ISSN: 2029-4913 / eISSN: 2029-4921

TECHNOLOGICAL and ECONOMIC DEVELOPMENT of ECONOMY

2025
Volume 31
Issue 4
Pages 1206–1236
https://doi.org/10.3846/tede.2025.24346

BIVARIATE GRID SCALE BASED MULTIPLE ATTRIBUTE EVALUATION TECHNIQUE (GAMETE) WITH INCOMPLETE INFORMATION ON WEIGHTS

Mohamed SOUISSI[1]✉, Edmundas Kazimieras ZAVADSKAS[2], Salem CHAKHAR[3]

[1]Modelling and Optimisation for Decisional, Industrial and Logistic Systems Laboratory, Faculty of Economics and Management of Sfax, University of Sfax, Sfax, Tunisia
[2]Institute of Sustainable Construction, Faculty of Civil Engineering, Vilnius Gediminas Technical University, Vilnius, Lithuania
[3]Portsmouth Business School & Centre for Operational Research and Logistics, University of Portsmouth, Portsmouth, UK

Article History:
- received 07 March 2025
- accepted 01 July 2025

Abstract. In this paper, we have devised a novel Multiple Attribute Decision Making (MADM) method referred to as the bivariate Grid Scale based Multiple Attribute Evaluation Technique (GAMETE) method to deal with MADM decision problems involving tangible and intangible attributes under incomplete weight information. The proposed method innovatively incorporates an Attractiveness GRID Scale (AGRIDS) to evaluate intangible attributes, grounded in cognitive psychological principles – particularly the separability and independence of positive and negative aspects in human judgement. Additionally, a new bi-dimensional positional advantage operator (bi-pao) is introduced to compute the intangible attractiveness index. Further, linear programming models are formulated in order to construct the pairwise dominance matrix. Afterwards, we rank alternatives using a dominance intensity measure and the Boolean matrix. Furthermore, the proposed method is illustrated through a logistics center location problem. We also perform a comparison with several state-of-the-art linguistic Intuitionistic Fuzzy Sets (LIFS) and linguistic Pythagorean Fuzzy Sets (LPFS) based MADMs with the aim of showing the applicability and feasibility of the method suggested. Notably, GAMETE provides a multidimensional decision-making framework suitable for addressing complex technological and economic challenges where both quantitative and qualitative factors coexist. Its flexibility and interpretability make it a promising tool for real-world strategic decision scenarios.

Keywords: dominance measure, intangible attributes, incomplete weight information, Grid scale, Multiple Attribute Decision Making.

JEL Classification: C41, C61, D81.

✉Corresponding author. E-mail: ***mouhamed.souissi@yahoo.com***

1. Introduction

1.1. General context

The Multiple Attribute Decision Making (MADM) refers to a process of evaluating a set of alternatives described with respect to multiple (usually conflicting) attributes. A large number of MADM methods have been designed and used in various application areas (Filip et al., 2017; Zavadskas et al., 2016; Hisoğlu et al., 2025). For instance, Wen et al. (2020) have proposed a Mixed Aggregation by Comprehensive Normalization Technique (MACONT) to select sustainable third-party reverse logistics providers. Ecer and Hashemkhani Zolfani (2022) have used the removal effects of criteria (MEREC) and the double normalization-based multi-aggregation (DNMA) to specify the ranking of countries according to economic freedom. Hasheminasab et al. (2021) have designed an integrated model combining the extended stepwise weight assessment ratio analysis extended SWARA (E-SWARA) method and the Ranking according to COmpromise Solution (MARCOS) method for implementing the circular economic in fossil fuel development to minimize the unsustainable effects and ensure the environment's resiliency. Wang et al. (2022) combine the Best Worst Method (BWM) and the technique for ordering preference by similarity to the ideal solution (TOPSIS) methods. Zakeri et al. (2023) proposed a novel MCDM method called alternative ranking with elected nominee (ARWEN) to solve the supplier selection problem.

The attributes considered within the MADM decision problems are generally grouped into tangible (i.e., clearly defined) and intangible attributes. The decision-makers, through positive and negative aspects, generally subjectively assess the values of the intangible attributes, often imprecisely defined. Under these circumstances, decision-makers are usually inclined to use linguistic terms to represent their opinions, which are similar to natural language and close to human cognitive processes. These categories pose fundamental problems. Firstly, they resonate rather with the way humans think and communicate. Secondly, they tend to express subjective judgements that reduce a full spectrum into positive (favourable) and negative (unfavourable) dimensions. Such dual-sided assessments reflect the predominant double-judgment thinking pattern of human cognition, in which acceptance and rejection are not necessarily mutually exclusive, as they may coexist simultaneously (Zayas et al., 2017). Therefore, an effective decision-making method should not only support inherently language-bound, imprecise data but also facilitate the independent modelling of positive and negative evaluations so as to grasp the nuanced nature of human judgement with more accuracy.

1.2. Problem statement

Despite the remarkable progress in MADM, most methods still rely on unipolar evaluations that consider only the positive aspects of alternatives. Drawbacks are either ignored or inadequately accounted for. In real-life situations, decision-making depends on a series of overlapping circumstances with positive and negative perspectives or facets of information set in at the same time. Human decisions depend on double-sided judgement, considering positive as well as negative aspects. Positive evaluation suggests that the information is deemed satisfactory or desirable. Nonetheless, the negative evaluation expresses what is impossible, unfavourable, rejected, or forbidden (Al-Sharqi et al., 2024). According to several psychological studies, positive and negative information are not necessarily complementary and should be processed separately (Cacioppo et al., 2012; Norris et al., 2010). In fact, analyses of human neuroimaging literature provide evidence for the independence and separability of positive and negative evaluations. These studies show that positive and negative affect are supported by distinct brain systems (Lindquist et al., 2016).

To integrate positive and negative evaluations, there are basically two models: the univariate bipolar model and the bivariate unipolar model. The former conceptualizes evaluation as a single continuum with a neutral midpoint distinguishing between positive and negative evaluations (Franco et al., 2017). A major limitation of this model is its inability to process ambivalence where positivity and negativity exist simultaneously. With positive and negative evaluations being regarded as mutually exclusive, an alternative cannot be judged as both positive and negative at the same time, making the univariate bipolar model inadequate for mixed and two-sided judgements (Luo & Hu, 2023).

The bivariate unipolar model, by Cacioppo and Berntson (1994), supports both positivity and negativity through two distinct unipolar scales. Each scale ranges from a minimum positive (respectively negative) to a maximum positive (respectively negative). A concept may bear positive, negative, neither positive nor negative, or both positive and negative meanings at the same time. By handling two-sided evaluations, where decision-makers assess alternative aspects as both favorable and unfavorable simultaneously, this model accurately reflects the complexity of human cognition in situations involving conflicting information (Franco et al., 2015). This paper uses a multidimensional bivariate model of evaluative space introduced by Cacioppo and Berntson (1994) and Cacioppo et al. (2012) to evaluate intangible attributes under double-sided appraisal.

To cope with MADM problems comprising double-sided information, sophisticated models such as Intuitionistic Fuzzy Sets (IFS) (Atanassov, 1986) and Pythagorean Fuzzy Sets (PFS) (Yager & Abbasov, 2013) have been devised. IFS and PFS are particularly effective in handling uncertainty and responding to the dual nature of human reasoning, making it easier for decision-makers to select the most appropriate alternative. Both IFS and PFS feature information using a pair of values within the unit interval: a positive-membership degree (expressing preference or acceptance) and a negative-membership degree (indicating rejection or non-preference). The main distinction between the two lies in their mathematical constraints: in IFS, the sum of membership and non-membership degrees must not exceed one, while in PFS, the sum of their squares must remain within unity.

Nevertheless, these mathematical constraints limit their flexibility, especially when positive and negative evaluations coexist and exert generally antagonistic effects. As a result, existing decision-making methods often struggle to capture the nuances of human judgment, particularly in scenarios where alternatives have both favorable and unfavorable features. Furthermore, the realism and applicability of many models in cognitively complex decision contexts are limited due to the lack of an ex-ante psychological foundation.

1.3. Motivation

Decision-making tools that accurately reflect human cognitive processes are particularly needed in applications involving subjective evaluations – such as service quality, software feasibility, and policy impact assessments. However, current MADM methods often fail to capture human judgment accurately, especially when evaluating aspects that are simultaneously positive and negative. As a result, these methods do not effectively address the complexity of ambivalent or conflicting evaluations, which are common in real-world decision-making. Furthermore, as real-life problems become increasingly complex – particularly in the digital economy and service-based industries – models that recognize nuanced evaluations such as indifference, ambivalence, and partial favorability offer superior decision support and greater behavioral realism. Filling this methodological gap is both timely and critical for advancing theory and practice in decision science.

1.4. Aim, novelty, and contribution

This paper proposes a novel MADM method called Grid Scale based Multiple Attribute Evaluation Technique (GAMETE), with Bivariate Attractiveness GRID Scale (AGRIDS) that enables a dual-axis evaluation of alternatives based on the psychological separability of positive and negative judgments. GAMETE incorporates a newly developed operator – the Bidimensional Positional Advantage Operator (bi-PAO) – aimed to quantify intangible attractiveness indices. A dedicated ranking strategy is also introduced, relying on linear programming and interval dominance matrices.

GAMETE's contributions mainly include:

- Introducing a psychologically grounded evaluative model for MADM.
- Enabling the simultaneous and separate representation of positive and negative assessments.
- Providing a new ranking system adapted to interval-valued dominance relations.
- Offering flexibility to handle both tangible and intangible attributes with no restrictive assumptions on evaluative inputs.
- The validity of the newly defined approach is established by applying it to the MADM problem of logistics center location.
- We perform a comparison with several state-of-the-art LIF-MADMs and LPF-MADM with the aim of showing the applicability and feasibility of the approach suggested.

1.5. Technological and economic development of economy relevance

The proposed method enhances decision-making quality in tasks where subjective judgments prevail – such as logistics optimization, investment evaluation, human resource assessment, and sustainable planning. With optimized sensing of human judgment, GAMETE aims to guide organizations in selecting non-dominated alternatives that balance risk, desirability, and feasibility and eventually support more efficient resource allocation, reduced risk of implementation failure, and improved stakeholder satisfaction – critical for economic competitiveness and innovation.

1.6. Structure

The remainder of this paper is structured as follows. Section 2 reviews related work. Section 3 introduces some basic concepts. Section 4 presents the GAMETE method with relevant illustrations. Section 5 compares the proposed method with other recent models through two case studies. Section 6 discusses the main features and issues of GAMETE. Section 7 concludes the paper and outlines directions for future research.

2. Related work

The evaluation of attributes in decision-making is inherently vague and ambiguous, particularly when dealing with intangible attributes. It is difficult for decision-makers to express their preferences with precise numerical values. Decision-makers usually use natural language reflecting human cognitive processes. The words opinion or preference suggest positive (favorable) and negative (unfavorable) sides that coexist simultaneously. These assessments typically involve both favourable (positive) and unfavourable (negative) judgements.

Recent psychological research provides compelling evidence for the functional independence of positive and negative evaluations. The Evaluative Space Model (ESM), developed by Cacioppo and Berntson (1994) and Cacioppo et al. (2012), suggests that affective responses are best understood as occurring along two independent dimensions: positivity and negativity. Empirical studies (e.g., Larsen et al., 2009; Zayas et al., 2017) maintain that humans often give positive and negative appraisals simultaneously, particularly in subtle situations.

In the rest of this Section, we review the use of classical fuzzy set theory and its extensions in modeling positive and negative judgments.

2.1. Modeling positive and negative judgments with classical fuzzy set theory

The fuzzy set theory (FST), proposed by Zadeh (1965), is a useful tool to reflect human judgements. Integrated with MADM methods, FST theory has been applied in several real-world problems. For instance, Hashemkhani Zolfani et al. (2021) integrated F-SWARA and Fuzzy Multi-Attribute Border Approximation Area Comparison (F-MABAC) to select the best logistics village. Ulutaş et al. (2021) introduced three fuzzy MADM methods to cope with the transportation company selection problem. These methods include Fuzzy PIvot Pairwise RElative Criteria Importance Assessment (F-PIPRECIA) used to achieve the subjective weights of attributes; the fuzzy-PSI method to obtain the objective weights of attribute; and the Fuzzy-CoCoSo to rank alternative transportation companies according to their respective performances. Pamucar et al. (2022) employed the Fuzzy Measuring Attractiveness by a Categorical Based Evaluation TecHnique (MACBETH) and fuzzy Weight Aggregated Sum Product ASsessment (WASPAS) to select the most suitable location for a recovery center.

Rifle et al. (2024) proposed the fuzzy Simple Weight Calculation (F-SIWEC) and fuzzy COmpromise Ranking from Alternative SOlutions (CORASO) methods to select the most suitable spraying drone for a foodstuff business. The F-SIWEC method was used for determining the weights of the criteria and the fuzzy CORASO method to select the best drone. Demir and Ulusoy (2024) used F-WENSLO (Fuzzy Weight by Envelope and Slope) and the fuzzy Ranking Alternatives with Weights of Criterion (F-RAWEC) methods to evaluate potential locations for wind farms. Katranci et al. (2025) used the F-SIWEC-RAWEC to determine the most suitable sustainable solid waste disposal technology.

Despite their effectiveness in managing uncertainty, traditional fuzzy MADM methods predominantly focus only one side of the evaluation – most often the favorable aspect – while overlooking unfavorable dimensions. However, in many real-world decision-making problems, especially under ambiguous or complex environments, it is essential to consider both positive and negative assessments. For instance, the appraisal of a nation's economic performance requires the measurement of both positive and negative indicators. The traditional unidimensional assessment of FST – focused mainly on positive appraisals – was found insufficient (Ali et al., 2024).

FST is usually deployed within a bipolar univariate model where each alternative is represented by a single membership value $u(x) \in [0,1]$. This value stands for the degree of positive assessment. The negative assessment is tacitly derived as its complement, 1 – $u(x)$. This model poses a strict inverse relationship between positivity and negativity, implying that an increase in one dimension implies a decrease in the other. In other words, the bipolar univariate model inherently assumes mutual exclusivity and full complementarity between acceptance and rejection.

2.2. Modeling positive and negative judgments with orthopair fuzzy sets

Generalized orthopair fuzzy sets constitute effective tools to grasp the intricacy of human cognitive processes by comprising both positive membership and negative membership degrees (Campagner & Ciucci, 2017) and enabling a more comprehensive representation of vague or conflicting information. These sets fall under the bivariate unipolar model since they use two partially independent unipolar scales bounded in the interval [0,1]. Perhaps the most prominent orthopair fuzzy sets are the Intuitionistic Fuzzy Set (IFS) theory (Atanassov, 1986) and Pythagorean Fuzzy Set (PFS) theory (Yager & Abbasov, 2013).

A number of recent MADM methods have been designed to tackle intricate decision problems under the IFS environment. For example, Uyanik et al. (2020) combined DEMATEL with IF-TOPSIS methodology to determine the optimal location for a logistics centre. Patel et al. (2023) proposed an intuitionistic fuzzy version of the Elimination and Choice Translating Reality (EM) method followed by SWARA and TOPSIS to select the best medical waste treatment technique. Chakraborty et al. (2024) proposed intuitionistic fuzzy weighted sum product assessment (WASPAS), Combinative distance-based assessment (CODAS), and combined compromise solution (COCOSO) to determine the optimal healthcare supplier. Işik and Adalar (2024) introduced the intuitionistic fuzzy compromise ranking of alternatives from distance to ideal solution (IF-CRADIS) to appraise and rate the performance of non-life insurance companies. Yesilcayir et al. (2024) proposed IF-AHP-TOPSIS to select the most suitable transit warehouse location. Hezam et al. (2024) showcased the selection of hospital sites using the compromise solution (IF-MARCOS).

IFS imposes a limitation where the sum of membership and non-membership degrees must not exceed one. More advanced structures like PFS have been developed, where the squared sum of membership and non-membership degrees is restricted to one. This approach has been successfully applied in various fields. For example, Liao et al. (2020) utilized CoCoSo, the cumulative prospective theory, and a combined weight determining the methods under the PFS for the selection of a cold chain logistics distribution center. Rani et al. (2020) combined the SWARA with the VIKOR approach for panel selection in PFS environment. Ayyildiz (2022) merged SWARA with CODAS using the PF environment to determine the best location for an e-scooter charging station. Yalcin Kavus et al. (2023) combined the Bayesian Best Worst Method (B-BWM) with PF-WASPAS to evaluate five sites in Istanbul. Nila and Roy (2024) propose an enhanced PF-DEMATEL approach for logistics centers 4.0.

The Pythagorean fuzzy decision analysis and geographic information systems were combined by Çalış Boyacı and Şişman (2024) to determine optimal locations for the disposal of face masks and gloves waste boxes during the COVID-19 pandemic in Samsun city, Turkey.

Although IFS and PFS are effective in managing uncertainty and modelling dual aspects of human reasoning, they exhibit limitations when dealing with highly ambiguous or conflicting evaluations. For example, if a decision-maker assigns a membership degree of 0.8 and a non-membership degree of 0.7, the squared sum in PFS is $0.8^2+0.7^2=1.13>1$, violating the model's fundamental constraint requiring the sum of squares not to exceed unity. This example illustrates that PFS, despite advantages over IFS, fails to accommodate evaluations where strong positive and negative perceptions coexist. Consequently, these frameworks may not fully reflect the nuanced and often ambivalent nature of human judgement, especially in decision scenarios involving alternatives with both favourable and unfavourable characteristics.

Moreover, many existing MADM methods lack robust psychological grounding, which limits their realism and practical applicability in cognitively complex decision-making environments. To achieve higher fidelity in modelling real-world human evaluations, a more flexible and psychologically consistent framework is needed.

2.3. Summary

This review draws attention to the difficulties in assessing intangible qualities while making decisions, especially when both favorable and unfavorable assessments are present at the same time. Although it offers fundamental tools, classical fuzzy set theory is constrained by its one-dimensional methodology. More complex modeling of dual evaluations is possible using orthopair fuzzy sets, such as Intuitionistic and Pythagorean fuzzy sets. These frameworks are applied in a variety of real-world applications by different MADM techniques. However, when it comes to managing extremely unclear or contradictory judgments, both IFS and PFS have intrinsic limits. The need for more adaptable and realistic models that are in line with human cognitive processes is further highlighted by the fact that many present MADM techniques lack adequate psychological foundation.

3. Preliminaries

3.1. The evaluative space grid

The semantic differential scale, originating from the work of Osgood et al. (1967), is the traditional technique for attitude measurement. It is considered a simple way for measuring attitude where individuals are asked to indicate if a given object is perceived as being positive, neutral, or negative. It is a bipolar rating scale composed of a pair of antagonistic adjectives at two extremities of a continuum. This scale, which corresponds well to the typical bipolar univariate model (Montero et al., 2014), ranges from maximally positive (minimally negative) to maximally negative (minimally positive) and assumes that positive and negative attitudes are reciprocally activated. The negative side of the scale is the inverse mirror of the positive one; it cannot be positive and negative at the same time. This structure is a characteristic of a symmetric univariate bipolarity type that focuses on one side of judgment (negative/positive) and ignores the other. Moreover, psychological studies indicate that positive and negative attitudes are not complementary to each other (Cacioppo et al., 2012).

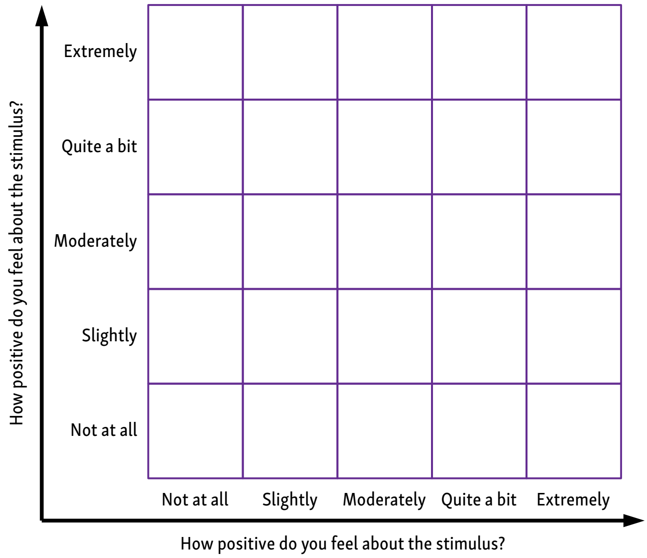

Figure 1. The evaluative space grid (GRID scale)

One way to model the non-complementary positive and negative attitudes is to use the ESG proposed by Larsen et al. (2009) (Figure 1), which is based on the Evaluative Space Model (Cacioppo et al., 2012; Zayas et al., 2017). The ESG suggests that assessing positivity and negativity involves two separable and partially distinct mechanisms. It comprises a 5×5 grid measuring both the degree of positivity and negativity of an evaluation within a two-dimensional matrix. In practice, a single dimension is assigned to measure the respondent's degree of negativity (from "not at all negative" to "extremely negative"). The other dimension is intended to measure the respondent's degree of positivity (from "not at all positive" to "extremely positive"). Considering these two dimensions, respondents are asked to choose which of the 25 cells best describes their evaluation.

The rationale of using ESG relies on the evidence substantiating that the dimensions of positivity and negativity are functionally separable. It also points out that an increase in one dimension does not necessarily lead to an equal reduction in the other dimension, which may or may not reduce at all. The ESG scale has been validated in psychology with respect to the unipolar measurement of positivity and negativity via combining the levels of positivity and negativity (Larsen et al., 2009). It provides a separate measure of indifference (low positivity, low negativity) and ambivalence (moderate to high positivity and moderate to high negativity).

3.2. The bounded-difference operation

Let us introduce some definitions and notation related to the bounded-difference operation.

Definition 1. Let a and b be two real numbers, then the *bounded-difference* of a and b, written $a \ominus b$, is defined by (Zadeh, 1975):

$$a \ominus b = \begin{cases} a-b, \text{ if } a>b \\ 0, \text{ otherwise} \end{cases}. \tag{1}$$

Definition 2. Let A and B be two alternatives with respective performance vectors $(a_1,a_2,a_3,\cdots,a_n)$ and $(b_1,b_2,b_3,\cdots,b_n)$, then

- The *favorable* bounded-difference vector to A, written $A \ominus B$, is defined by:

$$A \ominus B = (a_1 \ominus b_1, a_2 \ominus b_2, \cdots, a_n \ominus b_n). \tag{2}$$

- The *unfavorable* bounded-difference vector to A, written $B \ominus A$, is defined by:

$$B \ominus A = (b_1 \ominus a_1, b_2 \ominus a_2, \cdots, b_n \ominus a_n). \tag{3}$$

Definition 3. Let A and B be two alternatives with respective performance vectors $(a_1,a_2,a_3,\cdots,a_n)$ and $(b_1,b_2,b_3,\cdots,b_n)$. Alternative A will be said to dominate alternative B if and only if:

$$|A \ominus B| > |B \ominus A|, \tag{4}$$

where

$$|A \ominus B| = \sum_{j=1}^{n} w_j (a_j \ominus b_j), \tag{5}$$

and

$$|B \ominus A| = \sum_{j=1}^{n} w_j (b_j \ominus a_j), \tag{6}$$

with $w_j > 0 (j=1,\ldots,n)$ and $\sum_{j=1}^{n} w_j = 1$.

4. Presentation of GAMETE

Let $A = \{x\}_{i=1}^{i=n}$ be a finite set of n alternatives described with respect to a finite set $Q = \{q_j\}_{j=1}^{i=m}$ of *m*attributes. GAMETE has been designed to primordially dealt with intangible attributes but it can also deal with tangible attributes. Thus, we assume that set Q is divided into a subset Q_T of tangible attributes and a subset Q_I of intangible attributes, such that $Q_T \cup Q_I = Q$ and $Q_T \cap Q_I = \varnothing$. We also assume that there is at least one intangible attribute. Tangible attributes refer to regular attributes defined over one dimension. Each alternative will have one score with respect to each tangible attributes. Intangible attributes are assumed to be bi-dimensional. That is, the evaluations of any alternative with respect to the intangible attributes are of two different types: positive (i.e., favorable) evaluations and negative (i.e., unfavorable) ones. The ratings of each alternative x_i with respect to each intangible attribute Q_I are obtained by using the so-called attractiveness GRID described earlier.

4.1. The Attractiveness GRID Scale

Building on the ideas discussed in Section 2.1, GAMETE uses a new Attractiveness GRID Scale (AGRIDS) to assess intangible attributes. AGRIDS is defined as a two-dimensional 11 × 11 matrix (see Figure 2) with a view to obtain a joint measure of favorableness and unfavorableness.

Unlike the original ESG scale, AGRIDS is made up of two 11-point-semantic differential scales: one for rating favorableness and the other for rating unfavorableness. This manner yields an attribute-wise attractiveness joint index for each alternative.

From this perspective, AGRIDS offers advantages over the original semantic differential scale because it differentiates between four different types of evaluation reactions:

- *Indifference*. Exceptionally low to below the favorableness average and exceptionally low to below the unfavorableness average;
- *Ambivalence*. Average to exceptionally high favorableness and average to exceptionally high unfavorableness;
- *Positive attitude*. Average to exceptionally high favorableness and exceptionally low to below-average unfavorableness;
- *Negative attitude*. Exceptionally low to below the favorableness average and average to exceptionally high unfavorableness.

Hereinafter, favorableness and unfavorableness are treated as linguistic variables with a set of eleven terms S as follows:

$$S=\begin{Bmatrix} s_1: \text{Exeptionally Low}(\text{XL}) \;\; s_2: \text{Extremely Low}(\text{EL}) \;\; s_3: \text{Very Low}(\text{VL}) \;\; s_4: \text{Low}(\text{L}) \\ s_5: \text{Below Average}(\text{BA}), \;\; s_6: \text{Average}(\text{A}) \;\; s_7: \text{Above Average}(\text{AA}) \;\; s_8: \text{High}(\text{H}) \\ s_9: \text{Very High}(\text{VH}) \;\; s_{10}: \text{Extremely High}(\text{EH}) \;\; s_{11}: \text{Exceptionally High}(\text{XH}) \end{Bmatrix}. \quad (7)$$

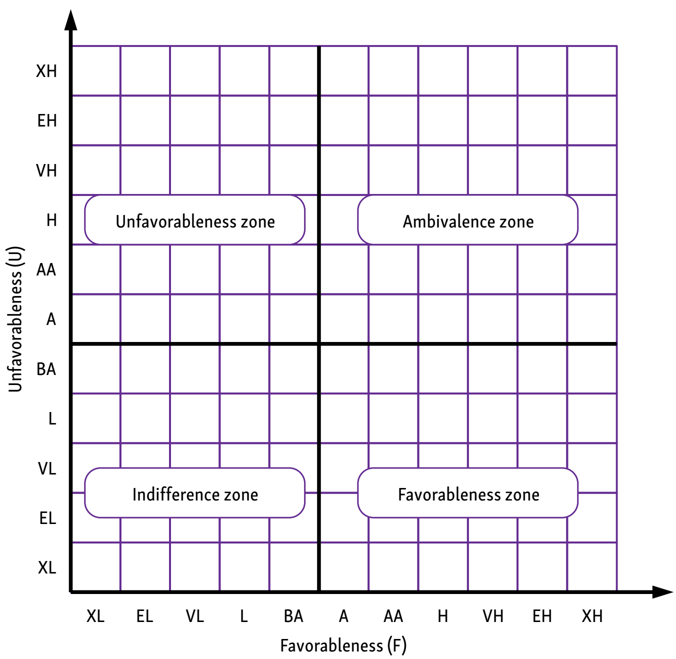

Figure 2. Attractiveness GRID Scale (AGRIDS)

4.2. Bidimensional positional advantage operator

In this subsection, we introduce the bidimensional positional advantage operator (bi-pao) as an extension of the definition of positional advantage operator (Rebai et al., 2006) to the bidimensional case.

Definition 4. Let $[L_1,U_1]$ and $[L_2,U_2]$ be two intervals of real numbers such that the endpoints L_1,L_2,U_1 and U_2 satisfy $0 \le L_1 < U_1 < +\infty$ and $0 \le L_2 < U_2 < +\infty$.

$$ⓟ : \left([L_1,U_1]\times[L_2,U_2]\right)^2 \to [0,1]. \tag{8}$$

Then the mapping is a bi-dimensional positional advantage operator (bi-pao) defined over the Cartesian product $[L_1,U_1]\times[L_2,U_2]$ if and only if it satisfies the following essential requirements:

- (U_1,U_2) ⓟ (L_1,L_2) = 1 (boundary condition),
- For a_1,a_1',b_1 and $b_1' \in [L_1,U_1]$, a_2,a_2',b_2 and $b_2' \in [L_2,U_2]$:
  - $a_1 < a_1' \Rightarrow (a_1,a_2)$ ⓟ $(b_1,b_2) < (a_1',a_2)$ⓟ(b_1,b_2) (i.e., ⓟ is strictly increasing in the first place of the first pair),
  - $a_2 < a_2' \Rightarrow (a_1,a_2)$ ⓟ $(b_1,b_2) < (a_1,a_2')$ⓟ(b_1,b_2) (i.e., ⓟ is strictly increasing in the second place of the first pair),
  - $b_1 < b_1' \Rightarrow (a_1,a_2)$ ⓟ $(b_1,b_2) < (a_1,a_2)$ⓟ(b_1',b_2) (i.e., ⓟ is strictly decreasing in the first place of the second pair),
  - $b_2 < b_2' \Rightarrow (a_1,a_2)$ ⓟ $(b_1,b_2) < (a_1,a_2)$ⓟ(b_1,b_2') (i.e., ⓟ is strictly decreasing in the second place of the second pair),
  - $[(b_1,b_2)ⓟ(a_1,a_2)] + [(a_1,a_2)ⓟ(b_1,b_2)] = 1$ (i.e., adding up to unity property)

The following are two bi-pao examples defined for a_1, $b_1 \in [L_1,U_1]$ and a_2, $b_2 \in [L_2,U_2]$:

$$(a_1,b_1)ⓟ(a_2,b_2) = \frac{a_1+a_2-b_1-b_2+U_1+U_2-L_1-L_2}{2(U_1+U_2-L_1-L_2)}, \tag{9}$$

$$(a_1,b_1)ⓟ(a_2,b_2) = \begin{cases} \dfrac{a_1+a_2-L_1-L_2}{a_1+a_2+b_1+b_2-2L_1-2L_2} & \text{if } (a_1,b_1)\ne(L_1,U_1) \text{ and } (a_2,b_2)\ne(L_2,U_2) \\ \dfrac{1}{2} & \text{otherwise.} \end{cases} \tag{10}$$

4.3. Intangible and tangible attractiveness indexes

4.3.1. Intangible attractiveness index

The assessment of attractiveness of an alternative x_i with respect to intangible attribute $I_j \in Q_I$ relies on the AGRIDS. Thus, each potential alternative x_i will be characterized by a pair of ratings $\left(R_j^F(x_i), R_j^U(x_i)\right)$, where $R_j^F(x_i)$ denotes the j-th favorableness rating and $R_j^U(x_i)$ denotes the j-th unfavorableness rating. One possible way to compute the intangible attractiveness index is to compare any alternative x_i to all other alternatives on each of the points of view. Following Rebai (1994), we define the superiority (resp. inferiority) score of alternatives x_i with respect to intangible attribute $I_j \in Q_I$ as the number of alternatives beaten by (resp. beating) x_i. Thus, for each alternative x_i, the superiority and inferiority scores with respect to the j-th favorableness rating $R_j^F(x_i)$ of intangible attribute $I_j \in Q_I$ are given by Equation (11) and Equation (12), respectively:

$$S_j^F(x_i) = \left|\left\{x_k \in L \,/ R_j^F(x_i) \succ R_j^F(x_k)\right\}\right|, \quad (11)$$

$$I_j^F(x_i) = \left|\left\{x_k \in L \,/ R_j^F(x_k) \succ R_j^F(x_i)\right\}\right|. \quad (12)$$

Analogically, the superiority and inferiority scores of alternative x_i with respect to j-th unfavorableness rating $R_j^U(x_i)$ of intangible attribute I_j are given by Equation (13) and Equation (14), respectively:

$$S_j^U(x_i) = \left|\left\{x_k \in L \,/ R_j^U(x_i) \succ R_j^U(x_k)\right\}\right|, \quad (13)$$

$$I_j^U(x_i) = \left|\left\{x_k \in L \,/ R_j^U(x_k) \succ R_j^U(x_i)\right\}\right|, \quad (14)$$

where the symbol $\succ$ stands for strict preference and | · | indicates cardinality.

The intangible attractiveness index $\rho_j(x_i)$ of alternative x_i with respect to intangible attribute can be computed using the bi-pao introduced in Section 3.2 as follows:

$$\rho_j(x_i) = \left(S_j^F(x_i), I_j^F(x_i), S_j^U(x_i), I_j^U(x_i)\right). \quad (15)$$

Example. (*Clarifying example*) let us consider six alternatives $\{x_i\}_{i=1}^{i=6}$ evaluated with respect to a given intangible attribute as shown in Figure 3.

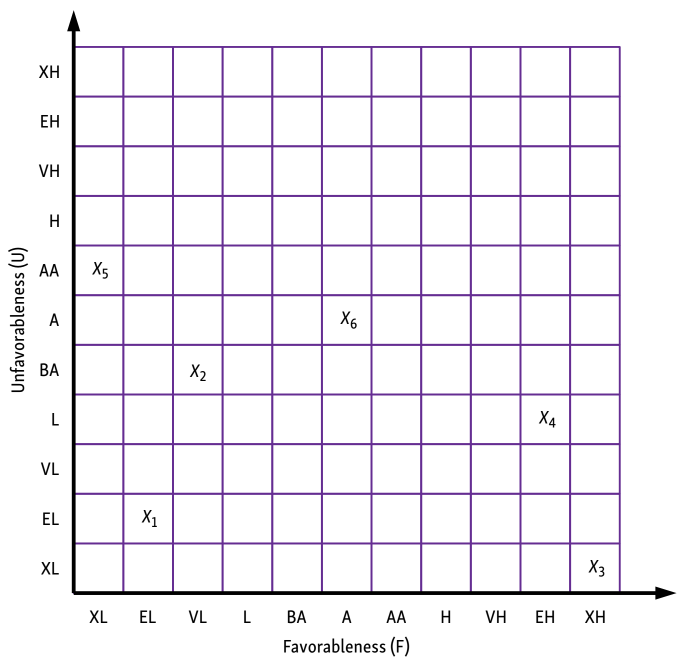

Figure 3. Evaluation of alternatives on intangible attribute

After eliciting the decision maker's judgment for each alternative using the Attractiveness Grid, each alternative is represented by two linguistic rating vectors. We then compute the superiority and inferiority scores for each alternative by comparing it with all other available alternatives across each point of view, as defined in Equations (11) to (14). The results are presented in the third to sixth columns of Table 1. Next, we calculate the intangible attractiveness index for each alternative with respect to each intangible attribute using Equation (16). These results are provided in the seventh column of Table 1.

$$\rho_I\left(x_i\right)=\frac{S_I^F(x_i)+S_I^u\left(x_i\right)-I_I^F(x_i)-I_I^u(x_i)+2\left(n-1\right)}{4\left(n-1\right)}. \tag{16}$$

Table 1. Intangible attractiveness indexes

x_i	$R_I^F\left(x_i\right), R_I^U\left(x_i\right)$	$S_I^F\left(x_i\right)$	$I_I^F\left(x_i\right)$	$S_I^U\left(x_i\right)$	$I_I^U\left(x_i\right)$	$\rho_I\left(x_i\right)$
x_1	(s_2, s_2)	1	4	4	1	0.5
x_2	(s_3, s_5)	2	3	2	3	0.4
x_3	(s_{11}, s_1)	5	0	5	0	1
x_4	(s_{10}, s_4)	4	1	3	2	0.7
x_5	(s_1, s_7)	0	5	0	5	0
x_6	(s_6, s_6)	3	2	1	4	0.7

4.3.2. Tangible attractiveness index

Let $T_j \in Q_T$ be a tangible attribute and $f_j\left(x_i\right)$ be the score of alternative x_i with respect to T_i. To avoid scaling issues, scores $f_j\left(x_i\right)$ $\left(T_j \in Q_T, i=1,\cdots,n\right)$ need to be normalized using a linear max transformation. Thus, the tangible attractiveness index $\gamma_j\left(x_i\right)$ of alternative x_i with respect to tangible attribute T_i is given by:

$$\gamma_j\left(x_i\right)=\begin{cases}\dfrac{f_{ij}}{\max_k f_j\left(x_k\right)}, & \text{for a benefit attribute}\\[2ex] 1-\dfrac{f_{ij}}{\max_k f_j\left(x_k\right)}, & \text{for a cost attribute.}\end{cases} \tag{17}$$

4.4. Ranking of alternatives on the basis of a dominance measure

To rank order the alternatives, GAMETE needs to compute a global overall score for all the alternatives. Generally, MADM methods use a pre-defined set of relative weights for the attributes in order to combine partial evaluations (i.e., with respect to a single attribute) into a global evaluation. Attribute weights are important parameters in decision-making problems because they directly influence the accuracy of the final results. However, due to the increasing complexity of many real decision situations, there are often challenges for decision-makers in providing precise and complete weight information. This may arise from several factors, including time pressure, lack of data, and human cognitive limitations. The decision-makers are not confident in providing exact value weights to all attributes. Thus, GAMETE assumes that information about attribute weights is either partially or incompletely known. This means that the decision maker provides only some linear relations to express incomplete information about the weights, such as ranking, interval description, and so on.

There are five basic forms of incomplete weights information (IWI) specification (Souissi & Hnich, 2022).

- Form 1. Weak ranking: $w_1 \geq \cdots \geq w_m$.
- Form 2. Strict ranking: $w_j - w_{j+1} \geq \alpha_j > 0$ for $j = 1$ to $m-1$.
- Form 3. Interval form: $\alpha_j \leq w_j \leq \alpha_j + \varepsilon_j$ for $j = 1$ to m.
- Form 4. Ranking with multiples: $w_j \geq \alpha_j w_{j+1}$ for $j = 1$ to $m-1$.
- Form 5. Ranking differences of adjacent weights: $w_1 - w_2 \geq w_2 - w_3 \geq \cdots \geq w_m - w_{m+1}$, with $w_{m+1} = 0$.

Forms 1–2 and 4–5 are well known types of IWI, whereas form 3 is a ranking of differences of adjacent weights obtained by ranking between two parameters that can be constructed based on form 1.

In the situations where the information on attribute weights provided by the decision makers is incomplete. We formulate two linear programming problems P1 and P2 to check the dominance relation between each pair of alternatives based on the bounded-difference operation. These are the following:

$$(P1) = \begin{cases} \min(\max) Z_1(Z_2) = \sum_{I_j \in Q_I} w_j \left[\rho_j(x_i) \ominus \rho_j(x_k)\right] + \sum_{T_j \in Q_T} w_j \left[\gamma_j(x_i) \ominus \gamma_j(x_k)\right] \\ \text{Subject to} \\ w = (w_1, \cdots, w_m) \in W \\ \sum_{j=1}^{m} w_j = 1 \\ w_j \in [0,1]; \end{cases} \tag{18}$$

$$(P2) = \begin{cases} \min(\max) Z_3(Z_4) = \sum_{I_j \in Q_I} w_j \left[\rho_j(x_k) \ominus \rho_j(x_i)\right] + \sum_{T_j \in Q_T} w_j \left[\gamma_j(x_k) \ominus \gamma_j(x_i)\right] \\ \text{Subject to} \\ w = (w_1, \cdots, w_m) \in W \\ \sum_{j=1}^{m} w_j = 1 \\ w_j \in [0,1], \end{cases} \tag{19}$$

where W stands for the feasible weight space defined based on the incomplete weight information provided by the user.

By solving the linear programming problems P(1) and P(2) for n (n – 1) ordered pairs of alternatives, we can assign to each pair of alternatives (x_i, x_k) with $i < k$, an interval of the form $ID(x_i, x_k) = \left[\underline{D}(x_i, x_k), \bar{D}(x_i, x_k)\right]$ such that: $\underline{D}(x_i, x_k) = Z_1^*$ and $\bar{D}(x_i, x_k) = Z_2^*$ with Z_1^* (resp. Z_2^*) is the optimal solution for the min (resp. max) version of linear program (P1).

Similarity, we assign to each pair of alternatives $\left(x_k, x_i\right)$ with $i > k$, an interval of the form $ID\left(x_k, x_i\right) = \left[\underline{D}\left(x_k, x_i\right), \bar{D}\left(x_k, x_i\right)\right]$ such that $\underline{D}\left(x_k, x_i\right) = Z_3^*$ and $\bar{D}\left(x_k, x_i\right) = Z_4^*$ with Z_3^* (resp. Z_4^*) is the optimal solution for the min (resp. max) version of linear program (P2).

The pairwise dominance matrix can then be defined using the intervals $ID\left(x_i, x_k\right)$ and $ID\left(x_k, x_i\right)$ as entries as follows:

$$ID = \begin{pmatrix} - & ID_{12} & \cdots & ID_{1(n-1)} & ID_{1n} \\ ID_{21} & - & \cdots & ID_{2(n-1)} & ID_{2n} \\ ID_{31} & ID_{32} & - & ID_{3(n-1)} & ID_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ ID_{n1} & ID_{n2} & \cdots & ID_{n(n-1)} & - \end{pmatrix}.$$

We notice that if $\underline{D}\left(x_k, x_i\right) > \bar{D}\left(x_i, x_k\right)$, then alternative x_k will be said to strictly dominate x_i while if $\underline{D}\left(x_k, x_i\right) > \underline{D}\left(x_i, x_k\right)$ and $\bar{D}\left(x_k, x_i\right) > \bar{D}\left(x_i, x_k\right)$, then alternative x_k will be said to weakly dominate x_i . If it is not possible to differentiate the alternatives on this basis, a dominance intensity measure is proposed to calculate the preference intensities. The dominance intensity measure is defined as follows (Li et al., 2018):

$$DI(x_i, x_k) = \max\left\{1 - \max\left\{\frac{\bar{D}\left(x_k, x_i\right) - \underline{D}\left(x_i, x_k\right)}{\left(\bar{D}\left(x_k, x_i\right) - \underline{D}\left(x_k, x_i\right)\right) + \left(\bar{D}\left(x_i, x_k\right) - \underline{D}\left(x_i, x_k\right)\right)}, 0\right\}, 0\right\}. \quad (20)$$

Afterwards, we can establish a Boolean matrix $B = \left(b_{ij}\right)_{m \times m}$ with following entries of B is defined as follows:

$$b_{ij} = \begin{cases} 1 & si\ DI(x_i, x_k) \geq 0.5 \\ 0 & si\ DI(x_i, x_k) < 0.5 \end{cases}. \quad (21)$$

Then, the global score $\pi\left(x_i\right)$ of alternative x_i can then be calculated simply by summing up the entries b_{ij} in i-th row of matrix B:

$$\pi\left(x_i\right) = \sum_{i=1}^{m} b_{ij} \ . \quad (22)$$

At last, we can rank the potential alternative according to the decreasing order of $\pi\left(x_i\right)$, where the most suitable potential location with the maximum value among all $\pi\left(x_i\right)$ values.

4.5. The GAMETE method steps

GAMETE is structured into five steps, as shown in Figure 4. The algorithm begins by evaluating each alternative x_i with respect to both tangible and intangible attributes. This is done by computing the measures $\rho_j\left(x_i\right)$ for each x_i using Equations (16) and (17), respectively (Step 1). Once the individual attribute scores are determined, the algorithm proceeds with pairwise dominance analysis (Step 2). For each pair of alternatives x_i, it assesses their relative strength by solving optimization problems that yield the lower and upper dominance bounds: $\underline{D}\left(x_i, x_k\right)$, and $\bar{D}(x_i, x_k)$ for $k > i$ and $\underline{D}\left(x_k, x_i\right)$ and $\bar{D}\left(x_k, x_i\right)$ for $i > k$. These values quantify the extent to which one alternative dominates another. Next, the dominance intensity measure $DI(x_i, x_k)$ is computed to quantify the strength of dominance between alternatives (Step 3).

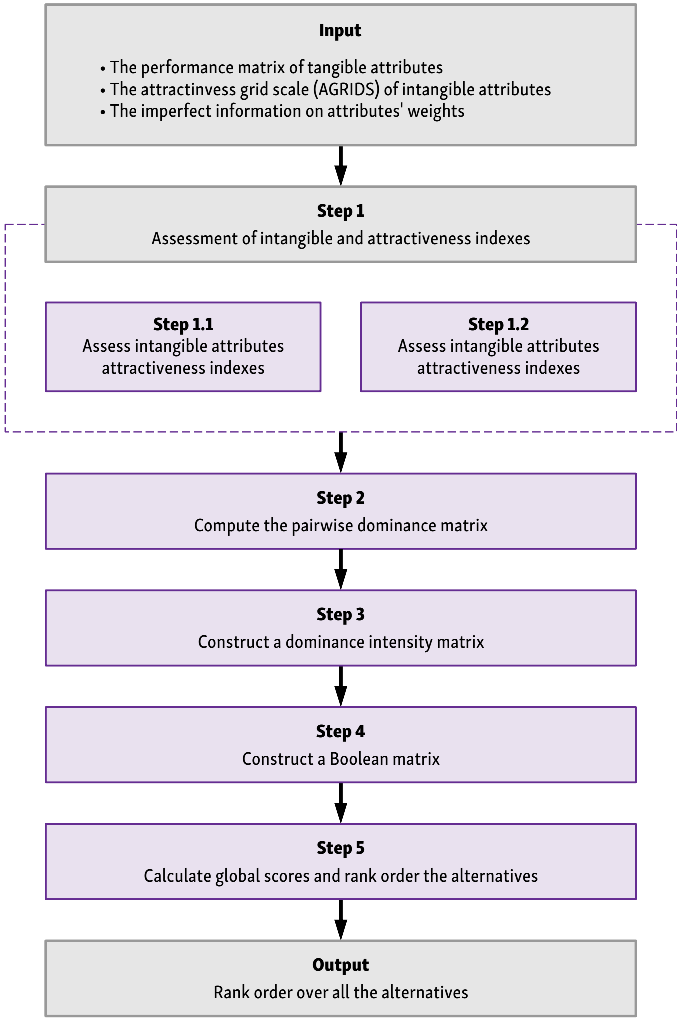

Figure 4. GAMETE five steps process

A Boolean matrix B is then constructed according to Equation (22), representing the pairwise comparisons (Step 4). Finally, the algorithm computes a global score $\pi(x_i)$ for each alternative x_i by aggregating the dominance relationships into a final ranking (Step 5). The alternatives are then sorted in descending order based on their global scores.

4.6. Illustrative application

A logistics center is a specific area that acts as the core of a specific area within which all activities related to logistics, such as distribution, storage, transportation, consolidation, handling, customs clearance, imports, exports, transit processes, infrastructural services, insurance, and banking, occur (Moroza & Jurgelane-Kaldava, 2019). A wide range of names has been used to refer to different versions of logistics centers. These include distribution center, freight village, dry port, inland port, load center, logistics node, gateway, central warehouse, freight/transport terminal, transport node, logistics platform, logistics depot, or Distri-park (Yazdani et al., 2018).

The logistic center location selection is one of the most critical issues to affect organizations survival, performance and competitiveness in the long term. It consists basically of determining the most suitable placement of infrastructural components in a considered area where a firm can perform its logistics, production, and procurement functions, keep its inventories, and sustain its economic objectives. The optimal logistic center location assures a good transport system performing in logistics activities and bringing benefit not only to the service quality but also to the company's competitiveness. The goal is to identify the location that operates at minimal cost and maximum efficiency while meeting operational and strategic requirements (Pamucar et al., 2018).

The location decision is critical for both short- and long-term planning, as it involves substantial costs and is difficult to reverse. Thus, the decision maker must select the location for a logistics center that will not only deliver high performance but also be flexible enough to accommodate the necessary future change. A bad choice of location might result in excessive transportation costs, a loss of competitive advantage, inadequate supplies of raw materials, or some similar conditions that would be detrimental to the operations. The evaluation and selection of a suitable logistics center location is a critical decision involving complexity due to the existence of numerous intangible and tangible attributes. Hence, it is considered as a multi-attribute decision making problem.

In this Section, we provide an illustrative application for logistic center location selection by using the proposed approach. There are six alternatives $\{x_i\}_{i=1}^{i=6}$ evaluated in terms of nine attributes including: size (q_1), investment cost (q_2) , proximity to industrial zone (q_3), proximity to airport (q_4), proximity to railroad system (q_5), proximity to harbor (q_6), safety and security (q_7), social attractiveness (q_8), and environmental friendliness (q_9). Attributes $\{q_j\}_{j=1}^{j=6}$ are tangible while attributes $\{q_j\}_{j=7}^{j=9}$ are intangible. In this problem, the attribute weights are ranked-ordered as follows: $w_2 > w_3 > w_8 > w_9 > w_5 > w_4 > w_7 > w_1 > w_6$.

4.6.1. Specification of input data

The initial scores of all tangible attributes (q_1) to (q_6) are shown in Table 2, along with the preference directions of these attributes.

Table 2. Initial scores of tangible attributes

Alternatives	q_1 (max)	q_2 (min)	q_3 (min)	q_4 (min)	q_5 (min)	q_6 (min)
x_1	38	100	18	21	23	22
x_2	47	200	2	12	4	1
x_3	47	900	3.4	10	1.3	1
x_4	30	100	6	29	4	4
x_5	100	300	6	55	59	47
x_6	100	300	0.35	65	141	3

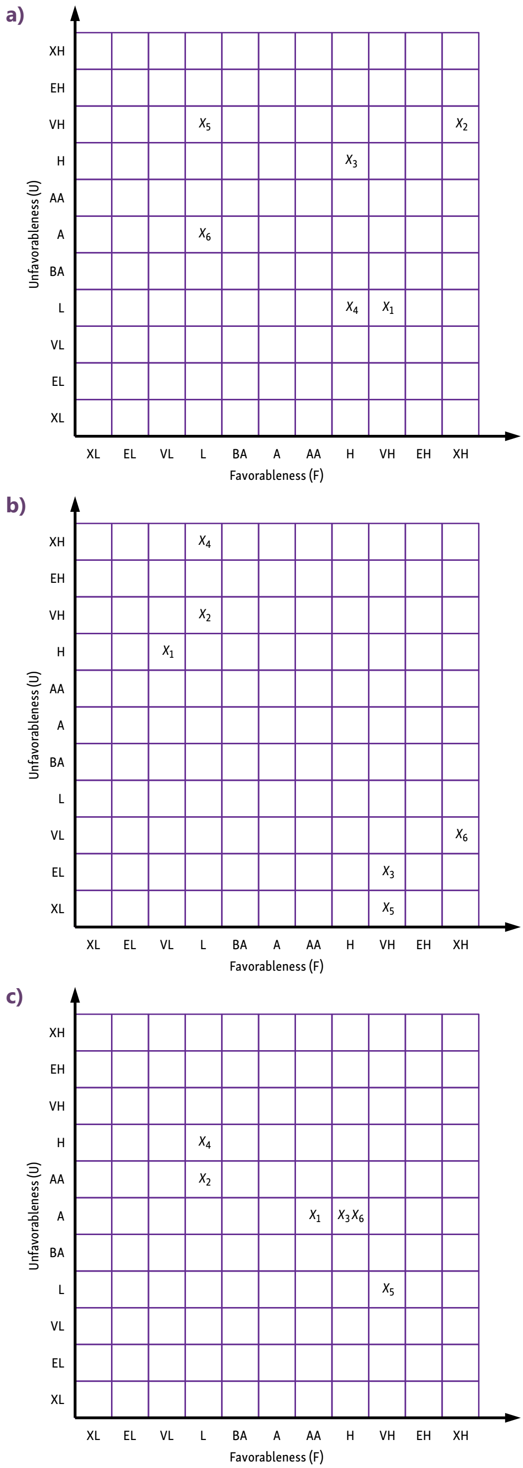

Figure 5. Attractiveness ratings with respect to intangible attributes

The attractiveness ratings with respect to intangible attributes Safety and Security (q_7), Social Attractiveness (q_8), and Environmental Friendliness (q_9) are given in the AGRIDSs shown in Figure 5a to 5c, respectively.

4.6.2. Application of GAMETE

The computational procedure of the proposed GAMETE using the data above is summarized as follows.

Step 1. Assessment of intangible and attractiveness indexes. The first step of GAMETE consists of calculating the attractiveness indexes for intangible and tangible attributes. The attractiveness indexes with respect to intangible attributes Safety and Security (q_7), Social attractiveness (q_8), and Environmental friendliness (q_9) are summarized in Tables 3–5, respectively. The second columns in these tables are directly extracted from the attractiveness ratings in Figure 5. The third to sixth columns in Tables 3–5, have been computed using Equations (11)–(14), respectively. The last columns in Tables 3–5 correspond to the attractiveness indexes of the alternatives computed through Equation (16).

Table 3. Intangible attractiveness indexes w.r.t. Safety and Security (q_7)

Alternatives	Safety and Security					
x_i	$\left(R_7^F(x_i), R_7^U(x_i)\right)$	$S_7^F(x_i)$	$I_7^F(x_i)$	$S_7^U(x_i)$	$I_7^U(x_i)$	$\rho_7(x_i)$
x_1	(s_9, s_4)	4	1	4	0	0.85
x_2	(s_{11}, s_9)	5	0	0	4	0.55
x_3	(s_8, s_8)	2	2	3	2	0.55
x_4	(s_8, s_4)	2	2	4	0	0.7
x_5	(s_4, s_9)	0	4	0	4	0.1
x_6	(s_4, s_6)	0	4	3	2	0.35

Table 4. Intangible attractiveness indexes w.r.t. Social Attractiveness (q_8)

Alternatives	Social Attractiveness					
x_i	$\left(R_8^F(x_i), R_8^U(x_i)\right)$	$S_8^F(x_i)$	$I_8^F(x_i)$	$S_8^U(x_i)$	$I_8^U(x_i)$	$\rho_8(x_i)$
x_1	(s_3, s_8)	0	5	2	3	0.2
x_2	(s_4, s_9)	1	3	0	5	0.15
x_3	(s_8, s_2)	3	2	4	1	0.7
x_4	(s_4, s_{11})	1	3	0	5	0.15
x_5	(s_9, s_1)	4	1	5	0	0.9
x_6	(s_{11}, s_3)	5	0	3	2	0.8

Table 5. Intangible attractiveness indexes w.r.t. Environmental Friendliness (q_9)

Alternatives	Environmental Friendliness					
x_i	$\left(R_9^F\left(x_i\right), R_9^U\left(x_i\right)\right)$	$S_9^F\left(x_i\right)$	$I_9^F\left(x_i\right)$	$S_9^U\left(x_i\right)$	$I_9^U\left(x_i\right)$	$\rho_9\left(x_i\right)$
x_1	(s_7, s_6)	2	3	2	1	0.5
x_2	(s_4, s_7)	0	4	1	4	0.15
x_3	(s_8, s_6)	3	1	2	1	0.65
x_4	(s_4, s_8)	0	4	0	5	0.05
x_5	(s_9, s_4)	5	0	4	1	0.9
x_6	(s_8, s_6)	3	1	2	1	0.65

Now, we compute the tangible attractiveness index by the normalization procedure linear max are provided in Table 6.

Table 6. Tangible attractiveness index

Alternatives	Tangible attributes					
x_i	q_1	q_2	q_3	q_4	q_5	q_6
x_1	0.35	0.889	0	0.677	0.837	0.532
x_2	0.47	0.778	0.889	0.815	0.972	0.979
x_3	0.47	0	0.811	0.846	0.991	0.979
x_4	0.3	0.889	0.667	0.554	0.972	0.915
x_5	1	0.667	0.667	0.154	0.582	0
x_6	1	0.667	0.981	0	0	0.936

The results of the Intangible and Tangible attractiveness indexes are summarized in Table 7 below.

Table 7. The attribute attractiveness indexes

Alternatives	Tangible attributes						Intangible attributes		
x_i	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9
x_1	0.35	0.889	0	0.677	0.837	0.532	0.85	0.2	0.5
x_2	0.47	0.778	0.889	0.815	0.972	0.979	0.55	0.15	0.15
x_3	0.47	0	0.811	0.846	0.991	0.979	0.55	0.7	0.65
x_4	0.3	0.889	0.667	0.554	0.972	0.915	0.7	0.15	0.05
x_5	1	0.667	0.667	0.154	0.582	0	0.1	0.9	0.9
x_6	1	0.667	0.981	0	0	0.936	0.35	0.8	0.65

Step 2. Compute the pairwise dominance matrix. The computation of the pairwise dominance matrix is the core step of GAMETE. This matrix is obtained by solving the linear programs (P1) and (P2) using the attractiveness indexes in Table 7. The obtained pairwise dominance matrix is given in Table 8. This Table 8 shows for each pair of alternatives $\left(x_i, x_k\right)$

the dominated and dominating degrees of the form $\left[\underline{D}\left(x_i,x_k\right),\bar{D}\left(x_i,x_k\right)\right]$ for $i<k$ and of the form $[\underline{D}\left(x_k,x_i\right),\bar{D}\left(x_k,x_i\right)]$ for $i>k$. The bounds of the first interval are optimal solutions of the min and the max versions of (P1), respectively; while the bounds of the second interval are the optimal solutions of the min and max versions of (P2), respectively.

Table 8. The pairwise dominance matrix

Alternatives	x_1	x_2	x_3	x_4	x_5	x_6
x_1	–	[0.07 0.117]	[0.162 0.614]	[0.032 0.113]	[0.094 0.221]	[0.117 0.286]
x_2	[0.073 0.357]	–	[0.128 0.565]	[0.03 0.101]	[0.122 0.269]	[0.094 0.279]
x_3	[0.107 0.386]	[0.06 0.228]	–	[0.084 0.291]	[0.056 0.246]	[0.069 0.265]
x_4	[0.052 0.265]	[0.024 0.083]	[0.148 0.643]	–	[0.093 0.237]	[0.115 0.272]
x_5	[0.115 0.407]	[0.088 0.328]	[0.185 0.51]	[0.945 0.351]	–	[0.046 0.166]
x_6	[0.116 0.453]	[0.076 0.274]	[0.162 0.503]	[0.098 0.348]	[0.027 0.134]	–

For instance, the dominance relations between alternatives x_1 and x_2 is formulated as follows:

- The dominated degrees $ID\left(x_2,x_1\right)$ is defined by:

$ID\left(x_1,x_2\right)=\min(\max)Z_1\left(Z_2\right)=0.111w_2+0.3\,w_7+0.05w_8+0.35w_9$.

Subject to:

$w_2-w_3\geq 0.01$; w_3 - $w_8\geq 0.01$; $w_8-w_9\geq 0.01$; $w_9-w_5\geq 0.01$; $w_5-w_4\geq 0.01$;

$w_4-w_7\geq 0.01$; $w_7-w_4\geq 0.01$; $w_4-w_6\geq 0.01$; $w_6>0$;

$w_1+w_2+w_3+w_4+w_5+w_6+w_7+w_8+w_9=1$.

- The dominating degrees $ID\left(x_2,x_1\right)$ is defined by:

$ID\left(x_2,x_1\right)=\min(\max)Z_3\left(Z_4\right)=0.12w_1+0.889\,w_3+0.138w_4+0.135w_5+0.447w_6$.

Subject to:

$w_2-w_3\geq 0.01$; w_3 - $w_8\geq 0.01$; $w_8-w_9\geq 0.01$; $w_9-w_5\geq 0.01$; $w_5-w_4\geq 0.01$;

$w_4-w_7\geq 0.01$; $w_7-w_4\geq 0.01$; $w_4-w_6\geq 0.01$; $w_6>0$;

$w_1+w_2+w_3+w_4+w_5+w_6+w_7+w_8+w_9=1$.

Step 3. Construct a dominance intensity matrix. The dominance intensity is computed by applying Equation (21). The calculations have resulted in the data shown in Table 9.

Table 9. Dominance intensity

Alternatives	x_1	x_2	x_3	x_4	x_5	x_6
x_1	0.5	0	1	0	0	0.336
x_2	1	0.5	1	1	0.468	1
x_3	0	0	0.5	0	0	0
x_4	1	0	1	0.5	0	0.427
x_5	1	0.532	1	1	0.5	1

Step 4. Construct a Boolean matrix. The Boolean matrix is computed using Equation (22). The Boolean matrix for the considered example is given in Table 10.

Table 10. Boolean matrix

Alternatives	x_1	x_2	x_3	x_4	x_5	x_6
x_1	0.5	0	1	0	0	0
x_2	1	0.5	1	1	0	1
x_3	0	0	0.5	0	0	0
x_4	1	0	1	0.5	0	0
x_5	1	1	1	1	0.5	1
x_6	1	0	1	1	0	0.5

Step 5. Calculate global scores and rank order the alternatives. GAMETE then calculates the global scores $\pi(.)$ for all alternative by applying Equation (23) using the data in the Boolean matrix. The result of this step is summarized in Table 11. According to the global scores, the final ranking of alternative location is: $x_5 > x_2 > x_6 > x_4 > x_1 > x_3$.

Table 11. Global scores and ranks

Alternatives	Global score $\pi(x_i)$	Rank
x_1	1.5	5
x_2	4.5	2
x_3	0.5	6
x_4	2.5	4
x_5	5.5	1
x_6	3.5	3

5. Comparative analysis

This section aims to compare the prescriptions of GAMETE against some other well-known MADM methods running on Linguistic Intuitionistic fuzzy and Linguistic Pythagorean fuzzy. The comparison made show**s** the applicability and feasibility through two real case studies.

5.1. Case study 1: building materials selection problem

This first case concerns the problem of selecting sustainable indoor flooring materials (Meng & Dong, 2022). Four alternative types of sustainable indoor flooring materials are considered, namely Terrazzo flooring (x_1), Solid hardwood flooring (x_2), Luxury vinyl planks (x_3), and Ceramic tiles (x_4). The four alternatives are evaluated using 17 intangible attributes: Initial cost (q_1), Maintenance cost (q_2), Disposal cost (q_3), Energy saving and thermal insulation (q_4), Potential for recycling and reuse (q_5), Energy and water consumption (q_6), CO_2 emission and air pollution (q_7), damage to natural resources (q_8), Usage of local material (q_9), Labor availability (q_{10}), Esthetics (q_{11}), Safety and health (q_{12}), Maintainability (q_{13}), Buildability (q_{14}), Life expectancy (q_{15}), Corrosion resistance (q_{16}) and Fire resistance (q_{17}). The ranking of weights of attributes is as follows: $q_{16} \geq q_5 \geq q_1 \geq q_{17} \geq q_2 \geq q_{15} \geq q_{12} \geq q_4 \geq q_{10} \geq q_{14} \geq q_9 \geq q_6 \geq q_{13} \geq q_{11} \geq q_7 = q_8 \geq q_3$. The decision matrix for this problem is displayed in Table 12.

Table 12. Ratings of alternatives

Attributes	x_1	x_2	x_3	x_4
q_1	(s_3, s_5)	(s_4, s_2)	(s_2, s_5)	(s_3, s_4)
q_2	(s_3, s_4)	(s_2, s_5)	(s_2, s_6)	(s_5, s_3)
q_3	(s_4, s_4)	(s_4, s_3)	(s_2, s_4)	(s_3, s_3)
q_4	(s_3, s_4)	(s_6, s_2)	(s_6, s_1)	(s_2, s_5)
q_5	(s_5, s_3)	(s_3, s_4)	(s_5, s_2)	(s_3, s_4)
q_6	(s_2, s_5)	(s_4, s_3)	(s_6, s_1)	(s_3, s_4)
q_7	(s_4, s_4)	(s_2, s_5)	(s_6, s_2)	(s_3, s_3)
q_8	(s_5, s_3)	(s_3, s_4)	(s_2, s_5)	(s_5, s_3)
q_9	(s_2, s_6)	(s_3, s_5)	(s_3, s_4)	(s_5, s_2)
q_{10}	(s_5, s_2)	(s_5, s_1)	(s_4, s_3)	(s_6, s_1)
q_{11}	(s_6, s_0)	(s_5, s_3)	(s_6, s_1)	(s_5, s_3)
q_{12}	(s_4, s_4)	(s_5, s_2)	(s_5, s_3)	(s_3, s_4)
q_{13}	(s_5, s_2)	(s_6, s_2)	(s_2, s_6)	(s_5, s_1)
q_{14}	(s_2, s_6)	(s_4, s_3)	(s_3, s_5)	(s_5, s_2)
q_{15}	(s_6, s_1)	(s_4, s_3)	(s_4, s_4)	(s_6, s_2)
q_{16}	(s_5, s_1)	(s_5, s_2)	(s_3, s_4)	(s_3, s_5)
q_{17}	(s_7, s_1)	(s_4, s_2)	(s_1, s_5)	(s_2, s_5)

5.1.1. Application of GAMETE

The results of the application of Step 1 to Step 4 are summarized in Tables 13–16. These Tables 13–16 show the attractiveness indexes, pairwise dominance matric, the dominance intensity matrix and the Boolean matrix.

Table 13. Attractiveness indexes

Attributes	x_1	x_2	x_3	x_4
q_1	0.333	1,000	0,083	0,583
q_2	0.667	0,250	0,083	1.000
q_3	0.500	0.833	0.083	0.750
q_4	0.333	0.750	0.917	–
q_5	0.750	0.167	0.917	0.167
q_6	–	0.667	1.000	0.333
q_7	0.500	–	1.000	0.500
q_8	0.833	0.333	–	0.833
q_9	–	0.417	0.583	1.000
q_{10}	0.417	0.667	–	0.916
q_{11}	0.917	0.333	0.750	–
q_{12}	0.417	0.917	0.750	0.083
q_{13}	0.500	0.750	–	0.750
q_{14}	–	0.667	0.333	1.000
q_{15}	0.917	0.250	0.083	0.750
q_{16}	0.917	0.750	0.250	0.083
q_{17}	1.000	0.667	0.083	0.250

Table 14. Pairwise dominance matrix

Alter.	x_1	x_2	x_3	x_4
x_1	–	$[0.174; 0.358]$	$[0.313; 0.63]$	$[0.241; 0.771]$
x_2	$[0.028; 0.25]$	–	$[0.26; 0.48]$	$[0.234; 0.622]$
x_3	$[0.0174; 0.212]$	$[0.016; 0.345]$	–	$[0.165; 0.434]$
x_4	$[0.023; 0.273]$	$[0.021; 0.214]$	$[0.041; 0.38]$	–

Table 15. Dominance intensity matrix

Alter.	x_1	x_2	x_3	x_4
x_1	–	0.813	1	0.95
x_2	0.187	–	0.845	1
x_3	0	0.155	–	0.646
x_4	0.04	0	0.354	–

Table 16. Boolean matrix

Alter.	x_1	x_2	x_3	x_4	$\pi(x_i)$
x_1	–	1	1	1	3
x_2	0	–	1	1	2
x_3	0	0	–	1	1
x_4	0	0	0	–	0

The Boolean matrix in Table 16 is used to compute the global scores, which are shown in Table 17 along with ranks of all alternatives. According to these global scores, the rank of alternatives is as follows: $x_1 > x_2 > x_3 > x_4$. This means that Terrazzo flooring (x_1) is the best sustainable indoor flooring materials to be used.

Table 17. Global scores and ranks

Alter.	$\pi(x_i)$	Rank
x_1	3	3
x_2	2	2
x_3	1	1
x_4	0	0

5.1.2. Analysis of results

To assess the effectiveness of GAMETE, we used the data in Section 4.2 to run the following methods: linguistic intuitionistic fuzzy Weighted Averaging (LIF-WA) method (Chen et al., 2015), LIF-TOPSIS (Ou et al., 2018), linguistic intuitionistic fuzzy Choquet-an acronym in Portuguese for iterative multi-criteria decision making (LIF-CTODIM) method (Liu & Shen, 2019), and linguistic intuitionistic fuzzy Preference Ranking Organization Method for Enrichment Evaluation (LIF-PROMETHE) (Meng & Dong, 2022), The obtained ranking and best solution for all these methods and GAMETE are shown in Table 18.

Table 18. Obtained ranking and best solution

Method	Reference	Rank order	Best alternative
LIF-WA	Chen et al. (2015)	$x_1 > x_2 > x_4 > x_3$	x_1
LIF-TOPSIS	Ou et al. (2018)	$x_1 > x_2 > x_4 > x_3$	x_1
LIF-CTODIM	Liu and Shen (2019)	$x_2 > x_1 > x_4 > x_3$	x_2
LIF-PROMETHEE	Meng and Dong (2022)	$x_2 > x_1 > x_4 > x_3$	x_2
GAMETTE	The current study	$x_1 > x_2 > x_4 > x_3$	x_1

It is noteworthy that in this illustrative application, GAMETE, LIF-WA, LIF-TOPSIS and LPF-TOPSIS produce the same ranking of the alternative. Besides, alternative x_1 is ranked firstly by GAMETE, LIF-WA, and LIF-TOPSIS. At the same time, x_1 is ranked second by LIF-CTODIM and LIF-PROMETHEE. In addition, LIF-CTODIM and LIF-PROMETHHE produce the same ranking and prescribe alternative x_2 as the preferred choice. In fact, the produced ranking which differ only in the positions assigned to alternative x_1 and x_2. However, all the MADM models favored x_1 and x_2 as the dominant alternatives. Thus, the comparison results demonstrate that the GAMETE method performs well.

5.2. Case study 2: Technology security evaluation of computers systems

The second case study, extracted from Lin et al. (2019), is about the ranking of technology security evaluation of computers systems. In this case study, GAMETE is compared to recent LPFs methods including: LPF-weighted averaging (LPF-WA) method (Garg, 2018), LPF-Weighted Geometric (LPF-WG) method (Garg, 2018), LPF-TOPSIS method (Lin et al., 2019; Han et al., 2020), LPF based on Interaction Portioned Bonferroni Mean (LPF-IPBM) method (Lin et al., 2018) and LPF based on Interaction Portioned Geometric Bonferroni Mean (LPF-IGBM) method (Lin et al., 2018). Four computer systems $\{x_i\}_{i=1}^{i=4}$ are evaluated using four intangible attributes, which are: Hardware security (q_1), System software security (q_2), Application software security (q_3) and Data security (q_4). In this case study, the ranking of weights of attribute is defined as follows $0.26 \le w_1 \le 0.29$, $0.3 \le w_2 \le 0.35$, $0.15 \le w_3 \le 0.22$, $0.23 \le w_4 \le 0.25$. The linguistic decision matrix is given in Table 19.

Table 20 depicts all the rankings prescribed by the various LPF- MADM methods being used. Noteworthy, in this illustration application, GAMETE, LPF-TOPSIS, LPF-WG, and LPF-WIPGM produce the same ranking of the alternatives. Besides this, as shown in Table 20, alternative A_2 is ranked first of all LPF-MADM methods being used, including GAMETE.

Table 19. Decision matrix for case study 2

	x_1	x_2	x_3	x_4
q_1	(S_5, S_3)	(S_7, S_1)	(S_5, S_2)	(S_6, S_2)
q_2	(S_7, S_1)	(S_7, S_3)	(S_6, S_2)	(S_6, S_2)
q_3	(S_6, S_2)	(S_5, S_1)	(S_6, S_2)	(S_6, S_1)
q_4	(S_6, S_1)	(S_6, S_2)	(S_6, S_1)	(S_5, S_1)

Table 20. Comparison results for case study 2

Method	Reference	Ranking orders	Best solution
LPF-TOPSIS	Lin et al. (2019)	$x_2 > x_1 > x_4 > x_3$	x_2
LPF-TOPSIS	Han et al. (2020)	$x_2 > x_4 > x_3 > x_1$	x_2
LPF-WG	Garg (2018)	$x_2 > x_4 > x_3 > x_1$	x_2
LPF-WA	Garg (2018)	$x_2 > x_1 > x_4 > x_3$	x_2
LPF-WIPBM	Lin et al. (2018)	$x_2 > x_1 > x_4 > x_3$	x_2
LPF-WIPGBM	Lin et al. (2018)	$x_2 > x_4 > x_1 > x_3$	x_2
GAMETE	The current study	$x_2 > x_4 > x_3 > x_1$	x_2

6. Discussion

The results obtained from both illustrative applications demonstrate that the application of GAMETE ensures sound and defensible solutions to MADM decision problems. In our view, the GAMETE method offers several clear advantages over the LIF-MADM and LPF-MADM approaches. First, GAMETE avoids the restrictive assumptions on membership and non-membership degrees, allowing decision makers to evaluate positive and negative information independently. It is particularly adept at handling conflicting positive and negative information, which is a common characteristic of real-world decision problems. Furthermore, GAMETE is sufficiently flexible to address practical MADM scenarios involving both tangible and intangible attributes, as well as evaluatively positive and negative information regarding the alternatives. The integration of the Attractiveness Grid scale to assess intangible attributes adds predictive validity to the evaluation process by distinguishing among four types of evaluative reactions: indifference, ambivalence, positive attitude, and negative attitude. This scale is particularly well-suited for evaluating both positive and negative aspects concurrently, and it reflects key psychological factors underlying human behavior. Finally, the ranking of alternatives, based on dominance relations, dominance intensity, and a Boolean matrix, ensures the selection of the most appropriate alternative.

We acknowledge that, compared to some traditional MADM methods, GAMETE may require relatively higher computational effort due to its formulation and optimization steps. However, several key points help mitigate this concern. First, GAMETE eliminates the need for explicit preference parameters, which are often required in other MCDM methods – such as membership functions in fuzzy methods or linguistic scales in PFS/IFS approaches. By employing a simplified scoring mechanism, GAMETE reduces the cognitive burden on decision-makers. Second, it avoids the need for precise and often subjective elicitation of decision parameters, a common source of complexity in comparable methods. Third, although the optimization component may initially appear demanding, it is designed to be efficiently solvable using standard mathematical programming solvers, ensuring that practical implementation does not present a significant barrier. Finally, the slightly higher computational effort is a reasonable trade-off for the enhanced robustness, flexibility, and reduced subjectivity that the method offers.

7. Conclusions and future work

This research work aims at developing a new Multiple Attribute Decision Making (MADM) method named Bivariate Grid scale based on a multiple attribute evaluation technique (GAMETE). This method pioneers the use of an Attractiveness GRID scale (AGRIDS). It aims to assess the intangible attributes, taking into consideration cognitive psychological imperatives with reference to the separability and independence of positive and negative aspects. Additionally, the authors use a new bi-dimensional positional advantage operator (bi-pao) with the intention of calculating the intangible attractiveness index. Further, linear programming models are formulated in order to construct the pairwise dominance matrix. Afterwards, we rank alternatives, using a dominance intensity measure and the Boolean matrix. Furthermore, we illustrate the applicability of the method proposed through a logistics center location problem. Eventually, we perform a comparison with several state-of-the-art LIF-MADMs and LPF-MADM with the aim of showing the applicability and feasibility of the approach suggested.

GAMETE has three main shortcomings despite its innovative features. To start with, it lacks an effective validation with a real-life decision problem, and it uses primary data. Secondly, the computational requirements of GAMETE increase rapidly with the number of alternatives and/or attributes. The computational issues arise from the pairwise comparisons necessary to assess decision alternatives. They also arise from the need to solve $2n(n-1)$ linear programs (where n is the number of alternatives) to compute the pairwise dominance matrix. Eventually, GAMETE is a failure in accounting for the interaction relationships between attributes.

Future research avenues are directly linked to these limitations. In the short term, we intend to develop a user-friendly decision tool supporting GAMETE in order to facilitate its application in real-life decision problems. In the medium and long terms, we plan to investigate the use of a reference point-based measure to reduce the computational demands of GAMETE. We also plan to design an extended version of GAMETE that takes into account the interaction relationships among the attributes.

References

Ali, Z., Hayat, K., & Pamucar, D. (2024). Analysis of coupling in geographic information systems based on WASPAS method for bipolar complex fuzzy linguistic Aczel-Alsina power aggregation operators. *PLoS ONE*, *19*(9), Article e0309900. https://doi.org/10.1371/journal.pone.0309900

Al-Sharqi, F., Al-Quran, A., & Md. Rodzi, Z. (2024). Multi-attribute group decision-making based on aggregation operator and score function of bipolar neutrosophic hypersoft environment. *Neutrosophic Sets and Systems*, *61*, Article 25. https://digitalrepository.unm.edu/nss_journal/vol61/iss1/25

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

Ayyildiz, E. (2022). A novel Pythagorean fuzzy multi-criteria decision-making methodology for e-scooter charging station location-selection. *Transportation Research Part D*: *Transport and Environment*, *111*, Article 103459. https://doi.org/10.1016/j.trd.2022.103459

Cacioppo, J. T., & Berntson, G. G. (1994). Relationship between attitudes and evaluative space: A critical review with emphasis on the separability of positive and negative substrates. *Psychological Bulletin*, *115*(3), 401–423. https://doi.org/10.1037/0033-2909.115.3.401

Cacioppo, J. T., Berntson, G. G., Norris, C. J., & Gollan, J. K. (2012). The evaluative space model. In P. A. M. Van Lange, A. W. Kruglanski, & E. T. Higgins (Eds.), *Handbook of theories of social psychology* (Vol. 1, pp. 50–73). Sage Publications. https://doi.org/10.4135/9781446249215.n4

Çalış Boyacı, A., & Şişman, A. (2024). Location selection for waste disposal boxes: A geographic information systems based Pythagorean fuzzy multi-criteria decision analysis. *International Journal of Environmental Science and Technology*, *21*, 8577–8592. https://doi.org/10.1007/s13762-024-05573-0

Campagner, A., & Ciucci, D. (2017). Measuring uncertainty in orthopairs. In A. Antonucci, L. Cholvy, & Papini, O. (Eds.), *Lecture notes in computer science*: *Vol. 10369*. *Symbolic and quantitative approaches to reasoning with uncertainty*. *ECSQARU 2017* (pp. 423–432). Springer. https://doi.org/10.1007/978-3-319-61581-3_38

Chakraborty, S., Raut, R. D., Rofin, T. M., & Chakraborty, S. (2024). On solving a healthcare supplier selection problem using MCDM methods in intuitionistic fuzzy environment. *OPSEARCH*, *61*, 680–708. https://doi.org/10.1007/s12597-023-00733-1

Chen, Z., Liu, P., & Pei, Z. (2015). An approach to multiple attribute group decision making based on linguistic intuitionistic fuzzy numbers. *International Journal of Computational Intelligence Systems*, *8*(4), 747–760. https://doi.org/10.1080/18756891.2015.1061394

Demir, G., & Ulusoy, E. I. (2024). Wind power plant location selection with fuzzy logic and multi-criteria decision-making methods. *Computer and Decision Making*: *An International Journal*, *1,* 211–234. https://doi.org/10.59543/comdem.v1i.10713

Ecer, F., & Hashemkhani Zolfani, S. (2022). Evaluating economic freedom via a multi-criteria MEREC-DNMA model-based composite system: Case of OPEC countries. *Technological and Economic Development of Economy*, *28*(4), 1158–1181. https://doi.org/10.3846/tede.2022.17152

Filip, F. G., Zamfirescu, C.-B., & Ciurea, C. (2017). *Computer-supported collaborative decision-making*. Springer. https://doi.org/10.1007/978-3-319-47221-8

Franco, C., Rodríguez, J. T., & Montero, J. (2015). Building the meaning of preference from logical paired structures. *Knowledge-Based Systems*, *83*, 32–41. https://doi.org/10.1016/j.knosys.2015.03.005

Franco, C., Rodríguez, J. T., & Montero, J. (2017). Learning preferences from paired opposite-based semantics. *International Journal of Approximate Reasoning*, *86*, 80–91. https://doi.org/10.1016/j.ijar.2017.04.010

Garg, H. (2018). Linguistic Pythagorean fuzzy sets and its applications in multi-attribute decision-making process. *International Journal of Intelligent Systems*, *33*(6), 1234–1263. https://doi.org/10.1002/int.21979

Han, Q., Li, W., Lu, Y., Zheng, M., Quan, W., & Song, Y. (2020). TOPSIS method based on novel entropy and distance measure for linguistic Pythagorean fuzzy sets with their application in Multiple Attribute Decision Making. *IEEE Access*, *8*, 14401–14412. https://doi.org/10.1109/ACCESS.2019.2963261

Hasheminasab, H., Hashemkhani Zolfani, S., Zavadskas, E. K., Kharrazi, M., & Skare, M. (2021). A circular economy model for fossil fuel sustainable decisions based on MADM techniques. *Economic Research – Ekonomska Istraživanja*, *35*(1), 564–582. https://doi.org/10.1080/1331677X.2021.1926305

Hashemkhani Zolfani, S., Görçün, Ö. F., & Küçükönder, H. (2021). Evaluating logistics villages in Turkey using hybrid improved fuzzy SWARA (IMF SWARA) and fuzzy MABAC techniques. *Technological and Economic Development of Economy*, *27*(6), 1582–1612. https://doi.org/10.3846/tede.2021.16004

Hezam, I. M., Mishra, A. K., Pamucar, D., Rani, P., & Mishra, A. R. (2024). Standard deviation and rank sum-based MARCOS model under intuitionistic fuzzy information for hospital site selection. *Kybernete*s, *53*(10), 3727–3753. https://doi.org/10.1108/K-01-2023-0136

Hisoğlu, S., Çömert, R., Antila, M., Aman, R., & Huovila, A. (2025). Towards solar-energy-assisted electric vehicle charging stations: A literature review on site selection with GIS and MCDM methods. *Sustainable Energy Technologies and Assessments*, *75*, Article 104193. https://doi.org/10.1016/j.seta.2025.104193

Işık, Ö., & Adalar, İ. (2025). A multi-criteria sustainability performance assessment based on the extended CRADIS method under intuitionistic fuzzy environment: A case study of Turkish non-life insurers. *Neural Computing and Applications*, *37*(5), 3317–3342. https://doi.org/10.1007/s00521-024-10803-0_

Katranci, A., Kundakci, N., & Arman, K. (2025). Fuzzy SIWEC and fuzzy RAWEC methods for sustainable waste disposal technology selection. *Spectrum of Operational Research*, *3*, 87–102. https://doi.org/10.31181/sor31202633

Larsen, J. T., Norris, C. J., McGraw, A. P., Hawkley, L. C., & Cacioppo, J. T. (2009). The evaluative space grid: A single-item measure of positivity and negativity. *Cognition and Emotion*, *23*(3), 453–480. https://doi.org/10.1080/02699930801994054

Li, D., Zeng, W., &., Yin, Q. (2018). Novel ranking method of interval numbers based on the Boolean matrix. *Soft Computing*, *22*, 4413–4122. https://doi.org/10.1007/s00500-017-2625-4

Lindquist, K. A., Satpute, A. B., Wager, T. D., Weber, J., & Barrett, L. F. (2016). The brain basis of positive and negative affect: Evidence from a meta-analysis of the human neuroimaging literature. *Cerebral Cortex*, *26*(5), 1910–1922. https://doi.org/10.1093/cercor/bhv001

Lin, M., Wei, J., Xu, Z., & Chen, R. (2018). Multiattribute group decision-making based on linguistic pythagorean fuzzy interaction partitioned Bonferroni mean aggregation operators. *Complexity*, *2018*, Article 9531064. https://doi.org/10.1155/2018/9531064

Lin, M., Huang, C., & Xu, Z. (2019). TOPSIS method based on correlation coefficient and entropy measure for linguistic Pythagorean fuzzy sets and its application to Multiple Attribute Decision Making. *Complexity*, *2019*, Article 6967390. https://doi.org/10.1155/2019/6967390

Liu, P., & Shen, M. (2019). An extended C-TODIM method with linguistic intuitionistic fuzzy numbers. *Journal of Intelligent* & *Fuzzy Systems*, *37*(3), 3615–3627. https://doi.org/10.3233/JIFS-182554

Luo, J., & Hu, M. (2023). A bipolar three-way decision model and its application in analyzing incomplete data. *International Journal of Approximate Reasoning*, *152*, 94–123. https://doi.org/10.1016/j.ijar.2022.10.011

Meng, F., &. Dong, B. (2022). Linguistic intuitionistic fuzzy PROMETHEE method based on similarity measure for the selection of sustainable building materials. *Journal of Ambient Intelligence and Humanized Computing*, *13*, 4415–4435. https://doi.org/10.1007/s12652-021-03338-y

Montero, J., Bustince, H., Franco, C., Rodriguez, J. T., Gómez, D., Pagola, M., Fernandez, J., & Barrenechea, E. (2014). *Paired structures and bipolar knowledge representation* (MSAP Working Paper Series No. 06/2014). University of Copenhagen, Department of Food and Resource Economics.

Moroza, N., & Jurgelane-Kaldava, I. (2019). Development and location of logistics centres: A systematic review of literature. *Economics and Business*, *33*, 264–272. https://doi.org/10.2478/eb-2019-0019

Nila, B., & Roy, J. (2024). Analysing the key success factors of Logistics Center 4.0 implementation using improved Pythagorean fuzzy DEMATEL method. *Arabian Journal for Science and Engineering*, *49*, 11883–11905. https://doi.org/10.1007/s13369-023-08398-0

Norris, C. J., Gollan, J., Berntson, G. G., & Cacioppo, J. T. (2010). The current status of research on the structure of evaluative space. *Biological Psychology*, *84*(3), 422–436. https://doi.org/10.1016/j.biopsycho.2010.03.011

Osgood, C. E., Suci, G. J., & Tannenbaum, P. H. (1967). *The measurement of meaning*. University of Illinois Press.

Ou, Y., Yi, L., Zou, B., & Pei, Z. (2018). The linguistic intuitionistic fuzzy set TOPSIS method for linguistic multi-criteria decision makings. *International Journal of Computational Intelligence Systems*, *11*, 120–132. https://doi.org/10.2991/ijcis.11.1.10

Pamucar, D. S., Tarle, S. P., & Parezanovic, T. (2018). New hybrid multi-criteria decision-making DEMATEL-MAIRCA model: Sustainable selection of a location for the development of multimodal logistics centre. *Economic Research-Ekonomska Istraživanja*, *31*(1), 1641–1665. https://doi.org/10.1080/1331677X.2018.1506706

Pamucar, D., Torkayesh, A. E., Deveci, M., & Simic, V. (2022). Recovery center selection for end-of-life automotive lithium-ion batteries using an integrated fuzzy WASPAS approach. *Expert Systems with Applications*, *206*, Article 117827. https://doi.org/10.1016/j.eswa.2022.117827

Patel, A., Jana, S., & Mahanta, J. (2023). Intuitionistic fuzzy EM-SWARA-TOPSIS approach based on new distance measure to assess the medical waste treatment techniques. *Applied Soft Computing*, *144*, Article 110521. https://doi.org/10.1016/j.asoc.2023.110521

Rani, P., Mishra, A. R., Mardani, A., Cavallaro, F., Štreimikienė, D., & Khan, S. A. R. (2020). Pythagorean fuzzy SWARA–VIKOR framework for performance evaluation of solar panel selection. *Sustainability*, *12*(10), Article 4278. https://doi.org/10.3390/su12104278

Rebai, A. (1994). Canonical fuzzy bags and bag fuzzy measures as a basis for MADM with mixed non cardinal data. *European Journal of Operational Research*, *78*(1), 34–48. https://doi.org/10.1016/0377-2217(94)90120-1

Rebai, A., Aouni, B., & Martel, J.-M. (2006). A multi-attribute method for choosing among potential alternatives with ordinal evaluation. *European Journal of Operational Research*, *174*(1), 360–373. https://doi.org/10.1016/j.ejor.2005.01.045

Rifle, A., Nedeljković, M., Božanić, D., Štilić, A., & Muhsen, Y. R. (2024). Evaluation of agricultural drones based on the COmpromise Ranking from Alternative SOlutions (CORASO) methodology. *Engineering Review*, *44*(4), 77–90. https://doi.org/10.30765/er.2653

Souissi, M., & Hnich, B. (2022). A safe sequential screening technique for solving multi-attribute choice problems under ranked weights. *Computational and Applied Mathematics*, *41*, Article 163. https://doi.org/10.1007/s40314-022-01843-0

Ulutaş, A., Popovic, G., Radanov, P., Stanujkic, D., & Karabasevic, D. (2021). A new hybrid fuzzy PSI-PIPRECIA-CoCoSo MCDM based approach to solving the transportation company selection problem. *Technological and Economic Development of Economy*, *27*(5), 1227–1249. https://doi.org/10.3846/tede.2021.15058

Uyanik, C., Tuzkaya, G., Kalender, Z. T., & Oguztimur, S. (2020). An integrated DEMATEL–IF-TOPSIS methodology for logistics centers' location selection problem: An application for Istanbul Metropolitan area. *Transport*, *35*, 548–556. https://doi.org/10.3846/transport.2020.12210

Wang, W.-Y., Yang, Y.-C., & Lin, C.-Y. (2022). Integrating the BWM and TOPSIS algorithm to evaluate the optimal token exchanges platform in Taiwan. *Technological and Economic Development of Economy*, *28*(2), 358–380. https://doi.org/10.3846/tede.2021.15935

Wen, Z., Liao, H., & Zavadskas, E. K (2020). MACONT: Mixed aggregation by comprehensive normalization technique for multi-criteria analysis. *Informatica*, *31*(4), 857–880. https://doi.org/10.15388/20-INFOR417

Yager, R. R., & Abbasov, A. M. (2013). Pythagorean membership grades, complex numbers, and decision making. *International Journal of Intelligent Systems*, *28*, 436–452. https://doi.org/10.1002/int.21584

Yalcin Kavus, B., Ayyildiz, E., Gülüm Taş, P., & Taşkın, A. (2023). A hybrid Bayesian BWM and Pythagorean fuzzy WASPAS-based decision-making framework for parcel locker location selection problem. *Environmental Science and Pollution Research*, *30*, 90006–90023. https://doi.org/10.1007/s11356-022-23965-y

Yazdani, M., Muñoz-Ocaña, Y., Fernández-Rodríguez, V., & Torres-Jiménez, M. (2018). Logistics center location decision using a multi-attribute analysis structure. In P. Chatterjee, M. Yazdani, & S. Chakraborty (Eds.), *Sustainability modeling in engineering* (pp. 1–26). World Scientific. https://doi.org/10.1142/9789813276338_0001

Yesilcayir, N., Ayvazoglu, G., Celik, S., & Peker, I. (2024). Transit warehouse location selection by IF AHP-TOPSIS integrated methods for disaster logistics: A case study of Turkey. *Research in Transportation Business & Management*, *57*, Article 101232. https://doi.org/10.1016/j.rtbm.2024.101232

Zadeh, L. A. (1965) Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

Zakeri, S., Chatterjee, P., Konstantas, D., & Shojaei Farr, A. (2023). Introducing alternatives ranking with elected nominee (ARWEN) method: A case study of supplier selection. *Technological and Economic Development of Economy*, *29*(3), 1080–1126. https://doi.org/10.3846/tede.2023.18789

Zavadskas, E. K., Mardani, B., Turskis, Z., Jusoh, A., & Nor, K. M. D. (2016). Development of TOPSIS method to solve complicated decision-making problems – an overview on developments from 2000 to 2015. *International Journal of Information Technology & Decision Making*, *15*(3), 645–682. https://doi.org/10.1142/S0219622016300019

Zayas, V., Surenkok, G., & Pandey, G. (2017). Implicit ambivalence of significant others: Significant others trigger positive and negative evaluations. *Social and Personality Psychology Compass*, *11*(11), Article e12360. https://doi.org/10.1111/spc3.12360