Technological and Economic Development of Economy
ISSN 2029-4913 / eISSN 2029-4921
2018 Volume 24 Issue 4: 1679–1694
https://doi.org/10.3846/20294913.2016.1275877

TOTAL AREA BASED ON ORTHOGONAL VECTORS (TAOV) AS A NOVEL METHOD OF MULTI-CRITERIA DECISION AID

Seyed Hossein RAZAVI HAJIAGHA[1], Hannan AMOOZAD MAHDIRAJI[2*], Shide Sadat HASHEMI[3]

[1]Department of Management, Faculty of Management and Finance, Khatam University, Tehran, Iran
[2]Faculty of Management, University of Tehran, Tehran, Iran
[3]Department of Management, Saramadan Andisheh Avina Co., Tehran, Iran

Received 10 May 2016; accepted 19 December 2016

Abstract. Multi criteria decision aid (MCDA) deals with the problem of evaluating a set of finite alternatives regard to a set of finite criteria. A remarkable volume of qualitative and quantitative researches are done on decision making methods and situations, indicating its important role for managers at different organizational levels. These types of problems are applied in many different fields of human life. A challenging feature of these problems is non-existence of an optimal solution due to considering multiple criteria and the proposed methods seeking to find a satisfactory solution called efficient of Pareto-optimal. In consideration of MCDA problem, in this paper a new method is proposed for solving DM problems, consisting three fundamental steps of initialization, orthogonalization, and comparison. Thus, a new MCDA method called *total area based on orthogonal vectors* (TAOV) is introduced. This method is constructed on orthogonality of decision criteria. Application of TAOV method is illustrated in a decision problem and its performance is evaluated regard to other MCDA methods. Furthermore, its features are explained around the features of a desirable MCDA method. The obtained results indicate that the TAOV method can be considered as an acceptable method of handling multi-criteria decision making problems.

Keywords: decision making, Pareto-optimal, multi-criteria decision aid, orthogonality, principal component analysis, TAOV.

JEL Classification: C02, C44, M19.

Introduction

Decision making (DM) problems differs from studying in various schools of selecting an appropriate supplier or a human resource manager. A remarkable volume of qualitative and quantitative researches are done on DM methods and situations, indicating its important role for managers at different organizational levels. Choosing the best way to assign resources reasonably and obtain benefits for corporate and the employees is the main function of DM for managers (Hashemi *et al.* 2016).

*Corresponding author. E-mail: *h.amoozad@ut.ac.ir*

Selecting a solution based on one criteria is not sufficient and may lead to risky solutions. The real world problems are usually complex and it is impossible to achieve an optimal decision considering only a single criterion (Zavadskas *et al.* 2014). The Multi Attribute Decision Making methods (MADM) are developed to evaluate the decision alternatives considering several criteria simultaneously and selecting the best solution. The role of MADM methods has been approved during recent decades in various fields. MADM methods in general, are very useful in many problems such as project selection, supplier selection, risk assessment, contractor evaluation, etc. Many studies have been made on MADM methods and applications (Keshavarz Ghorabaee *et al.* 2015). Some valuable and extensive reviews on MADM techniques can be found in Hwang and Yoon (1981), Tzeng and Huang (2011), and Köksalan *et al.* (2011).

MADM provides an effective tool for comparison, based on the evaluation of multiple conflict criteria. Different types of MADM methods have grown very fast among different areas of operational research by virtue of it is often found that many concrete problems can be represented by several conflicting criteria (Hashemi *et al.* 2016).

One of the first structured studies in the field of scientific decision theory is von Neumann – Morgenstern utility theory (Neumann, Morgenstern 1953) introducing the notion of utility theory in human decisions. Later, multi attribute utility theory (MAUT) is proposed to capture decision makers' utility about an alternative, upon different attributes (Keeney, Raiffa 1976). MAUT applied extensively in DM problems (Torrance *et al.* 1982; Min 1994). Considering axioms of MAUT, several methods are proposed for approximating decision makers' utilities. Hwang and Yoon (1981) called MAUT based techniques as scoring methods. Churchman and Ackoff (1954) introduced simple additive weighting (SAW), Saaty (1980) proposed analytic hierarchy process (AHP) for simple hierarchical problems and later extended it to network structures, known as analytic network process (ANP) (Saaty, Vargas 2006). ANP is a generalization of AHP considering the dependence between the elements of hierarchy. Actually, it considers the interactions among decision elements while making decision. Therefore, it is proposed on the basis of a network rather than a hierarchy. It is useful for dealing with complex decisions involving interdependent elements including some feedbacks among them in the context of benefit, cost, etc. Furthermore, it leads to valid results in real practice (Saaty 2005). Sivilevicius *et al.* (2008) applied additive assessment as a multi-criteria decision making method. Turskis *et al.* (2009) proposed a six step-algorithm including normalization, choosing an optimum criterion, and evaluating the alternatives for solving MADM problems. Peldschus *et al.* (2010) inspired game theory framework in solving multi-criteria decision making problems. Zavadskas *et al.* (2012) introduced weighted aggregated sum product assessment (WASPAS) method for evaluating a number of alternatives in terms of a number of decision criteria. They applied a joint method of Weighted Product Model (WPM) and Weighted Sum Model (WSM), two well-known methods, to create WASPAS to increase the accuracy of decision making. Accuracy of two rather well known methods and the accuracy of aggregating both methods is analyzed, demonstrating that accuracy of aggregated method is larger, comparing to accuracy of single one (Zavadskas *et al.* 2012). Recently, Best-Worst Method (BWM) originated by Rezaei (2015), deriving the weights upon pairwise comparison of the best and worst alternatives regarding other alternatives. Moreover, the resulted weights are more consistent than AHP results consequently combinable with other multi criteria decision making (MCDM) methods (Rezaei 2015). Beyond usual additive utility functions, Zavadskas *et al.* (2009) applied multiplicative utility function in decision making problems.

During these decades, another type of MCDM methods called as Compromise programming is introduced. In this regard, many methods are recommended. Zeleny (1974) proposed the notion of compromise programming to deal with multiple non-commensurable objectives in order to find a set of non-dominated solutions. Following the concept of compromise solution, Hwang and Yoon (1981) developed the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) to identify solutions from a finite set of alternatives. In this method, the desired solutions should indicate the shortest distance from positive ideal solution and simultaneously the farthest distance from the negative ideal solution. Zavadskas *et al.* (1994) proposed the COPRAS method as a compromising method evaluating the alternatives on the ratio to the ideal solution and the ratio to the anti-ideal solution. The VIKOR method is another MCDM method introduced by Opricovic (1998) as a multicriteria optimization of complex systems based on ranking and selecting the options from a set of alternatives against conflicting criteria. This method is widely employed in DM problems. Zavadskas and Turskis (2010) developed the ARAS (Additive Ratio Assessment) method capable for dealing with problems encompassing qualitative and quantitative criteria being hinged on different units of measurement and a different optimization direction. Brauers and Zavadskas (2006) presented the MOORA (Multi-Objective Optimization by Ratio Analysis) method. Later, Brauers and Zavadskas (2010) presented the MULTIMOORA method including MOORA plus the full multiplicative form.

The Outranking methods create the third category of MADM methods arose from European school of decision making techniques (Roy, Vanderpooten 1996). The problem is to build a preference relation on a set of multi-attribute alternatives on the basis of preferences expressed in each attribute and "inter attribute" information. Outranking methods were first developed in France in the late 60s. As a matter of fact, they were introduced following difficulties experienced with the value function approach in dealing with practical problems. The most well-known outranking methods are ELECTRE, ORESTE, and PROMETHEE (Bozbura *et al.* 2007). The ELECTRE method is one of the main outranking methods. The ELECTRE approach was introduced in 1968 (Roy 1968). The origin of ELECTRE methods refers to 1965 to the European consultancy company SEMA (Benayoun *et al.* 1966). Different versions of ELECTRE have been extended including ELECTRE I, IS, II, III, IV and TRI during recent decades, being comprehensively built on the same fundamental concepts (Marzouk 2011); however, might be different in way of defining the outranking relations between options and the way that they apply these relations to achieve the final ranking of the options (Wang, Triantaphyllou 2008). The ORESTE method was initially introduced by Roubens (1982). This method allows to rank the experiments in a complete order or in a partial order by considering incomparability. Abounding MCDM methods require some detailed information concerning different criteria's consisting of weights, order relation, preference functions, etc. In the vast majority of cases, it is pretty difficult to obtain this information in real world studies (Givescu 2007). Nonetheless, ORESTE deals with this situation where the alternatives are ranked against to criterion c_i, and the criteria themselves are ranked upon their importance (Dinçer 2011). PROMETHEE is different outranking method introduced by Brans (1982). A couple of years later, PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking) were developed consequently. About ten years coming after, two other versions of this method; PROMETHEE III (ranking based on intervals) and PROMETHEE IV (continuous case) were extended in order to solve the decision making problems (Albuquerque 2015). Some studies also applied hybrid methods including different techniques in real world applications (Zavadskas *et al.* 2013). Beyond the mentioned methods, there are a variety of different methods for solving MADM problems. Part of studies compared the performance of these methods. Saaty and Ergu (2015) reviewed the problem of choosing the best MADM method. They proposed a framework consist some features for a good MADM method including simplicity of execution, comprehensive structure, logical and mathematical procedure, justifiable axioms, scales of measurement, and ranking of tangibles and intangibles.

Usually, the above methods suppose that decision criteria are independent to avoid any over or underestimation of the scores. However, some researchers considered the problem of criteria independence. Antuchevičiene *et al.* (2010) proposed using Mahalanobis distance, considering correlation among criteria, in TOPSIS method. Wang, Z. and Wang, Y. (2014) improved classic TOPSIS method by proposing an improved relative closeness and using weighted Mahalanobis distance when criteria are correlated. Zhu *et al.* (2015) applied principal component analysis to eliminate correlation among criteria and transformed the initial decision matrix into an independent decision matrix. Then, the TOPSIS method is applied over the new obtained matrix.

In consideration of MADM problem, in this paper a new method is proposed for solving DM problems, consisting three fundamental steps of initialization, orthogonalization, and comparison which will be detailed in the next sections. To this end, the proposed method, entitled *total area based on orthogonal vectors* is detailed in section 2. A numerical example is solved with the proposed method in section 3 and the obtained result is compared with different methods. Finally, the advantages of the method on conclusion remarks are explained in section 4.

1. Total area based on orthogonal vectors (TAOV) method

Consider a multi-criteria decision making problem consist evaluating *m* discrete alternatives $A_1, A_2, \ldots, A_m$ based on *n* finite criteria of $C_1, C_2, \ldots, C_n$. Furthermore, the criteria weight vector $w = \left(w_1, w_2, \ldots, w_n\right)$ can be determined by using different methods, incorporating pairwise comparison (Saaty 1980), Entropy (Hwang, Yoon 1981), LINMAP (Srinivasan, Shocker 1973), SWARA (Kersuliene *et al.* 2010) or FARE (Ginevicius 2011).

Regarding the aforementioned situation, the problem can be formulated in the following form of a decision matrix:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}. \quad (1)$$

Where, x_{ij} is the performance of alternative $A_i, i=1,2,\ldots,m$ with regard to criterion C_j, $j=1,2,\ldots,n$. The first step of the proposed TAOV method, similar to many other MCDM methods, is to normalize decision matrix. Considering a monotonic increasing utility for decision criteria, the set of benefit criteria, B, i.e. the more is better, is normalized as:

$$r_{ij} = \frac{x_{ij}}{\max\limits_i x_{ij}}, j \in B. \tag{2}$$

For cost type criteria, C, i.e. the less is better, normalization is completed as below:

$$r_{ij} = \frac{\min\limits_i x_{ij}}{x_{ij}}, j \in C. \tag{3}$$

Subsequently, the normalized decision matrix R is constituted:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}. \tag{4}$$

The weighted normalized matrix $WN = \left[\overline{r_{ij}}\right]$ is afterwards constructed by multiplying each normalized element n_{ij} in its corresponding criterion weight w_j, i.e. $\overline{n_{ij}} = w_j \cdot n_{ij}$. Therefore,

$$WN = \begin{bmatrix} \overline{r_{11}} & \overline{r_{12}} & \cdots & \overline{r_{1n}} \\ \overline{r_{21}} & \overline{r_{22}} & \cdots & \overline{r_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{r_{m1}} & \overline{r_{m2}} & \cdots & \overline{r_{mn}} \end{bmatrix}. \tag{5}$$

It is worth noting here that a critical question in this phase is whether to assure that columns of matrix *WN* are not correlated with each other. Considering any two columns of $\overline{r_k} = \left(\overline{r_{1k}}, \overline{r_{2k}}, \ldots, \overline{r_{mk}}\right)$ and $r_l = \left(\overline{r_{1l}}, \overline{r_{2l}}, \ldots, \overline{r_{ml}}\right)$ of the above matrix, are they independent of each other or not? To assure this independency of weighted normalized decision matrix, since TAOV method acts on orthogonal vectors, the next step aims to transform current criteria vectors of $C_1, C_2, \ldots, C_n$ to an orthogonal vector of $Y_1, Y_2, \ldots, Y_n$. To find this orthogonal vector, principal component analysis (PCA) is applied on matrix *WN*. PCA finds a linear combination of vectors called principal components being independent. Each principal component, y_j, is a linear combination of vectors $\left(\overline{r_1}, \overline{r_2}, \ldots, \overline{r_n}\right)$ (Jolliffe 2013), i.e.

$$Y^T = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = AY^t = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix} \begin{bmatrix} \overline{r_1} \\ \overline{r_2} \\ \vdots \\ \overline{r_n} \end{bmatrix}. \tag{6}$$

Where, A includes the weights used in PCA to transform correlated criteria into independent ones. The PCA also can be fixed easily by SPSS package. Performing PCA, the values of each alternative can be found easily in the corresponding components. i.e. for an alternative A_i, its score in component y_j, illustrated as y_{ij}, can be calculated as $y_{ij} = a_{j1}x_{i1} + a_{j2}x_{i2} + \ldots + a_{jn}x_{in}$, where $\left(a_{j1}, a_{j2}, \ldots, a_{jn}\right)$ are the coefficients of variables in jth component.

Now, suppose that PCA is applied on *WN* matrix, using SPSS package, and the orthogonal decision matrix Y is computed:

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix}. \tag{7}$$

Remark that columns of matrix Y are orthogonal; thus, distance between different criteria can be calculated by Euclidean distance (according to Pythagorean Theorem). Consider an alternative A_i, whose coordinate in two orthogonal components y_k and y_l are $(0,0,\ldots,0,y_{ik},0,\ldots,0)$ and $(0,0,\ldots,0,y_{il},0,\ldots,0)$. Then the distant between these two points is:

$$d_{k,l}^i = \sqrt{y_{ik}^2 + y_{il}^2}. \tag{8}$$

Figure 1 illustrates the geometric interpretation of $d_{k,l}^i$.

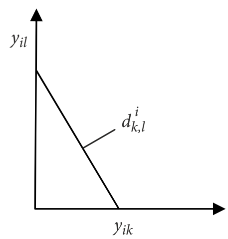

Figure 1. Geometric interpretation of $d_{k,l}^i$

Suppose that performance of an alternative on criteria is ordered according to indices of criteria. i.e. for alternative A_i as $(y_{i1}, y_{i2}, \ldots, y_{in})$. Next, total area of alternative A_i is computed as:

$$TA_i = \sum_{j=1}^{n-1} d_{j,j+1}^i. \tag{9}$$

Later, alternatives can be ranked according to their corresponding total area measure. If decision maker intends to a desirability measure, the normalized total area measure is defined as:

$$NTA_i = \frac{TA_i}{\sum_{k=1}^{m} TA_k}. \tag{10}$$

1.1. TAOV algorithm

In this subsection, an algorithmic scheme is presented for TAOV method of MCDM. The TAOV algorithm is organized in three phases of (1) initialization, (2) orthogonalization, and (3) comparison.

Phase 1. Initialization

Step 1. Identify decision alternatives $A_1, A_2, \ldots, A_m$.

Step 2. Identify decision criteria $C_1, C_2, \ldots, C_n$.

Step 3. Construct decision matrix $X = \left[x_{ij} \right]$ as shown in Eq. (1).
Step 4. Determine the criteria weight vector $w = \left(w_1, w_2, \ldots, w_n \right)$ using one of the mentioned methods.
Step 5. Normalize decision matrix applying Eq. (2) for benefit criteria and Eq. (3) for cost criteria.
Step 6. Compute the weighted normalized decision matrix $WN = \left[\overline{r}_{ij} \right]$ as shown in Eq. (5).

Phase 2. Orthogonalization

Step 7. Applying principal component analysis, transforms the *WN* matrix to the equivalent matrix of components *Y* as shown in Eq. (7).

Phase 3. Comparison

Step 8. Find the total area of each alternative using Eq. (9).
Step 9. Determine the alternatives desirability base on their normalized total area using Eq. (10). Figure 2 describes the proposed method.

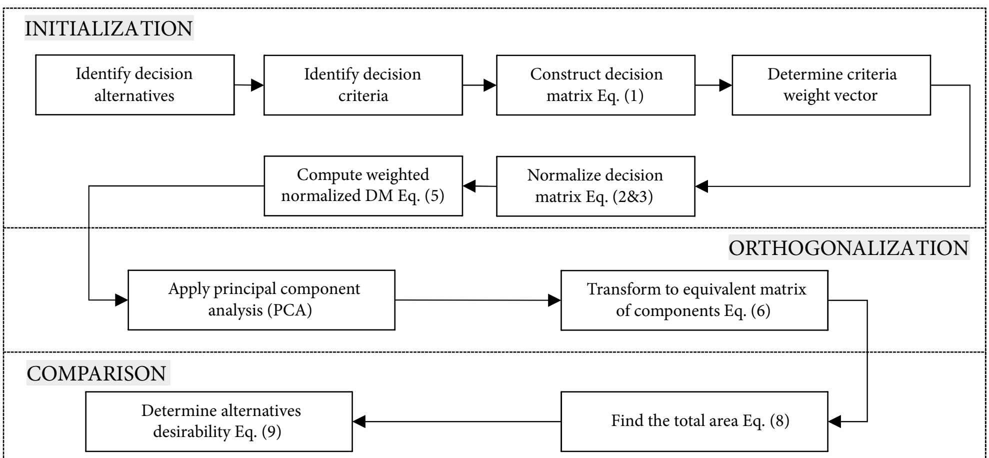

Figure 2. TAOV algorithm

2. Numerical example

In this section, a numerical example is solved employing TAOV method and the results are compared with other methods.

2.1. Problem description

Zavadskas and Turskis (2010) analyzed the following decision matrix in evaluating 14 rooms based on six criteria about appraising microclimate of rooms. The considered criteria's comprise:

X_1: The amount of air per head (benefit);

X_2: Relative air humidity (benefit);
X_3: Air temperature (benefit);
X_4: Illumination during work hours (benefit);
X_5: Rate of air flow (cost);
X_6: Dew point (cost).

The weight vector of criteria are determined as $w = (0.21, 0.16, 0.26, 0.17, 0.12, 0.08)$ using pairwise comparisons. Table 1 illustrates the constructed decision matrix.

Table 1. Decision matrix of the problem

Room No.	Criteria					
	X_1	X_2	X_3	X_4	X_5	X_6
1	7.6	46	18	390	0.1	11
2	5.5	32	21	360	0.05	11
3	5.3	32	21	290	0.05	11
4	5.7	37	19	270	0.05	9
5	4.2	38	19	240	0.1	8
6	4.4	38	19	260	0.1	8
7	3.9	42	16	270	0.1	5
8	7.9	44	20	400	0.05	6
9	8.1	44	20	380	0.05	6
10	4.5	46	18	320	0.1	7
11	5.7	48	20	320	0.05	11
12	5.2	48	20	310	0.05	11
13	7.1	49	19	280	0.1	12
14	6.9	50	16	250	0.05	10

2.2. Solving with TAOV method

In this subsection, the above problem is solved based on new TAOV method.

Phase 1. Initialization. The first three steps of this phase are summarized at the above decision matrix and the criteria weight vector w is also represented. At the 4^{th} step, decision matrix in Table 1 is normalized applying Eq. (2) on benefit criteria X_1, X_2, X_3 and X_4 and Eq. (3) on cost criteria of X_5 and X_6. Subsequently, the constructed normalized matrix elements are multiplied in the corresponding criterion weight. The weighted normalized matrix is formed and presented in Table 2.

Phase 2. Orthogonalization. At this step, principal component analysis is applied on Table 2. The principal component coefficients are demonstrated in Table 3 for each variable, applying SPSS package.

The orthogonal decision matrix Y is obtained by multiplying two matrices illuminated in Tables 2 and 3. This matrix is constructed as follows (Table 4).

Table 2. Weighted normalized matrix

Room No.	Criteria					
	X_1	X_2	X_3	X_4	X_5	X_6
1	0.1970	0.1472	0.2229	0.1658	0.0600	0.0364
2	0.1426	0.1024	0.2600	0.1530	0.1200	0.0364
3	0.1374	0.1024	0.2600	0.1233	0.1200	0.0364
4	0.1478	0.1184	0.2352	0.1148	0.1200	0.0444
5	0.1089	0.1216	0.2352	0.1020	0.0600	0.0500
6	0.1141	0.1216	0.2352	0.1105	0.0600	0.0500
7	0.1011	0.1344	0.1981	0.1148	0.0600	0.0800
8	0.2048	0.1408	0.2476	0.1700	0.1200	0.0667
9	0.2100	0.1408	0.2476	0.1615	0.1200	0.0667
10	0.1167	0.1472	0.2229	0.1360	0.0600	0.0571
11	0.1478	0.1536	0.2476	0.1360	0.1200	0.0364
12	0.1348	0.1536	0.2476	0.1318	0.1200	0.0364
13	0.1841	0.1568	0.2352	0.1190	0.0600	0.0333
14	0.1789	0.1600	0.1981	0.1063	0.1200	0.0400

Table 3. Component coefficient matrix

Component	Variable					
	1	2	3	4	5	6
X_1	0.709	0.589	−0.078	−0.026	−0.335	0.180
X_2	−0.038	0.847	−0.397	0.050	0.343	0.060
X_3	0.720	−0.566	0.079	−0.209	0.240	0.232
X_4	0.748	0.343	0.403	−0.302	0.084	−0.250
X_5	0.726	−0.185	−0.078	0.649	0.050	−0.089
X_6	−0.309	0.323	0.847	0.244	0.072	0.134

Table 4. Orthogonal decision matrix

Room No.	Criteria					
	X_1	X_2	X_3	X_4	X_5	X_6
1	0.4509	0.1721	0.0368	−0.0466	0.0575	0.0541
2	0.4747	0.0656	0.0519	−0.0124	0.0712	0.0481
3	0.4488	0.0524	0.0403	−0.0033	0.0705	0.0546
4	0.4289	0.0857	0.0346	0.0070	0.0664	0.0548
5	0.3463	0.0741	0.0406	−0.0256	0.0768	0.0573
6	0.3564	0.0800	0.0436	−0.0283	0.0758	0.0561

End of Table 4

Room No.	Criteria					
	X_1	X_2	X_3	X_4	X_5	X_6
7	0.3139	0.1154	0.0638	–0.0135	0.0782	0.0489
8	0.5118	0.1574	0.0633	–0.0072	0.0642	0.0585
9	0.5091	0.1575	0.0595	–0.0048	0.0617	0.0616
10	0.3653	0.1212	0.0486	–0.0305	0.0834	0.0499
11	0.4548	0.1132	0.0233	–0.0022	0.0826	0.0535
12	0.4425	0.1041	0.0227	–0.0006	0.0866	0.0522
13	0.4162	0.1486	0.0135	–0.0350	0.0640	0.0665
14	0.4177	0.1559	0.0055	0.0175	0.0603	0.0559
Ideal	0.5118	0.1721	0.0638	0.0175	0.0866	0.0665

Phase 3. Comparison. Subsequently, by applying Eq. (9), the TA measures are figured out. As a case in point, TA_1 is calculated as:

$$TA_1 = \Big[\sqrt{0.4509^2 + 0.1721^2} + \sqrt{0.1721^2 + 0.0368^2} + \sqrt{0.0368^2 + (-0.0466)^2} + \sqrt{(-0.0466)^2 + 0.0575^2} + \sqrt{0.0575^2 + 0.0541^2} \Big] = 0.8710.$$

Similarly, TA measures are performed for all alternatives. The final ranking of alternatives is obtained by arranging TA values in a decreasing manner. Table 5 illustrates the values of TA measure, along with the obtained ranking of alternatives.

Table 5. Total area measure

Room No.	TA_i	NTA_i	Ranking
1	0.8710	0.8823	3
2	0.7745	0.7845	6
3	0.7181	0.7274	10
4	0.7179	0.7273	11
5	0.6634	0.6721	14
6	0.6836	0.6925	13
7	0.7030	0.7121	12
8	0.9203	0.9323	1
9	0.9102	0.9220	2
10	0.7588	0.7687	9
11	0.7888	0.7991	5
12	0.7716	0.7816	7
13	0.7937	0.8041	4
14	0.7652	0.7752	8
Ideal	0.9872	1.0000	

2.3. Analyzing the results

To verify the acceptability of the TOAF method, in this subsection the aforementioned problem is solved with a variety of models. Table 6 illustrates the obtained rankings of different methods.

Table 6. Ranking of alternatives with different methods

Room No.	TAOV	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	ELECTREE
1	3	4	3	3	3	6	4	3
2	6	6	5	7	7	5	6	4
3	10	10	8	10	10	8	10	6
4	11	9	10	9	9	9	9	9
5	14	14	13	13	13	13	14	12
6	13	13	12	12	12	12	12	13
7	12	12	14	14	14	14	13	14
8	1	2	1	1	1	1	1	1
9	2	1	2	2	2	2	2	2
10	9	11	11	11	11	10	11	10
11	5	3	4	4	4	3	3	5
12	7	5	6	8	5	4	5	7
13	4	8	7	6	8	7	8	8
14	8	7	9	5	6	11	7	11

First of all, the Kendall's coefficient of concordance for the above results is equal to 0.935 indicating a great amount of concordance among different ranking. Additionally, the pairwise correlation coefficients are computed for different methods. Since the considered values are equal to ranks of alternatives, the Spearman's rank correlation is gauged between each pair in Table 7. The numbers in parenthesis delineate the percentage of matched rank between each pairs.

Table 7. Ranking similarity for different methods

	TAOV	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	ELECTREE
TAOV	–	0.92	0.94	0.93	0.91	0.88	0.92	0.87
ARAS		–	0.95	0.94	0.97	0.92	0.99	0.88
SAW			–	0.93	0.96	0.96	0.96	0.96
TOPSIS				–	0.97	0.84	0.95	0.85
COPRAS					–	0.90	0.99	0.87
VIKOR						–	0.93	0.93
WASPAS							–	0.89
ELECTREE								–

Considering TAOV, its correlation with other methods seems acceptable. Afterwards, the results in Table 4 are aggregated using Copeland method. The obtained result is: $8 > 9 > 1 > 11 > 2 > 12 > 13 \sim 14 > 3 \sim 4 > 10 > 6 > 5 > 7$. The Spearman's correlation among different methods with this aggregated one is presented at Table 8.

Table 8. Similarity with aggregated ranking method

	TAOV	ARAS	SAW	TOPSIS	COPRAS	VIKOR	WASPAS	ELECTREE
Aggregated	0.94	0.96	0.98	0.96	0.97	0.93	0.97	0.93

In view of the results of Tables 7 and 8, it implies that the results of new TAOV method have a great concordance with previous methods. According to Table 7, the least correlation of the TAOV method points 87% (with ELECTREE), while this method exhibits a correlation more than 90% with other methods. Moreover, TAOV method has a correlation of 94% with aggregated decision. All in all, it can be concluded that the TAOV method results can be considered acceptable.

Conclusions

Considering the importance of decision making in social and professional life, and necessity of making non-risky and well decisions in these areas, it can be justified the presence of a wide range of MADM methods in the literature. With this fact in mind, a new method called TAOV is introduced in this paper. This method is based upon the notion that for performing Pythagorean Theorem in computing distance between two points, their vectors should essentially act orthogonal to each other. Hence, employing principal component analysis, the classic decision matrix converted to an orthogonal decision matrix. Next, the total area of each alternative was computed as its overall performance over other alternatives. Consequently, A numerical example consists of evaluating 14 alternatives with regard to 6 criteria was solved with TAOV method and the obtained results were compared with seven different and well-known methods. The performance of TAOV method related to those methods indicated acceptable results.

Considering the features of a prominent decision making method, as proposed by Saaty and Ergu (2015), the following conclusions can be made regarding TAOV.

1) Simplicity of execution. Considering ease of use as a criterion for choosing an MADM method, the TAOV method's procedure can be simply performed for decision making.
2) Comprehensive structure: breadth and depth. Saaty and Ergu (2015) called an MADM method as broad if it contains a number of distinct criteria and deep if criteria can broke down to sub-criteria. It is clear that the TAOV method is broad since it contains several distinct criteria. Besides, using factor analysis method along with PCA, the structure of method can be broke down into sub-criteria.
3) Comprehensive structure consisting of merit substructures. A decision structure is said to be comprehensive if it represents a decision problem considering different

political, social, economic, legal, and etc. criteria. Considering the TAOV method, there isn't any limitation to define different criteria.

4) Logical, mathematical procedure. The TAOV method follows a logical procedure for solving the MADM problem.
5) Justification of the approach – justifiable axioms. The main axiom of the TAOV method declares that decision criteria must be independent of each other, since the additive utility function can be employed. Thus, it uses PCA to assure that decision criteria are independent.
6) Scales of measurement. The TAOV method doesn't have any limitation regarding the scale used for quantification of problem's criteria.
7) Synthesis of judgments with merging functions. For group decision making, the individual decision matrices can be synthesized by weighted averaging method. Hence, according to Saaty and Ergu (2015), the TAOV method ranked medium in this criterion.
8) Ranking of tangibles. Since the TAOV method provides a cardinal ranking of alternatives, it ranked high in this criterion.
9) Generalization to ranking of intangibles. The TAOV method rated low in this criterion since the intangible factors are simply assigned by arbitrary ordinal numbers.
10) Rank preservation and reversal. As illustrated in the numerical example, the TAOV method has an acceptable consistency with different methods.
11) Sensitivity analysis. The TAOV method includes criteria weighting vector and alternatives performance as its input parameters. Therefore, it rated medium in this criterion.
12) Validation of decision problems. The TAOV method is applicable in real-world situations with tangible and intangible factors.
13) Generalizability to dependence and feedback. Since the TAOV method used PCA to transform the initial decision matrix to an orthogonal decision matrix without any dependence; therefore, any form of dependence and feedback can be analyzed with this method.
14) Applicability to conflict resolution. By reason of TAOV method used weighted averaging to aggregate individual decision matrices, the method rated low in this criterion.
15) Trustworthiness and validity of the approach. The TAOV method deals with cardinal measurements with a mathematical logical procedure. Accordingly, it rated medium in this criterion.

Table 9 summarizes the performance of TAOV method regarding aforementioned criteria's.

In summary, the main advantage of the proposed method is that since current decision making methods usually are developed based on additive utility function, the independence of criteria is required for a better performance of these methods. While other methods suppose that this independency is established, the proposed method provides a formal method which guarantees this independency. This can be considered as the main advantage of the

Table 9. Performance of TAOV

	Criterion	TAOV method rated
1	Simplicity of execution	High
2	Comprehensive structure: breadth and depth	High
3	Comprehensive structure consisting of merit substructures	High
4	Logical, mathematical procedure	High
5	Justification of the approach – justifiable axioms	High
6	Scales of measurement	High
7	Synthesis of judgments with merging functions	Medium
8	Ranking of tangibles	High
9	Generalization to ranking of intangibles	Low
10	Rank preservation and reversal	High
11	Sensitivity analysis	Medium
12	Validation of decision problems	Medium
13	Generalizability to dependence and feedback	High
14	Applicability to conflict resolution	Low
15	Trustworthiness and validity of the approach	Medium

proposed method that adjusts its application. In fact, using PCA, any interrelationship among criteria is eliminated and neither over- nor underestimation of overall performance of alternatives is occurred. Considering the above table and examination of the numerical example results, it seems that TAOV method is an acceptable MADM method both practically and theoretically. Therefore, the method can be used as an MADM method for solving the real world problems without any concern regarding the dependence of decision criteria.

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