

A MULTI-VALUED NEUTROSOPHIC ARAS MODEL FOR ECONOMIC RESILIENCE IN UNCERTAIN OLIVE FARMING SYSTEMS

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Abstract. This study proposes the integration of Multi-Valued Neutrosophic Numbers (MVNNs) with the Additive Ratio Assessment (ARAS) method to conduct an economic evaluation of multi-criteria decision-making (MCDM) problems under high uncertainty. MVNNs enhance traditional neutrosophic approaches by allowing multiple degrees of truth, indeterminacy, and falsity, providing a richer representation of the ambiguous and often conflicting economic data prevalent in agricultural planning. The proposed MVN-ARAS framework is presented with formal definitions, aggregation operators, and a step-by-step decision-making procedure tailored for economic assessment. Its effectiveness is demonstrated through an illustrative investment case and a real-world application to prioritize and economically evaluate climate change adaptation strategies for smallholder olive growers in Chile's Coquimbo Region. Results show that drought-resistant olive cultivars and water-saving irrigation technologies achieve the highest relative utility values, indicating their dominant role in improving economic resilience under prolonged water scarcity. In contrast, diversification-oriented measures – such as value-added olive products and rural ecotourism – rank lower but still contribute to long-term income stability. These findings highlight the ability of the MVN-ARAS framework to deliver a structured economic evaluation, a robust ranking of alternatives based on cost-benefit and risk criteria, and improved handling of conflicting expert opinions, offering decision-makers a transparent and reliable tool for strategic investment and resource allocation to enhance economic resilience.

Keywords: multi-valued neutrosophic numbers, additive ratio assessment, neutrosophic decision-making, multi-criteria decision making, climate change adaptation, economic resilience.

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1. Introduction

In today's rapidly changing world, decision-making in complex and uncertain environments has become increasingly challenging. Information variability, global interconnectivity, geopolitical instability, and unpredictable outcomes all contribute to this difficulty.

These challenges are particularly pronounced in critical sectors. In the fields of medicine and public health, the emergence of new diseases, recurrent pandemics, and growing antibiotic resistance have intensified systemic vulnerabilities (Morehead & Scarbrough, 2018; Salam et al., 2023) demands innovative strategies and rapid responses. The COVID-19 pandemic highlighted how swiftly infectious diseases can spread, causing widespread morbidity, mortality, and socioeconomic disruption (V. Mishra et al., 2021). In the economy, geopolitical shifts,

digitization, and automation have introduced new layers of uncertainty to long-term planning (Zheng & Gong, 2024). At the same time, advances in AI and data science are reshaping decision-making by optimizing resources and enhancing efficiency, while social media accelerates information flows, influencing geopolitics and economic growth but also reducing productivity and shaping public opinion (Kamruzzaman, 2022). Agriculture faces equally pressing challenges, as climate change drives extreme and unpredictable weather patterns that disrupt water availability, alter production cycles, and threaten food security (Mirzabaev et al., 2023). These disruptions lower crop yields and farm incomes, yet studies suggest that strong climate action could mitigate such risks and even enhance global GDP by 2040.

Agriculture is a central pillar of Chile's economy, contributing significantly to exports and rural employment, yet it faces growing challenges from climate variability and water scarcity (Fernández et al., 2023). Prolonged droughts, declining river flows, and shifts in precipitation patterns have reduced water availability for irrigation, threatening both productivity and farmer livelihoods (Fuentes et al., 2021). These pressures are particularly acute in the country's arid and semi-arid regions, where irrigation is essential for sustaining crop production.

Within this context, olive cultivation stands out as a strategic activity, supplying both domestic and international markets with high-quality olive oil and earning recognition in global competitions (Herrera-Cáceres et al., 2017). However, the sector is under mounting pressure from prolonged droughts, erratic rainfall, and sustained declines in water availability – conditions exacerbated by climate change (Mirzabaev et al., 2023). These adverse factors directly affect irrigation management, reduce yields, and erode farmer incomes, making long-term planning increasingly uncertain. In regions such as Coquimbo, irrigation is indispensable for the commercial viability of olive groves, with recommended strategies including drip and deficit irrigation, alongside the adoption of drought-tolerant cultivars like Coratina and Leccino (Mora et al., 2007). Producers and policymakers are thus confronted with difficult decisions among competing adaptation strategies – ranging from investing in water-saving technologies to shifting toward drought-resistant varieties – often under conditions of incomplete information and conflicting expert opinions. In such settings, where uncertainty is high and multiple, sometimes contradictory, factors must be considered, structured analytical approaches become essential to guide robust and effective decision-making. Multi-Criteria Decision-Making (MCDM) methods provide such structure by offering systematic ways to evaluate alternatives under diverse criteria. Classical approaches such as AHP (Saaty, 1987), VIKOR (Opricovic & Tzeng, 2004), and TOPSIS (Hwang & Yoon, 1981) have proven effective in many applications but often struggle when uncertainty is high. To address this limitation, researchers have developed extensions that incorporate uncertainty modeling, particularly through fuzzy numbers, which better capture vagueness in decision problems (Yager, 2016). Methods like Fuzzy AHP (Demirel et al., 2008; Y. Liu et al., 2020; Ahmed & Kilic, 2019) and Fuzzy TOPSIS (Nădăban et al., 2016; Palczewski & Sałabun, 2019) improve decision accuracy by representing imprecise information more effectively.

The foundation of these methods lies in fuzzy set theory, introduced by Zadeh (1965), which used membership functions to handle linguistic uncertainty. Later, Atanassov (1986) expanded this framework with Intuitionistic Fuzzy Sets (IFS), incorporating both membership and non-membership functions. Subsequent advances included Interval-Valued Fuzzy Sets

(IVFS) (Gorzalczy, 1987), Pythagorean Fuzzy Sets (PFS) (Yager, 2014), and Neutrosophic Sets (NS) (Smarandache, 2004), which introduced three independent membership functions – truth, indeterminacy, and falsity. These developments aimed to better capture ambiguity and conflicting information in decision-making.

Further refinements include Single-Valued Neutrosophic Sets (SVNS) (H. Wang et al., 2010), Quadripartitioned Single-Valued Neutrosophic Sets (QSVNS) (Chatterjee et al., 2016), and, most recently, Multi-Valued Neutrosophic Numbers (MVNNs) (Peng & Wang, 2015). MVNNs extend traditional neutrosophic sets by allowing truth, indeterminacy, and falsity memberships to take multiple values, thereby improving their ability to represent conflicting and imprecise information. This makes them especially valuable in domains such as risk analysis (Peng et al., 2014; P. Liu et al., 2016) and medical diagnosis (Martina & Deepa, 2023), where uncertainty is pervasive. Table 1 summarizes key differences between MVNNs and other fuzzy set extensions.

In light of the increasing need for robust Fuzzy Multi-Criteria Decision-Making (FMCDM) models in dynamic environments (Shahmohammad et al., 2024), MVNNs offer a richer framework for handling uncertainty. Their integration into MCDM strengthens reliability and adaptability, allowing for more realistic and effective assessments.

Given the limitations of conventional MCDM methods in highly uncertain settings like Coquimbo's olive sector, a method that is both robust to uncertainty and practically interpretable is required. While MVNNs provide the necessary robust framework for handling indeterminacy and conflicting data, they lack an inherent, transparent ranking procedure. This presents a critical methodological gap: the need to pair MVNNs' advanced uncertainty modeling with a simple, interpretable decision-making structure for real-world application. To bridge this gap, this study integrates MVNNs with the ARAS method. The ARAS method is selected for its transparent additive structure and ease of interpretation, which are vital for stakeholder engagement in agricultural planning (Thakkar, 2021). While standard ARAS can struggle with high uncertainty, its framework integrates naturally with MVNN-based modeling. This synergy is key: the MVNN component directly overcomes ARAS's limitations in uncertain contexts by capturing a broader spectrum of expert opinions and ambiguity, while ARAS provides the clear, systematic ranking procedure that MVNNs lack. For these reasons, the MVN-ARAS hybrid is better suited for evaluating climate-adaptation strategies in the olive sector than other compensatory methods or a standalone fuzzy ARAS approach.

Accordingly, the objective of this article is to develop and validate this novel MVN-ARAS model, providing a more reliable and uncertainty-resilient framework for selecting climate change adaptation policies in Coquimbo's olive farming sector.

The remainder of the article is structured as follows. Section 2 reviews MVNNs, their operators, and their integration with MCDM methods, as well as extensions of ARAS in uncertain environments. Section 3 introduces the theoretical foundations of MVNNs, while Section 4 presents the proposed MVN-ARAS framework. Section 5 illustrates the approach through an example with sensitivity analysis, and Section 6 applies MVN-ARAS to evaluate adaptation policies for olive farmers in Coquimbo. Section 7 discusses economic development implications, while Section 8 discusses managerial and governance implications. Finally, Section 9 summarizes the key findings and outlines directions for future research.

Table 1. A comparative analysis of Multi-Valued Neutrosophic Sets (MVNSs) against other fuzzy set extensions based on key characteristics. The comparison framework is adapted from (Borah & Dutta, 2024).

Approach	Uncertainty Handling	Membership Components	Indeterminacy Consideration	Multiple Values Representation	Suitability for Complex Decision-Making
FSs	Handles vagueness but not full uncertainty	Membership function (μ)	No explicit representation	No	Moderate
IFS	Extends FSs by adding a hesitation degree	$\mu + \nu$ (Non-membership)	Indirectly modeled	No	Moderate
IVFSs	Provides a range of values to express uncertainty	Interval-valued μ	Not explicitly considered	No	Moderate
PFSSs	Allows greater flexibility than IFSs	$(\mu^2 + \nu^2 \leq 1)$	Not explicitly considered	No	Moderate
NSs	Handles indeterminacy and inconsistency	Three independent components: T , I , and F	Explicitly modeled	No	High
SVNSs	Simplified NSs where T , I , and F assume single values	T , I , F (Single values)	Explicitly modeled	No	High
QSVNSs	Further refines indeterminacy handling by partitioning it into two components	T , I (Contradiction & Ignorance), F	Provides a clearer distinction of indeterminacy	No	High
MVNSs	Extends NSs by allowing multiple values per component	Multiple values for T , I , and F	Provides the most detailed modeling of indeterminacy	Yes	Very High

2. Background

In this section, the study compiled and structured the fundamental theoretical contents and concepts related to MVNSs and the ARAS method, with the purpose of addressing problems involving uncertainty. This approach seeks to establish a solid theoretical basis for the article, which will allow effective progress in the following stages of the research.

2.1. Overview of multi-valued neutrosophic numbers

Decision-making in complex and uncertain environments often requires advanced computational techniques and structured evaluation frameworks. MVNSs have emerged as a significant advancement in the field of MCDM, offering a robust framework for handling uncertainty, indeterminacy, and inconsistency. Unlike traditional methods, MVNSs allow truth membership, indeterminacy-membership, and falsity-membership functions to take on a set

of crisp values between zero and one (Peng et al., 2014; Peng & Wang, 2015). This capability makes MVNNs particularly suitable for addressing complex decision-making scenarios where conventional approaches may fall short.

Recent research has focused on developing specialized operators and methods to enhance the applicability of MVNNs in MCDM. For instance, Peng et al. (2018) introduced the multi-valued neutrosophic geometric weighted Choquet integral Heronian mean (MVNGWCIHM) operator, which combines the Heronian mean and Choquet integral for MVNNs. This operator is designed to handle cases with unknown criteria weights and has been validated through illustrative examples, sensitivity analysis, and comparative studies. Similarly, P. Liu et al. (2016) proposed Bonferroni mean operators for MVNNs, including the Weighted Bonferroni Mean (WBM) and Weighted Geometric Bonferroni Mean (WGBM) operators. They further extended these to Multi-Valued Neutrosophic Weighted BONFERRONI Mean (MVNWBM) and Multi-Valued Neutrosophic Weighted Geometric Bonferroni Mean (MVNWGBM) operators, demonstrating their application in investment selection and analyzing the impact of parameter choices on decision outcomes.

In the real of linguistic approaches, Yang and Li (2018) introduced Multi-Valued Neutrosophic Linguistic Sets (MVNLSs) and defined operations for Multi-Valued Neutrosophic Linguistic Numbers (MVNLNs). They proposed two aggregation operators, the Multi-Valued Neutrosophic Linguistic Power Weighted Average (MVNLPWA) and Multi-Valued Neutrosophic Linguistic Power Weighted Geometric (MVNLPWG), and developed an MCDM method based on these operators. Their approach was validated through an illustrative example, showcasing its effectiveness. Building on this, Kamal et al. (2020) proposed a group decision-making framework using the Multi-Valued Interval Neutrosophic Linguistic Set Weighted Arithmetic Average (MVINLSWAA) and Multi-Valued Interval Neutrosophic Linguistic Set Weighted Geometric Average (MVINLSWGA) operators. These operators, combined with a score function, enable the aggregation and ranking of multi-valued interval neutrosophic linguistic information, facilitating the selection of optimal alternatives.

Correlation-based methods have also been explored to address challenges in MVNNs. For example, Ye et al. (2020) proposed a method to transform MVNNs into Consistency Single-Valued Neutrosophic Sets (CSVNSs) using average values and consistency degrees. This transformation enables consistent operations across sequences of varying lengths. The authors introduced two correlation coefficients for CSVNSs and developed a decision-making approach within the MVNNs framework. The method's effectiveness and rationality were demonstrated through illustrative examples and comparative analyses.

Despite these advancements, few studies have integrated MVNNs with established MCDM approaches such as ELECTRE (Roy, 1968), QUALIFLEX (Paelinck, 1978), and MULTI-MOORA (Brauers & Zavadskas, 2010). For instance, extensions of the ELECTRE method have been proposed to incorporate MVNNs, enhancing its ability to manage multi-valued neutrosophic information (Peng et al., 2017a). Similarly, the QUALIFLEX method has been adapted to utilize MVNNs, offering a flexible approach to MCDM problems (Peng & Tian, 2018; Peng et al., 2017b). In the work of Xiao et al. (2021) proposed an Improved Generalized Multi-Valued Neutrosophic Weighted Heronian Mean (IGMVNWHM) operator to better handle interactions among criteria, a new distance measure for deriving objective weights, and an

improved MULTIMOORA method integrated with prospect theory to account for decision-makers' bounded rationality.

The integration of MVNNs with these classical methods not only enriches the decision-making toolkit but also provides a more nuanced understanding of decision criteria and alternatives. These developments highlight the versatility and potential of MVNNs in addressing real-world problems, underscoring the need for further research to explore their full capabilities.

2.2. Overview of ARAS method

The ARAS method has been extensively reviewed and applied across various fields, showcasing its versatility. The ARAS method was initially developed to address the challenges posed by qualitative and quantitative information in Multi-Attribute Decision-Making (MADM) problems, particularly those involving different measurement units. It has since been extended to various information environments and application fields, as highlighted in a comprehensive review that discusses its theoretical development, application extensions, and future challenges (N. Liu & Xu, 2021).

In logistics, the ARAS method, specifically its grey version (ARAS-G), has been utilized to evaluate the logistics performance of OECD (Stević et al., 2024). This application demonstrates the method's ability to handle time-series data and compare it with existing indices like the World Bank's Logistics Performance Index, providing a robust alternative for performance evaluation. Similarly, in mineral prospectivity mapping, a hybrid approach combining the Best-Worst Method (BWM) and ARAS has been proposed, which effectively prioritizes and ranks mineralization prospects, outperforming traditional methods like TOPSIS (Bahrami et al., 2019).

The ARAS method, particularly when combined with fuzzy approaches, has been widely applied across diverse decision-making scenarios. For instance, the Fermatean fuzzy approach in food waste treatment technology selection demonstrates ARAS's ability to handle uncertainty and qualitative information by incorporating Fermatean fuzzy numbers to manage imprecision and strengthen decision-making robustness (Rani et al., 2021). In a similar vein, the hesitant fuzzy linguistic approach used in smart watch evaluation highlights ARAS's adaptability in contexts where preferences are uncertain and expressed linguistically, integrating hesitant fuzzy linguistic term sets to reflect the subjective nature of judgments (Büyükožkan & Güler, 2020). Building on these advances, Hu et al. (2022) developed a q-rung orthopair fuzzy SWARA-ARAS framework to assess IoT risks in supply chain management, illustrating ARAS's effectiveness in highly uncertain, information-intensive environments. By coupling SWARA for criteria weighting with ARAS under q-rung orthopair fuzzy sets, their work emphasizes the method's flexibility in addressing complex technological and organizational risks. Similarly, Liao et al. (2019) integrated the Best Worst Method (BWM) with ARAS under a hesitant fuzzy linguistic setting to support digital supply chain finance supplier selection. This HFL-BWM-ARAS framework demonstrates the method's capacity to accommodate multiple experts, ensuring both criteria weighting and alternative ranking capture hesitant linguistic information. Extending ARAS's applicability to the economic and technological domains, Ecer

(2018) proposed a Fuzzy AHP-ARAS model to evaluate mobile banking services, where FAHP derived the criteria weights and ARAS ranked the alternatives. This integration underscores ARAS's versatility in balancing qualitative perceptions, such as ease of use, risk, and system quality, with quantitative performance indicators, reinforcing its robustness across complex decision-making environments.

In the field of facilities management strategy selection, the ARAS method is combined with fuzzy MCDM models to address the fuzzy nature of market situations and stakeholder preferences. This integration allows for a comprehensive evaluation of management strategies by considering cost-effectiveness and stakeholder priorities (Zavadskas et al., 2017). Furthermore, the interval-valued intuitionistic fuzzy environment extends the ARAS methodology to digital supply chain management, facilitating supplier selection by incorporating decision-makers' expertise and handling uncertainty in the evaluation process (Büyüközkan & Göçer, 2018).

The interval-valued fuzzy extension of the ARAS method is also applied in the evaluation of oil and gas well drilling projects, highlighting its utility in balancing sustainable development goals with environmental and human well-being considerations. This approach underscores the method's flexibility in addressing complex, multi-attribute decision-making problems (Dahooie et al., 2018). Additionally, the fuzzy ARAS-H method, which incorporates hierarchical criteria, exemplifies the method's ability to manage linguistic variables and expert judgments in a structured decision-making framework (Ghram & Frikha, 2022). The ARAS method has also been extended in various directions. For instance, the integration of Z-numbers has led to the Z-ARAS method for FMEA applications, improving reliability under uncertain risk evaluations (Adalı & Tuş, 2023). Other enhancements, such as A-ARAS1 and A-ARAS2, were developed to address the rank reversal phenomenon and improve evaluation accuracy in performance appraisal settings (Karimi & Nikkhah-Farkhani, 2022). Although rank reversal can occasionally arise in compensatory methods, it is not a major threat in this study because the set of alternatives remains fixed and no alternatives are added or removed during analysis. For this reason, A-ARAS2 was not required; the standard ARAS formulation is sufficient and methodologically consistent with MVNN integration while avoiding unnecessary model complexity.

A. R. Mishra and Rani (2021) proposed a novel framework integrating ARAS with q-Rung Orthopair Fuzzy Sets (q-ROFSs) and information measures to evaluate Sustainable Recycling Partners (SRPs), demonstrating its effectiveness in uncertain decision-making environments. The study introduced new entropy and discrimination measures for q-ROFSs to determine criteria weights, balancing subjective expert inputs with objective data. Through a case study and sensitivity analysis, the framework's robustness was validated, showing consistent results despite variations in criteria weights. However, the authors acknowledge limitations in handling uncertain, imprecise, indeterminate, and inconsistent information, an area where MVNNs could provide a promising solution.

In the context of drought management, Dehkordi et al. (2025) developed an integrated model combining fuzzy Shannon entropy and the Fuzzy ARAS method. Unlike earlier works that treated risk indicators separately, this study compiled a comprehensive set of factors from the literature and applied them simultaneously to evaluate four management scenarios

in Chaharmahal and Bakhtiari province. Using fuzzy Shannon entropy, volume reliability, vulnerability, and sustainability emerged as the most critical indicators, while fuzzy ARAS ranked cropping pattern change as the top priority, followed by reducing agricultural demand and improving irrigation systems. The study highlights how integrated fuzzy MCDM approaches can strengthen decision-making in drought-prone regions by providing a structured and risk-oriented framework for water resource management.

2.3. Overview of fuzzy ARAS in agriculture under climate change uncertainty

Agricultural decision-making faces increasing complexity due to climate change impacts, such as erratic rainfall, temperature variability, and extreme weather events. Fuzzy MCDM methods, particularly the ARAS technique, have been employed to address uncertainty in crop selection, irrigation planning, and sustainable farming adaptation.

The foundational work of Turskis and Zavadskas (2010) introduced ARAS as a robust ranking tool, later extended to fuzzy environments Turskis et al. (2012) in the assessment of construction site alternatives for a non-hazardous waste incineration plant. Recent studies have applied Fuzzy ARAS to climate-resilient agriculture, such as Mardani et al. (2020), who evaluated drought-resistant crop alternatives using fuzzy linguistic scales, and Mishra et al. (2022), who integrated ARAS with interval-valued fuzzy sets to assess adaptive farming strategies under climatic volatility.

Overall, the ARAS method's adaptability to different contexts and its integration with other decision-making frameworks underscore its utility in diverse applications, from logistics and mineral mapping to risk assessment and performance appraisal. These reviews and applications highlight the method's potential for continued development and its capacity to provide reliable decision-making support across various domains.

Although ARAS has been extended to different information environments, it applies the ARAS method using triangular fuzzy sets, followed by gray sets. Some studies start with linguistic variables, which are then converted into triangular fuzzy numbers, interval triangular fuzzy numbers, hesitant triangular fuzzy numbers, or Z-numbers to enhance computability (N. Liu & Xu, 2021). However, approaches along the lines of neutrosophic numbers and MVNNs are not presented. MVNNs are highly effective in handling uncertainty and multi-valued data but lack a structured decision-making framework. ARAS provides a transparent and systematic approach to MCDM but struggles with dynamic or uncertain. While the integration of MVNNs and MCDM methods has shown promise, existing studies have not explored the combination of MVNNs and ARAS.

This gap presents a significant opportunity for innovation. By integrating MVNNs with ARAS, it is possible to develop a hybrid model that leverages the adaptive learning capabilities of MVNNs and the structured evaluation framework of ARAS. Such a model could address the limitations of both methods and provide a more robust solution for decision-making in complex, uncertain environments.

3. Preliminaries

In this section, the definitions and operators that will be used in the rest of the article are established.

3.1. Neutrosophic set

Definition 3.1 (Smarandache, 2006). Let U be a universe of discourse. A neutrosophic set A can be defined as:

$$A = \{ \langle y, T_A(y), I_A(y), F_A(y) \rangle \mid y \in U \}, \tag{1}$$

where, $T_A(y), I_A(y), F_A(y): U \rightarrow [-0, 1]^+$ define the degree of truth-membership, indeterminacy-membership, and falsity-membership, respectively. There is no restriction on their sum:

$$0 \leq T_A(y) + I_A(y) + F_A(y) \leq 3. \tag{2}$$

3.2. Single-valued neutrosophic set

Definition 3.2 (H. Wang et al., 2010). Let U be a universal set. A single-valued neutrosophic set (SVNS) A over U is defined as:

$$A = \{ \langle y, T_A(y), I_A(y), F_A(y) \rangle \mid y \in U \}, \tag{3}$$

where, for each $y \in U$, we have $T_A(y), I_A(y), F_A(y) \in [0, 1]$ and

$$0 \leq T_A(y) + I_A(y) + F_A(y) \leq 3. \tag{4}$$

3.3. Multi-valued neutrosophic set

Definition 3.3 (H. Wang et al., 2010). Let U be a space of points. A multi-valued neutrosophic set (MVNS) A over U is defined as:

$$A = \{ \langle y, T_A(y), I_A(y), F_A(y) \rangle \mid y \in U \}, \tag{5}$$

where:

$$T_A(y) = \{ T_A^1(y), T_A^2(y), \dots, T_A^q(y) \},$$

$$I_A(y) = \{ I_A^1(y), I_A^2(y), \dots, I_A^r(y) \},$$

$$F_A(y) = \{ F_A^1(y), F_A^2(y), \dots, F_A^s(y) \}$$

and $0 \leq T_A^i(y), I_A^j(y), F_A^k(y) \leq 1 \forall y \in U$.

This extension of neutrosophic sets accounts for situations where an entity may have multiple truth, indeterminacy, and falsity values rather than a single or interval-based value. This is particularly useful in scenarios involving multiple decision-makers or systems collecting input from multiple sources, such as multi-sensor fusion applications.

Definition 3.4. Let $A = \langle \overline{T}_A, \overline{I}_A, \overline{F}_A \rangle$ be a multi-valued neutrosophic number. Then, the complement set (A^C) is defined as follows:

$$A^C = \left\langle \bigcup_{\gamma \in I_A} \{1-\gamma\}, \bigcup_{\eta \in I_A} \{1-\eta\}, \bigcup_{\xi \in F_A} \{1-\xi\} \right\rangle.$$

Definition 3.5. To examine how variations in the weights influence the decision-making process within our proposed framework, we compute the weights using the metric introduced by (Biswas et al., 2015). Let $(A_j = \langle T_A^j, I_A^j, F_A^j \rangle)$ represent the multi-valued neutrosophic evaluation of the (j^{th}) criterion provided by the expert. Then, the weight (w_j) of the (j^{th}) criterion is defined as:

$$w_j = \bigcup_{\gamma \in T_A^j, \eta \in I_A^j, \xi \in F_A^j} \frac{1 - \sqrt{\frac{(1-\gamma)^2 + \eta^2 + \xi^2}{3}}}{\sum_{k=1}^n \bigcup_{\gamma' \in T_A^k, \eta' \in I_A^k, \xi' \in F_A^k} \left(1 - \sqrt{\frac{(1-\gamma')^2 + (\eta')^2 + (\xi')^2}{3}} \right)}$$

subject to

$$\sum_{j=1}^n w_j = 1.$$

To evaluate the influence of different criteria in the decision-making process under the proposed multi-valued neutrosophic framework, we adopt the weight computation metric introduced by Biswas et al. (2015). Subsequently, the aggregation of the multi-valued neutrosophic evaluations is performed using the Multi-Valued Neutrosophic Number Weighted Averaging (MVNNWA) operator proposed by Peng and Wang (2015).

Definition 3.6. Let $A_j = \langle T_A^j, I_A^j, F_A^j \rangle$ for $j=1,2,\dots,n$ be a collection of Multi-Valued Neutrosophic Numbers (MVNNs). The Multi-Valued Neutrosophic Number Weighted Averaging (MVNNWA) operator is defined as:

$$\text{MVNNWA} : \text{MVNN}^n \rightarrow \text{MVNN},$$

and expressed by:

$$\text{MVNNWA}(A_1, A_2, \dots, A_n) = \sum_{j=1}^n w_j A_j, \tag{6}$$

where, $W = w_1, w_2, \dots, w_n$ notes the weight vector, satisfying $w_j \geq 0$ and $\sum_{j=1}^n w_j = 1$.

Given a collection of MVNNs $A_j = \langle T_A^j, I_A^j, F_A^j \rangle$ for $j = 1, 2, \dots, n$, the aggregated result obtained using the MVNNWA operator is also an MVNN and is given by:

$$\text{MVNNWA}(A_1, A_2, \dots, A_n) = \left\langle \bigcup_{\gamma_j \in T_{A_j}} \left\{ 1 - \prod_{j=1}^n (1 - \gamma_j)^{w_j} \right\}, \bigcup_{\eta_j \in I_{A_j}} \left\{ \prod_{j=1}^n \eta_j^{w_j} \right\}, \bigcup_{\xi_j \in F_{A_j}} \left\{ \prod_{j=1}^n \xi_j^{w_j} \right\} \right\rangle, \tag{7}$$

where, $W = (w_1, w_2, \dots, w_n)$ is the weight vector with $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

The MVNNWA operator satisfies the following fundamental properties:

- 1. **Idempotency:** If all MVNNs are identical, i.e., $A_j = A$ for all j , then:

$$\text{MVNNWA}(A_1, A_2, \dots, A_n) = A.$$

2. **Boundedness:** For $A_j = \langle T_A^j, I_A^j, F_A^j \rangle$, the lower and upper bounds of the aggregation are defined as: $A^- = \langle \min_j T_A^j, \max_j I_A^j, \max_j F_A^j \rangle$, $A^+ = \langle \max_j T_A^j, \min_j I_A^j, \min_j F_A^j \rangle$, and the following inclusion relation holds:

$$A^- \subseteq \text{MVNNWA}(A_1, A_2, \dots, A_n) \subseteq A^+.$$

Definition 3.7. Let $(A = \langle T_A, I_A, F_A \rangle)$ be a MVNN, then the score function $s(A)$, accuracy function $a(A)$, and certainty function $c(A)$ of an MVNN are defined as follow (Peng & Wang, 2015):

$$s(A) = \frac{1}{I_{T_A} \cdot I_{I_A} \cdot I_{F_A}} \times \sum_{\gamma_i \in \widetilde{T}_A, \eta_j \in \widetilde{I}_A, \xi_k \in \widetilde{F}_A} \frac{\gamma_i + 1 - \eta_j + 1 - \xi_k}{3}; \tag{8}$$

$$a(A) = \frac{1}{I_{T_A} \cdot I_{F_A}} \sum_{\gamma_i \in \widetilde{T}_A, \xi_k \in \widetilde{F}_A} (\gamma_i - \xi_k); \tag{9}$$

$$c(A) = \frac{1}{I_{T_A}} \sum_{\gamma_i \in \widetilde{T}_A} \gamma_i; \tag{10}$$

where, $\gamma_i \in \widetilde{T}_A, \eta_j \in \widetilde{I}_A, \xi_k \in \widetilde{F}_A$, and $I_{T_A}, I_{I_A}, I_{F_A}$ denote the element numbers $\widetilde{T}_A, \widetilde{I}_A$ and \widetilde{F}_A , respectively.

The score function is an important index in ranking MVNNs. For an MVNN A , the greater the truth-membership \widetilde{T}_A , the greater the MVNN. Conversely, the lower the indeterminacy-membership \widetilde{I}_A , the greater the MVNN. Similarly, the lower the false-membership \widetilde{F}_A , the greater the MVNN.

For the accuracy function, if the difference between truth and falsity is greater, then the statement is more affirmative. That is, the larger the values of $\widetilde{T}_A, \widetilde{I}_A$, and \widetilde{F}_A , the more accurate the MVNN.

As for the certainty function, the greater the truth-membership \widetilde{T}_A , the more certainty in the MVNN.

The definitions and operators presented in this section form the theoretical foundation for the proposed MVN-ARAS framework. They will be consistently applied throughout the remainder of the article to model expert evaluations, aggregate multi-valued neutrosophic information, compute weights, and rank alternatives in the decision-making process.

The score function is an important index in ranking MVNNs. For an MVNN A , the greater the truth-membership \widetilde{T}_A , the greater the MVNN. Conversely, the lower the indeterminacy-membership \widetilde{I}_A , the greater the MVNN. Similarly, the lower the false-membership \widetilde{F}_A , the greater the MVNN.

For the accuracy function, if the difference between truth and falsity is greater, then the statement is more affirmative. That is, the larger the values of $\widetilde{T}_A, \widetilde{I}_A$, and \widetilde{F}_A , the more accurate the MVNN.

As for the certainty function, the greater the truth-membership \widetilde{T}_A , the more certainty in the MVNN.

The definitions and operators presented in this section form the theoretical foundation for the proposed MVN-ARAS framework. They will be consistently applied throughout the remainder of the article to model expert evaluations, aggregate multi-valued neutrosophic information, compute weights, and rank alternatives in the decision-making process.

4. Multi-value Neutrosophic ARAS

This section presents a new integrated decision-making framework, called MVN-ARAS, designed to address complex MCDM problems from the perspective of MVNNs. To this end, key concepts of MVNNs are incorporated, including definitions, aggregation methods, and expected values. The MVN-ARAS approach is based on a consistent comparison by weighted summation of the criteria values, which allows determining the degree of optimality of each alternative. The steps of the proposed approach are described below, and their graphical representation is shown in Figure 1.

Assume there are n available alternatives, denoted as $A = \{a_1, a_2, \dots, a_n\}$, and m criteria, represented as $C = \{c_1, c_2, \dots, c_m\}$. Each criterion c_j is assigned a weight w_j , forming the weight vector: $w = (w_1, w_2, \dots, w_m)$, where $w_j \geq 0$ for all $j=1, 2, \dots, m$, and the sum of the weights

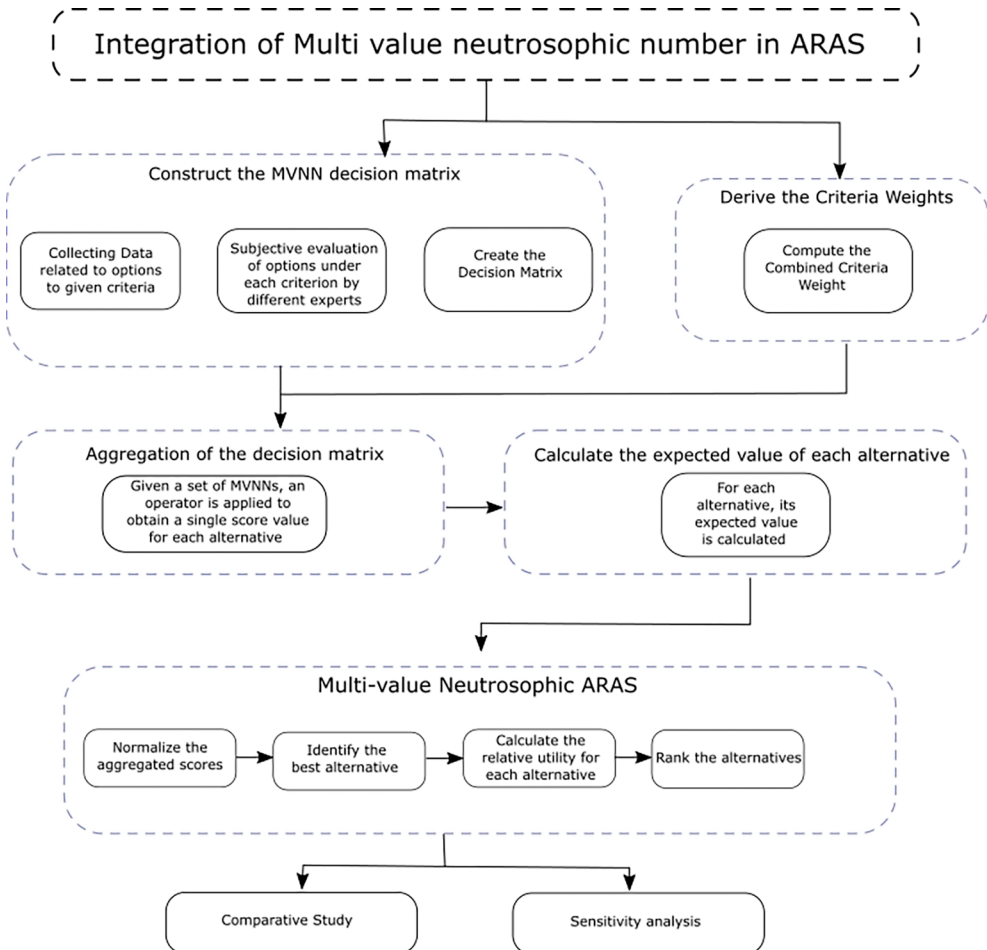


Figure 1. Schematic diagram of the proposed methodology for integrating MVNNs into the ARAS framework, illustrating the main stages from data collection and aggregation to the final ranking of alternatives and comparative analysis.

satisfies: $\sum_{j=1}^m w_j = 1$. Let $R = a_{ij_{n \times m}}$ define the neutrosophic decision matrix, where each criterion value a_{ij} is represented by a MVNN $a_{ij} = (T_{ij}, I_{ij}, F_{ij})$, where: T_{ij} denotes the degree of truth membership, indicating how strongly alternative a_i satisfies criterion c_j . I_{ij} represents the indeterminacy membership, reflecting the uncertainty in evaluating a_i under c_j . F_{ij} expresses the falsity membership, showing the extent to which a_i does not satisfy c_j .

The following methodology is proposed to assess and rank the alternatives, ultimately selecting the most suitable one(s).

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Step 1: Construct the initial decision matrix

- Define a *neutrosophic decision-making matrix* $\tilde{X}_{m \times n}$, where each alternative a_i ($i = 1, 2, \dots, m$) is evaluated based on criteria c_j ($j = 1, 2, \dots, n$).
- Each element of the matrix is represented as a MVNN:

$$\tilde{x}_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle,$$

where:

- T_{ij} represents the truth-membership;
- I_{ij} represents the indeterminacy-membership;
- F_{ij} represents the falsity-membership.

Step 2: Normalization of decision matrix

Normalize the decision information to obtain the standardized matrix considering both benefit and cost criteria (J. Wang & Li, 2015). To ensure consistency across criteria, cost criteria are typically transformed into benefit criteria using the following approach:

$$\beta_{ij}^k = \begin{cases} \tilde{x}_{ij}^k & \text{for benefit criteria } c_j \\ (\tilde{x}_{ij}^k)^c & \text{for cost criteria } c_j \end{cases}, (i = 1, 2, \dots, m; j = 1, 2, \dots, n),$$

where $(\tilde{x}_{ij}^k)^c$ is the complement of \tilde{x}_{ij}^k . The decision matrix $\tilde{X} = \tilde{X}_{m \times n}$ can be transformed into a normalized decision matrix $x^k = (\beta_{ij}^k)_{m \times n}$.

The detailed normalization methods are shown as follows.

For the cost criteria, the normalization equation is:

$$\beta_{ij}^k = (r_{ij}^k)^c = \left\{ \bigcup_{\nu_j \in \tilde{T}_{A_j}} \{1 - \nu_j\}, \bigcup_{\eta_j \in \tilde{I}_j} \{1 - \eta_j\}, \bigcup_{\xi_j \in \tilde{F}_j} \{1 - \xi_j\} \right\}, \tag{11}$$

for the benefit criteria, is

$$\beta_{ij}^k = r_{ij}^k = \left\{ \bigcup_{\nu_j \in \tilde{T}_{A_j}} \{\nu_j\}, \bigcup_{\eta_j \in \tilde{I}_j} \{\eta_j\}, \bigcup_{\xi_j \in \tilde{F}_j} \{\xi_j\} \right\}.$$

Step 3: Aggregation of decision matrix

Given a set of MVNNs, using the operator MVNNWA of the Equation (7) we can obtain an individual value of the alternative a_i ($i = 1, 2, \dots, n$).

Step 4: Calculate the expected value of each alternative

Using the Eq. (8), a score $s(A)$ is calculated for each alternative. The ARAS method is well-suited for ranking alternatives based on their proportional benefit relative to the best available option (Zavadskas & Turskis, 2010). Since each alternative a_i has a single aggregated value after the MVNN aggregation, ARAS can directly process this data.

Step 5: Normalize the aggregated scores

Each alternative a_i has an aggregated value $s(a_i)$, which is normalized as:

$$S(a_i)' = \frac{s(a_i)}{\sum_{i=1}^n s(a_i)}. \quad (12)$$

Step 6: Identify the best alternative

Define the ideal alternative S^+ as the maximum normalized score:

$$S^+ = \max(S(a_i)'). \quad (13)$$

This represents the best-performing alternative.

Step 7: Calculate the relative utility for each alternative

The relative utility of each alternative is calculated as:

$$U_i = \frac{S(a_i)'}{S^+}, \quad (14)$$

where: U_i represents the final ranking score for alternative a_i . Higher U_i values indicate better alternatives.

Step 8: Rank the alternatives

The alternatives are ranked in descending order based on U_i , where the highest-ranked alternative is the most suitable choice.

5. Illustrative example

To verify the effectiveness of the proposed method in this paper, an example from (Ye, 2014) is used for calculation, comparison, and analysis. An investment company is preparing to invest a sum of money to achieve maximum benefits. There are four companies to choose from: a_1 is an automobile company; a_2 is a food company; a_3 is a computer company; and a_4 is an arms company. When making decisions, the investment company needs to consider the following three criteria: C_1 is risk control capability; C_2 is growth potential; and C_3 is environmental impact. Among these, C_1 and C_2 are benefit-type criteria, while C_3 is a cost-type criterion. The weight vector for the three criteria is $w = (0.35, 0.25, 0.4)$. The decision-maker provides the decision values in the form of multivalued neutrosophic numbers, and the decision matrix \tilde{X} is as follows:

Step 1: Construct the initial decision matrix

$$\tilde{X} = \begin{matrix} \langle\{0.4,0.5\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.4\}, \{0.2,0.3\}, \{0.3\}\rangle & \langle\{0.2\}, \{0.2\}, \{0.5\}\rangle \\ \langle\{0.6\}, \{0.1,0.2\}, \{0.2\}\rangle & \langle\{0.6\}, \{0.1\}, \{0.2\}\rangle & \langle\{0.5\}, \{0.2\}, \{0.1,0.2\}\rangle \\ \langle\{0.3,0.4\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.5\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.5\}, \{0.2,0.3\}, \{0.2\}\rangle \\ \langle\{0.7\}, \{0.1,0.2\}, \{0.1\}\rangle & \langle\{0.6\}, \{0.1\}, \{0.2\}\rangle & \langle\{0.4\}, \{0.3\}, \{0.2\}\rangle \end{matrix} \quad (15)$$

Step 2: Normalization of decision matrix

Normalize the decision information to obtain the standardized matrix $X' = [x'_{ij}]_{m \times n}$. For benefit-type criteria, the corresponding decision information does not need to be changed. For cost-type criteria, take the complement of the decision information according to definition (3.4),

$$\tilde{X}' = \begin{matrix} \langle\{0.4,0.5\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.4\}, \{0.2,0.3\}, \{0.3\}\rangle & \langle\{0.2\}, \{0.2\}, \{0.5\}\rangle \\ \langle\{0.6\}, \{0.1,0.2\}, \{0.2\}\rangle & \langle\{0.6\}, \{0.1\}, \{0.2\}\rangle & \langle\{0.5\}, \{0.2\}, \{0.1,0.2\}\rangle \\ \langle\{0.3,0.4\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.5\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.5\}, \{0.2,0.3\}, \{0.2\}\rangle \\ \langle\{0.7\}, \{0.1,0.2\}, \{0.1\}\rangle & \langle\{0.6\}, \{0.1\}, \{0.2\}\rangle & \langle\{0.4\}, \{0.3\}, \{0.2\}\rangle \end{matrix} \quad (16)$$

Step 3: Aggregation of decision matrix

Using definition 3.6, we aggregated the MVNNs.

$$\text{MVNNWA } (A_1, A_2, \dots, A_n) = \begin{matrix} \{0.613\}, \{0.637\}, \{0.348, 0.385\}, \{0.368\} \\ \{0.563\}, \{0.230, 0.293\}, \{0.365, 0.348\} \\ \{0.438, 0.467\}, \{0.348, 0.330\}, \{0.444\} \\ \{0.638\}, \{0.218, 0.278\}, \{0.273\} \end{matrix} \quad (17)$$

Step 4: Calculate the expected value of alternatives

To the matrix of alternatives MVNNWA, using Eq. (8), then we have:

$$s_A = (0.630, 0.648, 0.556, 0.706). \quad (18)$$

Step 5: Normalize the aggregated scores

Each alternative a_i has an aggregate value $s(a_i)$, which is normalized according to the Eq. (12), to ensure that all alternatives are compared proportionally. The results for each $s(a_i)$ are presented in Table 2.

Step 6: Identify the best alternative

We define the ideal alternative s^+ with the maximum normalized score $s^+ = \max(s)$, according Eq. (13). Based on the normalized scores reported in Table 2., the highest value corresponds to alternative a_4 .

Step 7: Calculate the relative utility for each alternative

The relative utility U_i of each alternative is calculated according to Eq. (14). The results are presented in Table 2.

Step 8: Rank the alternatives

The alternative with the highest value of U_i is deemed the most suitable. The remaining alternatives are ranked in descending order based on the results of their utility functions. According to the results in Table 2, the ranking is as follows $a_4 > a_2 > a_1 > a_3$.

Table 2. Optimality values $s(a_i)$, $s(a_i)'$, U_i and ranking for alternatives

Alternative	$s(a_i)$	$s(a_i)'$	U_i	Ranking
a_1	0.630	0.248	0.893	3
a_2	0.648	0.255	0.918	2
a_3	0.556	0.219	0.788	4
a_4	0.706	0.278	1	1

5.1. Comparison and sensitivity analysis

To verify the feasibility of the proposed decision-making approach based on MVN-ARAS, a comparative analysis based on the same illustrative example is performed. Two papers are considered which use the same case study. One article in comparison is the work of (J. Wang & Li, 2015). In the paper assemble the TODIM method based in conjunction with MVNNs. The other one is the article proposed by (Peng & Wang, 2015). In their work they develop an approach for multi-criteria decision making by applying aggregation operators in MVNNs.

Table 3. The compared results utilizing the different methods with MVNNs

Methods	The final ranking	The best alternative(s)	The worst alternative(s)
J. Wang and Li (2015)	$a_4 > a_2 > a_1 > a_3$	a_4	a_3
Peng and Wang (2015)	$a_4 > a_2 > a_3 > a_1$	a_4	a_1
The proposed method	$a_4 > a_2 > a_1 > a_3$	a_4	a_3

Each methodology offers distinct computational and philosophical contributions. TODIM, as used by (J. Wang & Li, 2015), is based on prospect theory, which makes it particularly useful for capturing the risk attitudes and psychological biases of decision makers. However, it can be computationally intensive and very sensitive to variations in input data. In contrast, (Peng & Wang, 2015)'s approach focuses on aggregation functions, which provide a robust way to synthesize information. The proposed MVN-ARAS method adopts a compensatory perspective, evaluating alternatives based on their total utility for a transparent ranking process. In terms of computational demand, MVN-ARAS involves more intricate operations than classical fuzzy ARAS due to its handling of value sets, yet it remains computationally feasible for strategic-scale problems like the one presented here. Table 3 shows that while all three approaches identify a_4 as the best alternative, their intermediate rankings differ slightly. These differences indicate that the proposed ARAS-based MVNN approach is more in line with the aggregation-based method, while TODIM offers a unique behavior-based alternative. The MVN-ARAS approach is particularly suitable when a trade-off between criteria needs to be explicitly evaluated and when a ranking based on total utility is preferred.

To assess the robustness of the proposed MVNN-ARAS method, a comprehensive sensitivity analysis was conducted by systematically varying the criteria weights. Table 4 presents the results of ten different weight configurations, demonstrating the stability of the ranking results.

Table 4. Sensitivity analysis: ranking stability under different weight configurations

Weight Configuration ranking	C1	C2	C3
Original $a_4 > a_2 > a_1 > a_3$	0.35	0.25	0.40
Case 1 $a_4 > a_2 > a_1 > a_3$	0.30	0.40	0.30
Case 2 $a_4 > a_2 > a_1 > a_3$	0.20	0.40	0.40
Case 3 $a_4 > a_1 > a_2 > a_3$	0.20	0.30	0.50
Case 4 $a_4 > a_2 > a_1 > a_3$	0.10	0.40	0.50
Case 5 $a_4 > a_2 > a_1 > a_3$	0.40	0.20	0.40
Case 6 $a_4 > a_2 > a_1 > a_3$	0.25	0.50	0.25
Case 7 $a_4 > a_2 > a_1 > a_3$	0.50	0.25	0.25
Case 8 $a_4 > a_1 > a_2 > a_3$	0.25	0.25	0.50
Case 9 $a_4 > a_2 > a_1 > a_3$	0.333	0.333	0.333

5.1.1. Key findings

Ranking stability: a_4 remains the optimal alternative in all weight configurations;

Robustness: 8 out of 10 cases maintain the original ranking $a_4 > a_2 > a_1 > a_3$;

Sensitivity points: Only when C3 weight increases significantly (cases 3 and 8) does a_1 and a_2 swap positions;

Equal weights: The equal weight case (0.333 each) confirms the original ranking;

Implications: The sensitivity analysis demonstrates that the MVNN-ARAS method produces stable and reliable rankings across a wide range of weight preferences. The consistent identification of a_4 as the best alternative, despite substantial weight variations, confirms the robustness of the proposed approach for multi-criteria decision-making under uncertainty.

6. Selection of climate change adaptation strategies for smallholder olive growers in the Coquimbo region

This case study applies the proposed MVN-ARAS model to the prioritization of climate change adaptation strategies for small-scale olive producers in the arid and semi-arid zones of Chile's Coquimbo Region. The olive sector is a core economic and cultural activity in the

area; however, prolonged droughts, water scarcity, and increasing climate variability threaten both productivity and long-term viability (Mora et al., 2007). These challenges are particularly acute for smallholder farmers, who often operate with limited financial resources, restricted access to irrigation infrastructure, and constrained technical support. As a result, selecting feasible and cost-effective adaptation measures is a critical policy and management need.

In this context, the MVN-ARAS model is used not only as a mathematical tool but as a structured decision-support mechanism to translate expert knowledge into actionable priorities for farmers and local agencies. The method helps evaluate diverse adaptation options by integrating uncertainty in expert judgments and capturing trade-offs across agronomic, economic, and social dimensions. The evaluation focuses on identifying strategies that are both technically attainable for small producers and aligned with local resource constraints. Four domain experts familiar with olive production and regional climatic challenges were consulted. Their profiles, summarized in Table 5, include gender, years of experience, and educational background. These experts assessed each adaptation alternative with respect to six practical criteria:

Table 5. Profile of the experts involved in the evaluation

Attribute	Category	Number of Experts
Gender	Male	4
	Female	0
Years of experience	4–8 years	2
	8–12 years	2
Educational background	Ph. D.	2
	Master	1
	Bachelor	1

These experts assessed each adaptation alternative with respect to six practical criteria: *Economic Viability (C1)* – Cost-effectiveness, investment requirements, and expected returns; *Environmental Sustainability (C2)* – Impact on soil, water, and biodiversity; *Social Acceptability (C3)* – Alignment with local traditions and farmer willingness; *Technical Feasibility (C4)* – Ease of implementation and required technical knowledge; *Resilience Improvement (C5)* – Contribution to reducing climate-related risks; *Market Potential (C6)* – Opportunities for new income streams or market access.

The experts' evaluations were compiled using linguistic variables, which were transformed into MVNN for quantitative analysis (see Table 6). The experts' responses regarding the relative importance of each criterion are presented in Table 7. Subsequently, these assessments were aggregated using the *MVNNWA* operator (see Definition 3.6).

Once the aggregated weights were obtained in MVNN format, the definition 3.5 was applied to calculate the final crisp weights. The results of this process, including the aggregated values and the final weights associated with each criterion, are summarized in Table 8.

Table 6. Linguistic variables and their corresponding representations using MVNNs. The numerical mappings are adapted from the work of (Martina & Deepa, 2023)

Linguistic Variable	MVNN Representation
Very Low (VL)	$\langle\{0, 0.1\}, \{0.9, 1\}, \{1\}\rangle$
Low (L)	$\langle\{0.2\}, \{0.8\}, \{0.9, 1\}\rangle$
Medium Low (ML)	$\langle\{0.3, 0.4\}, \{0.6, 0.7\}, \{0.8\}\rangle$
Fair (F)	$\langle\{0.5\}, \{0.5\}, \{0.5\}\rangle$
Medium High (MH)	$\langle\{0.6, 0.7\}, \{0.3, 0.4\}, \{0.2\}\rangle$
High (H)	$\langle\{0.8\}, \{0.2\}, \{0, 0.1\}\rangle$
Very High (VH)	$\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$

Table 7. Expert evaluations using linguistic terms and their MVNN representations

Criterion	Expert 1	Expert 2	Expert 3	Expert 4
C1	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$
C2	MH → $\langle\{0.6, 0.7\}, \{0.3, 0.4\}, \{0.2\}\rangle$	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$	H → $\langle\{0.8\}, \{0.2\}, \{0, 0.1\}\rangle$
C3	ML → $\langle\{0.3, 0.4\}, \{0.6, 0.7\}, \{0.8\}\rangle$	H → $\langle\{0.8\}, \{0.2\}, \{0, 0.1\}\rangle$	F → $\langle\{0.5\}, \{0.5\}, \{0.5\}\rangle$	H → $\langle\{0.8\}, \{0.2\}, \{0, 0.1\}\rangle$
C4	H → $\langle\{0.8\}, \{0.2\}, \{0, 0.1\}\rangle$	H → $\langle\{0.8\}, \{0.2\}, \{0, 0.1\}\rangle$	H → $\langle\{0.8\}, \{0.2\}, \{0, 0.1\}\rangle$	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$
C5	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$	F → $\langle\{0.5\}, \{0.5\}, \{0.5\}\rangle$	H → $\langle\{0.8\}, \{0.2\}, \{0, 0.1\}\rangle$
C6	F → $\langle\{0.5\}, \{0.5\}, \{0.5\}\rangle$	F → $\langle\{0.5\}, \{0.5\}, \{0.5\}\rangle$	MH → $\langle\{0.6, 0.7\}, \{0.3, 0.4\}, \{0.2\}\rangle$	VH → $\langle\{0.9, 1\}, \{0, 0.1\}, \{0\}\rangle$

Average MVNN components:

$$C1 = \langle 0.95, 0.05, 0 \rangle,$$

$$C2 = \langle 0.915, 0, 0 \rangle,$$

$$C3 = \langle 0.665, 0.335, 0.125 \rangle,$$

$$C4 = \langle 0.915, 0.085, 0 \rangle \quad C5 = \langle 0.915, 0.09, 0 \rangle \quad C6 = \langle 0.84, 0.16, 0 \rangle.$$

Compute and normalize weights:

$$w_n = \frac{1 - \sqrt{\frac{(1 - \tilde{t}_n)^2 + \tilde{i}_n^2 + \tilde{f}_n^2}{3}}}{\sum_{n=1}^6 \text{Numerator}_n} \quad [\text{Total sum of numerators} = 5.524]. \quad (19)$$

Table 8. Calculation of weights for each criterion

Criterion	Numerator	Weight (w_n)
C1	0.977	0.177
C2	0.951	0.172
C3	0.784	0.142
C4	0.951	0.172
C5	0.948	0.172
C6	0.913	0.165

The weights are obtained by dividing each numerator by the total sum (5.524), ensuring $\sum w_n = 1$.

The ten strategies described in the Table 9 were selected for their potential to mitigate the most pressing challenges in the Coquimbo region: water scarcity, soil degradation, and economic vulnerability. Each strategy is supported by evidence in the literature and has demonstrated applicability in contexts facing similar climatic and socioeconomic conditions. Collectively, they offer practical and scalable pathways for smallholders, balancing immediate adaptive needs with the long-term goal of resilience. Techniques such as water-efficient irrigation and drought-tolerant cultivars directly target climate-induced water stress, while measures like soil amendments and agroforestry address land degradation. At the same time, diversification strategies – ranging from value-added products to rural ecotourism – enhance farmers' income stability and reduce dependence on a single commodity. Table 10 presents a summary of the experts' verbal evaluations of the proposed strategies across all assessment criteria. The detailed questionnaire used to collect these responses is provided in the Appendix.

Using Table 6, the linguistic terms for the alternatives and attributes in Table 10 are first converted into MVNNs. Following Step 1, the multi-valued neutrosophic decision matrix is constructed. In Step 2, this matrix would normally be normalized; however, since all criteria are of the benefit type, normalization is not required.

Once these conversions are completed, the *aggregated neutrosophic decision matrix* is generated. This matrix is computed for each alternative and criterion by aggregating the responses of all decision makers.

To illustrate the procedure, consider alternative a_1 evaluated solely under criterion c_1 , with an associated weight of $w_1 = 0.177$. The assessments provided by the four decision makers (DMs) are:

$$DM_1: \langle \{0.9, 1\}, \{0, 0.1\}, \{0\} \rangle,$$

$$DM_2: \langle \{0.9, 1\}, \{0, 0.1\}, \{0\} \rangle,$$

$$DM_3: \langle \{0.8\}, \{0.2\}, \{0, 0.1\} \rangle,$$

$$DM_4: \langle \{0.9, 1\}, \{0, 0.1\}, \{0\} \rangle,$$

Table 9. Climate adaptation strategies for olive cultivation

Adaptation strategy (Detailed description)	Key benefit / Rationale
1. <i>Drip or sprinkler irrigation systems</i> Installation of pressure-regulated drip lines or pivot/sprinkler systems to deliver water directly to the root zone, reducing evaporation, runoff, and deep percolation losses.	Enhances water-use efficiency by minimizing losses (Dehkordi et al., 2025)
2. <i>Drought-resistant olive cultivars</i> Replacement of traditional cultivars with varieties such as 'Arbequina', 'Koroneiki', or local drought-tolerant genotypes that maintain yields under water scarcity.	Better suited to arid conditions and climate variability (Mora et al., 2007)
3. <i>Rainwater harvesting and micro-reservoirs</i> Construction of small on-farm reservoirs, lined ponds, or storage tanks to capture winter rainfall for use during critical growth stages	Stores rainfall for supplemental irrigation (Abdalla, 2025)

End of Table 9

Adaptation strategy (Detailed description)	Key benefit / Rationale
4. <i>Agroforestry with olive-compatible species</i> Integration of legumes, aromatic shrubs (e.g., rosemary, lavender), or shade trees between rows to reduce erosion, improve biodiversity, and increase soil organic matter	Improves soil stability and ecosystem resilience (Mora et al., 2007)
5. <i>Organic soil amendments and compost</i> Application of compost, manure, biochar, or mulches to increase water-holding capacity, enhance soil structure, and supply slow-release nutrients	Boosts moisture retention and fertility (Chehab et al., 2020)
6. <i>Solar-powered irrigation pumps</i> Replacement of diesel or electric pumps with photovoltaic-powered pumping systems, reducing operational costs and greenhouse-gas emissions	Lowers energy costs and supports clean energy transition (van de Loo et al., 2024; Campana et al., 2022)
7. <i>Olive oil value-added products</i> Processing olives into premium or niche products such as cold-pressed extra virgin oil, olive-based cosmetics, cured table olives, soap, and artisanal gourmet items to diversify revenue streams.	Increases income through higher-value goods (Gullón et al., 2020)
8. <i>Community-managed water-sharing systems</i> Locally coordinated systems where farmers share irrigation turns, maintain shared canals or wells, and allocate water during drought through collective rules	Ensures equitable and reliable water distribution (Ghorbani et al., 2021)
9. <i>Beekeeping for pollination and income</i> Placement of beehives on olive farms to enhance pollination (especially in mixed cropping systems) while producing honey, propolis, and wax for added revenue.	Supports yields and generates additional income (Abro et al., 2022)
10. <i>Rural ecotourism linked to olive farms</i> Development of farm-based tourism activities such as olive oil tastings, harvest participation, guided orchard visits, and cultural experiences tied to local traditions.	Diversifies income and reinforces cultural heritage (Lin et al., 2025)

Table 10. Expert assessments of adaptation strategies (linguistic evaluations)

Alternative	D.M	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
a ₁	1	VH	ML	L	H	H	VH
	2	VH	VH	F	H	VH L	VH
	3	H	VH	H	MH	H	MH
	4	H	VH	MH	H		VH
a ₂	1	H	VH	ML	ML	H	VH
	2	VH	VH	L	H	VH	VH
	3	VH	H	H	L	L	MH
	4	VH	MH	H	H	VH	VH
a ₃	1	F	H	H	H	L	H
	2	VH	H	F	F	VH	VH
	3	L	VH	VH	MH	H	H
	4	VH	H	MH	VH	VH	VH
a ₄	1	F	F	F	H	H	F
	2	H	VH	VL	L	VH	H
	3	H	H	H	H	H	H
	4	VH	VH	H	H	H	VH

End of Table 10

Alternative	D.M	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
a ₅	1	H	F	H	H	L	F
	2	VH	VH	F	L	VH	H
	3	F	F	F	L	L	F
	4	H	VH	VH	H	H	VH
a ₆	1	F	H	H	L	F	F
	2	VH	H	L	H	VH	H
	3	H	H	H	MH	H	H
	4	H	H	H	H	MH	MH
a ₇	1	VH	L	F	H	F	VH
	2	VH	VH	F	H	VH	F
	3	MH	MH	H	MH	MH	MH
	4	VH	H	MH	VH	MH	VH
a ₈	1	F	F	F	F	L	L
	2	VH	H	F	H	H	H
	3	MH	H	H	F	F	F
	4	VH	VH	VH	MH	H	H
a ₉	1	H	H	F	F	H	H
	2	VH	H	VL	L	H	H
	3	H	H	H	MH	H	H
	4	VH	VH	MH	MH	H	VH
a ₁₀	1	VH	VH	VH	F	F	H
	2	VH	VH	VH	H	VH	H
	3	F	L	H	L	L	F
	4	VH	MH	VH	F	H	VH

Applying the MVNNA operator (Equation (7)) yields the following aggregated results:

1. Truth-membership aggregation

Using the combination {0.9, 0.9, 0.8, 0.9}:

$$T^* = 1 - \prod_{k=1}^4 (1 - T_k)^{w_k} = 1 - \left[(1 - 0.9)^{0.177} (1 - 0.9)^{0.177} (1 - 0.8)^{0.177} (1 - 0.9)^{0.177} \right] \approx 0.779. \quad (20)$$

A second combination {1, 1, 0.8, 1} yields

$$T^* \approx 1 - \left[(1 - 1)^{0.177} (1 - 1)^{0.177} (1 - 0.8)^{0.177} (1 - 1)^{0.177} \right] = 1. \quad (21)$$

Thus, the aggregated truth-membership set is

$$T^* = \{0.779, 1\}. \quad (22)$$

2. Indeterminacy-membership aggregation

Using {0, 0, 0.2, 0}:

$$I^* = 0^{0.177} \cdot 0^{0.177} \cdot 0.2^{0.177} \cdot 0^{0.177} = 0. \quad (23)$$

A second combination {0.1, 0.1, 0.2, 0.1} gives

$$I^* = 0.1^{0.177} \cdot 0.1^{0.177} \cdot 0.2^{0.177} \cdot 0.1^{0.177} \approx 0.221; \tag{24}$$

$$I^* = \{0, 0.221\}. \tag{25}$$

3. Falsity-membership aggregation

For {0, 0, 0, 0}:

$$F^* = 0^{0.177} \cdot 0^{0.177} \cdot 0^{0.177} \cdot 0^{0.177} = 0. \tag{26}$$

For {0, 0, 0.1, 0}:

$$F^* = 0^{0.177} \cdot 0^{0.177} \cdot 0.1^{0.177} \cdot 0^{0.177} = 0; \tag{27}$$

$$F^* = \{0\}. \tag{28}$$

Final Aggregated MVNN

$$MVNNWA = \langle \{0.779, 1\}, \{0, 0.221\}, \{0\} \rangle. \tag{29}$$

Once aggregation at the decision-maker level has been completed, with the aim of obtaining a representative value for their evaluations, the same procedure is applied to perform aggregation at the criterion level, in accordance with Step 3. The corresponding results are presented in Table 11.

Table 11. Aggregation value for each alternative

Alternative	<i>T</i>	<i>I</i>	<i>F</i>
<i>a</i> ₁	{0.670, 1}	{0, 0.330}	{0}
<i>a</i> ₂	{0.782, 1}	{0}	{0}
<i>a</i> ₃	{0.654, 1}	{0, 0.346}	{0}
<i>a</i> ₄	{0.635, 1}	{0}	{0}
<i>a</i> ₅	{0.577, 1}	{0}	{0}
<i>a</i> ₆	{0.598, 1}	{0, 0.399}	{0}
<i>a</i> ₇	{0.623, 1}	{0, 0.377}	{0}
<i>a</i> ₈	{0.567, 1}	{0}	{0}
<i>a</i> ₉	{0.627, 1}	{0, 0.373}	{0}

Following Step 4, the expected value (*s(a_i)*) of each alternative is calculated. The results corresponding to Steps 5 to 8, including the expected value (*s(a_i)*), the normalized score (*s(a_i)'*), the relative utility (*U_i*), and the final ranking of the alternatives, are presented in Figure 2. Based on these results, the ranking of the alternatives from most to least preferred is as follows:

$$a_2 > a_1 > a_3 > a_4 > a_{10} \approx a_9 > a_7 > a_6 > a_5 > a_8,$$

where *a*₂ achieves the highest score (*s(a*₂*) = 0.927*) and *a*₈ the lowest (*s(a*₈*) = 0.856*).

6.1. Discussion and recommendations

The evaluation of adaptation strategies using the MVN-ARAS method provides a structured and transparent framework for prioritizing actions under uncertainty. The results summarized in Figure 2 reveal clear differences in the performance of the ten proposed alternatives. The highest-ranked alternative is a_2 (*Drought-resistant olive cultivars*), achieving an expected score of $s(a_2) = 0.927$ and a relative utility of $U_2 = 1.000$. This strategy directly addresses the most critical climate-related risks for smallholder farmers in the Coquimbo region, as it enhances long-term resilience to water scarcity with moderate technical requirements and high acceptance among experts (Tugendhaft et al., 2016; Adi et al., 2025).

The second-best alternative, a_1 (*Drip or sprinkler irrigation systems*), follows closely with $s(a_1) = 0.890$. This strategy is highly effective in improving water-use efficiency and reducing

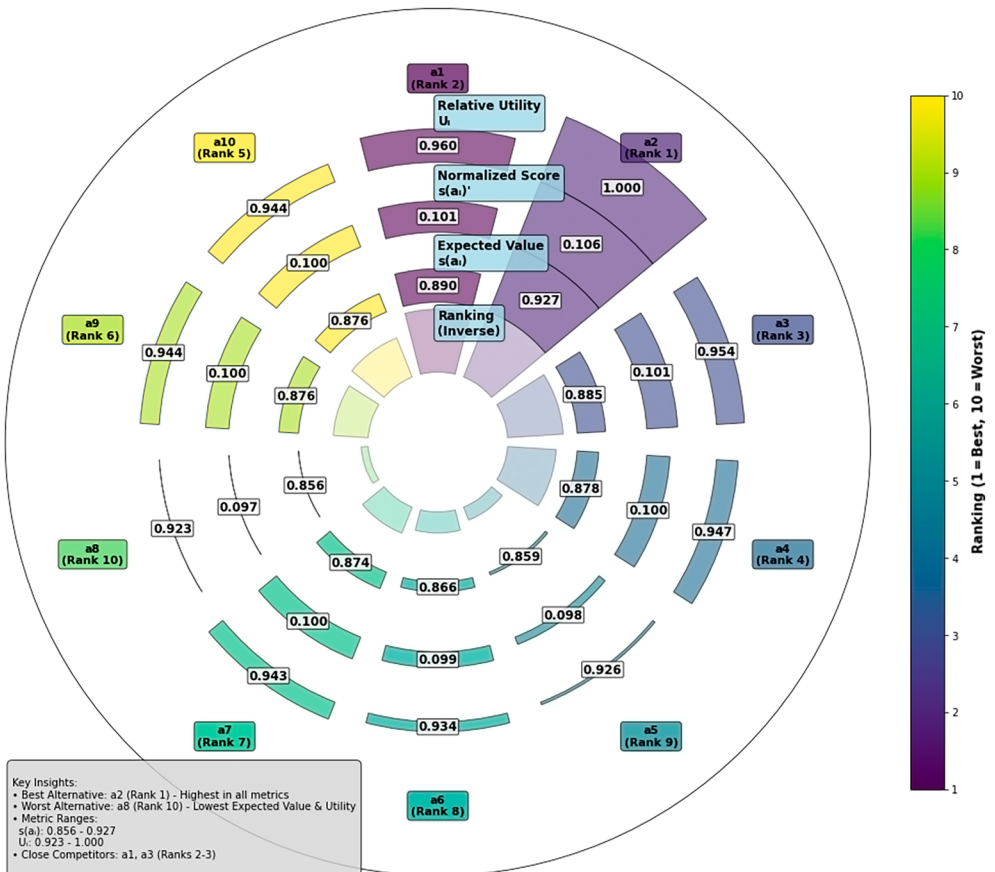


Figure 2. Comprehensive multi-metric analysis of alternatives using radial bar chart visualization. The plot displays four key metrics simultaneously for each alternative: (1) *Ranking* (inner ring, inverted scale where taller bars indicate better rank), (2) *Expected Value* $s(a_i)$ (second ring), (3) *Normalized Score* $s(a_i)$ (third ring), and (4) *Relative Utility* U_i (outer ring). Alternatives are arranged circularly with consistent color coding. The visualization clearly shows that alternative a_2 (Rank 1) performs best across all metrics, while a_8 (Rank 10) is the worst performer. Values are annotated above each bar segment for clarity

losses; however, its initial investment costs and technical requirements may limit adoption for farmers with lower financial capacity (Bhavsar et al., 2023).

At the opposite end of the ranking, a_8 (*Community-managed water-sharing systems*) is the least preferred option, with $s(a_8) = 0.856$. Although these systems promote equitable distribution and social participation in water governance (Raina & Longino, 2025), their practical limitations may explain the lower preference observed. Empirical studies in semi-arid regions of Latin America and South Asia show that community-based water management often faces difficulties in sustaining long-term cooperation, especially where social trust and institutional capacity are weak. The effectiveness of such systems relies on consistent participation, transparent rule enforcement, and external technical or financial support – conditions that are not always met in practice. In addition, when water demand intensifies or climatic variability increases, community-based systems may struggle to allocate water efficiently, leading to disputes and inequities among users. Another limitation frequently reported is their narrow operational scope: these initiatives typically prioritize agricultural users, excluding domestic and industrial stakeholders, which can exacerbate local tensions. Hence, despite their social and environmental appeal, the empirical evidence suggests that community-managed water-sharing schemes are less effective or sustainable under conditions of resource scarcity or institutional fragility.

6.1.1. Recommendations for decision-makers

In this study, “decision-makers” refers to agricultural authorities, water governance agencies, extension services, and smallholder olive producers. These actors are responsible for planning, financing, and implementing adaptation measures.

1. *Prioritize high-performing strategies*: Allocate immediate support to a_2 and a_1 due to their strong resilience benefits.
2. *Integrate technical and financial tools*: Combine subsidies or low-interest credit with targeted training, especially for irrigation technologies.
3. *Complement with diversification strategies*: Although a_7 and a_{10} are not core resilience measures, they provide valuable income diversification.
4. *Reconsider social strategies*: Options such as a_8 and a_5 may be strengthened through community programs or pilot initiatives.

High-ranked strategies such as drought-resistant cultivars (a_2) and drip irrigation (a_1) strengthen climate resilience while helping stabilize farm income. Lower-ranked options (e.g., a_8) provide important social benefits but yield less consistent financial returns, whereas diversification-oriented alternatives such as value-added products (a_7) and rural ecotourism (a_{10}) contribute to long-term income security. Overall, these results guide decision-makers in allocating resources toward measures that balance technical feasibility, resilience improvement, and economic viability. To address the need for more actionable guidance, we also include a concise workflow and supporting tools that illustrate how practitioners and policy-makers can apply the MVN–ARAS method in practice (see Figure 3).

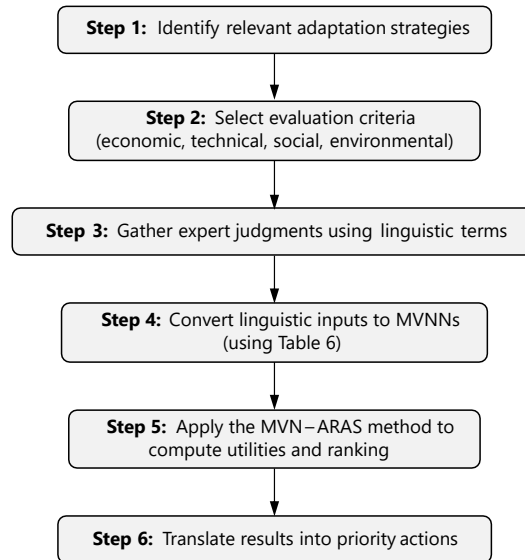


Figure 3. Step-by-step workflow for applying the MVN-ARAS method

7. Economic development implications

The results of this study have significant implications for farm business and agricultural governance, but also for broader economic development in semi-arid regions. Climate change adaptation in agriculture should not be viewed as an environmental or technical challenge, but rather as a primary driver of rural economic viability and competitiveness. In the Coquimbo region, for example, olive production is a key sector with direct implications for family livelihoods, rural incomes, and local social GDP. The decisions on adaptation strategies thus have spillover effects on local economies, value chains, and export markets.

7.1. Economic resilience, competitiveness, and diversification

The MVN-ARAS model identifies drought-resistant varieties and water-saving irrigation techniques as the most impactful measures for enhancing economic resilience. In this study, economic resilience refers to the capacity of olive producers to sustain stable production, maintain income levels, and remain competitive despite climatic variability, water scarcity, and market uncertainty. The selected strategies reinforce this resilience by reducing vulnerability to drought, stabilizing yields, and mitigating financial risks. In doing so, they provide both immediate and long-term benefits, helping to preserve Chile's reputation as a producer of high-quality olive oil while supporting employment across related processing, packaging, and distribution industries. Furthermore, the model highlights lower-ranked plans, such as rural ecotourism and value-added olive oil products, as vital channels for economic diversification. In regions highly vulnerable to climatic shocks like Coquimbo, developing these complementary activities builds resilience by creating new income sources. Initiatives like producing artisanal olive-based cosmetics or promoting heritage tourism contribute to value-added growth in the local GDP and generate employment in the hospitality, service,

and creative sectors. This strategic diversification aligns with Chile's broader policy goals of reducing the exposure of rural economies to commodity price risks and climatic variability. To operationalize these high-impact strategies, it is essential to articulate the specific policy mechanisms and institutional arrangements that can support their adoption. Regional and national authorities could introduce targeted subsidy schemes to reduce the financial barriers associated with drought-resistant varieties or the installation of advanced irrigation systems. Likewise, cooperative-based training and extension programs can foster knowledge transfer among smallholders, enabling the diffusion of best practices identified by the MVN-ARAS model. Partnerships with NGOs and private-sector actors can further strengthen implementation, particularly by mobilizing technical assistance and community-level engagement. Additionally, pilot initiatives in selected olive-producing areas of Coquimbo can serve as testing grounds for adaptation packages, allowing stakeholders to evaluate performance under real operational conditions before scaling up. These concrete pathways increase the practical relevance of the findings and provide actionable guidance for policymakers and practitioners.

7.2. Employment and income distribution

Rural economic development is not only measured in cumulative growth but also in benefit distribution. The adaptive choices promoted by MVN-ARAS also have significant implications for income equity. Technologies such as drip irrigation, with the promotion of focused subsidies or credit programs, can be adopted by small farms without economic polarization between the poor and the affluent resource controllers. In addition, adoption schemes under cooperative frameworks, where smallholders cooperate to pool resources to invest in technology or joint marketing, enable economic benefits to be distributed more evenly across farm communities. This approach reduces the probability of rural exodus and guarantees social stability, both of which are prerequisites for sustainable economic development.

7.3. Policy and investment implications

Adaptation planning must be incorporated in overall rural development plans, as suggested by the economic results of this study to policymakers. Government expenditure on infrastructure (e.g., irrigation systems, solar power for pumping stations) reduces the unit price of adaptation technology, which is made cheaper for small-scale farmers. Similarly, merging climate adaptation with financial instruments such as microcredit, crop insurance, or green bonds speeds up adoption while spreading the financial risk. Macro-economically, these interventions strengthen Chile's agricultural export potential and reduce the economic risk of climate shocks to national accounts.

Worldwide, the case study demonstrates how decision-making tools like MVN-ARAS can inform development agencies and multilateral organizations in designing investment programs. For instance, regional development banks and the Food and Agriculture Organization (FAO) are increasingly emphasizing climate-smart agriculture as a means not only to protect livelihoods but also to drive inclusive economic growth. By demonstrating a transparent and inclusive ranking of strategies, MVN-ARAS presents a replicable model to analyze investment priorities in other semi-arid agricultural economies.

7.4. Regional competitiveness and sustainable development

At the regional level, the integration of economic feasibility into adaptation decisions raises competitiveness in the long term. Coquimbo's olive production competes with other Mediterranean producers such as Spain, Italy, and Greece. In the competitive export market, the integration of efficient, resistant, and market-oriented approaches ensures that Chilean products maintain their niche position in high-end export markets. Additionally, diversification seeking, as low as they rank on the resilience scale, supports the agenda of sustainable development in the region by situating agriculture within a diversified rural economy. This fosters synergies between agriculture, tourism, services, and manufacturing that transcend farm-level benefits.

7.5. Contribution to national economic development

Finally, the economic aspect of this research stresses that agricultural climate adaptation cannot be divorced from national development planning. The Chilean government's emphasis on regional competitiveness, equity, and sustainability finds explicit endorsement in the findings of this study. By prioritizing successful adaptation interventions, policymakers not only protect a precarious sector but also enjoy earnings on foreign exchange, rural employment, and food security. Moreover, by triggering diversification of value-added products and tourism, adaptation sparks innovation and entrepreneurship, the driving forces of long-term economic transformation.

Briefly, the MVN-ARAS method offers decision-makers more than a technical ranking tool; it offers them actionable data on adaptation's economic trade-offs and opportunities. By explicitly linking adaptation policies to income security, employment generation, diversification, and competitiveness, this research brings agricultural adaptation within the scope of agricultural development as a tool for sustainable economic growth. Results show that adaptation planning, in light of good decision-support practices, can not only enhance resilience but also contribute to broader local, regional, and national economic change.

7.6. Extended economic perspectives

While Sections 7.1–7.6 highlight the direct consequences of adaptation for development and competitiveness, further examination of economic dimensions strengthens the analytical coverage and increases the policy contribution of the MVN-ARAS findings.

Cost-benefit and return on investment. Adaptation measures differ in their costliness and payback periods. Drip irrigation schemes, for instance, require high investment but provide measurable water savings and yield stability with investment return normally repayable within a medium-term horizon. Drought-tolerant crop varieties, on the other hand, entail moderate replanting expenses but ensure long-term stability with relatively low maintenance costs. Integration of such cost-benefit assessments into policy-making enables policymakers to balance technical feasibility with fiscal realism.

Macroeconomic linkages and balance of trade. Macroeconomic stability from olive production shields export revenues, anchors rural incomes, and sustains regional GDP shares. The spillovers extend down the value chain, generating employment and incomes in processing,

packaging, transportation, and export services. These multiplier responses ensure that adaptation investments yield spillover effects through the broader economy.

Risk economics under uncertainty. Adaptation measures can be defined as real options, which enable farmers to cope with the loss of finance during adverse climate shocks. Without adaptation, climate shocks would result in high year-to-year income uncertainty, while the use of resilient technologies reduces exposure to yield loss and improves predictability in incomes. Through this perspective, adaptation is consistent with financial risk management strategies.

Financing and incentive mechanisms. The efficient utilization of adaptation also hinges on the availability of funds. Frameworks such as microcredit programs, cooperative investment frameworks, green bonds, and carbon credits tied to climate-resilient action can facilitate adoption barriers. Their integration into adaptation policy frameworks not only increases viability to smallholders but also leverages private capital for climate-resilient agriculture.

Distributional and equity effects. Economic resilience must encompass welfare distribution. In the absence of protection, larger producers may dominate newer technologies, making inequality worse. Policies connecting subsidies, cooperative pooling, and technical support ensure inclusive adaptation and prevent smallholders from being marginalized. This reduces rural-urban migration pressures and sustains rural solidarity.

International benchmarking. Experience from Mediterranean producers such as Spain, Italy, and Tunisia establishes the potential of diversification into agrotourism and value-added olive products as alternative income sources. Positioning the olive industry of Coquimbo within this global frame of reference highlights both strengths, including niche branding and organic certification, and weaknesses, including scale disadvantages, to be mobilized to improve competitiveness.

Dynamic long-term impacts. Finally, the adaptation period is also a very important factor. Procrastination results in cumulative costs, including forgone yields, higher irrigation fees, and lower export competitiveness. Early investment, however, raises compound advantages, sustaining growth trends and stabilizing Coquimbo's olive sector as a consistent driver of regional and national development.

8. Managerial and governance implications

The outcomes of this research provide concrete implications for olive farm managers, associations, and policymakers in Chile's Coquimbo region. For managers and associations, the MVN-ARAS model serves as a critical decision-support tool. It reduces uncertainty in allocating scarce resources amidst a severe, multi-decadal drought. For instance, the model's rankings can guide collaborative procurement of drought-resistant varieties like Arbequina or Arbosana, which are better suited to the region's arid conditions than traditional Picual (J.-W. Wang et al., 2018). Furthermore, the model can help managers justify investments in specific irrigation technologies – such as localized subsurface drip irrigation – by quantifying their resilience payoff, thereby strengthening their negotiations with providers and applications for support from INDAP's (*Instituto de Desarrollo Agropecuario*) *Programa de Riego Intrapredial*.

For policy actors, the study underscores the necessity of moving from generic support to evidence-based, targeted interventions. Public agencies, notably INDAP and the *Comisión*

Nacional de Riego (CNR), can utilize the MVN-ARAS framework to design and justify policies specific to the olive sector. For example, the model can directly inform the prioritization criteria within the CNR's *Ley N° 18.450 de Fomento a la Inversión Privada en Obras de Riego y Drenaje* (Congress of Chile, 1985), ensuring subsidies are directed towards the irrigation efficiency strategies (e.g., precision irrigation, soil moisture sensors) that the model ranks highest for resilience. The methodology's inclusion of pluralistic expert views aligns perfectly with the participatory governance mandated by Chile's *Ley N° 20.417* (Congress of Chile, 2010) that created the Ministry of the Environment and its regional secretariats, ensuring that policies balance productivity with the *Environmental Integrity* of fragile semi-arid ecosystems like the Coquimbo Region. This evidence-based approach enhances accountability, a critical factor in a region where water allocation, governed by the *Código de Aguas* (Congress of Chile, 1981), is often politically contentious.

On a broader scale, the results are directly tied to enhancing the competitiveness of the *olive oil sector in the Coquimbo Region* as a whole. Building climate resilience is a strategic economic imperative for a region known for its premium, early-harvest oils. While the model may rank foundational measures like irrigation efficiency and drought-resistant varieties highest, it also provides a transparent method to evaluate complementary diversification strategies. For instance, lower-ranked but viable options like developing value-added olive oil-based cosmetics or integrating olive groves into *rural ecotourism* routes – such as those promoted by regional initiatives – can be strategically supported by *CORFO (Corporación de Fomento de la Producción)* programs. This holistic approach helps diversify income, create jobs in rural areas of the Limarí and Choapa provinces, and ultimately enhance the socio-economic resilience of the entire regional olive value chain.

By merging state-of-the-art decision science with the specific managerial and policy landscape of Chilean olive cultivation, the proposed MVN-ARAS methodology offers a scalable framework for semi-arid and arid economies worldwide. It is not only methodologically robust but also yields actionable governance suggestions, rendering adaptation planning a worthwhile investment that pays dividends in both social stability and economic resilience.

9. Conclusions

This study introduced and validated a novel hybrid method, MVN-ARAS, which integrates Multi-Valued Neutrosophic Numbers (MVNNs) with the Additive Ratio Assessment (ARAS) approach. This integration directly addresses a critical gap in the literature: the lack of rigorously developed ARAS extensions capable of handling multi-valued indeterminacy. The result is a robust framework for efficient decision-making under profound uncertainty.

By systematically incorporating expert judgments, the model offers a structured and transparent means to evaluate adaptation alternatives in the face of highly variable climatic, agronomic, and economic conditions. Application of the model consistently identified drought-resistant cultivars and water-saving irrigation technologies as the most effective measures for enhancing economic resilience. These high-priority strategies stabilize yields, reduce vulnerability to climatic stress, and safeguard the competitiveness of regional olive production. In contrast, lower-ranked diversification strategies – such as ecotourism or

value-added olive products – were shown to offer complementary, long-term benefits by expanding income streams and mitigating systemic economic risk.

The implications of these findings extend beyond individual farms to broader governance and policy. The rankings generated by MVN-ARAS can inform targeted subsidy programs, guide cooperative investments, and shape public-private initiatives for technological modernization. By pinpointing the measures with the highest resilience returns, the model provides an evidence-based foundation for prioritizing regional irrigation policies and agricultural development programs.

To further enhance the scope and scalability of this research, several future directions are proposed:

Dynamic scenario analysis: Integrating MVN-ARAS with climate projection models would enable the evaluation of adaptation pathways under various future scenarios of temperature, precipitation, and drought.

Digital twin integration: Linking the framework to digital replicas of olive farms could facilitate real-time simulations of water use, crop performance, and economic outcomes.

IoT and real-time data: Coupling the model with IoT platforms (e.g., soil moisture sensors, remote sensing) would allow for continuous, data-driven updates to criteria weights and performance scores.

Machine learning enhancement: Incorporating machine learning could refine expert input and autonomously identify emerging patterns of risk and performance.

In conclusion, this work establishes MVN-ARAS not merely as a robust decision-support tool, but as a scalable foundation for next-generation agricultural planning. By explicitly embracing uncertainty and delivering clear, defensible priorities, the approach makes a substantive contribution toward building climate resilience, ensuring economic stability, and promoting sustainable development in the world's vulnerable semi-arid agricultural regions.

References

- Abdalla, M. A. (2025). GIS-based identification of optimal rainwater harvesting sites to support irrigation in Egypt's northwestern coastal region. *Sustainable Geosciences: People, Planet and Prosperity*, 1, Article 100004. <https://doi.org/10.1016/j.susgeo.2025.100004>
- Abro, Z., Kassie, M., Tiku, H. A., Taye, B., Ayele, Z. A., & Ayalew, W. (2022). The impact of beekeeping on household income: evidence from north-western Ethiopia. *Heliyon*, 8(5), Article e09492. <https://doi.org/10.1016/j.heliyon.2022.e09492>
- Adali, E. A., & Tuş, A. (2023). ARAS method based on Z-numbers in FMEA. *Quality and Reliability Engineering International*, 39(7), 3059–3081. <https://doi.org/10.1002/qre.3416>
- Adi, B., Dag, A., Ben-Dor, E., Gabay, G., & Barazani, O. (2025). Exploring drought tolerance in wild and traditional olive varieties from the Southern Levant. *Frontiers in Plant Science*, 16, Article 1547174. <https://doi.org/10.3389/fpls.2025.1547174>
- Ahmed, F., & Kilic, K. (2019). Fuzzy analytic hierarchy process: A performance analysis of various algorithms. *Fuzzy Sets and Systems*, 362, 110–128. <https://doi.org/10.1016/j.fss.2018.08.009>
- Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96. [https://doi.org/10.1016/S0165-0114\(86\)80034-3](https://doi.org/10.1016/S0165-0114(86)80034-3)
- Bahrami, Y., Hassani, H., & Maghsoudi, A. (2019). BWM-ARAS: A new hybrid MCDM method for CU prospectivity mapping in the Abhar area, NW Iran. *Spatial Statistics*, 33, Article 100382. <https://doi.org/10.1016/j.spasta.2019.100382>

- Bhavsar, D., Limbasia, B., Mori, Y., Imtiyazali Aglodiya, M., & Shah, M. (2023). A comprehensive and systematic study in smart drip and sprinkler irrigation systems. *Smart Agricultural Technology*, 5, Article 100303. <https://doi.org/10.1016/j.atech.2023.100303>
- Biswas, P., Pramanik, S., & Giri, B. C. (2015). TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. *Neural Computing and Applications*, 27, 727–737. <https://doi.org/10.1007/s00521-015-1891-2>
- Borah, G., & Dutta, P. (2024, December). Fuzzy risk analysis in crop selection using information measures on quadripartitioned single-valued neutrosophic sets. *Expert Systems with Applications*, 255, Article 124750. <https://doi.org/10.1016/j.eswa.2024.124750>
- Brauers, W. K. M., & Zavadskas, E. K. (2010). Project management by MULTIMOORA as an instrument for transition economies. *Technological and Economic Development of Economy*, 16(1), 5–24. <https://doi.org/10.3846/tede.2010.01>
- Büyüközkan, G., & Göçer, F. (2018). An extension of ARAS methodology under interval valued intuitionistic fuzzy environment for digital supply chain. *Applied Soft Computing*, 69, 634–654. <https://doi.org/10.1016/j.asoc.2018.04.040>
- Büyüközkan, G., & Güler, M. (2020). Smart watch evaluation with integrated hesitant fuzzy linguistic SAW-ARAS technique. *Measurement*, 153, Article 107353. <https://doi.org/10.1016/j.measurement.2019.107353>
- Campana, P. E., Papic, I., Jakobsson, S., & Yan, J. (2022). Photovoltaic water pumping systems for irrigation: Principles and advances. In S. Gorjian & P. E. Campana (Eds.), *Solar energy advancements in agriculture and food production systems* (pp. 113–157). Academic Press. <https://doi.org/10.1016/B978-0-323-89866-9.00007-9>
- Chatterjee, R., Majumdar, P., & Samanta, S. (2016). On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets. *Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology*, 30 (4), 2475–2485. <https://doi.org/10.3233/IFS-152017>
- Chehab, H., Tekaya, M., Hajlaoui, H., Abdelhamid, S., Gouiaa, M., Sfina, H., Chihaoui, B., Boujnah, D., & Mechri, B. (2020). Complementary irrigation with saline water and soil organic amendments modified soil salinity, leaf Na⁺, productivity and oil phenols of olive trees (cv. Chemlali) grown under semiarid conditions. *Agricultural Water Management*, 237, Article 106183. <https://doi.org/10.1016/j.agwat.2020.106183>
- Congress of Chile. (1981, August 13). *Código de aguas*. [The Water Code of Chile, DLF 1122] <https://www.bcn.cl/leychile/navegar?idNorma=5605>
- Congress of Chile. (1985, October 22). *Ley de fomento a la inversión privada en obras de riego y drenaje* [Approves rules for the promotion of private investment in irrigation and drainage works (Law No. 18450)]. <https://www.bcn.cl/leychile/navegar?idNorma=29855>
- Congress of Chile. (2010, January 12). *Crea el ministerio, el servicio de evaluación ambiental y la superintendencia del medio ambiente* [Creates the Ministry, the environmental assessment service and the superintendence of the environment (Law No. 20417)] <https://www.bcn.cl/leychile/navegar?idNorma=1010459>
- Dahooie, J. H., Zavadskas, E., Abolhasani, M., Vanaki, A., & Turskis, Z. (2018). A novel approach for evaluation of projects using an interval-valued fuzzy additive ratio assessment (ARAS) method: A case study of oil and gas well drilling projects. *Symmetry*, 10(2), Article 45. <https://doi.org/10.3390/sym10020045>
- Dehkordi, M. F., Hatefi, S. M., & Tamošaitienė, J. (2025). An integrated Fuzzy Shannon entropy and Fuzzy ARAS model using risk indicators for water resources management under uncertainty. *Sustainability*, 17(11), Article 5108. <https://doi.org/10.3390/su17115108>
- Demirel, T., Demirel, N. Ç., & Kahraman, C. (2008). Fuzzy analytic hierarchy process and its application. In C. Kahraman (Ed.), *Fuzzy multi-criteria decision making: Theory and applications with recent developments* (pp. 53–83). Springer. https://doi.org/10.1007/978-0-387-76813-7_3

- Ecer, F. (2018). An integrated Fuzzy AHP and ARAS model to evaluate mobile banking services. *Technological and Economic Development of Economy*, 24(2), 670–695. <https://doi.org/10.3846/20294913.2016.1255275>
- Fernández, F. J., Vásquez-Lavín, F., Ponce, R. D., Garreaud, R., Hernández, F., Link, O., Zambrano, F., & Hanemann, M. (2023). The economics impacts of long- run droughts: Challenges, gaps, and way forward. *Journal of Environmental Management*, 344, Article 118726. <https://doi.org/10.1016/j.jenvman.2023.118726>
- Fuentes, I., Fuster, R., Avilés, D., & Vervoort, W. (2021). Water scarcity in central Chile: The effect of climate and land cover changes on hydrologic resources. *Hydrological Sciences Journal*, 66(6), 1028–1044. <https://doi.org/10.1080/02626667.2021.1903475>
- Ghorbani, M., Eskandari-Damaneh, H., Cotton, M., Ghoochani, O. M., & Borji, M. (2021). Harnessing indigenous knowledge for climate change-resilient water management – lessons from an ethnographic case study in Iran. *Climate and Development*, 13(9), 766–779. <https://doi.org/10.1080/17565529.2020.1841601>
- Ghran, M., & Frikha, H. M. (2022). Multiple hierarchically structured criteria in ARAS method under fuzzy environment. *International Journal of Fuzzy System Applications*, 11(1), 1–19. <https://doi.org/10.4018/IJFSA.315013>
- Gozdzalczy, M. B. (1987). A method of inference in approximate reasoning based on interval-valued fuzzy sets. *Fuzzy Sets and Systems*, 2 (1), 1–17. [https://doi.org/10.1016/0165-0114\(87\)90148-5](https://doi.org/10.1016/0165-0114(87)90148-5)
- Gullón, P., Gullón, B., Astray, G., Carpena, M., Fraga-Corral, M., Prieto, M. A., & Simal-Gandara, J. (2020). Valorization of by-products from olive oil industry and added-value applications for innovative functional foods. *Food Research International*, 137, Article 109683. <https://doi.org/10.1016/j.foodres.2020.109683>
- Herrera-Cáceres, C., Pérez-Galarce, F., Álvarez Miranda, E., & Candia-Véjar, A. (2017). Optimization of the harvest planning in the olive oil production: A case study in Chile. *Computers and Electronics in Agriculture*, 141, 147–159. <https://doi.org/10.1016/j.compag.2017.07.017>
- Hu, Y., Al-Barakati, A., & Rani, P. (2022). Investigating the internet-of- things (IoT) risks for supply chain management using q-rung orthopair fuzzy-SWARA-ARAS framework. *Technological and Economic Development of Economy*, 30(2), 376–401. <https://doi.org/10.3846/tede.2022.16583>
- Hwang, C.-L., & Yoon, K. (1981). *Multiple attribute decision making: Methods and applications a state-of-the-art survey*. Springer. <https://doi.org/10.1007/978-3-642-48318-9>
- Kamal, N. L. A. M., Abdullah, L., Abdullah, I., & Saqlain, M. (2020). Multi-valued interval neutrosophic linguistic soft set theory and its application in knowledge management. *CAAI Transactions Intelligence Technology*, 5(3), 200–208. <https://doi.org/10.1049/trit.2020.0036>
- Kamruzzaman, M. (2022). Impact of social media on geopolitics and economic growth: Mitigating the risks by developing artificial intelligence and cognitive computing tools. *Computational Intelligence and Neuroscience*. <https://doi.org/10.1155/2022/7988894>
- Karimi, H., & Nikkiah-Farkhani, Z. (2022). Performance appraisal of knowledge workers using augmented additive ratio assessment (A-ARAS) method: A case study. *IEEE Transactions on Engineering Management*, 69, 2285–2295. <https://doi.org/10.1109/TEM.2020.3009134>
- Liao, H., Wen, Z., & Liu, L. (2019). Integrating BWM and ARAS under hesitant linguistic environment for digital supply chain finance supplier section. *Technological and Economic Development of Economy*, 25(6), 1188–1212. <https://doi.org/10.3846/tede.2019.10716>
- Lin, M.-P., Lin, C.-H., Llonch-Molina, N., & Marine-Roig, E. (2025). The impact of olive oil tourism on multisensory experiences and tourist loyalty. *International Journal of Gastronomy and Food Science*, 40, Article 101195. <https://doi.org/10.1016/j.ijgfs.2025.101195>
- Liu, N., & Xu, Z. (2021). An overview of ARAS method: Theory development, application extension, and future challenge. *International Journal of Intelligent Systems*, 36, 3524–3565. <https://doi.org/10.1002/int.22425>

- Liu, P., Zhang, L., Liu, X., & Wang, P. (2016). Multi-valued neutrosophic number Bonferroni mean operators with their applications in multiple attribute group decision making. *International Journal of Information Technology Decision Making*, 15(5), 1181–1210. <https://doi.org/10.1142/S0219622016500346>
- Liu, Y., Eckert, C. M., & Earl, C. (2020). A review of fuzzy AHP methods for decision-making with subjective judgements. *Expert Systems with Applications*, 161, Article 113738. <https://doi.org/10.1016/j.eswa.2020.113738>
- Mardani, A., Jusoh, A., Zavadskas, E. K., Khalifah, Z., & Nor, K. M. (2020). Fuzzy MCDM for agricultural sustainability: A review. *Journal of Cleaner Production*, 276, Article 124187.
- Martina, D. J. S., & Deepa, G. (2023, 09). Application of multi-valued rough neutrosophic set and matrix in multi-criteria decision-making: Multi-valued neutrosophic rough set and matrix. *Mathematics in Applied Sciences and Engineering*, 4(3), 227–248. <https://doi.org/10.5206/mase/16636>
- Mirzabaev, A., Bezner Kerr, R., Hasegawa, T., Pradhan, P., Wreford, A., Cristina Tirado von der Pahlen, M., & Gurney-Smith, H. (2023). Severe climate change risks to food security and nutrition. *Climate Risk Management*, 39, Article 100473. <https://doi.org/10.1016/j.crm.2022.100473>
- Mishra, A. R., & Rani, P. (2021). A q-rung orthopair fuzzy ARAS method based on entropy and discrimination measures: An application of sustainable recycling partner selection. *Journal of Ambient Intelligence and Humanized Computing*, 14, 6897–6918. <https://doi.org/10.1007/s12652-021-03549-3>
- Mishra, A. R., Rani, P., Saha, A., & Hezam, I. M. (2022). Interval-valued fuzzy ARAS for climate-smart agriculture. *Environmental Science and Pollution Research*, 29(15), 21234–21251.
- Mishra, V., Seyedzenouzi, G., Almohtadi, A., Chowdhury, T., Khashkusha, A., Axiq, A., Wong, W. Y. E., & Harky, A. (2021). Health inequalities during COVID-19 and their effects on morbidity and mortality. *Journal of Healthcare Leadership*, 13, 19–26. <https://doi.org/10.2147/JHL.S270175>
- Mora, F., Tapia, F., Scapim, C. A., & Martins, E. N. (2007). Vegetative growth and early production of six olive cultivars in southern Atacama Desert, Chile. *Journal of Central European Agriculture*, 8 (3), 269–276.
- Morehead, M. S., & Scarbrough, C. (2018). Emergence of global antibiotic resistance. *Primary Care: Clinics in Office Practice*, 45(3), 467–484. <https://doi.org/10.1016/j.pop.2018.05.006>
- Nădăban, S., Dzitac, S., & Dzitac, I. (2016). Fuzzy TOPSIS: A general view. *Procedia Computer Science*, 91, 823–831. <https://doi.org/10.1016/j.procs.2016.07.088>
- Opricovic, S., & Tzeng, G.-H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156(2), 445–455. [https://doi.org/10.1016/S0377-2217\(03\)00020-1](https://doi.org/10.1016/S0377-2217(03)00020-1)
- Paelinck, J. (1978). Qualiflex: A flexible multiple-criteria method. *Economics Letters*, 1(3), 193–197. [https://doi.org/10.1016/0165-1765\(78\)90023-X](https://doi.org/10.1016/0165-1765(78)90023-X)
- Palczewski, K., & Sałabun, W. (2019). The fuzzy topsis applications in the last decade. *Procedia Computer Science*, 159, 2294–2303. <https://doi.org/10.1016/j.procs.2019.09.404>
- Peng, J.-j., Wang, J.-q., Wu, X.-h., Wang, J., & Chen, X.-h. (2014). Multi-valued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems. *International Journal of Computational Intelligence Systems*, 8, 345–363. <https://doi.org/10.1080/18756891.2015.1001957>
- Peng, J., Wang, J.-q., Hu, J., & Tian, C. (2018). Multi-criteria decision-making approach based on multi-valued neutrosophic geometric weighted choquet integral heronian mean operator. *Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology*, 35(3), 3661–3674. <https://doi.org/10.3233/JIFS-18249>
- Peng, J., Wang J.-q., & Wu, X.-h. (2017a). An extension of the electre approach with multi-valued neutrosophic information. *Neural Computing and Applications*, 28, 1011–1022. <https://doi.org/10.1007/s00521-016-2411-8>
- Peng, J., Wang, J.-q., & Yang, W.-E. (2017b). A multi-valued neutrosophic qualitative flexible approach based on likelihood for multi-criteria decision-making problems. *International Journal of Systems Science*, 48(2), 425–435. <https://doi.org/10.1080/00207721.2016.1218975>

- Peng, J., & Tian, C. (2018). Multi-valued neutrosophic distance-based QUALIFLEX method for treatment selection. *Information*, 9(12), Article 327. <https://doi.org/10.3390/info9120327>
- Peng, J., & Wang, J.-Q. (2015). Multi-valued neutrosophic sets and its application in multi-criteria decision-making problems. *Neutrosophic Sets and Systems*, 10, 3–17.
- Raina, R. S., & Longino, H. (2025). Community-led institutional innovation: Groundwater sharing, values and relationships in India's rainfed farming systems. *Studies in History and Philosophy of Science*, 112, 102–111. <https://doi.org/10.1016/j.shpsa.2025.06.011>
- Rani, P., Mishra, A., Saha, A., Hezam, I. M., & Pamučar, D. (2021). Fermatean fuzzy Heronian mean operators and MEREC-based additive ratio assessment method: An application to food waste treatment technology selection. *International Journal of Intelligent Systems*, 37, 2612–2647. <https://doi.org/10.1002/int.22787>
- Roy, B. (1968). Classement et choix en présence de points de vue multiples (la méthode electre) [Classification and selection in the presence of multiple viewpoints (the ELECTRE method)]. *Revue Française d'Informatique et de Recherche Opérationnelle*, 2(8), 57–75. <https://doi.org/10.1051/ro/196802V100571>
- Saaty, R. W. (1987). The analytic hierarchy process – what it is and how it is used. *Mathematical Modelling*, 9(3–5), 161–176. [https://doi.org/10.1016/0270-0255\(87\)90473-8](https://doi.org/10.1016/0270-0255(87)90473-8)
- Salam, M. A., Al-Amin, M. Y., Salam, M. T., Pawar, J. S., Akhter, N., Rabaan, A. A., & Alqumber, M. A. A. (2023). Antimicrobial resistance: A growing serious threat for global public health. *Healthcare*, 11(13), Article 1946. <https://doi.org/10.3390/healthcare11131946>
- Shahmohammad, F. N., Pourrahimian, Y., & Akbari-Gharalari, N. (2024). Synthesizing complexity: Trends, challenges, and future directions in fuzzy- based multicriteria decision-making (FMCDM) methods. *Applied Soft Computing*, 167, Article 112362. <https://doi.org/10.1016/j.asoc.2024.112362>
- Smarandache, F. (2004). Neutrosophic set – a generalization of the intuitionistic fuzzy set. *International Journal of Pure and Applied Mathematics*, 24.
- Smarandache, F. (2006, May 10–12). Neutrosophic set – a generalization of the intuitionistic fuzzy set. In *Proceedings of the 2006 IEEE International Conference on Granular Computing* (pp. 38–42). Atlanta, GA, USA. IEEE. <https://doi.org/10.1109/GRC.2006.1635754>
- Stević, Ž., Ersoy, N., Başar, E. E., & Baydaş, M. (2024, November). Addressing the global logistics performance index rankings with methodological insights and an innovative decision support framework. *Applied Sciences*, 14(22), Article 10334. <https://doi.org/10.3390/app142210334>
- Thakkar, J. J. (2021). Additive ratio assessment method (ARM/ARAS). In *Multi-criteria decision making* (Vol. 336, pp. 239–252). Springer. https://doi.org/10.1007/978-981-33-4745-8_14
- Tugendhaft, Y., Eppel, A., Kerem, Z., Barazani, O., Ben-Gal, A., Kadereit, J. W., & Dag, A. (2016). Drought tolerance of three olive cultivars alternatively selected for rain fed or intensive cultivation. *Scientia Horticulturae*, 199, 158–162. <https://doi.org/10.1016/j.scienta.2015.12.043>
- Turskis, Z., Lazauskas, M., & Zavadskas, E. K. (2012). Fuzzy multiple criteria assessment of construction site alternatives for non-hazardous waste incineration plant in Vilnius city, applying ARAS-F and AHP methods. *Journal of Environmental Engineering and Landscape Management*, 20(2), 110–120. <https://doi.org/10.3846/16486897.2011.645827>
- van de Loo, M., Camacho Poyato, E., van Halsema, G., & Rodríguez Díaz, J. A. (2024). Defining the optimization strategy for solar energy use in large water distribution networks: A case study from the Valle Inferior irrigation system, Spain. *Renewable Energy*, 228, Article 120610. <https://doi.org/10.1016/j.renene.2024.120610>
- Wang, H., Smarandache, F., & Sunderraman, R. (2010). Single valued neutrosophic sets. In F. Smarandache (Ed.), *Multispace & multistructure. Neutrosophic transdisciplinary (100 Collected Papers of Sciences)* (Vol. IV, pp. 410–413). North-European Scientific Publishers.
- Wang, J., & Li, X. (2015). TODIM method with multi-valued neutrosophic sets. *Control and Decision*, 30(6), 1139–1145.

- Wang, J.-W., Ma, L.-y., Gómez del Campo, M., Zhang, D.-s., Deng, Y., & Jia, Z.-k. (2018). Youth tree behavior of olive (*Olea europaea* L.) cultivars in wudu, China: Cold and drought resistance, growth, fruit production, and oil quality. *Scientia Horticulturae*, 236, 106–122. <https://doi.org/10.1016/j.scienta.2018.03.033>
- Xiao, F., Wang, J., & Wang, J.-Q. (2021). An improved MULTIMOORA method for multi-valued neutrosophic multi-criteria group decision-making based on prospect theory. *Scientia Iranica*.
- Yager, R. R. (2014). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, 22 (4), 958–965. <https://doi.org/10.1109/TFUZZ.2013.2278989>
- Yager, R. R. (2016). Uncertainty modeling using fuzzy measures. *Knowledge-Based Systems*, 92, 1–8. <https://doi.org/10.1016/j.knosys.2015.10.001>
- Yang, L., Li B., & Xu, H. (2018). Multi-valued neutrosophic linguistic power operators and their applications. *Engineering Letters*, 26(4), 518–525.
- Ye, J. (2014). A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent and Fuzzy Systems*, 26(5), 2459–2466. <https://doi.org/10.3233/IFS-130916>
- Ye, J., Song, J., & Du, S. (2020). Correlation coefficients of consistency neutrosophic sets regarding neutrosophic multi-valued sets and their multi-attribute decision-making method. *International Journal of Fuzzy Systems*, 24, 925–932. <https://doi.org/10.1007/s40815-020-00983-x>
- Zadeh, L. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- Zavadskas, E. K., & Turskis, Z. (2010). A new additive ratio assessment (ARAS) method in multicriteria decision-making. *Technological and Economic Development of Economy*, 16(2), 159–172. <https://doi.org/10.3846/tede.2010.10>
- Zavadskas, E. K., Turskis, Z., Vilutienė, T., & Lepkova, N. (2017). Integrated group fuzzy multi-criteria model: Case of facilities management strategy selection. *Expert Systems with Applications*, 82, 317–331. <https://doi.org/10.1016/j.eswa.2017.03.072>
- Zheng, Y., & Gong, B. (2024). Nexus between natural resources and digital economy: The role of geopolitical risk. *Resources Policy*, 89, Article 104600. <https://doi.org/10.1016/j.resourpol.2023.104600>

APPENDIX

A. Expert evaluation questionnaire

Objective: To assess climate change adaptation strategies for smallholder olive farmers in the Coquimbo Region using expert judgment on economic, environmental, social, and technical criteria.

Instructions: Please evaluate each adaptation strategy based on the criteria below using the following linguistic scale (Table A1). Also, please assign the importance (weight) of each criterion using the same scale.

Table A1. Linguistic terms and their meanings for contribution/importance assessment

Linguistic term	Meaning
Very Low (VL)	Very negative or negligible contribution / importance
Low (L)	Limited or below-average contribution / importance
Medium Low (ML)	Slightly below average contribution / importance
Fair (F)	Average or neutral contribution / importance
Medium High (MH)	Slightly above average contribution / importance
High (H)	Significant contribution / importance
Very High (VH)	Excellent or transformative contribution / importance

B. Criteria and weight assignment

Please indicate the importance of each criterion by selecting one of the linguistic terms (VL, L, ML, F, MH, H, VH).

Table B1. Criterion weights using the linguistic scale

Criterion	Assigned weight (VL–VH)
C1. Economic viability	
C2. Environmental sustainability	
C3. Social acceptability	
C4. Technical feasibility	
C5. Resilience improvement	
C6. Market potential	

C. Adaptation strategies for olive growers

Please rate each strategy against the criteria using the same linguistic terms (VL, L, ML, F, MH, H, VH).

Table C1. Evaluation of the strategy against criteria

Strategy	C1	C2	C3	C4	C5	C6
Implementation of drip or sprinkler irrigation systems						
Crop substitution with drought-resistant olive cultivars						
Rainwater harvesting and micro-reservoirs						
Agroforestry with olive-compatible species						
Use of organic soil amendments and compost						
Solar-powered irrigation pumps						
Diversification through olive oil value-added products (e.g., cosmetics, artisanal oil)						
Community-managed water-sharing systems						
Beekeeping for pollination services and income						
Participation in rural ecotourism linked to olive farms						

Comments or Suggestions: _____

Name of Expert: _____

Date: _____