

UNDERSTANDING MATERIAL DEPRIVATION IN SPANISH HOUSEHOLDS USING THE OWA AGGREGATION OPERATOR AND 2-TUPLE LINGUISTIC MODEL

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Abstract. Poverty has long been defined by income, but Sen (1995) broadened it to include deprivation of basic capabilities that prevent a decent life. This perspective has reshaped poverty assessment, leading institutions such as the European Union's Social Protection Committee and Eurostat to adopt household deprivation indicators to capture poverty and social exclusion. Such deprivation is usually measured by the lack of a certain quantity of essential goods or services, though further study is needed on how demanding decision-makers define poverty and social exclusion. Therefore, this work aims to define various scenarios of material deprivation, considering how demanding decision-makers are when measuring such deprivation. It employs three methods: (i) the Ordered Weighted Averaging (OWA) aggregation operator to simulate decision-makers' attitudes, (ii) the 2-tuple linguistic model to improve the interpretability of material deprivation results without information loss, and (iii) hierarchical clustering to summarize material deprivation in Spanish households by region and degree of urbanization. Results show that households may be economically poor based on income, but still have access to basic goods and services necessary for a decent life. The proposed model offers a more comprehensive, dynamic measure of material deprivation, enabling clearer insights into regional disparities through multiple scenarios.

Keywords: poverty, material deprivation, ordered weighted averaging aggregation operator, linguistic quantifiers, decision-making.

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1. Introduction

Poverty is a multidimensional concept that can emerge due to several economic and social factors (Alkire et al., 2015; Ünver et al., 2025). Research on relative poverty has basically focused on monetary indicators such as income or expenditure. This perspective dates back to Townsend's (1979) pioneering work in "Poverty in the United Kingdom", which examined the causes, effects, and manifestations of poverty in the UK, as well as policies and strategies to address it. Inspired by this work and other subsequent studies, the Statistical Office of the European Union (Eurostat) defined poverty as the lack of sufficient income and resources to

achieve an acceptable standard of living in society, reflecting a close relationship between poverty and inequality (Eurostat, n.d.). In addition, variables such as marital status, education level, age, gender, household size, homeownership, and social assistance usage were found to have significant relationships with relative poverty (Doğan et al., 2025).

The capability approach developed by Sen (1993) offered a new understanding of poverty. Since then, many authors have argued that poverty should also be understood not merely as a lack of income, but as being deprived of the basic capabilities that people need to live the kind of life they would like (Sen, 1999; Schokkaert, 2008; Nussbaum, 2011). The concept of capability-based poverty (i.e., material deprivation) does not contradict the view that low income is a major cause of poverty. A lack of income can be a primary factor that deprives a person of basic capabilities. The concept of material deprivation recognizes the variation in living conditions among individuals with seemingly similar resources and identifies those unable to achieve a minimum standard of well-being, a situation that is closely related to social exclusion. Material deprivation of services such as education and health, or social participation, limits individual welfare and human development, and has implications for the design of public policies (Alkire & Deneulin, 2009; Alkire & Foster, 2011).

Although purely income-based poverty and material deprivation are distinct concepts, they can be used together to analyze different aspects of household and individual living conditions. Income-based poverty and material deprivation are clearly associated (Fusco et al., 2011); measurements of both concepts aim to identify groups or collectives in precarious situations, and both are targeted by social policies. European strategies, such as the Europe 2020 Strategy and the 2030 Agenda (European Commission, 2010; United Nations, n.d.), define specific goals to tackle poverty and social exclusion, aligned with the principles of human development and equity. Under the definition used in these strategies, individuals are considered to be in relative poverty when their incomes fall below the poverty threshold. This threshold is calculated annually based on the income distribution of the previous year. Following the criteria recommended by Eurostat, it is established at 60% of the median income per consumption unit of individuals (Instituto Nacional de Estadística [INE], 2023).

Furthermore, efforts have been made to measure material deprivation by assessing the proportion of the population living in households that lack a minimum number of items reflecting the household's economic capacity. According to Eurostat, severe material and social deprivation is defined as the enforced lack of at least 7 out of 13 items, including both household-level and individual-level indicators (Eurostat, n.d.). Specifically, the indicator is based on a list of 13 items, seven related to the household and six related to the individual. In this work, only the seven household-related items are considered. Following the criteria defined by the Spanish National Statistics Institute (INE), a household is considered materially deprived if it presents at least three deficiencies among these seven items (Domènech-Abella et al., 2025). However, a stricter standard is applied to define severe material deprivation, with the threshold set at four deficiencies. Many studies have explored approaches to defining and measuring material deprivation (Guio et al., 2009, 2012, 2016, 2017; Marlier et al., 2010; García-Pérez et al., 2017).

Analyzing material deprivation, whether severe or not, provides an understanding of household living conditions, allowing for a more precise assessment of material deficiencies and their impact on people's quality of life. Since 2010, the European Union (EU) has

required member states to calculate and monitor the At Risk of Poverty or Social Exclusion (AROPE) rate. This rate is essential for evaluating poverty and social exclusion in member countries and monitoring the EU 2030 targets. According to the definition revised by the Subgroup of Indicators of the European Union Social Protection Committee and Eurostat in 2021, the AROPE rate is defined as the population experiencing at least one of these three components: risk of poverty, severe material and social deprivation, and low employment intensity (INE, 2023). The 2021 revision prioritizes the latter two components, underscoring their significance. Eurostat gathers information from the member states to form the European Union Statistics on Income and Living Conditions (EU-SILC) database, facilitating the analysis and comparison of the AROPE rate among member states.

The EU-SILC survey is a cross-sectional and longitudinal survey conducted in all 27 EU member states, as well as in Albania, the United Kingdom, Norway, Switzerland, North Macedonia, Montenegro, Serbia, Turkey, and Iceland. It collects harmonized and comparable data on income, living conditions, and various demographic and socioeconomic characteristics at individual and household levels. The survey methodology is harmonized across participating countries, allowing for comparisons of their social and living conditions. For instance, the INE conducts the Living Conditions Survey to gather data on the income level, and the composition of poverty and social exclusion in Spain. This survey includes questions about a series of items to assess the material deprivation experienced by households and individuals (INE, 2023). This deprivation indicator is determined by counting and summing the number of items from the list provided that a person or their household cannot afford. Regarding low employment intensity, a new definition aligned with the EU 2030 targets refers to the number of individuals living in a household where working-age members worked less than 20% of their total combined work-time potential during the reference year (INE, 2023).

In the European context, numerous studies highlight the importance of using indicators to measure and visualize deprivation and social exclusion, with the goal of better understanding household living conditions (Verbunt & Guio, 2019; Israel & Spannagel, 2019; Notten & Guio, 2019; Fabrizi et al., 2023, 2025). In Spain, research has focused on child material deprivation (Moro-Egido & Navarro, 2023) and the determinants of material deprivation risk in households across various regions (Ayala et al., 2021; Mussida et al., 2023). Beyond Europe, these indicators have been used in Australia, Canada, New Zealand, and Hong Kong (China) to assess social welfare and the impact of public policies (Saunders & Wong, 2011; Perry, 2016; Saunders & Tang, 2019; Notten & Kaplan, 2021). These countries have recognized the importance of measuring and understanding material deprivation as a fundamental aspect of evaluating living conditions. The use of material deprivation indicators in these contexts demonstrates a growing global concern about addressing social and economic disparities.

Recent literature demonstrates the use of various models in research on material deprivation, such as regression models (Notten & Guio, 2019; Verbunt & Guio, 2019; Fabrizi & Mussida, 2020; Wolf et al., 2022; Moro-Egido & Navarro, 2023; Mussida et al., 2023; Šoltés et al., 2023; Calejari et al., 2024), the graded response model (Ferrão et al., 2021), the latent class model (Ayala et al., 2021), the two-parameter logistic item response model (Ferrão & Vieira, 2023), and dimensionality reduction models (Fabrizi et al., 2023). These tools enable researchers to better understand the complex interactions between different variables and

their impact on the socioeconomic conditions of households. However, to the best of our knowledge, no previous study has explored the possibility of measuring material deprivation based on how demanding decision-makers are when identifying such deprivation. Although the concept of deprivation is inherently normative, it is ultimately the attitudes of policymakers, social planners, and researchers that determine the final classification. These decision-makers should establish cut-off points that reflect political priorities, budgetary constraints, and varying socio-economic realities. Consequently, while the same material deprivation variables are used in this work to evaluate household conditions, adjusting how “demanding” the decision-maker is allows for the generation of different scenarios about material deprivation. Such adaptable measurements are crucial for accurately assessing material deprivation across different contexts, as the number of deprivation criteria met reflects the consideration of various factors and preferences. For instance, comparing European countries with African nations such as Somalia, Mozambique, and Niger highlights the need for contextual flexibility. Due to differences in economic development, poverty rates in these African countries are considerably higher, and households more frequently lack access to basic goods and services. In such cases, policymakers cannot apply a rigid standard; instead, they need to adjust the threshold used to identify material deprivation to generate realistic scenarios that reflect the specific conditions of each national context and support appropriate policy decisions.

Therefore, this work aims to define various scenarios of household material deprivation using the same material deprivation variables as the INE. The identification of material deprivation can be interpreted as a Multiple Criteria Decision Making (MCDM) problem, in which several deprivation indicators act as criteria, and a rule is required to determine the degree to which households should satisfy these criteria to be classified as deprived. While a household is materially deprived if it presents at least three deficiencies among these seven items, this work generates alternative scenarios by considering the varying levels of demand applied in measuring such deprivation. To this end, this work uses a three-step methodological approach:

- The Ordered Weighted Averaging (OWA) aggregation operator: The linguistic quantifiers of this operator are used to simulate different decision-makers’ attitudes regarding the deprivation indicators that must be present to classify a household as deprived. These quantifiers (e.g., Few, Some, Most) allow the modelling of alternative policy perspectives, ranging from less demanding approaches to stricter interpretations of deprivation, thereby extending the conventional “three-out-of-seven” rule.
- The 2-tuple linguistic model: This model is applied to improve the interpretability of the results while avoiding information loss. This representation allows the outcomes of the aggregation process to be expressed in linguistic terms that are easier to interpret for policymakers while preserving the numerical precision of the analysis.
- Hierarchical clustering: This approach is used to summarize patterns of material deprivation among Spanish households by region and degree of urbanization. By grouping territories with similar deprivation profiles, this technique facilitates the identification of structural differences across regions and supports the design of more targeted social policies.

To demonstrate the practicality of the proposed model, it is applied to microdata from the 2023 Living Conditions Survey conducted by the INE to identify material deprivation in economically disadvantaged households. To the best of our knowledge, this is the first work to combine these models in research on material deprivation. This work proposes a novel, complementary perspective by modelling how household material deprivation may change depending on how demanding decision-makers are when identifying such deprivation, or on their attitudes toward material deprivation. In doing so, this work contributes to the literature on material deprivation measurement by providing a flexible analytical framework that allows researchers and policymakers to explore how deprivation patterns change under different measurement scenarios. By allowing the exploration of multiple scenarios rather than relying on a single fixed threshold, the proposed approach offers a broader perspective on the identification and analysis of materially deprived households.

The rest of this work is structured as follows: Section 2 reviews the literature on material deprivation, surveying studies conducted from 2019 to 2024. Section 3 introduces several fundamental concepts on which the proposed model is based. Section 4 demonstrates the application of the proposed model to microdata from the 2023 Living Conditions Survey conducted by the INE. Section 5 analyzes the results of the proposed model across various scenarios, including sensitivity and comparative assessments, and discusses their policy implications. Section 6 presents the conclusions of this work, highlights its limitations, and explores possible directions for future research.

2. Literature review

A systematic search of the Web of Science and Google Scholar databases for studies on material deprivation published between 2019 and 2024 revealed that a wide variety of statistical techniques have been employed. Studies published in 2025 were not included, as they were not yet fully available in the databases at the time this research was conducted. The following terms were used in the literature search: "material deprivation", "measurement of deprivation", and "European Union Statistics on Income and Living Conditions", along with synonyms and abbreviations (e.g., "EU-SILC" for European Union Statistics on Income and Living Conditions). Table 1 lists 26 articles relevant to the search topic, after excluding papers that were either not accessible in full or not pertinent to the research question and outcomes.

Some studies in the literature on material deprivation have focused on examining household characteristics, such as family structure and the presence of children, as these factors are key to understanding inequality and poverty dynamics. Family structure can greatly influence a household's resources and needs. For instance, single-parent households often face greater economic and social challenges due to the reliance on a single income and the need to balance work and childcare (Šoltés et al., 2023; Calejari et al., 2024). Likewise, the presence of children is a crucial factor, as their needs can significantly increase a household's financial burden and affect quality of life (Fabrizi & Mussida, 2020; Ferrão et al., 2021; Guio et al., 2022; Pérez-Corral & Moreno Mínguez, 2022; Ferrão & Vieira, 2023; Karagiannaki & Burchardt, 2024; Pérez-Corral et al., 2024). In contrast, studies suggest that older adults are at a lower risk of experiencing subjective economic hardship, especially in lower income brackets, due to their ability to rely on savings to sustain their standard of living (Palomäki et al., 2025).

Table 1. Studies related to material deprivation

Author(s) (year)	Data source (studied country/countries)	Methods and techniques
Notten and Guio (2019)	EU-SILC (Germany, Greece, Poland, and the United Kingdom)	Multivariate regression
Verbunt and Guio (2019)	EU-SILC (the 27 EU member states, plus Iceland, Norway, Switzerland, and the United Kingdom)	Single-level and multilevel multinomial logistic regression
Bárcena-Martín et al. (2020)	EU-SILC (Switzerland, the United Kingdom, the 27 EU member states, excluding Denmark, Finland, the Netherlands, Slovenia, and Sweden)	Linear estimation method
Karagiannaki and Burchardt (2020)	EU-SILC (the 27 EU member states, plus Norway, Serbia, Switzerland, and the United Kingdom)	Regression analysis, multivariate analysis
Fabrizi and Mussida (2020)	EU-SILC (Italy)	Correlated random effects probit models
Łuczak and Kalinowski (2020)	EU-SILC (the 27 EU member states, plus Iceland, Macedonia, Norway, Serbia, Switzerland, and the United Kingdom)	Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS)
Ayala et al. (2021)	EU-SILC (Spain)	Latent class model and decomposition analysis
Ferrão et al. (2021)	EU-SILC (Portugal)	Graded response model
Mussida and Parisi (2021)	EU-SILC (Italy and Spain)	Multinomial logit model
Pérez Corral and Moreno Mínguez (2021)	INE Living Conditions Survey* (Spain)	Descriptive analysis and logistic regression
Guio et al. (2022)	EU-SILC (the 27 EU member states, plus Iceland, Serbia, Switzerland, and the United Kingdom)	Multilevel negative binomial model
Pérez-Corral and Moreno Mínguez (2022)	EU-SILC (Italy and Spain)	Multiple overlapping deprivation analysis and logistic regression
Wolf et al. (2022)	EU-SILC and cross-national macro-level data from the Organisation for Economic Co-operation and Development, Eurostat and United Nations Economic Commission for Europe (Norway, the United Kingdom, the 27 EU member states, excluding Romania, Bulgaria, Malta, Cyprus)	Random intercepts models and linear models
Fabrizi et al. (2023)	EU-SILC (Switzerland, Serbia, the 27 EU member states, excluding Czech Republic, Denmark, Estonia, Finland, Germany, Lithuania, Luxembourg, and the Netherlands)	Factor analysis and regression analysis
Ferrão and Vieira (2023)	EU-SILC (Portugal)	Two-parameter logistic item response model
Moro-Egido and Navarro (2023)	EU-SILC (Spain)	Multilevel regression model
Mussida et al. (2023)	EU-SILC (Spain)	Multinomial logit regression model
Šoltés et al. (2023)	EU-SILC (Slovakia)	Logistic regression and contrast analysis

End of Table 1

Author(s) (year)	Data source (studied country/countries)	Methods and techniques
Calegari et al. (2024)	EU-SILC (Norway, Switzerland, Serbia, the 27 EU member states, excluding Austria, Belgium, Bulgaria, Luxembourg, Poland, Portugal, Slovakia)	Three-level random intercept dynamic probit model
Mussida and Sciulli (2024)	EU-SILC (France, Germany, Italy, and the United Kingdom)	Multi(three)-level dynamic panel model
Pérez-Corral et al. (2024)	EU-SILC (Portugal and Spain)	Logistic regression
Karagiannaki and Burchardt (2024)	EU-SILC (the 27 EU member states, plus Serbia, Switzerland, and the United Kingdom)	Descriptive analysis and multivariate analysis
Sedefoğlu and Dudek (2024)	Eurostat database and EU-SILC (the 27 EU member states)	Different weighting and normalization methods, Spearman's rank correlation
Traversa et al. (2024)	EU-SILC (the 27 EU member states, excluding Croatia)	Adjusted Mazziotta and Pareto Index, influence analysis, and cluster analysis
Palomäki et al. (2025)	EU-SILC 2018 (the 27 EU member states, plus Norway, and Switzerland)	Multinomial logistic regression
Wolf (2025)	EU-SILC (the 27 EU member states, plus Iceland, Norway, Serbia, Switzerland, and the United Kingdom)	Logistic multi-level random intercept models and regression analyses
This work	INE Living Conditions Survey* (Spain)	OWA aggregation operator, 2-tuple linguistic model, and hierarchical clustering

Note: * Form part of EU-SILC but can be downloaded directly from the INE.

Moreover, some studies examine how social policies and economic transfers affect poverty and material deprivation, as these interventions can help alleviate the challenging conditions faced by the most vulnerable households (Notten & Guio, 2019; Bárcena-Martín et al., 2020; Karagiannaki & Burchardt, 2020; Wolf et al., 2022; Traversa et al., 2024; Wolf, 2025). The impact of disability is also considered: it significantly increases the risk of poverty, although cash benefits for disabled citizens can mitigate this impact (Mussida & Sciulli, 2024). As EU-SILC data provide up-to-date, comparable information on income and living conditions across many European countries, various previous studies have conducted cross-country comparisons of living conditions, such as those by Verbunt and Guio (2019), Łuczak and Kalinowski (2020), Sedefoğlu and Dudek (2024). Several studies focused on Spain conduct within-country comparisons (Ayala et al., 2021; Pérez Corral & Moreno Mínguez, 2021; Moro-Egido & Navarro, 2023; Mussida et al., 2023), while others present analyses of differences between Spain and other European countries (Mussida & Parisi, 2021; Pérez-Corral & Moreno Mínguez, 2022; Pérez-Corral et al., 2024). Among studies on material deprivation in Spain, research by Ayala et al. (2021), Moro-Egido and Navarro (2023), and Mussida et al. (2023) highlights the importance of considering how deprivation changes across different areas and degrees of urbanization. An understanding of these differences is crucial for developing more effective policies that are tailored to regional realities.

Regarding the methods and techniques employed in studies on material deprivation, as shown in Table 1, researchers have used a range of approaches, including regression models, descriptive analysis, multivariate analysis, and dimensionality reduction models, among others. These tools allow researchers to gain a deeper understanding of the complex interactions between variables and their impact on the socioeconomic conditions of households. However, to the best of our knowledge, no previous research has explored the possibility of measuring material deprivation by taking into account how demanding decision-makers may be when identifying such deprivation, or their attitudes towards it. Such measurements can be crucial for assessing material deprivation across different scenarios, as the number of deprivation criteria that are met may reflect the consideration of various factors and preferences. To make these measurements, the OWA aggregation operator, the 2-tuple linguistic model, and hierarchical clustering are used in this work. The linguistic quantifiers of the OWA aggregation operator are employed to simulate decision-makers' attitudes. The 2-tuple linguistic model is applied to improve the interpretability of material deprivation results without loss of information. Hierarchical clustering is used to summarize material deprivation in Spanish households by region and degree of urbanization, enabling the identification of regions with similar characteristics and the exploration of differences. To the best of our knowledge, these models have not been used together in previous research on material deprivation, indicating the originality of the approach proposed in this work.

3. Methods applied to quantify material deprivation

This section presents the methodological framework employed in this work to measure material deprivation. The OWA aggregation operator enables a flexible aggregation of deprivation criteria, considering how demanding decision-makers are when measuring such deprivation. The 2-tuple linguistic model preserves interpretability of the results without any loss of information, while hierarchical clustering identifies groups of regions with similar deprivation patterns, highlighting potential targets for differentiated policy interventions. Together, these methods form an integrated pipeline that connects theoretical decision models with practical measurement of household material deprivation. The specific procedure for applying these methods is shown in Figure 2 (see Section 4).

3.1. The Ordered Weighted Averaging (OWA) aggregation operator

Introduced by Yager (1988), the OWA aggregation operator is a tool for aggregating information, creating an overall decision function by combining multiple criteria into one value. This operator can calculate decisions ranging from the minimum (i.e., all criteria must be satisfied) to the maximum (i.e., satisfaction of at least one criterion is sufficient). It is especially useful when the decision-maker needs to find a balance between criteria. The OWA aggregation operator is suitable for modeling decision-makers' attitudes (Serrano-Guerrero et al., 2022). This operator, together with its extensions, has been applied across multiple fields (Xu, 2006; Merigó, 2011; Emrouznejad & Marra, 2014; Gupta et al., 2019; Shu et al., 2022, 2023, 2024; Jin et al., 2023; Malik et al., 2024). The OWA aggregation operator is defined as follows:

Definition 1. (Yager, 1988) An OWA aggregation operator of dimension n is a mapping of

OWA: $R^n \rightarrow R$, with an associated weighting vector W of dimension n , such that $\sum_{j=1}^n w_j = 1$ and $w_j \in [0,1]$, then:

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j, \quad (1)$$

where, $a_1, a_2, \dots, a_n$ are the input values; b_j is the j th largest element of the input values; and w_j represents the ordered weights. Note that the ordered weights, unlike the importance weights associated with criteria, are associated with the criterion values on the location-by-location basis (Yager & Alajlan, 2016; Mokarram & Hojati, 2017). These weights are assigned to the criterion value at the j th position in descending order. Therefore, the OWA aggregation operator is also called the descending OWA (DOWA) aggregation operator.

The ordered weights are often calculated using linguistic quantifiers (e.g., *At least one*, *Some*, *Half*, *Many*, *All*, etc.), as these quantifiers can translate natural language statements into mathematical expressions (Zadeh, 1996). The weights calculated using distinct linguistic quantifiers can be used to represent decision-makers' attitudes in information aggregation (Yager, 1988, 1995; Ahn, 2010). For example, less cautious decision-makers may consider it sufficient if any of the criteria are fulfilled. In such cases, the weights are determined using the quantifier *At least one*. In the context of material deprivation, traditional thresholds (e.g., "3 out of 7 items") can be viewed as implicit quantifiers. Using linguistic quantifiers generalizes this idea, introducing flexibility and allowing the modeling of different attitudes toward deprivation severity.

The ordered weights using different linguistic quantifiers are calculated according to the following definitions:

Definition 2. (Yager, 1996) Regular increasing monotone (RIM) quantifiers are applied to generate a parameterized subset in the unit interval, as shown in Eq. (2):

$$Q_q(p) = p^\lambda, \quad \lambda > 0, \quad (2)$$

where, Q_q is a linguistic quantifier, expressed as a fuzzy subset over the unit interval $[0, 1]$; for each p in the unit interval, the grade of membership $Q_q(p)$ indicates the compatibility of p with the concept denoted by Q_q . Table 2 shows the parameter λ proposed by Boroushaki and Malczewski (2008) for each linguistic quantifier, along with the corresponding decision-makers' attitudes represented. Considering the context of this work, Table 2 also describes how demanding decision-makers are or the attitude that they have when measuring material deprivation.

Definition 3. (Yager, 1996) The ordered weights w_j ($w_j \in [0,1]$) are calculated using Eq. (3):

$$w_j = Q_q\left(\frac{j}{n}\right) - Q_q\left(\frac{j-1}{n}\right) = \left(\frac{j}{n}\right)^\lambda - \left(\frac{j-1}{n}\right)^\lambda, \quad \lambda > 0, \quad (3)$$

where, λ is the value related to each linguistic quantifier Q_q . The higher the value of λ , the more cautious the attitude of the decision makers. $\lambda = 1$ represents a neutral attitude.

Table 2. Linguistic quantifiers with their associated parameters λ and decision-makers' attitudes, adapted from Malczewski (2006) and Boroushaki and Malczewski (2008)

Linguistic Quantifier	λ	Attitude	Description
$Q_1 = \text{At least one}$	0.0001 ($\lambda \rightarrow 0$)	Extremely incautious (least demanding): decision-makers consider it sufficient if any of the criteria are met.	A situation of material deprivation is identified if at least one of the criteria used to measure such deprivation is met.
$Q_2 = \text{Few}$	0.1	Very incautious (less demanding): decision-makers consider it sufficient if a few criteria are met.	A situation of material deprivation is identified if a few of the criteria used to measure such deprivation are met.
$Q_3 = \text{Some}$	0.5	Somewhat incautious (slightly less demanding): decision-makers consider it sufficient if some criteria are met.	A situation of material deprivation is identified if some criteria used to measure such deprivation are met.
$Q_4 = \text{Half}$	1	Neutral: decision-makers consider it sufficient if several criteria are met.	A situation of material deprivation is identified if several criteria used to measure such deprivation are met.
$Q_5 = \text{Many}$	2	Somewhat cautious (slightly more demanding): decision-makers consider it sufficient if many criteria are met.	A situation of material deprivation is identified if many of the criteria used to measure such deprivation are met.
$Q_6 = \text{Most}$	10	Very cautious (more demanding): decision-makers consider it sufficient only if most criteria are met.	A situation of material deprivation is identified if most of the criteria used to measure such deprivation are met.
$Q_7 = \text{All}$	1000 ($\lambda \rightarrow \infty$)	Extremely cautious (most demanding): decision-makers consider it sufficient only if all the criteria are met.	A situation of material deprivation is identified only if all criteria used to measure such deprivation are met.

As mentioned above, the OWA aggregation operator is effective in achieving a balance among criteria. The *tradeoff* levels between criteria are directly controlled by the ordered weights (Malczewski, 1999). Moreover, based on the ordered weights, the *orness* can be calculated to represent the decision-makers' attitudes (i.e., how demanding they are). The *tradeoff* and *orness* define the decision strategy space (Malczewski, 2006). The following equations demonstrate the procedure for computing *orness* (Yager, 1988) and *tradeoff* (Jiang & Eastman, 2000) using the ordered weights.

$$orness = \frac{1}{n-1} \sum_{j=1}^n (n-j) \cdot w_j; \quad (4)$$

$$tradeoff = 1 - \sqrt[n]{\frac{n \sum_{j=1}^n \left(w_j - \frac{1}{n} \right)^2}{n-1}}, \quad (5)$$

where, $0 \leq orness \leq 1$ and $0 \leq tradeoff \leq 1$. Higher values of *orness* suggest that the decision-makers have a less cautious attitude (i.e., less demanding). For example, if ordered weights are calculated using the linguistic quantifier *At least one* (i.e., Q_1), resulting in *orness* being one and *tradeoff* being zero, this reflects an extremely incautious attitude of the decision-maker, who considers it sufficient if at least one of the evaluated criteria is met. If the quantifier *All* (i.e., Q_7) is used, *orness* and *tradeoff* are both equal to zero, reflecting an extremely cautious attitude of the decision-maker, who considers it sufficient only if all the criteria evaluated are met. If the quantifier *Half* (i.e., Q_4) is used, *orness* equals 0.5 and *tradeoff* equals 1, representing the neutral attitude of the decision-maker.

3.2. The 2-tuple linguistic model

Introduced by Herrera and Martínez (2000), the 2-tuple linguistic model addresses the issue of information loss in the fusion of linguistic information that affects other linguistic computing models. One advantage of this model is that it allows a continuous representation of the linguistic information within its domain, whereas this information is considered discrete in other conventional linguistic computing models (Malhotra & Gupta, 2023). Many authors have employed the 2-tuple linguistic model because it yields results that are more readily comprehensible and interpretable for decision-makers compared to those obtained solely through numerical scales (Labella et al., 2021; Monje et al., 2022; Shu et al., 2022, 2024; Malhotra & Gupta, 2023).

The 2-tuple linguistic model, founded on fuzzy logic and linguistic variables, represents the linguistic information using a pair of values known as a 2-tuple value (s_i, α) , where $s_i \in S$ is a linguistic term, and $\alpha \in [-0.5, 0.5]$ is a numerical value that represents the symbolic translation. It is defined as follows.

Definition 4. (Herrera & Martinez, 2000) Let $S = \{s_0, \dots, s_g\}$ be a set of linguistic terms and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, where $g + 1$ is the cardinality of S . The function $\Delta: [0, g] \rightarrow S \times [-0.5, 0.5]$ is used to convert β into the 2-tuple value (s_i, α) , as shown in Eq. (6):

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} i = \text{round}(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5] \end{cases} \quad (6)$$

where, $\text{round}(\cdot)$ is the rounding operation; s_i has the closest index label to β ; and α is the numerical value of the symbolic translation.

The 2-tuple linguistic model defines functions Δ and Δ^{-1} that enable the transformation of numerical values into 2-tuples and vice versa without loss of information, thereby facilitating the extension of any numerical aggregation operator to handle linguistic 2-tuple values (Herrera & Martinez, 2000). The function Δ is bijective, whose inverse function $\Delta^{-1}: S \times [-0.5, 0.5] \rightarrow [0, g]$ is used to return an equivalent numerical value β , calculated as $(s_i, \alpha) \rightarrow \Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$. Details about the negation operator of a 2-tuple linguistic value and aggregation operators for 2-tuple linguistic computation can be found in Herrera-Viedma et al. (2004). The following definition shows the comparison of linguistic information represented by 2-tuple linguistic values.

Definition 5. Let (s_K, α_1) and (s_V, α_2) be two 2-tuple values, with these 2-tuple values compared as follows:

- If $K < V$, then (s_K, α_1) is smaller than (s_V, α_2) .
- If $K > V$, then (s_K, α_1) is larger than (s_V, α_2) .
- If $K = V$, then:
 - a. $\alpha_1 = \alpha_2$, (s_K, α_1) and (s_V, α_2) represent the same information.
 - b. $\alpha_1 < \alpha_2$, (s_K, α_1) is smaller than (s_V, α_2) .
 - c. $\alpha_1 > \alpha_2$, (s_K, α_1) is larger than (s_V, α_2) .

Example 1. The set of linguistic terms used to evaluate the ranking in terms of material deprivation is $S = \{s_0 = \text{Very Low} = VL, s_1 = \text{Low} = L, s_2 = \text{Medium} = M, s_3 = \text{High} = H, s_4 = \text{Very High} = VH\}$, with $s_K < s_V$ for $K < V$, whose cardinality is $g+1=5$. The distribution of this set S is shown in Figure 1. If $\beta = 2.8$, its 2-tuple value will be $\Delta(2.8) = (s_{\text{round}(2.8)}, 2.8 - \text{round}(2.8)) = (s_3, -0.2) = (H, -0.2)$. Its numerical transformation is done with the function Δ^{-1} , which is $\Delta^{-1}(s_3, -0.2) = 3 - 0.2 = 2.8$. If $\beta = 3$, its 2-tuple value will be $\Delta(3) = (s_{\text{round}(3)}, 3 - \text{round}(3)) = (s_3, 0) = (H, 0)$. The difference between β and that linguistic term is zero, that is $(H, 0) = H$.

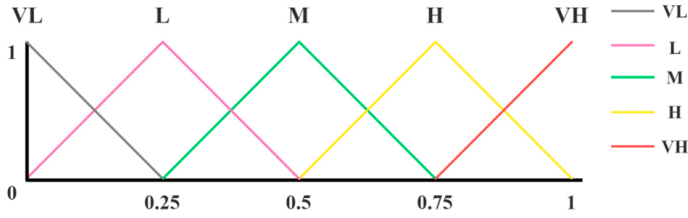


Figure 1. A linguistic term set S

3.3. Hierarchical clustering

Hierarchical clustering (Bridges, 1966) is a popular unsupervised machine learning algorithm that organizes data points into a hierarchy of clusters according to their similarity or dissimilarity. It has found widespread application in various fields, such as biology (Seo & Shneiderman, 2002; Wei et al., 2012; Subramaniyan et al., 2020), the environment (Govender & Sivakumar, 2020), and finance (Dai et al., 2010; Liu et al., 2021; Jaroonchokanan et al., 2022), among others. One advantage of hierarchical clustering is that it does not require the user to specify the number of clusters beforehand, unlike partitioning methods such as K-Means, although it can be time-consuming when handling large datasets. Hierarchical clustering produces a dendrogram, a tree-like structure that is easy to interpret. The dendrogram can then be cut at the appropriate level to obtain the desired number of clusters.

Hierarchical clustering has two main variants: agglomerative hierarchical clustering (also known as a bottom-up approach) and divisive hierarchical clustering (also known as a top-down approach) (Murtagh, 1983). In agglomerative hierarchical clustering, each data point begins as its own cluster. Then, it iteratively merges the two closest clusters based on a chosen distance metric until all data points belong to a single cluster. This forms a dendrogram,

with each merge corresponding to a node in the tree. On the contrary, divisive hierarchical clustering starts with all data points in a single cluster and recursively splits them into smaller clusters based on a chosen criterion until each data point has its own cluster. This results in a dendrogram similar to that produced by agglomerative clustering, but with the clusters being divided rather than merged.

In practice, the choice between agglomerative hierarchical clustering and divisive hierarchical clustering depends on factors such as the nature of the data, ease of implementation, interpretability of results, and the desired clustering structure. Agglomerative hierarchical clustering was chosen for this work due to its superior ability to handle outliers compared to divisive clustering, as well as offering more available packages for agglomerative clustering in Python and R. When conducting agglomerative hierarchical clustering, selecting the appropriate distance metric and linkage function is crucial because they can impact the clustering results. The distance metric (e.g., Euclidean distance, Squared Euclidean distance measure, Manhattan distance, etc.) determines how similarity or dissimilarity between data points is measured, affecting how clusters are formed. In Python and R, the default distance measure is the Euclidean distance (Müllner, 2013).

The linkage function determines how distances between clusters are calculated during the merging process, further influencing the structure and composition of the resulting clusters. The common linkage functions include:

- *Single linkage*: the distance between two clusters is calculated as the minimum distance between any single data point in the first cluster and any single data point in the second cluster.
- *Complete linkage*: the distance between two clusters is calculated as the maximum distance between any single data point in the first cluster and any single data point in the second cluster.
- *Average linkage*: the distance between two clusters is calculated as the average distance between data points in the first cluster and data points in the second cluster. It is considered superior to single and complete linkages as it facilitates the minimization of within-cluster variance and maximization of between-cluster variance (Kalkstein et al., 1987).
- *Centroid linkage*: the distance between two clusters is calculated as the distance between their centroids.
- *Ward linkage*: an ANOVA-based approach, defined as the smallest increase in the within-cluster sum of squares resulting from the merging of two clusters.

4. Proposed model

This section presents the application of the proposed model, using the concepts introduced in Section 3, to generate various scenarios for measuring the material deprivation of poor households. Figure 2 shows the procedure for each step of the proposed model, illustrating how each methodological component contributes to the measurement, aggregation, and interpretation of material deprivation.

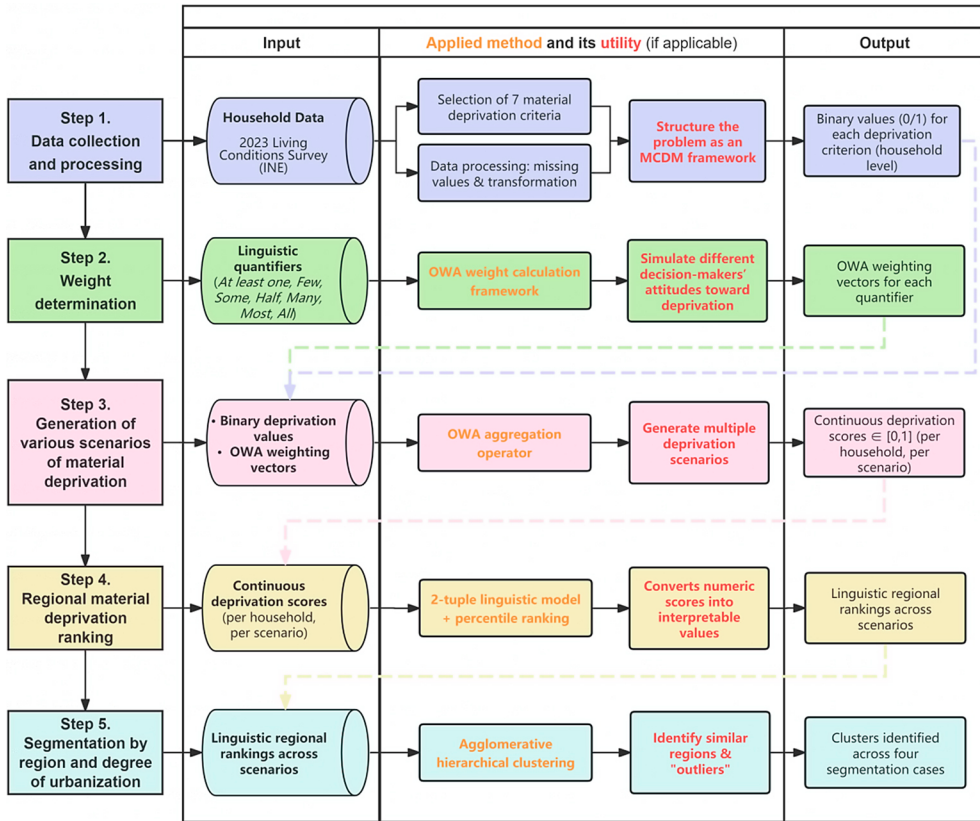


Figure 2. Step-by-step procedure of the proposed model

4.1. Data collection and processing

This step involves the collection of microdata from the 2023 Living Conditions Survey conducted by INE for 27,227 households (INE, n.d.). According to the methodology employed by INE in the Living Conditions Survey (INE, 2023), material deprivation is measured based on seven criteria, and households are considered deprived if they meet at least three of them. Thus, of all the variables in the survey, this work includes only the variables related to these criteria and basic household data (e.g., household identification, autonomous community, etc.). Table 3 provides descriptions of these variables and their corresponding criteria.

During data processing, households with missing data are excluded, and only economically poor households are included for analysis, ultimately resulting in a sample of 5079 households. On the other hand, as shown in Table 3, the variables related to the seven criteria used to measure material deprivation are binary questions, and do not directly indicate whether respondents meet the corresponding criterion. Therefore, data transformation is applied to convert all related variable values into 1 (if they meet the criterion) or 0 (if they do not). Table 4 displays some household data from the dataset after data processing. For instance, household 518 struggles to maintain an adequate temperature in the home (i.e., C_3), cannot cope with unforeseen expenses (i.e., C_4), and has experienced delays in paying bills related to the main residence in the past 12 months (i.e., C_5).

Table 3. Variable description

Variable	Description	Criteria
HouseID	Household identification number.	–
AutoC	The Autonomous Community or Autonomous City to which the household belongs.	–
UrbanD	The degree of urbanization of the region to which the household belongs.	–
HVacat	Can the household afford to go on vacation away from home for at least one week per year?	C ₁ : Cannot afford to go on holidays for at least one week a year
HMeal	Can the household afford a meal of meat, chicken, or fish (or vegetarian equivalent) at least every two days?	C ₂ : Cannot afford a meal of meat, chicken, or fish at least every two days
HTemp	Did the household benefit from any public subsidies to cover expenses related to electricity, heating, gas, etc.?	C ₃ : Cannot afford to keep the home at an adequate temperature
HExpen	Can the household handle unexpected expenses?	C ₄ : Unable to cope with unforeseen expenses
HDelay1	Have there been delays in paying the mortgage or rent of the household in the last 12 months?	C ₅ : Delays in paying expenses related to the main residence (mortgage or rent, gas bills, community fees, etc.) in the last 12 months
HDelay2	Have there been delays in paying the bills in the last 12 months?	
HDelay3	Have there been delays in paying deferred purchases or other loans in the last 12 months?	
HCar	Does the household have a car?	C ₆ : Cannot afford a car
HComputer	Does the household have a computer?	C ₇ : Cannot afford a computer

Table 4. Processed data example

HouseID	AutoC	UrbanD	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇
30	Basque Country	densely populated area	0	0	0	1	1	1	1
161	Castilla-La Mancha	sparsely populated area	0	0	1	0	0	0	0
268	Valencian Community	moderately populated area	0	0	0	0	0	0	0
518	Extremadura	densely populated area	0	0	1	1	1	0	0
2119	La Rioja	moderately populated area	1	0	0	1	0	0	0
2298	Community of Madrid	densely populated area	0	0	0	0	0	1	0

4.2. Weight determination

In this step, Eq. (3) is used to calculate the ordered weights for distinct linguistic quantifiers. Table 5 presents these ordered weights, along with the corresponding *orness* and *tradeoff* values computed using Eqs. (4)–(5).

4.3. Generation of various scenarios of material deprivation

In this step, Eq. (1) is employed to generate various scenarios of material deprivation for each household. Table 6 presents some examples of the generated scenarios for households' material deprivation. As shown in Table 6, aggregating the criteria values while considering the

Table 5. Weighting vector for each linguistic quantifier with the corresponding *orness* and *tradeoff*

Linguistic Quantifier	Weighting vector							<i>orness</i>	<i>tradeoff</i>
$Q_1 = \text{At least one}$	0.9998	0.0001	0	0	0	0	0	1	0
$Q_2 = \text{Few}$	0.8232	0.0591	0.0365	0.0268	0.0213	0.0178	0.0153	0.92	0.21
$Q_3 = \text{Some}$	0.3780	0.1566	0.1201	0.1013	0.0892	0.0807	0.0742	0.68	0.72
$Q_4 = \text{Half}$	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.50	1
$Q_5 = \text{Many}$	0.0204	0.0612	0.1020	0.1429	0.1837	0.2245	0.2653	0.31	0.77
$Q_6 = \text{Most}$	0	0	0.0002	0.0035	0.0309	0.1795	0.7859	0.04	0.23
$Q_7 = \text{All}$	0	0	0	0	0	0	1	0	0

attitudes or opinions of decision-makers can result in various values of material deprivation ranging between 0 and 1. These values reflect the varying degrees of material deprivation that a household may experience. For example, if a decision-maker considers that meeting even a single criterion is enough to identify a household as materially deprived, then households are more likely to be classified as materially deprived unless none of the criteria are met (e.g., household 268 in Table 6 is economically poor but does not experience material deprivation).

Table 6. Scenarios of material deprivation for a selection of households

	AutoC	UrbanD	Material deprivation						
			Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
30	Basque Country	densely populated area	0.9999	0.9456	0.7560	0.5716	0.3265	0.0037	0
161	Castilla-La Mancha	sparsely populated area	0.9998	0.8232	0.3780	0.1429	0.0204	0	0
268	Valencian Community	moderately populated area	0	0	0	0	0	0	0
518	Extremadura	densely populated area	0.9999	0.9188	0.6547	0.4287	0.1836	0.0002	0
2119	La Rioja	moderately populated area	0.9999	0.8823	0.5346	0.2858	0.0816	0	0
2298	Community of Madrid	densely populated area	0.9998	0.8232	0.3780	0.1429	0.0204	0	0

4.4. Regional material deprivation ranking

In this step, the arithmetic mean ($\overline{X_f}$) is applied to summarize the material deprivation situation of households in each region, and the material deprivation ranking is calculated using $\Delta(\text{percent_rank}(\overline{X_f}))$, where Δ has been defined in Eq. (6) and $\text{percent_rank}(\cdot) = \frac{\text{rank}_f(\cdot) - 1}{m - 1}$ with $\text{rank}_f(\cdot)$ representing the ranking of the f th region in ascending order, $f = 1, 2, \dots, m$, and m representing the total number of regions.

Table 7 shows the rankings of material deprivation by region, expressed in 2-tuple values. These values offer a more intuitive way to understand the ranking results for various scenarios

of material deprivation. For instance, in one scenario, a decision-maker considers a household to be experiencing material deprivation if any of the deprivation criteria are met (i.e., using the quantifier Q_1). As shown in Table 7, Ceuta is the region where households are most likely to experience material deprivation, while Cantabria is the region where households are least likely to experience it. Based on this table, it is evident that, regardless of the linguistic quantifier used to measure material deprivation, the rankings for Ceuta and Cantabria barely change. However, in other regions, such as the Valencian Community and the Basque Country, their rankings of material deprivation change when using different linguistic quantifiers in the measurement. This necessitates the use of clustering techniques to explore the changing patterns. Further results, including segmentation by the degree of urbanization of each region, will be presented in the next step.

Table 7. Material deprivation ranking by region

AutoC	Material deprivation ranking						
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
Andalusia	(H, -0.11)	(H, +0.11)	(H, +0.33)	(H, +0.33)	(H, +0.11)	(M, -0.44)	(M, -0.44)
Aragon	(VL, +0.22)	(VL, +0.22)	VL	VL	VL	(VL, +0.44)	VL
Canary Islands	(VH, -0.22)	(VH, -0.22)	(VH, -0.22)	(VH, -0.22)	(VH, -0.22)	(VH, -0.22)	(H, +0.11)
Cantabria	VL	VL	(VL, +0.44)	(VL, +0.44)	(VL, +0.22)	VL	VL
Castile and León	(M, +0.22)	(M, +0.22)	(M, -0.22)	(M, -0.22)	(M, -0.44)	(L, +0.11)	M
Castilla-La Mancha	(L, -0.33)	(L, -0.11)	(L, -0.33)	(L, -0.11)	(L, -0.33)	(L, -0.33)	VL
Catalonia	(H, -0.33)	(H, -0.33)	(M, +0.22)	(M, +0.22)	(M, +0.44)	(M, +0.22)	(H, -0.33)
Ceuta	VH	VH	VH	VH	VH	VH	(VH, -0.22)
Melilla	(VH, -0.44)	(H, +0.33)	(H, -0.33)	(M, +0.44)	(M, +0.22)	(VL, +0.22)	VL
Navarre	(M, -0.44)	M	(H, -0.11)	(H, +0.11)	(H, +0.33)	(H, -0.11)	VL
Valencian Community	M	(M, -0.22)	(M, -0.44)	(L, +0.33)	(L, +0.33)	(L, +0.33)	(M, -0.22)
Community of Madrid	(M, +0.44)	(M, +0.44)	(M, +0.44)	(H, -0.33)	(H, -0.33)	(H, +0.11)	(H, -0.11)
Extremadura	(H, +0.11)	(H, -0.11)	(H, +0.11)	(H, -0.11)	(H, -0.11)	M	(L, +0.11)
Galicia	(L, +0.11)	(L, +0.11)	(L, +0.11)	(L, +0.11)	(L, +0.11)	(H, -0.33)	(H, +0.33)
Balearic Islands	(L, -0.11)	(L, -0.33)	(VL, +0.22)	(VL, +0.22)	(VL, +0.44)	(H, +0.33)	VH
La Rioja	(L, +0.33)	(L, +0.33)	(L, -0.11)	(L, -0.33)	(L, -0.11)	(M, -0.22)	(M, +0.44)
Basque Country	(VL, +0.44)	(VL, +0.44)	(L, +0.33)	(M, -0.44)	M	(M, +0.44)	(M, +0.22)
Asturias	(H, +0.33)	(VH, -0.44)	(VH, -0.44)	(VH, -0.44)	(VH, -0.44)	(VH, -0.44)	(VH, -0.44)
Region of Murcia	(M, -0.22)	(M, -0.44)	M	M	(M, -0.22)	(L, -0.11)	(L, +0.33)

4.5. Segmentation by region and degree of urbanization

This step involves segmenting the regions using agglomerative hierarchical clustering, which requires the use of the function Δ^{-1} to convert the 2-tuple values generated in the previous step into numbers. The segmentation is conducted for four cases: one where regions are separated without considering the degree of urbanization, and three others where the degree of urbanization is considered (i.e., densely populated area, moderately populated area, and sparsely populated area).

To determine the appropriate number of clusters, both the Dunn index and the bootstrap-based stability measure were considered. As shown in Table 8, the Dunn index rises substantially for larger numbers of clusters, reaching high values between seven and ten, indicating well-separated and compact clusters. The standard deviation of the bootstrap-based stability measure evaluates robustness by measuring how consistent clusters remain across resampled datasets. Lower values indicate less variability and greater reliability. At nine clusters, the stability is acceptable (Stability standard deviation = 0.098), and the Dunn index is high (0.646), suggesting a favorable balance between cluster cohesion, separation, and robustness.

Table 8. Validation metrics for cluster number selection

Number of cluster	Standard deviation	Dunn index
2	0.097	0.137
3	0.137	0.265
4	0.092	0.315
5	0.067	0.382
6	0.082	0.541
7	0.091	0.603
8	0.086	0.603
9	0.098	0.646
10	0.109	0.754
11	0.099	0.481
12	0.076	0.532

Selecting nine clusters represents a balanced compromise, capturing meaningful regional heterogeneity while enabling clear comparisons and the identification of “outlier” countries across all four cases. This choice ensures that clusters are both interpretable and statistically defensible. Table 9 shows the corresponding cluster for each region in the four cases. Figure 3 displays radar maps for the following cases: a) Case 1: based on region, b) Case 2: based on densely populated area, c) Case 3: based on moderately populated area, and d) Case 4: based on sparsely populated area. It should be noted that the same labels in different cases do not necessarily imply identical clustering characteristics (e.g., Cluster 1 in Case 1 might differ from Cluster 1 in other cases; conversely, clusters with the same label might coincide in some cases).

Table 9. Cluster result

AutoC	Case 1: based on region	Case 2: based on densely populated area	Case 3: based on moderately populated area	Case 4: based on sparsely populated area
Andalusia	Cluster 4	Cluster 7	Cluster 4	Cluster 2
Aragon	Cluster 8	Cluster 8	Cluster 1	Cluster 7
Canary Islands	Cluster 5	Cluster 5	Cluster 4	Cluster 9
Cantabria	Cluster 8	Cluster 8	Cluster 8	Cluster 7
Castile and León	Cluster 7	Cluster 9	Cluster 5	Cluster 2
Castilla-La Mancha	Cluster 8	Cluster 8	Cluster 7	Cluster 7
Catalonia	Cluster 6	Cluster 7	Cluster 6	Cluster 1
Ceuta	Cluster 5	Cluster 5	– *	– *
Melilla	Cluster 1	Cluster 8	– *	– *
Navarre	Cluster 2	Cluster 2	Cluster 2	Cluster 4
Valencian Community	Cluster 7	Cluster 7	Cluster 9	Cluster 8
Community of Madrid	Cluster 6	Cluster 9	Cluster 8	Cluster 2
Extremadura	Cluster 4	Cluster 1	Cluster 7	Cluster 5
Galicia	Cluster 9	Cluster 3	Cluster 9	Cluster 2
Balearic Islands	Cluster 3	Cluster 6	Cluster 3	Cluster 3
La Rioja	Cluster 9	Cluster 6	Cluster 9	Cluster 7
Basque Country	Cluster 9	Cluster 4	Cluster 4	Cluster 8
Asturias	Cluster 5	Cluster 9	Cluster 5	Cluster 2
Region of Murcia	Cluster 7	Cluster 8	Cluster 6	Cluster 6

Note: * Ceuta and Melilla are the autonomous cities where there are no households belonging to moderately or sparsely populated areas in the sample.

5. Analysis and discussion

This section analyzes and compares the results generated by the proposed model and discusses the practical policy implications of these findings.

5.1. Sensitivity analysis

A sensitivity analysis was conducted to assess the impact of variations in the OWA weighting parameter λ on the final deprivation values. For each linguistic quantifier, the original λ values demonstrated in Table 2 were adjusted by relative changes of approximately 0.1 % to 10 %, generating alternative weighting vectors. For each scenario, deprivation scores were recalculated, and relative differences from the baseline scores were computed and aggregated into a sensitivity index.

The average sensitivity across all scenarios is 0.239 %, indicating that the deprivation patterns are highly robust. The ranking of households by region expressed in the 2-tuple linguistic values of material deprivation changes only minimally. Even under the largest tested

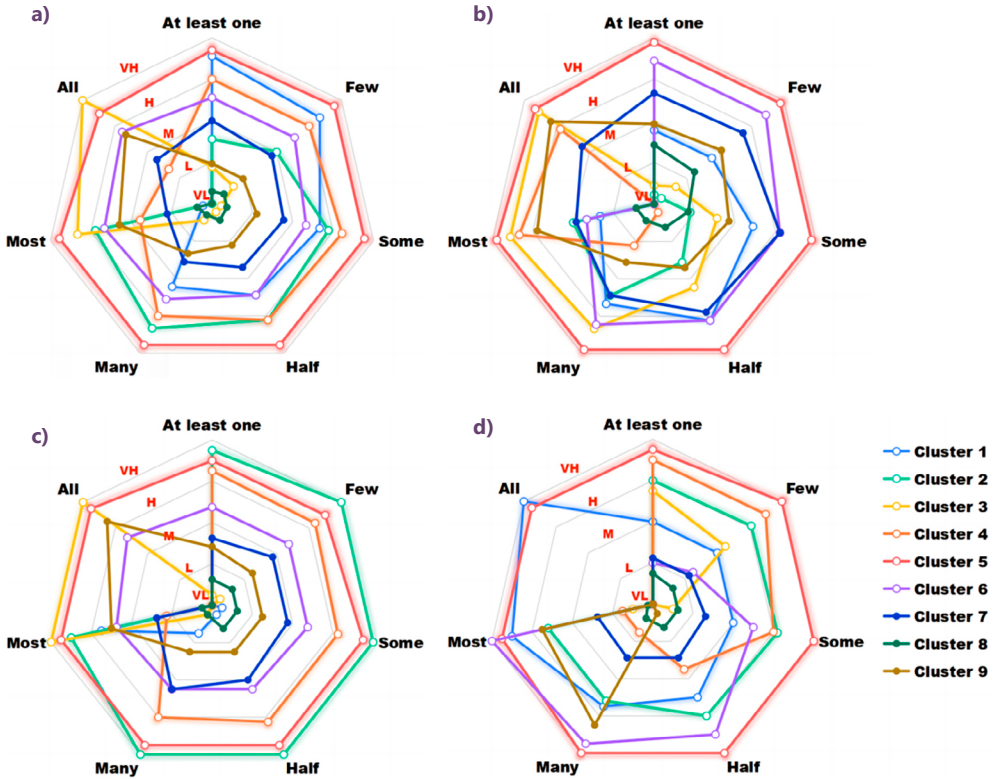


Figure 3. Cluster center radar map: a) Case 1: based on region; b) Case 2: based on densely populated area; c) Case 3: based on moderately populated area; d) Case 4: based on sparsely populated area

perturbations of λ , the relative differences in deprivation scores remained below 0.5%, and the material deprivation ranking by region was largely preserved. This confirms that the results presented in this work are stable and that the proposed framework provides a reliable assessment of multiple material deprivation scenarios.

5.2. Comparative analysis

As shown in Figure 3, some clusters with the same labels have almost the same characteristics across all four cases (i.e., Clusters 5 and 8), while others vary. For example, Ceuta consistently falls within Cluster 5, regardless of whether segmentation is conducted based on the degree of urbanization. This suggests that, despite using different linguistic quantifiers to measure material deprivation, Ceuta consistently ranks very high in terms of material deprivation (with only slight fluctuations), indicating that poor households in Ceuta experience a significant lack of resources. The Canary Islands show a similar pattern in Cases 1 and 2, but shift to Cluster 4 when segmentation is based on moderately populated areas. Their ranking remains high when using the first five linguistic quantifiers (i.e., Q_1 to Q_5), but falls to low when using the quantifier *Most* (i.e., Q_6), and to very low when using quantifier *All* (Q_7). This implies that for

moderately populated areas of the Canary Islands, if decision-makers consider a household to be materially deprived only if most or all of the relevant criteria are met, households in this area are less likely to be identified as such, although they are more likely to experience material deprivation in other scenarios. Furthermore, when segmentation is conducted based on sparsely populated areas, the Canary Islands belong to Cluster 9, indicating that households in this area are more likely to experience material deprivation if decision-makers consider a household to be materially deprived only when many or most of the deprivation criteria are met. In other scenarios, they are less likely to experience such deprivation.

Regarding the regions belonging to Cluster 8, Cantabria consistently belongs to Cluster 8, except in Case 4 (sparsely populated area), where it shifts to Cluster 7; however, both clusters display similar low deprivation levels. This indicates that Cantabria maintains a consistently low level of material deprivation across scenarios. This said, despite using different linguistic quantifiers to measure material deprivation, Cantabria consistently ranks low in this regard, indicating that poor households there are less likely to experience such deprivation. A similar pattern is observed for Castilla-La Mancha in Cases 1, 2, and 4. Although in Cases 3 and 4, Castilla-La Mancha falls into Cluster 7, as mentioned earlier, clusters with the same labels do not necessarily have the same characteristics across all four cases. In Case 3, Cluster 7 shows a medium ranking when using the first five linguistic quantifiers (i.e., Q_1 to Q_5), but drops to a medium-low ranking when using the quantifier *Most* (i.e., Q_6), and to a very low ranking when using the quantifier *All* (Q_7). This implies that for moderately populated areas of Castilla-La Mancha, if decision-makers consider a household to be materially deprived only if most or all of the relevant criteria are met, households in this area are less likely to experience material deprivation, although they are more likely to experience it in other scenarios. Cluster 7 in Case 4 evidently does not exhibit the same characteristics (see also Figure 3).

A similar conclusion to Cantabria applies to the Community of Madrid in Case 3, but its material deprivation ranking is generally medium to high in other cases, except in one scenario of Case 4, where it is very low. This means that in Cases 1, 2, and 4, except in sparsely populated areas of Madrid, where decision-makers consider a household to be materially deprived only if all criteria are met, poor households are least likely to experience such deprivation; in other scenarios, they have a medium-high likelihood of experiencing material deprivation. As shown in Table 9, Catalonia and the Community of Madrid both belong to Cluster 6 in Case 1, but are not in the same cluster in other cases. This suggests that, in general, when the degree of urbanization is not accounted for, poor households in these two regions have a medium-high likelihood of experiencing material deprivation (see also Figure 3). This can be attributed to the high cost of living in the Community of Madrid and Catalonia, even though these regions have higher average incomes.

Considering the degree of urbanization, in Case 2, Catalonia belongs to Cluster 7, and the Community of Madrid belongs to Cluster 9. As shown in Figure 3, the ranking of Cluster 7 in material deprivation decreases from nearly high (when using Q_1) to nearly medium (when using Q_7), while the deprivation ranking of Cluster 9 is consistently nearly medium, except for a high ranking when using the last two linguistic quantifiers (i.e., Q_6 and Q_7) to measure material deprivation. This implies that, in densely populated areas, if decision-makers identify a household as materially deprived only when most or all of the relevant measurement

criteria are met, households in the Community of Madrid are more likely to experience such deprivation than those in Catalonia. In other scenarios of Case 2, the opposite conclusion is reached. In Case 3, regardless of the linguistic quantifier used to measure material deprivation, households in moderately populated areas of Catalonia show a higher likelihood of experiencing such deprivation than those in the corresponding areas of the Community of Madrid. In Case 4, if decision-makers identify a household as materially deprived only when many or most or all of the measurement criteria are met, households in sparsely populated areas of the Community of Madrid are less likely to experience such deprivation than those in the corresponding areas of Catalonia. In other scenarios of Case 4, the opposite conclusion can be drawn.

In summary, the regions belonging to Clusters 5 and 8 consistently maintain their rankings in material deprivation, despite the use of different linguistic quantifiers for measurement (i.e., Cluster 5 consistently ranks very high, and Cluster 8 consistently ranks very low). Regions in other clusters exhibit varying rankings depending on the linguistic quantifiers used. Additionally, each region's material deprivation ranking can change based on the degree of urbanization. The comparison between the Community of Madrid and Catalonia highlights that, although both regions generally exhibit a medium to high likelihood of experiencing material deprivation, significant intra-regional differences exist depending on the degree of urbanization, indicating that some areas require greater policy attention than others.

5.3. Policy implications

The divergence in results across different linguistic quantifiers and urbanization levels offers critical guidance for policy design. The finding that regions such as Ceuta and Cantabria maintain stable rankings regardless of measurement strictness suggests that, for these areas, standardized national interventions are sufficient; their status as highly deprived or highly resilient is robust to changes in definition. However, for regions with variable rankings, such as the Community of Madrid and Catalonia, a more nuanced and granular approach is required. The analysis reveals that aggregating data at the regional level can mask significant internal disparities. For example, in moderately populated areas of the Community of Madrid, households are less likely to experience material deprivation compared to other areas within the region. This may be due to higher living costs in densely populated areas, while sparsely populated areas may face lower average incomes and limited access to services and job opportunities.

The dynamic nature of the results offers actionable insights for policymakers and social planners, as follows:

- The framework allows policymakers to “stress-test” social interventions by simulating different definitions of deprivation. By adjusting the linguistic quantifiers, authorities can estimate how the pool of eligible households expands or contracts if the criteria for aid are relaxed (e.g., “At least one” criterion) or tightened (e.g., “All” criteria). This helps with budget forecasting and ensures that policies are robust enough to cover vulnerable populations under varying economic pressures without being overly inclusive or exclusive.

- The finding that deprivation levels vary significantly by degree of urbanization within the same region challenges one-size-fits-all regional policies. By leveraging the degree of urbanization and adjusting the strictness of deprivation measurements, decision-makers can achieve a more precise allocation of resources, ensuring that aid reaches the specific pockets of vulnerability that aggregate statistics often overlook.
- The model highlights that economic impoverishment (low income) does not always equate to material deprivation (lack of specific goods/services), particularly in high-cost areas like Madrid and Catalonia, where high incomes may coexist with high deprivation due to living costs. Policymakers should therefore complement income-based support with specific measures addressing access to services and essential goods, tailored to the specific deprivation profile of each urbanization cluster. Dense areas may benefit more from housing subsidies or cost-of-living adjustments, whereas sparse areas may require improvements in infrastructure, transport, and service accessibility.

6. Conclusions and future work

This work presents a novel approach for generating various scenarios of material deprivation, taking into account how demanding decision-makers are when measuring such deprivation, or their attitudes to material deprivation. The proposed model uses the OWA aggregation operator, the 2-tuple linguistic model, and hierarchical clustering. Within this framework, the linguistic quantifiers of the OWA aggregation operator are employed to simulate decision-makers' attitudes, which range from considering a household materially deprived if any of the criteria are met, to identifying it as such only if all criteria are met. In the context of household material deprivation measurement, traditional thresholds (e.g., "3 out of 7 items") can be viewed as implicit quantifiers. The use of linguistic quantifiers extends this conventional approach by introducing greater flexibility, thereby allowing the modeling of different attitudes toward deprivation severity. The 2-tuple linguistic model is used to improve the linguistic interpretability of material deprivation results, helping to gain a clearer understanding of the issue without loss of information. Since the comparison of material deprivation rankings across various regions should be made under the same linguistic qualifier (e.g., not comparing deprivation levels indicated by "*At least one*" in Madrid with those indicated by "*Most*" in Catalonia), hierarchical clustering is employed to summarize the material deprivations of Spanish households by region and degree of urbanization. This facilitates the identification of regions with similar characteristics and the exploration of differences.

The practicability of the proposed model was assessed using microdata from the 2023 Living Conditions Survey conducted by the INE. The results indicate that the proposed model contributes by offering a more comprehensive and dynamic approach to measuring households' material deprivation, which enables decision-makers to adjust the measurement of material deprivation according to their specific needs. It has been observed that the material deprivation ranking of certain Spanish regions varies depending on the adjustments made by decision-makers (e.g., if they consider a household to be materially deprived when some of the relevant criteria are met, or only when all criteria are met, etc.), although some Spanish regions do not show any such variation. For instance, despite the use of various linguistic

quantifiers to measure material deprivation, Ceuta consistently ranks very high in this regard, indicating that impoverished households in this region face a significant problem of material deprivation. In the moderately populated areas of the Community of Madrid, the material deprivation ranking is consistently at very low levels, while in other cases, its ranking varies depending on the linguistic quantifiers used. The analysis further reveals that the material deprivation ranking of each region can vary depending on the degree of urbanization. In the Community of Madrid, households in densely and sparsely populated areas are more likely to experience material deprivation than those in moderately populated areas. Additionally, the findings highlight that not all economically impoverished households face the problem of material deprivation. In such cases, regardless of the attitudes of decision-makers, these households would not be considered materially deprived.

The primary contributions of this work can be summarized as follows:

- It combines the OWA aggregation operator, the 2-tuple linguistic model, and hierarchical clustering, providing a more comprehensive analysis of material deprivation than traditional approaches based on single thresholds or isolated methods.
- It improves the interpretability of results by translating complex numerical outputs into intuitive and policy-relevant scenarios.
- It introduces a flexible framework for measuring material deprivation that enables decision-makers to examine multiple scenarios and how deprivation patterns evolve under different conditions.
- It supports more precise targeting of interventions, enabling decision-makers, like policymakers and social planners, to allocate resources more efficiently across different areas within a region, thereby facilitating more targeted and evidence-based policy design.

Despite its contributions, several limitations should be acknowledged. The analysis is constrained by the absence of household size data, which may affect the assessment of deprivation. In addition, this work relies on binary deprivation indicators. Although these are officially provided and no alternative measures are available, this approach may overlook the intensity or gradation of deprivation within specific items. Moreover, this work relies on a single clustering method, which limits the ability to assess whether alternative algorithms might offer greater accuracy or computational efficiency. The clustering process is largely exploratory, and no uncertainty analysis has been conducted. As the empirical application is based on Spanish data, the findings may not be directly generalizable to other national contexts without appropriate adaptation. Although the framework is flexible, it does not include an interactive visualization tool to reproduce the results or to allow decision-makers to determine the number of clusters based on their specific objectives or statistical criteria. Finally, while the model successfully identifies that some economically impoverished households do not experience material deprivation, the scope of the current analysis does not capture the underlying causal factors driving this phenomenon.

Therefore, future research should address these limitations in several ways. First, additional demographic variables, non-binary or graded indicators, and uncertainty analysis could be incorporated into the analysis to improve measurement precision. Applying the framework to datasets from other countries would also help validate its generalizability and adaptability. Further investigation is needed to understand why some economically

impoverished households do not experience material deprivation. The development of an interactive visualization tool could facilitate the reproduction of results, allow users to adjust clustering parameters, and support more effective decision-making. Finally, alternative clustering techniques could be explored to compare results and further improve the accuracy and efficiency of the analysis.

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Disclosure statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability statement

Dataset related to this article can be found at https://www.ine.es/dyngs/INEbase/en/operacion.htm?c=Estadistica_C&cid=1254736176807&menu=resultados&iidp=1254735976608#_tabs-1254736195153

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