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THE TIME-COST TRADE-OFF ANALYSIS IN CONSTRUCTION PROJECT USING COMPUTER SIMULATION AND INTERACTIVE PROCEDURE

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Abstract. Several criteria must be considered while preparing the schedule of a construction project. The completion time and project cost are analyzed in most cases. Additionally, the risk related to the criteria has to be taken into account as well. Thus, project planning problem can be defined as a multicriteria decision problem under risk. In this paper, a project scheduling problem including time-cost trade-offs is analyzed. We assume that various resource allocations can be considered. A new technique based on computer simulation and interactive approach is proposed. In the first step, simulation experiments are performed to evaluate decision alternatives with respect to the criteria. An interactive technique INSDECM is employed for generating the final solution of the problem. The procedure uses stochastic dominance rules for comparing decision alternatives with respect to the criteria. A numerical example is presented to illustrate the applicability of the technique.

Keywords: project planning, multi-criteria decision aiding, time-cost trade-off, interactive approach, simulation.

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1. Introduction

As far as projects are concerned, one of their specific aspects is the current need to make decisions which will result in an unpredictable state in the future. The project's systematic nature requires each decision analysis which concerns selected objects in a project – system, taking into account all interactions with other objects and, eventually, the change of their parameters. Several objects may be characterized by multiple parameters which may influ-

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ence the attributes of the whole project on a different level. One of the examples is a link between the time required to complete a certain activity and to start/finish dates of the activities related to it by preceding/proceeding relations. The expansion of the scheduled time of an activity (especially when it is located on a critical path) results in a necessity of time compression in proceeding activities, which will ensure the scheduled project completion date. Such a compression, if possible, requires the utilization of additional resources and that results in project cost increase. The cost increase obviously causes negative and undesirable effects. It also can happen in the opposite direction – going beyond the budget results, at a certain stage, with a need to save in a further realization, which may cause delays in schedule. That is why, in most common cases, we have to deal with criteria conflicts when two objectives (time compression and cost cutting) are subjects to be reached. This kind of problem, referred to as time-cost trade-off, was recognized by Fulkerson (1961) and Kelley (1961) in the early 1960s, just after CPM and PERT network approaches, and since then it has been analyzed. The problem has generally been formulated on the basis of different time-cost relations, which could be linear or nonlinear, concave or convex, continuous or discrete, or hybrid. Linear continuous cases, built with linear and dynamic programming models are widely described; however, their practical usability is rather limited. The more in line with practice models with discrete cost-duration functions were proposed in the late 1970s by Harvey and Patterson (1979) and Hindelang and Muth (1979). These formulations make use of enumeration algorithms and dynamic programming approaches. However, the dynamic programming models are still popular. Recently, the latest results have been obtained in addition to the network decomposition support. Many of them were investigated by Akkan *et al.* (2005). There was also some approximation (Skutella 1998) and heuristic algorithms for larger problems. Different heuristical approaches were presented by Siemens (1971) and Moselhi and Deb (1993). Metaheuristics such as genetic algorithms were exploited in solving this problem by Feng *et al.* (1997) and Chua *et al.* (1997). Leu *et al.* (2001) introduced uncertainty issues in their approach by using fuzzy numbers to represent possible activity durations. This idea was formerly researched by several authors, for example Chanas and Kamburowski (1981) and Hapke and Słowiński (1996). Other approaches to quantify the uncertainty of durations were built by using the regular PERT technique, which assumes the beta distribution of activity durations; Monte Carlo simulation which was exploited in researches of Ang (1975), Elmaghraby (1977), and other techniques such as Goldratt's Critical Chain approach (1997). The interesting approach for the time-cost trade-offs in construction project was presented by Chen (2008), who used the resource matrices and elements of Earned Value method in his model, optimized in spreadsheet.

In the most general form of the completed project or milestone evaluation, its success measure is the deviations from schedule and budget. The uncertainty and risk, influencing all assumptions and system behaviour forecasts, cause the need to analyze the sensitivity of project schedule and budget and to insure the project against potential threats. Practical experience proves that reaction costs in the execution phase with expected loss are usually incommensurably higher than additional cost of risk and alternative analysis during the planning phase. That is why the main emphasis is put on the complexity and profundity of a priori analyses during the planning phase.

The decision alternative is an acceptable solution to the decision problem, different from other acceptable solutions. In project planning processes the decision alternative is usually referred to as mutually excluding project solutions, characterized by specific vectors of the evaluation criteria. Project alternatives may be both repetitions of similar solutions used in the past with its adjustment and actualization to the current problem and products of the team members' creativity. The generation of innovative solutions is an essential problem of project planning related, for instance, to its realization in an unusual environment or research and development issues.

In the chronology of decision making process the alternatives formulation proceeds just after defining the decision criteria. Such an order allows for taking into account eventual system changes with respect to multi-criteria optimization model.

Multicriteria techniques are widely used in project selection problems. The evaluation of each project is usually a multidimensional problem. On the one hand, financial analysis is very important; on the other, however, technical, social, and ecological factors are also taken into account. In recent years numerous procedures have been proposed for evaluating construction projects based on an established set of objectives. Moselhi (1993), Moselhi and Deb (1993), Wong *et al.* (2000) use the utility concept for solving project selection problems. They propose techniques for estimating single-criteria utility functions and aggregating them into a multi-attribute utility function. For a similar problem Nowak (2005) proposed a technique based on simulation model, stochastic dominance rules and a multicriteria aggregation procedure PROMETHEE II.

This paper proposes a new approach for a project scheduling problem including time-cost trade-offs. We assume that various resource allocations can be considered. Our technique is based on computer simulation and interactive approach. The paper is organized as follows: section 2 gives the problem formulation. In section 3 we present the methodology that we propose to solve the problem. Section 4 gives an example. The last section is the conclusion.

2. Problem formulation

In this paper we consider the project planning problem. Various resources can be used to complete project activities. Here we assume that only a finite number of resource allocations can be considered. For example, one, two or three workers can be employed to complete an activity. Thus, we face a discrete decision making problem, in which the decision alternatives are defined by the resource allocations.

The completion time depends on the resources allocated to the activity. We assume that for each activity and for each alternate resource allocation three completion time estimates are known: optimistic, most probable and pessimistic. We also suppose that the relations between the time and cost are recognized for each activity. For example, if we know the wage per hour paid to a worker and the completion time, we are able to calculate labour cost of the activity. Similarly, the cost of other resources can be estimated. As the activity times are uncertain, the project completion time and project costs are uncertain as well.

The decision situation considered in this paper may be conceived as a problem (A, X, E) where A is a finite set of alternatives a_i, $i = 1, 2, \ldots, m$, X is a finite set of criteria X_k, $k = 1, 2, \ldots, n$ and E is a set of evaluations of alternatives with respect to the criteria:

$$E = \begin{bmatrix} X_{11} & \cdots & X_{1k} & \cdots & X_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ X_{i1} & \cdots & X_{ik} & \cdots & X_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ X_{m1} & \cdots & X_{mk} & \cdots & X_{mn} \end{bmatrix}$$

In this case, the decision alternatives are evaluated with respect to two criteria: the project completion time and the total cost. Performances of each alternative with respect to the criteria are evaluated by the distribution functions. The simulation model is used to obtain the knowledge base used for the construction of these functions. For each alternative, a series of simulation experiments is carried out. The results are used for generating distributional evaluations of alternatives with respect to the criteria.

Various techniques can be used for solving a discrete decision making problem under risk. Keeney and Raiffa (1976) suggest the multi-attribute utility function approach. They show that if the additive independence condition is verified, the multi-attribute comparison of two alternatives can be decomposed into one-attribute comparisons. In practice, however, both the estimation of one-attribute utility functions and the assessment of the synthesis function are difficult.

In this paper stochastic dominance (SD) rules are used for comparing distributional evaluations. Huang *et al.* (1978) show that if the additive independence condition is verified, the necessary condition for multi-attribute stochastic dominance (MSD) is the verification of stochastic dominance with respect to each criterion. In practice the MSD rule is very rarely verified. Zaras and Martel (1994) suggest weakening the unanimity condition and accepting a majority criteria condition. They propose MSD_r – multi-attribute stochastic dominance for a reduced number of criteria. This approach is based on the observation that people tend to simplify the multi-attribute problem by taking into account only the most important criteria. The procedure consists of two steps. Firstly, the SD relations are verified for each pair of alternatives with respect to all criteria. Secondly, the multi-attribute aggregation is realized – the ELECTRE I methodology is used to obtain the final ranking of alternatives.

The solution of the multiple criteria decision making problem is possible if the decision maker is able to provide information about his/her preferences with respect to the set of objectives under consideration. Procedures listed above assume that the preference information is collected prior to calculating the final solution. The analysis is therefore based on an a priori basis. In many situations, however, the decision maker is unable or unwilling to provide all required information at the same time.

A methodology known as the interactive approach is very useful in such cases. This technique assumes that the decision maker is able to provide the preference information with respect to a given solution or a given set of solutions (the local preference information). Two main advantages are usually mentioned for employing interactive techniques. Firstly, such methods need much less information on the preferences of the decision maker. Secondly, since the decision maker is closely involved in all phases of the problem solving process, he or she put much reliance in the generated solution, and, as a result, the final solution has a better chance of being implemented. Numerous interactive techniques have been proposed in recent years. Most of them are applicable in circumstances of certainty, although the methods devised for the case of risk are also proposed. The INSDECM technique, which was presented in Nowak (2006), combines the interactive approach and the risk analysis based on stochastic dominance and mean-risk analysis. In this work we use this method for solving the project planning problem.

3. Methodology

The procedure we propose here consists of three steps. First, evaluations of alternatives with respect to the criteria are generated. Next, these evaluations are compared with respect to the criteria. Finally, the interactive technique is used for the selection of the most desirable resource allocation. The steps required to perform the analysis are described below.

Step 1. The generation of project evaluations

Our approach uses the simulation model for generating evaluations of alternatives with respect to the criteria. One of the most important elements of simulation modeling is identifying appropriate probability distributions for the input data. Usually, it requires analyzing empirical or historical data and fitting these data to distributions. Sometimes, however, such data are not available and an appropriate distribution has to be selected according to the decision maker's or expert's judgment. In such case triangular distributions are usually used to illustrate variability of input data. In the example presented below we use this technique for estimating probability distributions for the activity completion time. Nevertheless, our approach is able to utilize another type of data, if only they can be transformed to probability distributions.

The model we use for simulating the project assumes that in each run following steps are performed:

- The completion time for each activity is generated using the inverse-transform method.
- Taking into account the relation between time and cost of activity the cost for each activity is calculated.
- Finally, assuming each activity to start as-soon-as-possible, the completion time of the project is identified, and the cost of the project is calculated as a sum of activity costs.

The simulation model is used for generating probability distributions of output variables. For each alternative a sequence of simulations is run. In each experiment values of criteria are recorded. As a result a sequence of replications is obtained for each criterion. This data are used for constructing the probability distributions of output variables.

Step 2. Comparing the alternatives with respect to the criteria

Once the projects' evaluations are obtained, the relations between projects with respect to the criteria can be analyzed. Two methods are usually used for comparing uncertain outcomes: mean-risk analysis and stochastic dominance. The former is based on two criteria: one measuring expected outcome and another representing variability of outcomes. In the stochastic dominance approach random variables are compared by pointwise comparison of their distribution functions. In this paper both techniques are used. While the stochastic dominance is employed for constructing rankings of alternatives with respect to each criterion, mean-risk technique is used when a final solution is chosen.

Let $F_{ik}(x)$ and $F_{jk}(x)$ be right-continuous cumulative distribution functions that represent evaluations of a_i and a_j respectively over criterion X_k:

$$F_{ik}(x) = \Pr\{X_{ik} \le x\},$$

$$F_{jk}(x) = \Pr\{X_{jk} \le x\}.$$

Definitions of the first and the second stochastic dominance relations are as follows:

Definition 1. (FSD – First Stochastic Dominance)

X_{ik} dominates X_{jk} by FSD rule ($X_{ik} \succ_{\mathrm{FSD}} X_{jk}$) if and only if
$F_{ik}(x) \ne F_{jk}(x)$ and $H_1(x) = F_{ik}(x) - F_{jk}(x) \le 0$ for $x \in \mathrm{R}$.

Definition 2. (SSD – Second Stochastic Dominance)

X_{ik} dominates X_{jk} by SSD rule ($X_{ik} \succ_{\mathrm{SSD}} X_{jk}$) if and only if
$F_{ik}(x) \ne F_{jk}(x)$ and $H_2(x) = \int_{-\infty}^{x} H_1(y)dy \le 0$ for $x \in \mathrm{R}$.

Hadar and Russel (1969) show that the FSD rule is equivalent to the expected utility maximization rule for all decision makers preferring larger outcomes, while the SSD rule is equivalent to the expected utility maximization rule for the risk-averse decision makers preferring larger outcomes. In this paper we assume that the decision maker is risk averse in relation to both criteria and use both FSD and SSD rules for analysing relations between decision alternatives with respect to the criteria.

Step 3. Selecting the final solution

As soon as the relations between the alternatives with respect to each criterion are identified, we are able to select the efficient alternatives. We assume that alternative a_i is efficient if, and only if, for no other alternative a_j the following condition is fulfilled:

$$\forall k = 1, \ldots, n \; X_{jk} \succ_{\mathrm{SD}} X_{ik},$$

where $\succ_{\mathrm{SD}}$ stands for the stochastic dominance relation (FSD/SSD). Thus, we assume that alternative a_i is efficient if there is no other alternative that dominates a_i according to stochastic dominance rules with respect to all criteria. The set of efficient alternatives A' can be identified by pairwise comparisons.

We suggest using interactive procedure INSDECM for the selection of the final solution. Each iteration of this method includes the following steps:

– presentation of the data,
– asking the decision maker to provide the preference information by specifying additional requirements,
– identifying the alternatives satisfying the decision maker's aspirations.

For each criterion the decision maker may choose one or more distribution characteristics to be presented (mean, median, standard deviation, quantilles). The best and the worst values of these measures, attainable within the set of alternatives are identified and presented to the decision maker. Additional requirements are defined by specifying minimum or maximum values of the distribution characteristics. They can be defined, for example, in the following form:

– mean of the distributional evaluation with respect to the criterion X_k should not exceed ξ: $\mu_{ik} \le \xi$,
– probability that the criterion X_k will be greater than ξ should not exceed α: $P\{X_{ik} \ge \xi\} \le \alpha$.

Such restrictions are in general not consistent with stochastic dominance rules (Ogryczak and Ruszczyński 1999, 2001). We say that decision maker's requirement is not consistent with these rules if the following conditions are fulfilled simultaneously:

– the evaluation of a_i with respect to X_k does not satisfy the requirement,
– the evaluation of a_j with respect to X_k satisfies the requirement,
– $X_{ik} \succ_{SD} X_{jk}$.

If such situation takes place, the implementation of the decision maker's requirement would result in rejecting an alternative evaluated better than an alternative that satisfies this requirement according to stochastic dominance rules. The reason for this is that the decision maker defines his or her requirements by specifying fixed values of distribution characteristics, while stochastic dominance rules take into account the whole information contained in probability distributions.

Let us consider the following example. The decision maker has defined the following requirement: the probability that the cost of the project (criterion X_2) will exceed 100,000 EUR should not be higher than 0.05. Two alternate resource allocations represented by alternatives a_i and a_j are considered. Following data have been accessed:

– $P\{X_{i\,2} \ge 100{,}000\} = 0.051$,
– $P\{X_{j\,2} \ge 100{,}000\} = 0.049$,
– $X_{i\,2} \succ_{SSD} X_{j\,2}$.

Taking into account the requirement of a decision maker results in the rejection of alternative a_i. In such case alternative a_j is still considered as a candidate for the final solution, although it is worse than a_i according to stochastic dominance rules. In fact a small change of the requirement may result in a quite different recommendation.

We propose to verify whether the constraint defined by the decision maker is consistent with the stochastic dominance rules or not and to suggest some methods of redefining constraint, if the inconsistency is found for any pair of alternatives. Let us assume that inconsistency has been verified for alternatives a_i and a_j. Any inconsistent constraint should be redefined in a way that results in accepting or rejecting both a_i and a_j. The former can be achieved by relaxing the constraint, the latter by making the constraint more restrictive.

The INSDECM procedure operates as follows:

1. Let $l = 1$, $B_l = A'$.
2. Ask the decision maker to specify the distribution characteristics that should be presented during the conversational phase of the procedure.
3. Identify the best and the worst values of distribution characteristics attainable for $a_i \in B_l$; present the data to the decision maker.
4. Ask the decision maker whether he/she is satisfied with the data presented or not; if the answer is *no*, go to (2).
5. Ask the decision maker whether the worst values of distribution characteristics are satisfactory or not; if the answer is *yes*, go to (10).
6. Ask the decision maker to choose the characteristic to be improved and to specify its minimal or maximal acceptable value.
7. Verify the consistency of the constraint specified by the decision maker with stochastic dominance rules. If the inconsistency is found, go to (8), otherwise go to (9).
8. Present to the decision maker the ways in which the constraint can be redefined and ask him/her to choose one of the suggestions. If he/she does not accept any proposal, go to (6).
9. Generate B_{l+1} – the set of alternatives $a_i \in B_l$ satisfying the decision maker's constraint. If $B_{l+1} = \varnothing$, notify the decision maker and go to (6), else assume $l = l + 1$ and go to (3).
10. Present the list of considered alternatives to the decision maker. If he/she is able to choose the final solution, then end the procedure, otherwise go to (6).

The procedure iterates until the decision maker is able to accept one of the considered alternatives as the final solution. Although the procedure does not limit the number of distribution characteristics to be presented, the decision maker is usually not able to analyze too many of them. If the number of criteria is large, it is practical to limit the number of the measures for each criterion to one. Usually, the central tendency measures provide beneficial information. The measures based on the probability of getting outcomes above or below the specified target value are also interesting, as they are intuitively comprehensible for the decision maker.

Our procedure allows the decision maker to define a single constraint at each iteration. Nevertheless, it is also possible to permit him (or her) formulating multiple restrictions. In particular, if the decision maker has all constraints ready at the beginning of the interactive decision making process, they have to be taken into account. We must remember, however, that in many cases such restrictions cannot be satisfied simultaneously. If none alternative satisfies all constraints, we have to inform the decision maker of that and ask him (or her) to reformulate his (or her) restrictions.

The final solution is made in step (10). Assuming that the worst values of all distribution characteristics under consideration are satisfactory, the decision maker is asked to choose one of alternatives satisfying constraints defined so far. A following question arises: what should be done if more than one alternative are favoured by the decision maker? In such case we can return to the dialog phase of the procedure and try to provide additional information to the decision maker presenting values of other distribution characteristics (e.g. probability of meeting another target value).

4. Numerical example

A manager of the carpentry service considers making a bid for sudden work in delayed investment of the major real estate investor. The invitation for the project has been issued, as the previous contract has been scrapped due to subcontractor's delay. The offer provides for a single service, but it is quite possible that it will result in starting a long-term cooperation. As the contractor is a leading company on the market, the success in tendering is considered to be of a primary importance. Thus, the overall goal of the manager is to win the tendering, even if the contract would not make a profit. The invitation for the project specifies all the tasks that should be realized. The answer should specify the proposed price and the total time in which the project will be completed. The project consists of ten time-consuming activities (Fig. 1).

Two experienced craftsmen – carpenters may be engaged. While the first one (E1) is able to complete all the activities, the latter (E2) specializes in tasks that are described by activities b, c, e, h and i. Only one carpenter can be engaged for each activity.

As other contracts are also realized, the constraints related to the accessibility of employees have to be taken into account. Thus, when the answer for tendering is prepared, additional costs arising from the tardiness of other projects have to be considered.

The decision maker is not sure how long each activity will take. However, three estimates for each activity have been obtained: optimistic time (a), most probable time (m) and pessimistic time (b) (Table 1).

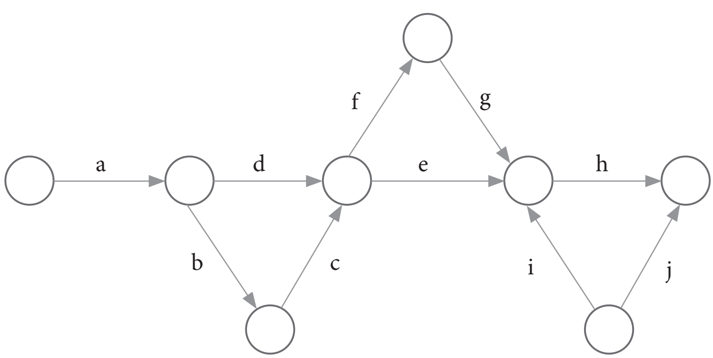

Fig. 1. Project network

Table 1. List of activities

Activity	Optimistic time [hours]	Most probable time [hours]	Pessimistic time [hours]	Employee	Cost if the activity is realized by employee E1 [EUR/h]	Cost if the activity is realized by employee E2 [EUR/h]
a	9	12	18	E1	30	0
b	6	8	12	E1 or E2	30	45
c	3	4	6	E1 or E2	35	50
d	9	12	18	E1	30	0
e	6	8	12	E1 or E2	45	65
f	3	4	6	E1	30	0
g	3	4	6	E1	25	0
h	18	24	36	E1 or E2	30	45
i	9	12	18	E1 or E2	50	65
j	6	8	12	E1	40	0

The cost of each activity depends on the time it takes and the employee that is employed. The differences in the labour costs arise from the fact that, at this moment, the carpenters are engaged in other projects. Thus, the cost of the project has to be increased by the cost of the delays of the other projects.

Taking into account the information about the possible employees' assignments, the set of alternate resource allocations (alternatives) has been generated (Table 2).

Table 2. The set of alternatives

Activity	Alternative															
	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
a	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1
b	E1	E2	E1	E1	E1	E1	E2	E2	E2	E2	E1	E1	E1	E1	E1	E1
c	E1	E1	E2	E1	E1	E1	E2	E1	E1	E1	E2	E2	E2	E1	E1	E1
d	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1
e	E1	E1	E1	E2	E1	E1	E1	E2	E1	E1	E2	E1	E1	E2	E2	E1
f	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1
g	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1
h	E1	E1	E1	E1	E2	E1	E1	E1	E2	E1	E1	E2	E1	E2	E1	E2
i	E1	E1	E1	E1	E1	E2	E2	E1	E1	E2	E1	E1	E2	E1	E2	E2
j	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1
a	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1
b	E2	E2	E2	E2	E2	E2	E1	E1	E1	E1	E2	E2	E2	E2	E1	E2
c	E2	E2	E2	E1	E1	E1	E2	E2	E2	E1	E2	E2	E2	E1	E2	E2

Continuation of **Table 2**

	a_{17}	a_{18}	a_{19}	a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}
d	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1
e	E2	E1	E1	E2	E2	E1	E2	E2	E1	E2	E2	E2	E1	E2	E2	E2
f	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1
g	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1
h	E1	E2	E1	E2	E1	E2	E2	E1	E2	E2	E2	E1	E2	E2	E2	E2
i	E1	E1	E2	E1	E2	E2	E1	E2	E2	E2	E1	E2	E2	E2	E2	E2
j	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1	E1

To solve the problem, simulations have been run for each alternate resource allocations represented by decision alternatives. We used MS-EXCEL assuming that the uncertainty in activity times can be described by triangular distributions. For each alternative 10,000 simulation runs have been performed. Thus, samples consisting of 10,000 observations have been obtained for project completion time and project cost. We have used this data for constructing the probability distributions of output variables. Table 3 presents means and standard deviations of these distributions.

Table 3. Results of simulation experiments

Alternative	Time		Cost	
	Mean	Standard deviation	Mean	Standard deviation
a_1	105.17	4.94	3629.69	177.23
a_2	96.49	4.92	3758.78	173.92
a_3	100.44	4.96	3692.82	167.10
a_4	96.85	4.95	3807.21	186.95
a_5	83.41	4.49	4024.44	208.98
a_6	91.54	4.58	3835.99	189.40
a_7	92.45	4.82	3834.51	180.32
a_8	88.43	4.69	3944.54	187.88
a_9	74.51	4.27	4148.37	212.40
a_{10}	82.86	4.45	3961.77	186.43
a_{11}	92.89	4.95	3869.55	185.09
a_{12}	78.83	4.40	4080.17	214.30
a_{13}	87.65	4.78	3889.67	184.01
a_{14}	75.18	4.47	4193.57	211.86
a_{15}	83.61	4.68	4013.77	193.98
a_{16}	96.29	4.94	4227.24	211.08
a_{17}	84.35	4.59	3999.68	190.11

Continuation of **Table 3**

Alternative	**Time**		**Cost**	
	Mean	**Standard deviation**	**Mean**	**Standard deviation**
a_{18}	70.96	4.20	4207.50	212.96
a_{19}	79.27	4.41	4035.38	199.33
a_{20}	66.60	3.97	4319.55	207.13
a_{21}	74.86	4.49	4132.47	196.53
a_{22}	87.76	4.84	4349.29	218.83
a_{23}	71.06	4.30	4266.35	218.02
a_{24}	79.40	4.57	4069.64	201.48
a_{25}	91.92	4.94	4280.08	216.85
a_{26}	88.21	4.70	4395.29	222.66
a_{27}	63.08	4.05	4390.83	224.24
a_{28}	71.37	4.27	4207.90	205.23
a_{29}	84.14	4.53	4435.24	222.84
a_{30}	79.41	4.74	4526.85	237.88
a_{31}	83.89	4.61	4468.38	228.42
a_{32}	76.16	4.56	4592.72	220.43

In the second stage we generate the set of efficient alternatives A'. We compare evaluations of the alternatives employing stochastic dominance rules. The efficient set consists of 18 alternatives:

$$A' = \{a_1, a_2, a_3, a_6, a_7, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{27}, a_{28}\}.$$

For example alternative a_4 is dominated by alternative a_2, as:

$$X_{2\,1} \succ_{\mathrm{SSD}} X_{4\,1} \quad \text{and} \quad X_{2\,2} \succ_{\mathrm{SSD}} X_{4\,2}.$$

Finally, the interactive procedure is used for generating the solution of the problem.

$$l = 1, B_1 = A'.$$

Iteration 1

1) The decision maker specifies distribution characteristics to be presented during the conversational phase of the procedure:

– criterion X_1: mean and probability that the completion time will not exceed 95 hours,

– criterion X_2: mean and probability that the cost will not exceed 4400 EUR.

2) The best and the worst values of the distribution characteristics are presented to the decision maker (Table 4).

Table 4. Data presented to the decision maker at iteration 1

	X_1		X_2	
	mean	**P{$X_{i1} \leq 95$}**	**mean**	**P{$X_{i2} \leq 4400$}**
the best value	63.08	1.000	3629.69	1.000
the worst value	105.17	0.014	4390.83	0.537

3) The decision maker is not satisfied with the worst values of the distribution characteristics and specifies the following requirement: "the probability that the completion time will not exceed 95 should not be less than 0.90": $\mathrm{P}\{X_{i1} \leq 95\} \geq 0.90$.

4) The restriction is consistent with the stochastic dominance rules.

5) The set of alternatives satisfying the restriction is generated:

$$B_2 = \{a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{27}, a_{28}\}.$$

6) $l = 2$.

Iteration 2

1) The best and the worst values of the distribution characteristics are presented to the decision maker (Table 5).

Table 5. Data presented to the decision maker at iteration 2

	X_1		X_2	
	mean	**P{$X_{i1} \leq 95$}**	**mean**	**P{$X_{i2} \leq 4400$}**
the best value	63.08	1.000	3889.67	0.961
the worst value	87.65	0.922	4390.83	0.537

2) The decision maker is not satisfied with the worst values of the distribution characteristics and specifies the following requirement: "the probability that the cost will not exceed 4400 should not be less than 0.95": $\mathrm{P}\{X_{i2} \leq 4400\} \geq 0.95$.

3) The restriction is not consistent with the stochastic dominance rules:

$$\mathrm{P}\{X_{10\,2} \leq 4400\} = 0.949 \quad \text{and} \quad \mathrm{P}\{X_{17\,2} \leq 4400\} = 0.950 \quad \text{and} \quad X_{10\,2} \succ_{\mathrm{SSD}} X_{17\,2}.$$

4) The ways in which the requirement can be redefined are presented to the decision maker:

$$(1)\ \mathrm{P}\{X_{i\,2} \leq 4400\} \geq 0.949, \qquad (2)\ \mathrm{P}\{X_{i\,2} \leq 4405\} \geq 0.95,$$

$$(3)\ \mathrm{P}\{X_{i\,2} \leq 4400\} \geq 0.951, \qquad (4)\ \mathrm{P}\{X_{i\,2} \leq 4396\} \geq 0.95.$$

Proposals (1) and (2) accept both a_{10} and a_{17}, while proposals (3) and (4) eliminate each of them. The decision maker accepts proposal (1).

5) The set of alternatives satisfying the restriction is generated:

$$B_3 = \{a_{10}, a_{13}, a_{17}\}.$$

6) $l = 3$.

Iteration 3

1) The best and the worst values of the distribution characteristics are presented to the decision maker (Table 6).

Table 6. Data presented to the decision maker at iteration 3

	X_1		X_2	
	mean	**P{$X_{i1} \leq 95$}**	**mean**	**P{$X_{i2} \leq 4400$}**
the best value	82.86	0.997	3889.67	0.961
the worst value	87.65	0.922	3999.68	0.949

2) The decision maker is satisfied with the worst values of distribution characteristics.

3) Alternatives satisfying restrictions formulated in iterations 1 and 2 are presented to the decision maker (Table 7).

Table 7. Alternatives satisfying restrictions specified by the decision maker

Alternative	X_1		X_2	
	mean	**P{$X_{i1} \leq 95$}**	**mean**	**P{$X_{i2} \leq 4400$}**
a_{10}	82.86	0.997	3961.77	0.949
a_{13}	87.65	0.922	3889.67	0.961
a_{17}	84.35	0.987	3999.68	0.950

The decision maker selects alternative a_{13} as the final solution. As a result, it was decided to prepare an answer to the tendering assuming that employee E1 would be engaged in activities a, b, d, e, f, g, h and j, while employee E2 for activities c and i.

5. Conclusions

In the paper, the discrete time-cost trade-off problem has been considered. We have assumed the uncertain completion times of activities. As the finite number of alternate resource allocations has been considered, we have faced the discrete decision-making problem under risk.

We have proposed a new method for such a problem. It uses the simulation technique for evaluating decision alternatives with respect to the criteria and interactive procedure for identifying the final solution of the problem. The interactive approach is one of the leading methodologies in multi-criteria decision making. Several motivations have been mentioned for implementing this approach. It is usually pointed out that the limited amount of a priori preference information is required from the decision maker, as compared to other techniques. The interactive procedure may be considered as a learning process. By observing the results of succeeding iterations of the procedure, the decision maker extends their knowledge of the decision problem. On the other hand, as the decision maker actively participates in all phases of the problem solving procedure, he (or she) puts much reliance on the final solution that is obtained. As a result, the solution of the procedure has a better chance of being implemented.

Although we have used our procedure in the bi-criteria problem, it can be employed with more criteria as well. In some cases, the dates of reaching milestones of the project are of primary importance. Thus, the completion times for parts of projects can be considered as separate criteria.

The procedure is designed for problems with up to moderate number of discrete alternatives (not more than hundreds). In real-life problems, the number of alternate resource allocations may be very large. In such case, a small subset of alternatives that differ in criteria values can be considered first. During the initial phase of the procedure the area for searching the final solution should be identified. Then the search for final solution can be focused on that area. We plan to propose a procedure based on this approach.

The applicability of the procedure presented here is not limited to the construction project planning problems. It may be useful for various types of problems in which uncertain outcomes are compared. It can be also applied, for example, in inventory models, evaluation of investment projects, production process control, and many others.

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STATYBOS PROJEKTO LAIKO IR KAINOS SUDERINAMUMO ANALIZĖ, GRINDŽIAMA KOMPIUTERINIU MODELIAVIMU IR INTERAKTYVIU METODU

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Santrauka

Rengiant statybos projekto įvykdymo grafiką reikia įvertinti keletą kriterijų. Daugeliu atvejų analizuojamas projekto baigimo laikas ir sąmatinė kaina. Taip pat gali būti įvertinama rizika. Taigi projekto planavimo problema gali būti apibūdinama kaip daugiakriterinė sprendimo problema įvertinant riziką. Straipsnyje analizuojama projekto planavimo problema, suderinant projekto įvykdymo laiką ir kainą. Remiamasi prielaida, kad galimi įvairūs išteklių paskirstymai. Pasiūlyta nauja metodologija, pagrįsta kompiuteriniu modeliavimu ir interaktyvaus metodo taikymu. Pirmuoju etapu imitaciniais modeliais įvertinamos sprendimo alternatyvos. Antruoju etapu galutiniam problemos sprendimui taikoma interaktyvi INSDECM metodologija. Šioje procedūroje, siekiant palyginti sprendimo alternatyvas pagal kriterijus, taikomos stochastinės dominavimo taisyklės. Naujos metodologijos taikymą iliustruoja skaitmeninis pavyzdys.

Reikšminiai žodžiai: projekto planavimas, daugiakriterinių sprendimų priėmimo automatizavimas, laiko ir kainos suderinamumas, interaktyvi metodologija, modeliavimas.

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