020-0255(94)00030-F

Yager, R. R. (2016). Multicriteria decision making with ordinal/linguistic intuitionistic fuzzy sets for mobile apps. *IEEE Transactions on Fuzzy Systems, 24*(3), 590-599. https://doi.org/10.1109/TFUZZ.2015.2463740

Yano, Y. E., Sueyoshi, E., Shinohara, I., & Kato, T. (2003, December). Development of a recommendation system with multiple subjective evaluation process models. In *International Conference on Cyberworlds*. Singapore: IEEE Press. https://doi.org/10.1109/CYBER.2003.1253474

Yu, J. Q. (2013). *Research on benefit distribution among stakeholders of Public-Private Partnership project based on Shapley value*. Tianjin: Tianjin University.

Zadeh, L. A. A. (1968). Probability measures of fuzzy events. *Journal of Mathematical Analysis & Applications, 28*(2), 421-427. https://doi.org/10.1016/0022-247X(68)90078-4

Zhang, K. J., Kluck, C., & Achari, G. (2009). A comparative approach for ranking contaminated sites based on the risk assessment paradigm using fuzzy PROMETHEE. *Environmental Management, 44*(5), 952-967. https://doi.org/10.1007/s00267-009-9368-7

Zhu, B., Xu, Z. S., Zhang, R., & Hong, M. (2015). Generalized analytic network process. *European Journal of Operational Research, 244*(1), 277-288. https://doi.org/10.1016/j.ejor.2015.01.011