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A PARETO MULTI-OBJECTIVE OPTIMIZATION APPROACH FOR SOLVING TIME-COST-QUALITY TRADEOFF PROBLEMS

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Abstract. Speeding up a project's duration will definitely increase the cost and decrease the quality. The previous literatures were mainly related to project planning and controlling which mainly focus on cost-time tradeoff. However, limited researches have been referred to project quality based on mathematical methodologies. This paper proposes a tradeoff problem on time-cost-quality performance. A computer-based Pareto multi-objective optimization approach is utilized for solving the tradeoff problems. The approach can help searching near the reality Pareto-optimal set while not receiving any information on the stakeholders' preference for time, cost and quality. Based on the developed approach, decision-making can become easy according to the sorted non-dominated solutions and project preferences.

Keywords: Time-cost-quality, tradeoff, Pareto optimal set, project management, genetic algorithm.

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JEL Classification: C63, L23, M11.

1. Introduction

Each project involves a multidimensional problem (Blaszczyk and Nowak 2009). A project should be managed with the consideration of scope, time, cost and quality (Tam *et al.* 2002). Generally, project scope is defined in the contract; time and activity sequences are provided on project scheduling; cost is planned on project budget; and quality is controlled on the base of industry criterion. Successful project management should insure project completion in time, within budget, and to the project specifications (Babu and Suresh 1996; Ustinovichius *et al.* 2009; Zavadskas *et al.* 2010). To compress project duration for practical considerateness, it is necessary for stakeholders to consider how to arrange resources and reduce budget. Thus, in the existing literatures, many researchers were inclined to pay more attention for time-cost tradeoff problems (Law and Chu 1987; Lee 1989; Reda and Carr 1989; Deckro *et al.* 1995; Chua *et al.* 1997; Li *et al.* 1999; Leu *et al.* 2001; Jahan-Shahi *et al.* 2002; Laslo 2003; Wang 2004; Vanhoucke 2005; Abdallah 2007; Senouci and AI-Derham 2008; Blaszczyk and Nowak 2009; Ke *et al.* 2009). Some mathematical and heuristic models had been developed for solving these problems. Project quality is seldom considered on purpose with the time-cost tradeoff problems. Though in some studies (Khang and Myint 1999; Gardiner and Stewart 2000; Tareghian and Taheri 2006, 2007), project scheduling with time, cost and quality considerations had been mentioned, researchers did not provide an all-around design among the three but bounding one or two variables as constant. This paper develops a multi-objective optimization with across-the-board considering time, cost and quality.

The time-cost-quality tradeoff problem is formulated as a kind of multi-objective optimization problem. This paper aims to minimize time, minimize cost and maximize quality in managing a project according to some constraint conditions if any.

In practice, it is often considered to perform a tradeoff among different sub-objectives especially for those incompatible sub-objectives in multi-objective problems. Studies on multi-objectives have received in close attention tracing back to some literatures (Hwang and Yoon 1981; Steuer 1986; Dev 1995; Zavadskas 2008). The solution of multi-objective problem is generally named as Pareto optimal solution, which was first put forward by Pareto. The early related studies could be found in references (Stadler 1979, 1981). Among them, more attention is paid on searching solution by genetic algorithm. The main advantages of using genetic algorithm are:

1. capacity which efficiently search complex solution spaces with less possibility getting local optimum;
2. randomization which adding a probability mechanism of iterative operation;
3. suitable for combining with other methods and concurrent operation in computers;
4. robustness which means the solutions are similar while solving the same problem many times.

Thus, this paper develops a time-cost-quality tradeoff framework in view of two points:

1. the existing researches seldom consider time, cost and quality of a project as a whole, especially for the situation where stakeholders have not any preference among them;
2. the existing researches seldom mention to solve a construction problem by Pareto multi-objective genetic algorithm.

Therefore, this paper mainly focuses on the following objectives:

1. to provide a multi-objective scheme on project time, cost and quality;
2. to form a multi-objective optimization solution methodology;
3. to supply decision support according to the analysis of simulation results.

2. Problem descriptions

Generally, project manager should have a good application of knowledge, skills, tools and techniques for meeting the need of stakeholders in the construction. The primary purpose of this section is to formulate a robust optimization model which aims to resolve time-cost-quality tradeoff problems. The formulated model mainly refers to minimize time, minimize cost and maximize quality of a project according to some constraint conditions if any.

2.1. Minimizing project time

For each construction activity in a project, it is assumed that it has a normal completion time and a crash completion time. The present objective is try to reduce the project time. It is described as follows:

$$\text{Min } \mathrm{T}_{total} = \sum_{i=1}^{N} \mathrm{T}_i \,, \tag{1}$$

$$\text{S. t. } \; \mathrm{T}_{crash}(i) \le \mathrm{T}_i \le \mathrm{T}_{normal}(i) \,, \tag{2}$$

where T_i is the feasible duration of activity *i*; $\mathrm{T}_{crash}(i)$ the crash duration of activity i expected to complete in the least time; and $\mathrm{T}_{normal}(i)$ the normal duration of activity i expected to complete at the least cost.

Equation (2) defines the lower limit and the upper limit for every activity time.

2.2. Minimizing project cost

Expediting the time of a project will no doubt increase the cost which is attributed to the cost improvement of every activity. For each activity, cost is comparatively relative to time. If getting abundance of original data is easy, the possible best choice to gain the relation between cost and time is to perform a curve fitting by statistical softwares. The previous literatures mainly mentioned two models: a linear model (Law and Chu 1987; Babu and Suresh 1996; Khang and Myint 1999; Laslo 2003; Senouci and AI-Derham 2008; Ke *et al.* 2009) and a quadratic model (Reda and Carr 1989; Deckro *et al.* 1995; Li *et al.* 1999). More researchers used the linear model between time and cost, possibly because the linear model is easy to deal with. In this paper, considering the economic concept of marginally increasing or decreasing productivity (or utility) for different levels of input, the latter is chosen. The detailed models are represented as:

$$\text{Min } \mathrm{C}_{total} = \sum_{i=1}^{N} \mathrm{C}(\mathrm{T}_i) \,, \tag{3}$$

$$\text{S.t. } \; \mathrm{C}(\mathrm{T}_i) = a_i(\mathrm{T}_i)^2 + b_i \,, \tag{4}$$

$$\mathrm{C}(\mathrm{T}_{normal}(i)) \le \mathrm{C}(\mathrm{T}_i) \le \mathrm{C}(\mathrm{T}_{crash}(i)) \,, \tag{5}$$

where $\mathrm{C}(\mathrm{T}_i)$ is the feasible cost of activity *i*; $\mathrm{C}(\mathrm{T}_{crash}(i))$ the crash cost of activity i during the crash duration; and $\mathrm{C}(\mathrm{T}_{normal}(i))$ the normal cost of activity i during the normal duration.

The relation between cost and time of each activity is clearly defined in Equation (4). Thus, it is quite convenient to get constant values, a_i and b_i, which are given by:

$$a_i = \frac{\mathrm{C}(\mathrm{T}_{crash}(i)) - \mathrm{C}(\mathrm{T}_{normal}(i))}{(\mathrm{T}_{crash}(i))^2 - (\mathrm{T}_{normal}(i))^2},$$

$$b_\mathrm{i} = \frac{\mathrm{C}(\mathrm{T}_{normal}(i))(\mathrm{T}_{crash}(i))^2 - C(\mathrm{T}_{crash}(i))(\mathrm{T}_{normal}(i))^2}{(\mathrm{T}_{crash}(i))^2 - (\mathrm{T}_{normal}(i))^2}.$$

and Equation (5) defines the lower limit and the upper limit for every activity cost.

2.3. Maximizing project quality

Apparently, crashing the time of a project will definitely reduce project quality (Zeng *et al.* 2005, 2007). Similar the relation between cost and time of each activity, the relation between quality and time of each activity should be gained by fitting the original data based on statistical analysis. Most of the previous literatures mainly provided the linear model (Babu and Suresh 1996; Khang and Myint 1999; Tareghian and Taheri 2006, 2007). It is not an exception for this paper in adopting the linear model on the relation between quality and time. The total objective in the section is to maximize the project quality. Considering the quality level has a value between 0 and 1, arithmetic mean, geometric mean and the minimum of the individual activity quality are often chosen as objective functions for measuring project quality. Babu and Suresh (1996) used the three different ways as experiments and concluded that the three trends observed are similar without major differences. Tareghian and Taheri (2006) used arithmetic mean of every activity quality as a criterion for evaluating project quality. Tareghian and Taheri (2007) thought that geometric mean is a good option because it can deal with situations where quality of individual activity is dispersed. Though in this paper, arithmetic mean of each activity quality is chosen as an optimized objective, the methodology presented will not restrict its wide applications to other assumptions. The choice of arithmetic mean is mainly defined according to data characteristics. Geometric mean is seldom affected by an extreme value, thus it is suitable for calculating the dynamic mean value. But arithmetic mean can reflect the effect of extreme value. The detailed models are shown as follows:

$$\text{Max } \mathrm{Q}_{total} = \frac{1}{N}\sum_{i=1}^{N} \mathrm{Q}(\mathrm{T}_i), \tag{6}$$

$$\text{Or max } \mathrm{Q}_{total} = \sqrt[N]{\prod_{i=1}^{N} \mathrm{Q}(\mathrm{T}_i)}, \tag*{(6)*}$$

$$\text{Or max } \mathrm{Q}_{total} = \min\{\mathrm{Q}(\mathrm{T}_1),\cdots,\mathrm{Q}(\mathrm{T}_i),\cdots,\mathrm{Q}(\mathrm{T}_N)\}, \tag*{(6)**}$$

$$\text{S.t. } \mathrm{Q}(\mathrm{T}_\mathrm{i}) = m_i(\mathrm{T}_i) + n_i, \tag{7}$$

$$\mathrm{Q}(\mathrm{T}_{crash}(i)) \le \mathrm{Q}(\mathrm{T}_i) \le \mathrm{Q}(\mathrm{T}_{normal}(i)), \tag{8}$$

where $\mathrm{Q(T}_i)$ is the feasible quality of activity *i*; $\mathrm{Q(T}_{crash}(i))$ the crash quality of activity *i* during the crash duration; and $\mathrm{Q(T}_{normal}(i))$ the normal quality of activity i during the normal duration. By utilizing Equation (7), m_i and n_i can be calculated by:

$$m_i = \frac{\mathrm{Q(T}_{crash}(i)) - \mathrm{Q(T}_{normal}(i))}{\mathrm{T}_{crash}(i) - \mathrm{T}_{normal}(i)},$$

$$n_\mathrm{i} = \frac{\mathrm{Q(T}_{normal}(i))(\mathrm{T}_{crash}(i)) - \mathrm{Q(T}_{crash}(i))(\mathrm{T}_{normal}(i))}{\mathrm{T}_{crash}(i) - \mathrm{T}_{normal}(i)}.$$

Equation (8) defines the lower limit and the upper limit for every activity quality.

3. Pareto optimization design

The specific objective used in the paper is to optimize time-cost-quality tradeoff, which is formulated as a multi-objective optimization problem. To search for an optimal tradeoff among time, cost and quality, an improved non-dominated sorting genetic algorithm (NSGA) is designed for implementing the multi-objective optimization. The primary reason using evolutionary algorithm is its ability to find Pareto-optimal solution at each simulation run. Up to now, for a multi-objective optimization problem, it is quite difficult to find a single solution for simultaneously optimizing all objectives. The best option is to gain a large number of solutions lying on or near the Pareto-optimal front. The previous theories and methods related to the NSGA have been analyzed and discussed in details (Goldberg 1989; Fonseca and Fleming 1993; Srinivas and Deb 1995). Thereafter, with the ceaseless improvement in more useful operators, the NSGA has got better promotion, especially for the appearance of NSGAII advanced by Deb *et al.* (2002). Meanwhile, the NSGA has also gained wide varieties of applications to multidisciplinary aspects (Inamdar *et al.* 2004; Kuriakose and Shunmugam 2005; Kumar *et al.* 2006).

To find a diverse set of solutions and in converging near the reality Pareto-optimal set, the designed procedure with improved NSGA is explained with the following main phases:

Phase 1 – Initialization:

1. Parameters initialization on optimization model: set the number of objectives; set the number of constraints if any; and set the number of independent variables.
2. Parameters initialization on project, including: set the number of activities N in the whole project; input time T_{crash}, cost C_{crash} and quality Q_{crash} for crashing an activity; input normal time T_{normal}, normal cost C_{normal} and normal quality Q_{normal} of every activity; define precedence relationship if any.
3. Parameters initialization on genetic algorithm: set the size of population N_{pop}; set the number of generations N_{Gen}; set the probability of crossover operation p_c; set the probability of mutation p_m; and create a random parent population $\mathrm{P(t)}$ $(\mathrm{t}=0)$ of size N_{pop} based on the problem range and constraints if any.

Phase 2 – Forming Pareto front based on domination relation:

Time T, cost C, and quality Q for every solution are computed in $\mathrm{P(t)}$ according to equations (1), (3) and (6). Then the population $\mathrm{P(t)}$ is sorted based on non-domination algorithm into each front F(t) in criterion space. The detailed description can be gained in the study of Deb *et al.* (2002). Individuals in first front are given a fitness value of 1 and individuals in second are assigned fitness value as 2 and so on. The first front is also called Pareto front which will include the best solutions.

Phase 3 – Calculating crowding distance and defining partially ordered set:

When the non-dominated sort is completed, a quick sort is used for sorting the population based on every objective chosen. The crowding distance between two individuals in different fronts can then be calculated. Considering that the boundary points in a front are always selected, it is assigned infinite distance (Deb *et al.* 2002). The crowding distance of the other points in the sorted list except for the boundary values can be shown as:

$$\mathrm{I}_{i,distance} = (\sum_{k=1}^{3} (I_{i+1} \cdot f_k - I_{i-1} \cdot f_k)) / (f_k^{\max} - f_k^{\min}),$$

where $I_{i+1} \cdot f_k$ denotes the k-th objective function value of the $(i+1)$ th individual, and $I_{i-1} \cdot f_k$ the k th objective function value of the $(i-1)$ th individual; $f_k^{\max}$ and $f_k^{\min}$ the maximum and minimum values of the k th objective function.

After completing the crowding distance, it is formed a partially ordered set as described by Deb *et al.* (2002). Those individuals in the fronts with small serial numbers are considered first into the population when they are in different fronts; while the serial numbers of the fronts are the same, the individuals with larger crowding distance will be a priority.

Phase 4 – Genetic operation:

Genetic algorithm (GA) is a procedure for searching the optimized objective functions by the principles of natural genetics and natural selection. The main operation is related to selection, crossover and mutation.

First, a binary tournament selection operator is used in this procedure for updating population P(t). The selection criterion is now based on partial ordering relation which has been mentioned in the context.

Then, a simulated binary crossover operator (Deb and Agrawal 1995; Beyer and Deb 2001) is adopted for forming the population Q(t). The operation is basically based on the principle of a single point crossover operator on binary strings. The children solutions can be calculated as below:

$$\mathrm{c}_{1,gen} = \frac{1}{2}((P_{1,gen} + P_{2,gen}) + (P_{1,gen} - P_{2,gen})\,\beta_{gen}),$$

$$\mathrm{c}_{2,gen} = \frac{1}{2}((P_{1,gen} + P_{2,gen}) + (P_{2,gen} - P_{1,gen})\,\beta_{gen}),$$

where $\mathrm{c}_{\mathrm{i},gen}$ is the child solution, $P_{i,gen}$ the selected parent ($\mathrm{i} = 1,2$) and β_{gen} a random variable with the distribution of the following form:

$$\beta_{gen} = \begin{cases} (2u)^{\frac{1}{\eta_c+1}} & \text{if} \quad 0 \le u \le 0.5, \\ (2(1-u))^{-\frac{1}{\eta_c+1}} & \text{if} \quad u > 0.5 \end{cases}$$

u can be obtained from a random variable with the range between 0 and 1, and η_c the constant crossover distribution index. The detailed procedure can be seen from Appendix 1.

Finally a polynomial mutation operator (Deb and Agrawal 1995; Raghuwanshi and Kakde 2004) is applied for creating the population $Q'(t)$. In this operation, the children solutions can be calculated as below:

$$c_{gen} = P_{gen} + (P_{gen}^{upper} - P_{\text{gen}}^{lower})\delta_{gen},$$

where c_{gen} is the child solution, P_{gen} the selected parent who has the upper bound P_{gen}^{upper} and the lower bound P_{gen}^{lower}, and δ_{gen} can be obtained from a polynomial distribution, that is,

$$\delta_{gen} = \begin{cases} (2r_{gen})^{\frac{1}{\eta_m+1}} - 1 & \text{if} \quad 0 \le r_{gen} \le 0.5, \\ 1-(2(1-r_{gen}))^{-\frac{1}{\eta_m+1}} & \text{if} \quad r_{gen} > 0.5, \end{cases}$$

where r_{gen} is a random number, $r_{gen} \in (0,1)$; and η_m the constant mutation distribution index. The detailed can be obtained from Appendix 2.

Once end criterion is not satisfied, the iteration process with Phases 2 to 4 is kept on or the operation is stopped and the results needed are output. The whole repeated process has the ability of keeping the trend of speciation without any loss of good solutions if being found and the individuals will converge at an optimal area at the last generation.

Genetic algorithm does not adopt at the beginning of arithmetic until the second generation operates. The designed procedure with improved NSGA is shown in Fig. 1.

4. Illustrative examples and simulation results

In this section, the propositional technique in Section 3 and the problem description in Section 2 are evaluated from a particular example problem. The example presented by Li *et al.* (1999) is used to illustrate the optimization approach. Considering that the quality of a project may be an affected factor, the example has been modified and listed in Table 1.

The objective functions mention the three aspects stated in Equations (1), (3) and (6). The functions will form the criterion space $\{T_{total}, C_{total}, Q_{total}\}$. Considering the specific problem itself in this study, that is, existing in a functional relationship between cost and time, and between quality and time, constraints can be reduced to the least. The constraints are shown in the referred Equations which includes $N = 11$ (the number of activities) in equations. The total 11 independent variables are relative to time constitute the decision space $\{T_1, T_2, \cdots, T_N\}$. A population size of 40 is chosen with crossover probability of 0.9 and mutation probability of 0.1. And the crossover distribution index η_c and the mutation distribution index η_m are set to be 10. Especially program package written in the "C" has been implemented on a personal computer with Intel® Atom™ CPU N270. All simulations are completed in less than 1 minute. After the 100th generation, the Pareto optimal solutions in decision space are shown in Appendix 3. The 3D non-dominated Pareto optimal fronts after the 1st, 5th, 10th, 20th, 50th, and 100th are shown in Figures 2–6 and 7 respectively.

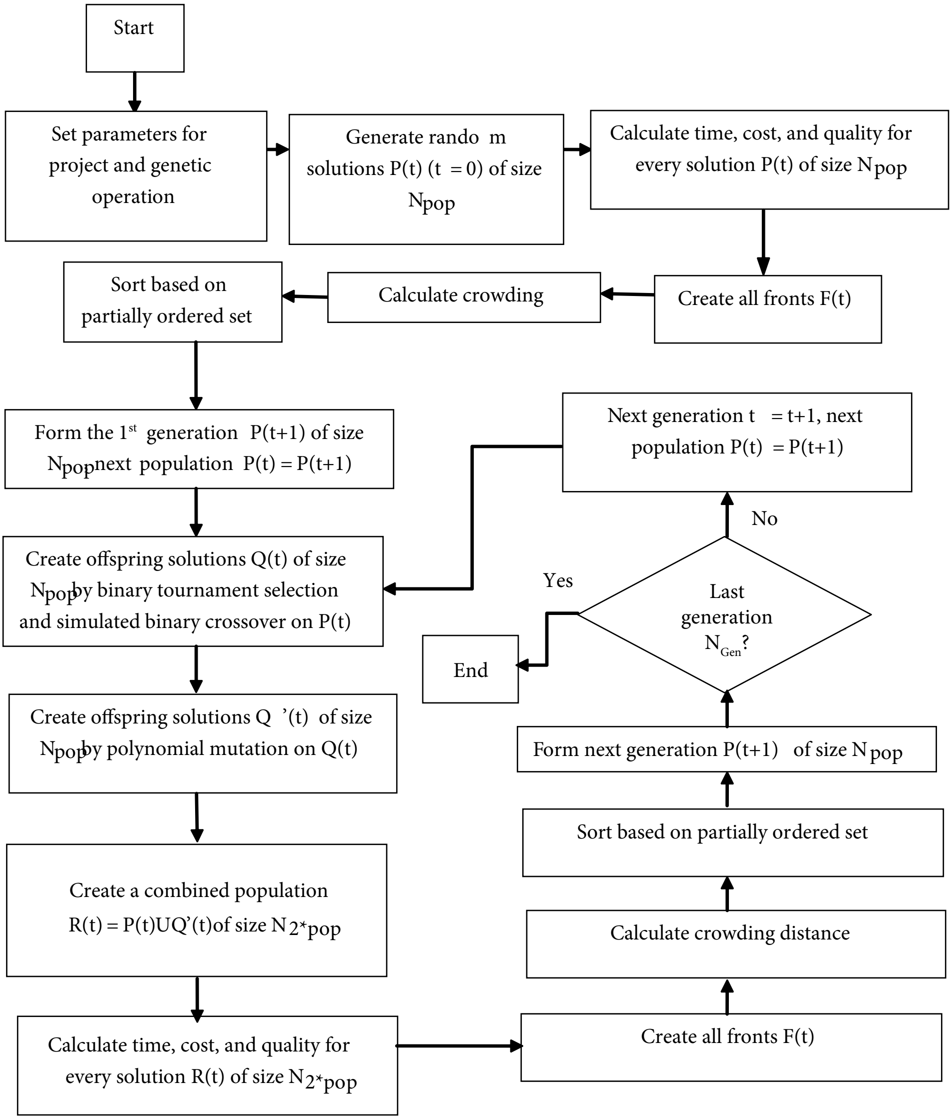
Start
Set parameters for project and genetic operation
Generate rando m solutions P(t) (t = 0) of size Npop
Calculate time, cost, and quality for every solution P(t) of size Npop
Create all fronts F(t)
Calculate crowding
Sort based on partially ordered set
Form the 1st generation P(t+1) of size Npop next population P(t) = P(t+1)
Create offspring solutions Q(t) of size Npop by binary tournament selection and simulated binary crossover on P(t)
Create offspring solutions Q '(t) of size Npop by polynomial mutation on Q(t)
Create a combined population R(t) = P(t)UQ'(t)of size N2*pop
Calculate time, cost, and quality for every solution R(t) of size N2*pop
Create all fronts F(t)
Calculate crowding distance
Sort based on partially ordered set
Form next generation P(t+1) of size Npop
Last generation NGen?
Yes
End
No
Next generation t = t+1, next population P(t) = P(t+1)

Fig. 1. A Pareto optimization framework for time-cost-quality problems

From Figs 2 to 7, it can be seen that 14 Pareto optimal solutions are obtained after the 1st generation, 31 after the 5th generation, 32 after the 10th generation, 40 after the 20th generation, and then keep 40 invariable in the next generation. In the 1st generation, about 35% Pareto optimal solutions can be gained. Up to the 20th generation, the percentage of Pareto optimal solutions has been improved to 100%. It means with the increase in the number of generation, the dominated solutions are continually eliminated and replaced by better solutions and the optimal speciation is formed.

Table 1. Example of time-cost-quality data from a project

Activity (*i*)	Crash Point			Normal Point		
	T(*i*)	C(*i*)	Q(*i*)	T(*i*)	C(*i*)	Q(*i*)
1. Substructure	30	6752.10	0.9	60	6432.20	1
2. Superstructure	300	4197.00	0.7	450	2842.00	1
3. Carpenter joinery	12	2358.18	0.8	30	2335.95	1
4. Window glazing, metal works	36	10538.24	0.8	66	10398.44	1
5. Tiling	39	15716.85	0.7	69	15701.35	1
6. Painting	39	1971.42	0.7	69	1964.82	1
7. Plumbing and drainage	60	6831.20	0.3	102	6776.74	1
8. Electrical installations	9	11637.42	0.75	36	11625.70	1
9. Lift	15	6746.85	0.4	42	6744.78	1
10. Fire service installation	18	1091.32	0.5	45	1090.29	1
11. Preliminaries	18	8789.79	0.9	42	8782.17	1

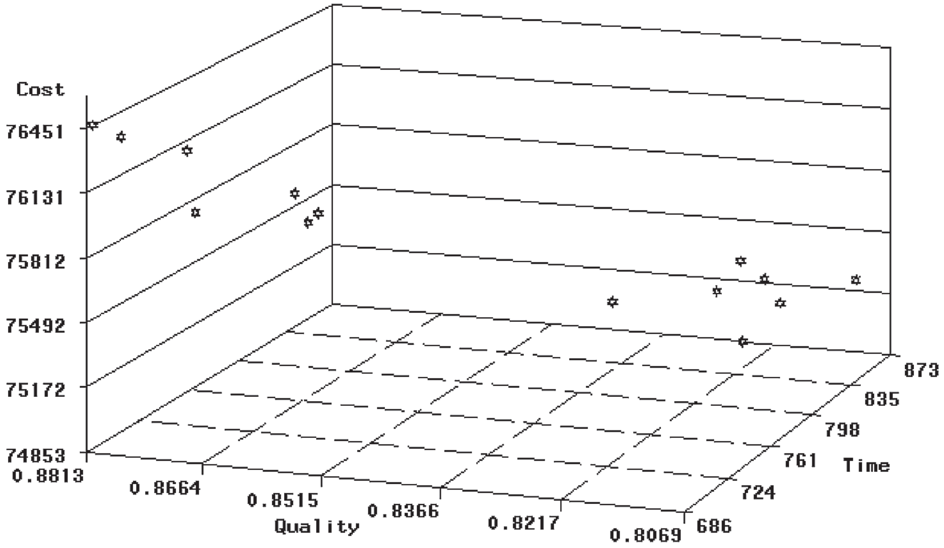

Fig. 2. 3D Pareto optimal front after the 1st generation

From Figure 7, it is shown how the distribution for 40 non-dominated points in the criterion space after the 100th generation will be formed. In Appendix 3, corresponding to these points of the criterion space shown in Figure 7, there are 40 efficient points in decision space which shows how to rationally arrange the time of every activity.

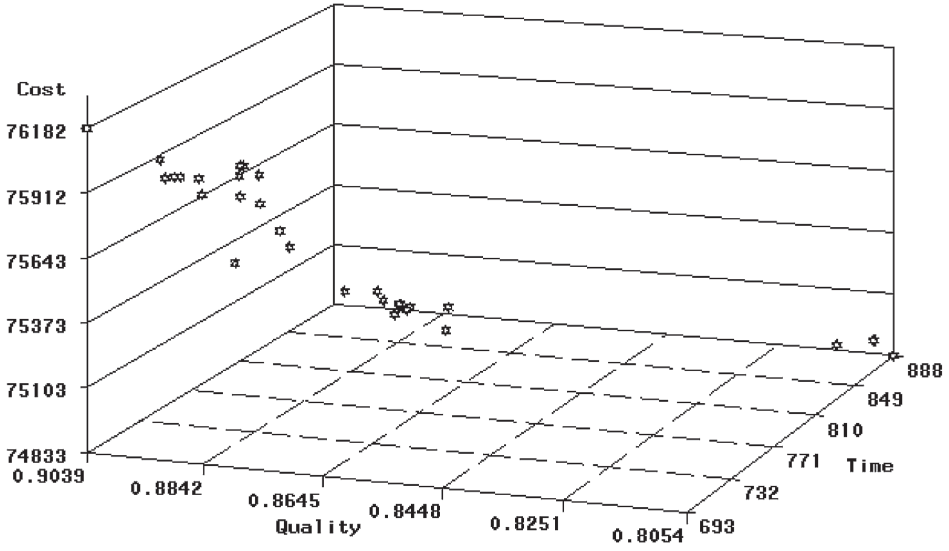

Fig. 3. 3D Pareto optimal front after the 5th generation

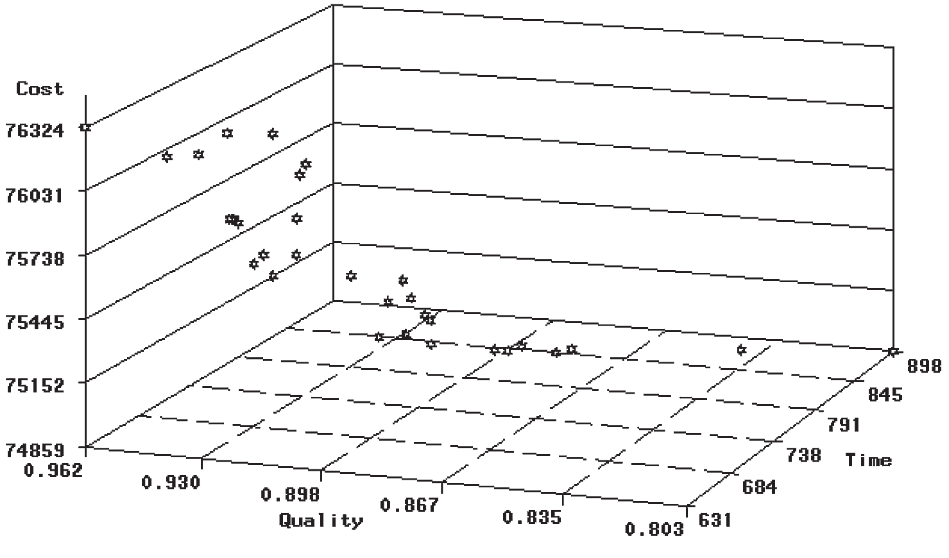

Fig. 4. 3D Pareto optimal front after the 10th generation

In this paper, solving the existing problems is based on the assumption of nothing in particular preference on time, cost and quality. In real life for making decision, it is usually demanded to choose the definite solution for a specified problem. Thus, three assumptions on stakeholders' preferences will be demonstrated for providing an optimal decision. Table 2 shows Pareto optimal solutions sorted according to three different preferences.

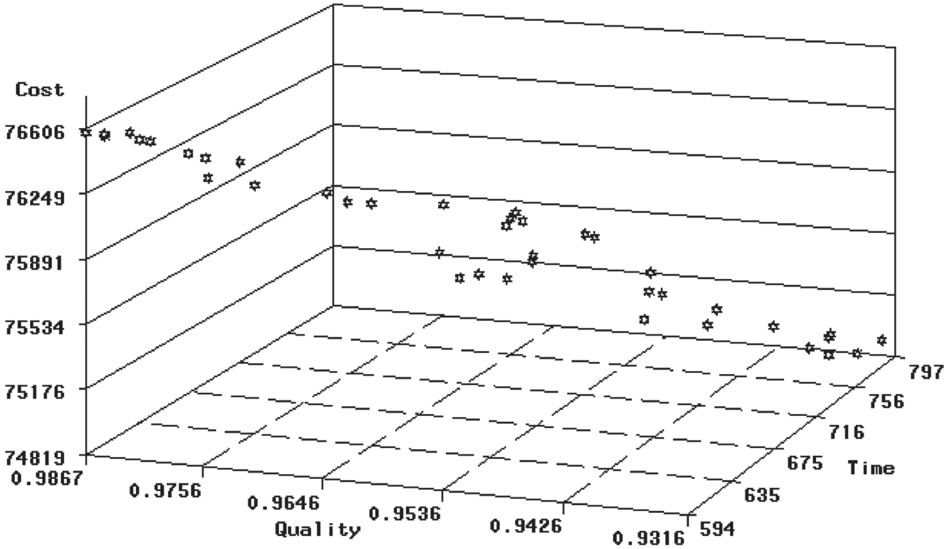

Fig. 5. 3D Pareto optimal front after the 20th generation

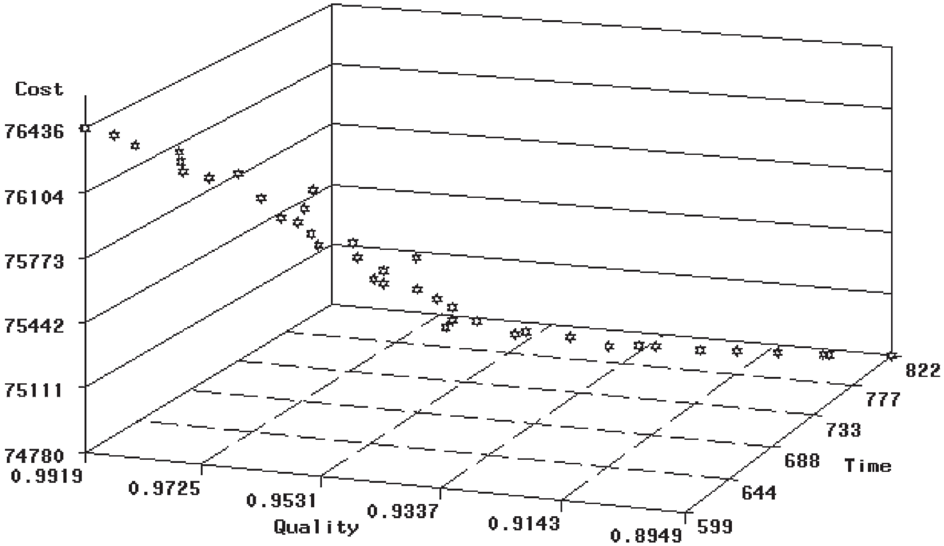

Fig. 6. 3D Pareto optimal front after the 50th generation

From Table 2, it is revealed that if time or quality is much more worth of concern, the solution $S2$ is preferred, the corresponding solution (see Appendix 3) is assigning the duration 31.42, 300.13, 13.25, 36.08, 39.01, 39.06, 60.14, 9.63, 15.29, 18.03 and 18.07 for activities A1,

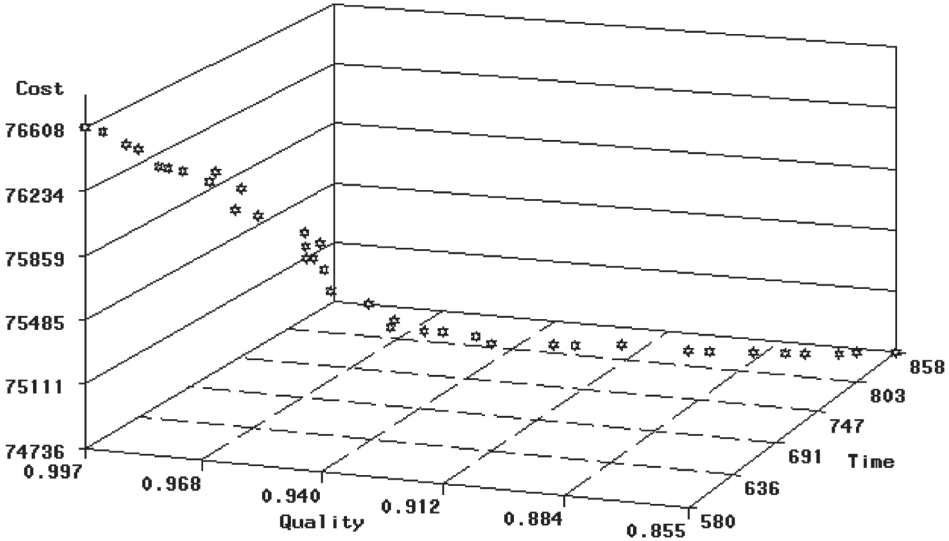

Fig. 7. 3D Pareto optimal front after the 100th generation

Table 2. Pareto optimal solutions sorted according to preferences

Time	Cost	Quality
S2	S1	S2
S18	S13	S18
S8	S16	S8
S11	S20	S11
S17	S22	S17
S38	S31	S38
S30	S36	S30
S29	S21	S12
S12	S10	S19
S7	S4	S29
S19	S9	S40
S40	S5	S7
S25	S6	S23
S33	S39	S14
S35	S37	S3
S23	S27	S35
S15	S32	S15
S3	S28	S25
S14	S34	S33
S24	S24	S26
S26	S26	S24
S28	S14	S32

The end of Table 2

Time	Cost	Quality
S34	S3	S28
S32	S15	S34
S27	S23	S27
S37	S35	S37
S39	S33	S39
S6	S25	S6
S5	S40	S5
S9	S19	S9
S4	S7	S4
S10	S12	S10
S21	S29	S21
S31	S30	S31
S36	S38	S36
S22	S17	S22
S20	S11	S20
S16	S8	S16
S13	S18	S13
S1	S2	S1

A2, A3, A4, A5, A6, A7, A8, A9, A10 and A11 respectively; if cost is quite important for the stakeholders, $S1$ is a good option, the corresponding solution (see Appendix 3) is assigning the duration 59.99, 449.99, 28.93, 65.83, 47.86, 48.32, 96.47, 9.00, 15.62, 18.07 and 18.09 for activities A1, A2, A3, A4, A5, A6, A7, A8, A9, A10 and A11 respectively. From the calculation results of crowding distance, it is not difficult to find that $S1$ and $S2$ are just boundary points with infinite crowding distance. Thus, once the stakeholders' preference is explicit, the solution points are often found on the border. In fact, Table 2 supplies a decision scheme among the tradeoff on time, cost and quality. According to the illustrative sample, the decision schemes on preference of time and quality are almost the same, this does not mean that will be the same for all samples.

5. Conclusion

Speeding up a project's duration will definitely increase the cost and decrease the quality. In this paper, a suggestion was provided that time, cost and quality should be taken into a whole consideration. To solve time-cost-quality tradeoff problems in construction, a Pareto multi-objective optimization approach was developed. A series of Pareto optimal solutions was gained from the approach. It was showed no difference if no preferences on time, cost and quality for the stakeholders are selected. If the stakeholders have definite preference on time, cost and quality, it can quickly make decision according to the sorted specific factors from the approach. It was noted that the stakeholders can provide guidelines on how to arrange time, cost and quality of every activity in the project. On the other hand, because of the wide popularity in using computers, the whole solution procedures can expediently be integrated to some project management or virtual construction softwares. Visualization in making decision can also be realized by designing user-interface.

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Appendix 1

```
/* Procedure for real variable simulated binary crossover */
//pc is crossover probability
//eps is an extreme small positive number
// parent is designated as the parent population
//child is specified as child population
// P[j] means the j-th decision variable
//ymin means the minimum in decision variable P[j]
//ymax means the maximum in decision variable P[j]

    generate a random number with the range between 0 and 1: rand_1
     if (rand_1 <= pc)
     {
       for each i, i is the serial number of decision variables
      if (rand_1<=0.5)
      {
         if (fabs(parent1.P[i]-parent2.P[i]) > eps)
         {
            if (parent1.P[i] < parent2.P[i])
            {
               y1 = parent1.P[i];
               y2 = parent2.P[i];
            }
            else
            {
               y1 = parent2.P[i];
               y2 = parent1.P[i];
            }
```

```
          generate a random number with the range between 0 and 1: rand_2
              if (rand _2<= 0.5)
              {
                value=2*rand_2;
                betagen = pow (value,(1.0/(eta_c+1.0)));
                                       c1 = 0.5*((y1+y2)-betagen*(y2-y1));
                                       c2 = 0.5*((y1+y2)+betagen*(y2-y1));
              }
              else
              {
                                       value=1/(2-2*rand_2);
                betagen = pow (value,(1.0/(eta_c+1.0)));
                                       c1 = 0.5*((y1+y2)-betagen*(y2-y1));
                                       c2 = 0.5*((y1+y2)+betagen*(y2-y1));
              }

              if (c1<ymin) then c1=ymin;
              if (c2<ymin) then c2=ymin;
              if (c1>ymax) then c1=ymax;
              if (c2>ymax) then c2=ymax;
        generate a random number with the range between 0 and 1: rand_3
              if (rand_3<=0.5)
              {
                 child1.C[i] = c2;
                 child2.C[i] = c1;
              }
              else
              {
                 child1.C[i] = c1;
                 child2.C[i] = c2;
              }
           }
           else
           {
              child1.C[i] = parent1.P[i];
              child2.C[i] = parent2.P[i];
           }
        }
        else
        {
           child1.C[i] = parent1.P[i];
           child2.C[i] = parent2.P[i];
        }
     end for i
  }
   else
  {
    for each i=0, i is the serial number of the decision variables
    {
      child1.C[i] = parent1.P[i];
      child2.C[i] = parent2.P[i];
    } end for i
  }
```

Appendix 2

```
/* Procedure for real polynomial mutation of an individual */
//pm is mutation probability
// parent is designated as the parent population
// P[j] means the j-th decision variable
// ymin means the minimum in decision variables P[j]
// ymax means the maximum in decision variables P[j]

      generate a random number with the range between 0 and 1: rand_1
      for each j, j is the serial number of decision variables
    if (rand_1 <= pm)
    {
      y = parent.P[j];
      mut_pow = 1.0/(eta_m+1.0);

     generate a random number with the range between 0 and 1: rand_2
      if (rand_2 <= 0.5)
      {
        val = 2.0*rand_2;
        deltagen = pow(val,mut_pow) – 1.0;
      }
      else
      {
        val =1/(2.0*(1.0-rand));
        deltagen = 1.0 – (pow(val,mut_pow));
      }
      y = y + deltagen*(ymax-ymin);
      if (y<ymin) then y = ymin
      if (y>ymax) then y = ymax
      parent.P[j] = y;
    } end for j
```

Appendix 3

The following table shows Pareto optimal solutions in decision space {T_1, T_1,..., T_N} after 100th generation.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
S1	59.99	449.99	28.93	65.83	47.86	48.32	96.47	9.00	15.62	18.07	18.09
S2	31.42	300.13	13.25	36.08	39.01	39.06	60.14	9.63	15.29	18.03	18.07
S3	34.03	432.58	13.20	36.02	39.12	39.42	60.63	9.91	15.02	18.94	19.28
S4	59.94	449.91	29.93	65.93	45.54	41.87	61.39	9.20	15.62	18.09	18.09
S5	59.18	450.00	12.98	65.99	39.09	39.01	70.15	9.01	15.00	18.01	19.36
S6	59.99	449.99	13.03	64.58	39.67	39.00	61.38	9.20	15.01	18.08	18.10
S7	59.51	323.01	13.33	36.20	39.14	39.38	65.87	9.93	15.06	19.23	19.52
S8	34.03	316.91	13.25	37.07	39.04	39.01	60.63	9.91	15.16	18.06	18.07
S9	59.94	449.92	29.67	65.99	39.07	39.30	61.40	9.20	15.03	18.08	18.10
S10	59.93	450.00	12.98	65.99	39.01	39.41	89.24	9.01	15.00	18.01	19.46
S11	33.91	322.70	13.18	36.16	39.04	39.38	61.36	9.90	15.05	18.06	18.07
S12	59.58	316.91	15.97	37.07	39.01	39.20	60.18	9.78	15.16	19.07	21.50
S13	59.99	449.99	28.42	65.47	39.29	48.32	96.52	9.00	15.65	18.24	18.09
S14	31.62	449.95	12.13	36.00	39.13	39.02	60.84	9.06	15.09	18.27	18.19
S15	59.93	391.56	12.09	38.13	39.33	39.41	61.32	9.08	15.08	18.52	19.30
S16	59.86	449.98	29.26	63.99	39.04	39.26	100.78	9.20	15.16	18.08	18.05
S17	57.86	301.65	13.28	36.09	39.02	39.06	61.81	9.08	15.61	18.03	18.07
S18	31.42	304.98	13.25	37.45	39.84	39.06	60.14	9.03	15.29	18.47	18.07
S19	59.83	345.95	12.62	36.22	39.04	39.64	60.81	9.45	15.72	18.48	18.01
S20	59.93	449.98	13.11	65.99	39.00	39.01	100.57	13.45	17.10	18.08	20.47
S21	59.93	450.00	13.11	65.99	41.56	39.40	91.08	9.01	15.00	18.01	18.02
S22	59.93	449.98	12.98	65.99	39.00	39.41	100.57	13.45	15.01	18.08	19.38
S23	59.93	391.56	12.09	36.00	39.33	39.41	61.32	9.08	15.08	18.52	19.30
S24	59.47	430.37	12.98	36.00	39.00	39.01	61.24	9.11	15.80	18.09	18.10
S25	59.94	369.12	12.60	36.00	46.16	39.41	61.39	9.08	15.08	18.53	19.30
S26	59.47	430.27	12.98	36.00	39.08	39.01	61.24	9.11	15.77	18.01	18.23
S27	59.99	449.99	13.04	38.30	41.30	39.17	61.38	9.20	16.27	18.00	18.09
S28	59.16	443.83	13.06	36.18	39.10	39.02	61.40	10.11	15.28	18.47	18.04
S29	59.95	302.34	13.06	40.56	39.13	42.65	61.98	9.09	16.65	18.70	19.39
S30	59.95	302.44	13.06	36.14	39.02	42.65	60.08	9.09	16.65	18.68	18.07
S31	59.93	450.00	15.99	65.99	39.00	39.66	95.13	9.07	15.17	18.97	19.41
S32	59.96	447.44	12.98	37.03	39.09	39.01	60.65	9.01	15.20	18.09	18.10
S33	59.94	377.00	13.26	37.19	39.06	39.69	64.94	9.03	15.27	18.94	18.00
S34	59.16	443.83	13.06	36.18	39.10	39.02	61.40	10.11	15.28	18.47	18.04
S35	59.97	378.71	13.06	43.86	39.14	39.05	60.83	9.01	15.03	18.28	18.07
S36	59.93	450.00	15.99	65.99	39.00	39.66	95.13	9.07	15.17	18.97	19.41
S37	59.93	450.00	15.79	42.45	39.03	39.67	61.34	9.07	15.18	18.94	19.54

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
S38	58.92	300.77	13.25	36.96	39.02	39.06	61.99	9.89	15.68	18.03	18.12
S39	59.25	446.59	13.04	62.49	39.28	39.01	61.38	9.13	15.03	18.08	18.10
S40	59.95	349.86	13.06	40.56	39.67	39.16	61.44	9.21	15.00	18.62	18.17

Note: A means activity, and S means solution; A1– Substructure; A2 – Superstructure; A3 – Carpenter joinery; A4 – Window glazing, metal works; A5 – Tiling; A6 – Painting; A7 – Plumbing and drainage; A8 – Electrical; A9 – Lift; A10 – Fire service installation; and A11 – Preliminaries.

PARETO DAUGIATIKSLIS OPTIMIZAVIMO METODAS KOMPROMISINEI LAIKO, IŠLAIDŲ IR KOKYBĖS PROBLEMAI SPRĘSTI

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Santrauka. Projektų eigos skubinimas neabejotinai padidina išlaidas ir sumažina kokybę. Ankstesni tyrimai daugiausiai buvo susiję su projektų planavimu ir kontrole, kuriuose daugiausiai dėmesio skiriama išlaidų ir laiko kompromisui. Tačiau tik nedaugelyje tyrimų buvo matematiškai nagrinėjama projekto kokybė. Šiame straipsnyje formuluojama laiko, išlaidų ir kokybės kompromiso problema. Jai spręsti sukurtas kompiuterizuotas Pareto daugiatikslio optimizavimo metodas. Pareto metodas gali pagelbėti ieškant geriausio sprendinio netgi neturint informacijos apie suinteresuotų šalių pirmenybę laiko, išlaidų ir kokybės atžvilgiu. Toks metodas palengvina sprendimų priėmimo procesą.

Reikšminiai žodžiai: laikas, išlaidos ir kokybė, kompromisas, Pareto optimalumo aibė, projektų valdymas, genetinis algoritmas.

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