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A ROBUST OPTIMIZATION MODEL WITH TWO UNCERTAINTIES APPLIED TO SUPPLIER SELECTION

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Abstract. Under intense industry competition, decision makers must ensure that products demanded by consumers can be quickly produced with minimum production cost. However, because uncertainties are unavoidable and inevitably affect decision makers, numerous studies have discussed how to control uncertainties or minimize their effects. Multiple uncertainties that interact simultaneously may cause a combined effect in actual systems. Therefore, this study presents a robust optimization model with two uncertainties, extending the method of robust optimization with one uncertainty. To demonstrate the applicability of the proposed model with two uncertainties, this study uses the supplier selection problem with component purchase quantity allocation in supply chain management as an analysis case. This considers the reliability of production and transportation and develops a multi-objective robust optimization model with two uncertainties. In addition, a nondominated sorting genetic algorithm is proposed for solving the proposed multi-objective robust optimization model. The relationship between price of robustness and budget parameters is explored by considering the robust optimization model with production and transportation uncertainties proposed in this study. Finally, there is a comparative analysis between the results for price of robustness in the proposed two-uncertainty model and in the one-uncertainty model.

Keywords: robust optimization, multiple uncertainties, supplier selection, quantity allocation, supply chain, nondominated sorting genetic algorithm.

JEL Classification: C44, C61, D81, L11, M11.

Introduction

Many unpredictable uncertainties occur in real life, and these risks can make appropriate decisions more difficult. Uncertainties arise when the result of a certain event or a certain decision cannot be known in advance or when a decision may have more than one result. For

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enterprises, uncertainties may affect only one operation or they can even lead to bankruptcy. Therefore, uncertainties can render decision makers unable to implement long-term planning and investment, resulting in missed opportunities. Uncertainties often lead to suboptimal solutions for optimization problems. However, decision makers usually ignore data uncertainties and base their decisions on a determined scenario.

A system based on deterministic data may not effectively address uncertain scenarios, so uncertainties are an unavoidable but complex problem. Numerous scholars have proposed methods to control the effects of uncertainties. To address this issue, Soyster (1973) first proposed the concept of robust optimization (RO), based on scenario analysis and goal planning to optimize systems, variables, and penalty costs. With refinements by several scholars and application in various fields, RO can now effectively control uncertainty effects. However, these RO methods have deficiencies.

Most RO studies have discussed only the optimization of a single uncertainty. However, in real conditions, more than one uncertainty can affect real-life decision-making, and these factors affect one another, making the consequences extremely complex for decision makers. For instance, the coronavirus has caused major interference to global production and transportation systems, necessitating the consideration of multiple uncertainties. However, the current RO model cannot provide sufficient decision-making assistance. Therefore, this study extends the original RO model with a single uncertainty to a RO model with two uncertainties as the basis for model development. This more closely represents reality, and enables further consideration of the model's applications to determine optimal supplier combinations and component purchase quantity. In addition, the heuristic algorithm concept is introduced to identify an approximate solution, reducing the time required for a solution, thereby improving timeliness of decision-making.

The main contributions in this paper are as follows: (1) Presenting a robust optimization model with two uncertain parameters. (2) Developing a new multi-objective optimization mathematical model for supplier selection, based on the proposed two-uncertainty robust optimization model. The considered objectives include minimum ordering cost and minimum completion time, while the uncertainties are production and transportation reliabilities. (3) Proposing a nondominated sorting genetic algorithm for solving the multi-objective robust optimization mathematical model. (4) Presenting a real case to demonstrate the effectiveness of the proposed robust optimization mathematical model and solving approach. (5) Discussing the results of the robust planning to allow a decision maker to handle greater risks in a scenario with two uncertainties.

The remainder of this paper is as follows: Section 1 reviews the literature on uncertainty, robust planning, and multi-objective genetic algorithm. Section 2 presents basic concepts on RO used in this study. Section 3 introduces the proposed robust optimization model with two uncertain parameters. Section 4 describes a multi-objective robust optimization mathematical model for supplier selection and outlines the procedure of the proposed NSGA-II algorithm for solving the mathematical model. Section 5 demonstrates the effectiveness of the proposed methodology through a real case application and discusses the robust planning. Finally, conclusions and suggestions for future studies are given in the last Section.

1. Literature review

1.1. Uncertainty and robust planning

The two approaches were proposed by Bertsimas and Sim (2003) to resolve the effects of uncertainties in optimization problems were stochastic programming and RO. Stochastic programming focuses on minimizing uncertainty effects and incorporating them into a deterministic model. This is done because uncertain parameters render solutions uncertain, thereby increasing the difficulty of obtaining solutions. However, Bertsimas and Sim (2003) stated that stochastic programming has two disadvantages. First, the probability distribution of data is assumed to conform to actual scenarios, but the so-called probability distribution is usually difficult to obtain. Another disadvantage is that the data are assumed to increase in quantity along with the number of scenarios.

The RO method includes several approaches to protect decision makers from vague and random uncertainties. Its main model is constructed using analysis of the worst-case scenario, so its solutions are based on the most unfavorable uncertainties for decision makers. RO is based on scenario analysis and goal planning, which identify optimization subject to system, variable, and penalty costs, while also considering cost and flexibility. Although the analysis model may not provide the optimal solution, it is definitely more robust. Mulvey et al. (1995) explained the purpose of RO and explored possible problems through scenario assumptions in pursuit of a robust solution for a scenario, signifying that the solution identified is the optimal decision for various situations. Pan and Nagi (2010) used RO to analyze the uncertainties of new markets, Leung et al. (2007) also used RO to solve multiple production-planning problems, and Li and Liu (2013) applied RO to solve problems associated with the bullwhip effect. Beyond applying RO to optimize supply chains and scheduling, Zeferino et al. (2012) also applied it to the planning of sewage treatment systems; Gülpinar and Pachamanova (2013) used it to solve the problems of asset and liability management subject to random time changes; Bohle et al. (2010) applied it to plan wine grape harvest scheduling. Zhao et al. (2020) proposed a tri-level RO model to plan the resources of distributed energy storage. Baringo et al. (2020) established an adaptive RO approach is for the energy planning problem of a distribution system for electric vehicles. A RO approach is developed by Izadpanahi et al. (2022) to handle demand uncertainty for energy transition planning in manufacturing firms. In addition, Majewski et al. (2017a, 2017b), and Moret et al. (2020) also handled the energy planning with uncertain inputs using RO. Thevenin et al. (2022) provided a RO formulation for the dynamic demand purchasing and supplier selection problem under lead time uncertainty. Chu et al. (2019) applied the RO approach to a multi-period, single-station inventory problem with supply and demand uncertainties. Thorsen and Yao (2017) and Hnaien and Afsar (2017) considerd lot-sizing problems with lead-time uncertainties in their RO models. In addition, a RO approach was provided by Liu et al. (2021) to hendle uncertain sailing times and uncertain waiting times at ports in a maritime inventory routing problem. These studies show that RO is commonly used to resolve the effects of uncertainties in a wide range of fields.

As can be seen, these studies demonstrate that most RO studies focused on the optimization problems with a single uncertainty and a few studies discussed the multiple uncertain parameters problems. However, as far as I know, no other study explors the multi-objective problem which has two uncertain parameters with multiplication relationship. Our present model extends from the original RO (Bertsimas & Sim, 2004), where the multiplication relationship between two uncertain parameters in the constraints are defined before optimization is executed.

1.2. Multi-objective genetic algorithm

Optimization has long been an essential management tool. Early multi-objective problems usually entailed simplifying objective functions into a single-objective function and then applying it to a genetic algorithm (GA). However, for increasingly complex problems, single-objective functions no longer satisfy current needs, so multi-objective optimization has become a major field in recent research. Mirzapour Al-e-hashem et al. (2011) also used an RO method to solve multi-objective problems.

Multi-objective optimization is a regional multiple criteria decision-making problem that can optimize multiple mathematical functions simultaneously and has the following two requirements: 1) The optimization model has multiple objective functions that must be maximized or minimized, and their constraints must be met. 2) A solution set and a nondominated solution set exist in the problem search space, where no solution in the nondominated solution set is dominated by any other solution in the solution set, and the mechanism achieves a balance between two or more conflicting objectives. No single solution exists for a multi-objective optimization problem, because each objective is optimized simultaneously, leading to a Pareto optimal solution. In certain cases, the Pareto optimal solution may be infinite, and one of the solutions is nondominated. Without the influence of subjective preferences, all Pareto solutions are optimal solutions.

Various scholars have proposed multi-objective evolutionary algorithms for multi-objective problems, and Schaffer (1985) was the first to propose a vector-evaluated genetic algorithm. Although the respective method is relatively simple, the model is flawed, and the weight depends on the current generation. The vector-evaluated genetic algorithm cannot guarantee that desirable individual traits can be successfully passed on to the next generation. Holland (1992) proposed GAs, based on the phenomenon of natural selection and survival from the theory of evolution. The optimal selection rule of genetic crossover and mutation permits the retention of desirable genes and their transferal to the next generation through evolution. This enables the approximate optimal solution to be sought. As mentioned, GA cannot effectively solve problems, but a multiple-objective genetic algorithm (MOGA) has been developed, according to which GA groups are used to classify nondominated solutions that are ranked to enhance the effectiveness of the search for nondominated solutions. Hajela and Lin (1992) used a weighted GA to obtain the sum of all objective values, integrated multiple objectives into a single objective, and performed calculations. However, weights are easily affected by the decision maker, rendering them nonobjective and reducing the solution quality. Fonseca and Fleming (1993) also used a MOGA that ranked all individuals. Higher ranked individuals were transferred to the next generation, resolving the multi-objective optimization problem, but the quality of the solution was limited by an imperfect sorting mechanism.

Since early MOGAs lacked a complete multi-objective algorithm, Srinivas and Deb (1994) proposed the nondominated sorting genetic algorithm (NSGA) to provide more satisfactory solutions and higher adaptive values on the basis of the dominance relationship between solutions to identify the Pareto front. However, the adaptation values of many solutions may be equal, which affects convergence speed. To overcome this shortcoming, Deb et al. (2002) proposed an improved nondominated sorting genetic algorithm II (NSGA-II), which exhibits elitism and a clear mechanism to maintain differences, and effectively improves the comparison mechanism for nondominated solutions to enhance algorithm effectiveness. A crowded comparison operator was also added to ensure even distribution of the solution on the Pareto front, and the elitism can effectively maintain the stability of more satisfactory solutions of subsequent generations.

NSGA-II has been applied in many fields. Yusoff et al. (2011) applied NSGA-II to solve the parameter settings of the machine production process and showed that it identified suitable parameter settings better than conventional methods. Lin and Yeh (2012) combined NSGA-II with the technique for order of preference by similarity to ideal solution to solve the problem of computer networks, and it outperformed the original algorithm. Wang and Hsu (2010) used NSGA-II to minimize the cost of supply chain partners, and Rezaei and Davoodi (2011) used NSGA-II to solve the supplier selection problem. Majumder et al. (2019) used NSGA-II to solve a multi-objective Chinese postman problem under maximizing the total profit earned and minimizing the total travel time for the tour of a postman. Majumder et al. (2020) employed NSGA-II to solve an uncertain multi-objective shortest path problem for a weighted connected directed graph. Dutta et al. (2020) proposed a multi-objective open set orienteering model in which the goals are the maximization of the profit score and the maximization of customer satisfaction and the model is solved by NSGA-II. Barma et al. (2021) adopted NSGA-II to deal with a multi-objective model of the capacitated vehicle routing problem to minimize both the quality degradation of the perishable items to be delivered and delivery costs. Che et al. (2021) proposed a multi-objective genetic algorithm based on NSGA-II for dealing with the multi-objective optimization mathematical model of supplier selection and assembly planning and Che et al. (2022) constructed a multi-objective optimization model with maximum facility coverage, minimum facility overlap, and minimum total idle capacity for planning the service areas of smart parcel lockers utilized, which is solved by NSGA-II. These studies show that NSGA-II provides excellent solutions to multi-objective problems.

The MOGA can coordinate the relationship between the objective functions and obtain the optimal solution set whith a relatively quality value of each objective function. The calculation process of the proposed model becomes complicated in the context of multiple objectives, this study uses the MOGA of a nondominated solution to assist the solution. The NSGA-II enhances the satisfactory optimal performance of NSGA with the fast non-dominated sorting, density value estimation strategy and elitist selection strategy for individuals. It has been widely and successfully utilized in many studies in various fields and has become one of the famous algorithms in multi-objective optimization fields. Therefore, to solve the supplier selection problem through consideration of the RO model with two uncertainties, this study uses NSGA-II as the solution for the entire decision-making system to seek the optimal combination of supply chain partners.

2. Preliminaries

This section presents some basic concepts for RO used in this study. RO can be achieved by using one of two models: RO and robust counterpart optimization. The RO model was first proposed by Soyster (1973), and its model is shown in Eqs (1)–(3):

$$\text{maximize } c'x; \tag{1}$$

$$\text{subject to} \sum_j a_{ij}x_j + \sum_{j\in M_i} \hat{a}_{ij}x_j \le b_i,\ \forall i; \tag{2}$$

$$x_j \ge 0,\ \forall j, \tag{3}$$

where M_i – a set of uncertain coefficients a_{ij} under the i^{th} constraint; a_{ij} – a random parameter whose value falls in $\left[\overline{a}_{ij} - \hat{a}_{ij}, \overline{a}_{ij} + \hat{a}_{ij}\right]$, where $\overline{a}_{ij}$ is the mean of the random variable, and $\hat{a}_{ij}$ is the maximum standard deviation.

This method is used mainly to achieve performance when the worst-case scenario occurs, that is, to maximize the gap between the performance achieved and the worst-case scenario. However, this method is overly conservative. Although it achieves complete robustness, it often involves the greatest price to pay for the most complete budget level from the actual scenario, and the probability of the worst-case scenario occurring is usually extremely low. Therefore, the optimal solution is sacrificed in assuming the worst-case scenario, reducing the effectiveness of the overall model.

Ben-Tal and Nemirovski (1999) improved the model of Soyster (1973) and proposed an effective algorithm for solving the problem of excessive conservatism. The uncertain parameter sets were introduced into an RO in box and ellipsoidal forms to solve the portfolio problem. This method used a safety parameter (θ) mechanism to control the risk through adjustment of the degree of conservatism, and the model changed from Eq. (2) to Eq. (4):

$$\sum_j a_{ij}x_j + \sum_{j\in M_i} \hat{a}_{ij}x_j + \theta_i \sqrt{\sum_{j\in M_i} \hat{a}_{ij}^2 x_j^2} \le b_i,\ \forall i, \tag{4}$$

where θ_i – The budget parameter under i^{th} constraint.

The robust counterpart optimization model proposed by Ben-Tal and Nemirovski (2000) guarantees that the probability of the i^{th} constraint being violated is at most $\exp\left(-\theta_i^2/2\right)$, as expressed by Eq. (5):

$$\Pr\left(\sum_j a_{ij}x_j + \sum_{j\in M_i} \hat{a}_{ij}x_j\right) \le e^{-\theta_i^2/2},\ \forall i. \tag{5}$$

This model uses the safety coefficient θ_i and the probability that the constraint violation of $\exp\left(-\theta_i^2/2\right)$ occurs to adjust the overall degree of conservatism of the optimization model. This model is more flexible than RO, but the entire model becomes nonlinear, increasing the complexity of the solution as well as involving quadratic programming. Therefore, this method is not applicable to the problem of integer programming (Bertsimas & Sim, 2003).

Bertsimas and Sim (2004) proposed another RO model, which first represents uncertain data as shown in Eq. (6):

$$U_i = \left\{ a_{ij} \middle| a_{ij} \in \left[\overline{a}_{ij} - \hat{a}_{ij}, \overline{a}_{ij} + \hat{a}_{ij} \right], \forall j \in M_i; \sum_{j \in M_i} \frac{\left| a_{ij} - \overline{a}_{ij} \right|}{\hat{a}_{ij}} \le \Gamma_i \right\}, \forall i, \tag{6}$$

where Γ_i – budget parameter is used to control the degree of conservatism of uncertainty, and the range of change is $\Gamma_i \in \left[0, \left|M_i\right|\right]$; $\overline{a}_{ij}$ – the mean of the number of data; $\hat{a}_{ij}$ – the maximum standard deviation of data.

Because the likelihood is low that the worst-case scenarios occur for all uncertain data, this method employs the concept of budget parameters used by Ben-Tal and Nemirovski (2000), allowing decision makers themselves to determine the degree of conservatism. By contrast, Bertsimas and Sim (2004) proposed a budget function, and its formula is as seen in Eq. (7):

$$\sum_j a_{ij} x_j + \beta_i \left(x, \Gamma_i \right) \le b_i, \ \forall i, \tag{7}$$

where $\beta_i \left(x, \Gamma_i \right)$ – the budget function is composed of the decision variable x and the budget parameter Γ_i, and its detailed structure is expressed as Eq. (8):

$$\beta_i \left(x, \Gamma_i \right) = \max_{\{m_i \cup \{t_i\} | m_i M_i, \, |m_i| = \lfloor \Gamma_i \rfloor, t_i \in M_i / m_i\}} \left\{ \sum_{j \in m_i} \hat{a}_{ij} x_j + \left(\Gamma_i - \Gamma_i \right) \hat{a}_{it_i} x_{t_i} \right\}, \ \forall i. \tag{8}$$

When the value of Γ_i in the budget function is set to 0, this mathematical model does not consider robustness and qualifies as a general optimized mathematical model. When Γ_i equals $\left|M_i\right|$, this mathematical model has the highest degree of conservatism, which is similar to the optimized mathematical model proposed by Soyster (1973). The decision-maker can decide the value of Γ_i, so the decision-making process is more flexible. This model remains a linear optimization mathematical model, and adding an RO model can effectively increase the feasibility of a solution.

Li and Ierapetritou (2008) used three RO models proposed by Soyster (1973), Ben-Tal and Nemirovski (2000), and Bertsimas and Sim (2004) to solve processing and scheduling problems when processing time, product demand, and market prices were uncertain. The results verified that the RO model proposed by Bertsimas and Sim (2004) effectively controls the degree of conservatism and is the most suitable method for the development of uncertainty models. Moreover, Omrani (2013) also employed this method with data envelopment analysis to solve the problem of how its weight is formulated subject to uncertainty. Ang et al. (2012) also improved this method for use on the problem of multiperiod inventory subject to uncertain demand. Therefore, this study extends the RO model proposed by Bertsimas and Sim (2004) to further discuss the development of the RO model with two uncertainties.

3. Development of robust optimization model with two uncertain parameters

Most scholars have studied RO from the perspective of single-uncertainty analysis, and this model cannot effectively solve problems involving two uncertain scenarios. Therefore, this study establishes an RO model for two uncertainties. In this section, an RO model with two uncertain parameters is constructed. In the RO model proposed by Bertsimas and Sim

(2004), the sampling range of the uncertain parameter set was determined using the mean and standard deviation of uncertain parameter data, per Eq. (6). This characteristic resembles the concept of a 67% confidence interval obtained through addition or subtraction of a normal distribution mean and one-time standard deviation, so a single source of uncertainty is subject to normal distribution. According to the characteristics of data subject to normal distribution, an RO model with two uncertainties is constructed, as in Eqs (9)–(14):

$$\text{maximize } c'x; \tag{9}$$

$$\text{subject to} \sum_j a_{ij} b_{ij} x_j + \beta_i \left(x, \Gamma_i\right) \le k_i\,,\ \forall i; \tag{10}$$

$$\beta_i\left(x,\Gamma_i\right) = \max_{\{m_i \cup \{t_i\} | m_i M_i,\ |m_i| = \lfloor \Gamma_i \rfloor, t_i \in M_i / m_i\}} \left\{ \sum_{j \in m_i} \widehat{ab}_{ij} x_j + \left(\Gamma_i - \Gamma_i\right) \widehat{ab}_{it_i} x_{t_i} \right\}, \forall i; \tag{11}$$

$$U_a = \left\{ a_{ij} | a_{ij} \in \left[\overline{a}_{ij} - \hat{a}_{ij}, \overline{a}_{ij} + \hat{a}_{ij}\right], \forall j \in M_i; \sum_{j \in M_i} \frac{\left|a_{ij} - \overline{a}_{ij}\right|}{\hat{a}_{ij}} \le \Gamma_i \right\},\ \forall i; \tag{12}$$

$$U_b = \left\{ b_{ij} | b_{ij} \in \left[\overline{b}_{ij} - \hat{b}_{ij}, \overline{b}_{ij} + \hat{b}_{ij}\right], \forall j \in M_i; \sum_{j \in M_i} \frac{\left|b_{ij} - \overline{b}_{ij}\right|}{\hat{b}_{ij}} \le \Gamma_i \right\},\ \forall i; \tag{13}$$

$$U_{ab} = \left\{ ab_{ij} \middle| \begin{array}{c} ab_{ij} \in \left[\overline{ab}_{ij} - \widehat{ab}_{ij}, \overline{ab}_{ij} + \widehat{ab}_{ij}\right], \\ \forall j \in M_i; \displaystyle\sum_{j \in M_i} \frac{\left|ab_{ij} - \overline{ab}_{ij}\right|}{\widehat{ab}_{ij}} \le \Gamma_i \end{array} \right\},\ \forall i, \tag{14}$$

where U_a – original set of uncertain parameter a; U_b – original set of uncertain parameter b; U_{ab} – new data distribution set generated through multiplication of two uncertain parameters; $\overline{ab}_{ij}$ – the mean obtained through multiplication of two uncertain parameters; $\widehat{ab}_{ij}$ – the expected standard deviation obtained through multiplication of two uncertain parameters.

For the interaction of two uncertainties, the existing parameters can be used to calculate the new data parameters because the uncertainties alone are a random parameter. Assuming that the two original parameters are X and Y, the expected value of the new data XY can be decomposed using the covariance, as shown in Eq. (15):

$$Cov\left(X,Y\right) = E\left(XY\right) - E\left(X\right)E\left(Y\right). \tag{15}$$

After rearrangement of the equation, Eq. (16) is obtained:

$$E\left(XY\right) = Cov\left(X,Y\right) + E\left(X\right)E\left(Y\right). \tag{16}$$

This is the expected mean obtained after multiplication of the two uncertain parameters. $E(XY)$ can be substituted into the $\overline{ab}_{ij}$ of Eq. (14). The definition of covariance is as in Eq. (17):

$$\begin{aligned} Var\left(XY\right) &= E\left(X^2Y^2\right) - \left[E\left(XY\right)\right]^2 = \\ E\left(X^2\right)E\left(Y^2\right) &+ Cov\left(X^2,Y^2\right) - \left[E\left(X\right)E\left(Y\right) + Cov\left(X,Y\right)\right]^2, \end{aligned} \tag{17}$$

where the expected value and the variance can be converted into Eqs. (18)–(20):

$$Var(X) = E(X^2) - E(X)^2; \tag{18}$$

$$E(X^2) = Var(X) + E(X)^2; \tag{19}$$

$$E(Y^2) = Var(Y) + E(Y)^2. \tag{20}$$

Subsequently, the conversion method is used to obtain $Cov(X^2, Y^2)$. Let

$$X = \mu_x + r\sigma_x Z + \sqrt{1-r^2}\,\sigma_x E_1; \tag{21}$$

$$Y = \mu_y + r\sigma_y Z + \sqrt{1-r^2}\,\sigma_y E_2; \tag{22}$$

$$Z, E_i \sim N(0,1),\ \ i = 1,2; \tag{23}$$

$$Cov(Z, E_i) = 0,\ \ i = 1,2; \tag{24}$$

$$\mathrm{Cov}(E_1, E_2) = 0, \tag{25}$$

then

$$E(X) = \mu_x; \tag{26}$$

$$E(Y) = \mu_y; \tag{27}$$

$$Var(X) = \sigma_x^2; \tag{28}$$

$$Var(Y) = \sigma_y^2; \tag{29}$$

$$Cov(X, Y) = r^2\sigma_x\sigma_y. \tag{30}$$

When the data are distributed symmetrically, the coefficient of skewness is $\gamma_1 = E(Z^3) = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0$. When the data are distributed normally, its coefficient of kurtosis is $\gamma_2 = E(Z^4) = E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = 3$. Therefore,

$$Var(Z^2) = E(Z^4) - \left[E(Z^2)\right]^2 = 3 - 1 = 2; \tag{31}$$

$$Cov(Z^2, Z) = E(Z^3) = 0; \tag{32}$$

$$X^2 = \left(\mu_x + r\sigma_x Z + \sqrt{1-r^2}\,\sigma_x E_1\right)\times \left(\mu_x + r\sigma_x Z + \sqrt{1-r^2}\,\sigma_x E_1\right) = \mu_X^2 + 2\mu_x r\sigma_x Z + 2\mu_x\sqrt{1-r^2}\,\sigma_x E_1 + (r\sigma_x Z)^2 + 2r\sigma_x Z\sqrt{1-r^2}\,\sigma_x E_1 + \left(\sqrt{1-r^2}\,\sigma_x E_1\right)^2; \tag{33}$$

$$Y^2 = \left(\mu_y + r\sigma_y Z + \sqrt{1-r^2}\,\sigma_y E_2\right)\times\left(\mu_y + r\sigma_y Z + \sqrt{1-r^2}\,\sigma_y E_2\right) = \mu_Y^2 + 2\mu_y r\sigma_y Z + 2\mu_y\sqrt{1-r^2}\,\sigma_y E_2 + (r\sigma_y Z)^2 + 2r\sigma_y Z\sqrt{1-r^2}\,\sigma_y E_2; \tag{34}$$

$$Cov\left(X^2,Y^2\right)=$$

$$Cov\left(\begin{array}{l}\mu_X^2+2\mu_x r\sigma_x Z+2\mu_x\sqrt{1-r^2}\sigma_x E_1+\left(r\sigma_x Z\right)^2+\\ 2r\sigma_x Z\sqrt{1-r^2}\sigma_x E_1+\left(\sqrt{1-r^2}\sigma_x E_1\right)^2,\mu_Y^2+2\mu_y r\sigma_y Z+\\ 2\mu_y\sqrt{1-r^2}\sigma_y E_2+\left(r\sigma_y Z\right)^2+2r\sigma_y Z\sqrt{1-r^2}\sigma_y E_2+\left(\sqrt{1-r^2}\sigma_y E_2\right)^2\end{array}\right)=$$

$$Cov\left(\mu_X^2,\mu_Y^2\right)+Cov\left(\mu_X^2,2\mu_y r\sigma_y Z\right)+Cov\left(\mu_X^2,2\mu_y\sqrt{1-r^2}\sigma_y E_2\right)+$$

$$Cov\left(\mu_X^2,\left(r\sigma_y Z\right)^2\right)+Cov\left(\mu_X^2,2r\sigma_y Z\sqrt{1-r^2}\sigma_y E_2\right)+$$

$$Cov\left(\mu_X^2,\left(\sqrt{1-r^2}\sigma_y E_2\right)^2\right)+Cov\left(2\mu_x r\sigma_x Z,\mu_Y^2\right)+$$

$$Cov\left(2\mu_x r\sigma_x Z,2\mu_y r\sigma_y Z\right)+Cov\left(2\mu_x r\sigma_x Z,2\mu_y\sqrt{1-r^2}\sigma_y E_2\right)+$$

$$Cov\left(2\mu_x r\sigma_x Z,\left(r\sigma_y Z\right)^2\right)+Cov\left(2\mu_x r\sigma_x Z,2r\sigma_y Z\sqrt{1-r^2}\sigma_y E_2\right)+$$

$$Cov\left(2\mu_x r\sigma_x Z,\left(\sqrt{1-r^2}\sigma_y E_2\right)^2\right)+Cov\left(2\mu_x\sqrt{1-r^2}\sigma_x E_1,\mu_Y^2\right)+$$

$$Cov\left(2\mu_x\sqrt{1-r^2}\sigma_x E_1,2\mu_y r\sigma_y Z\right)+$$

$$Cov\left(2\mu_x\sqrt{1-r^2}\sigma_x E_1,2\mu_y\sqrt{1-r^2}\sigma_y E_2\right)+$$

$$Cov\left(2\mu_x\sqrt{1-r^2}\sigma_x E_1,\left(r\sigma_y Z\right)^2\right)+$$

$$Cov\left(2\mu_x\sqrt{1-r^2}\sigma_x E_1,2r\sigma_y Z\sqrt{1-r^2}\sigma_y E_2\right)+$$

$$Cov\left(2\mu_x\sqrt{1-r^2}\sigma_x E_1,\left(\sqrt{1-r^2}\sigma_y E_2\right)^2\right)+Cov\left(\left(r\sigma_x Z\right)^2,\mu_Y^2\right)+$$

$$Cov\left(\left(r\sigma_x Z\right)^2,2\mu_y r\sigma_y Z\right)+Cov\left(\left(r\sigma_x Z\right)^2,2\mu_y\sqrt{1-r^2}\sigma_y E_2\right)+$$

$$Cov\left(\left(r\sigma_x Z\right)^2,\left(r\sigma_y Z\right)^2\right)+Cov\left(\left(r\sigma_x Z\right)^2,2r\sigma_y Z\sqrt{1-r^2}\sigma_y E_2\right)+$$

$$Cov\left(\left(r\sigma_x Z\right)^2,\left(\sqrt{1-r^2}\sigma_y E_2\right)^2\right)+Cov\left(2r\sigma_x Z\sqrt{1-r^2}\sigma_x E_1,\mu_Y^2\right)+$$

$$Cov\left(2r\sigma_x Z\sqrt{1-r^2}\sigma_x E_1,2\mu_y r\sigma_y Z\right)+$$

$$Cov\left(2r\sigma_x Z\sqrt{1-r^2}\sigma_x E_1,2\mu_y\sqrt{1-r^2}\sigma_y E_2\right)+$$

$$Cov\left(2r\sigma_x Z\sqrt{1-r^2}\sigma_x E_1,\left(r\sigma_y Z\right)^2\right)+$$

$$Cov\left(2r\sigma_x Z\sqrt{1-r^2}\sigma_x E_1, 2r\sigma_y Z\sqrt{1-r^2}\sigma_y E_2\right)+$$
$$Cov\left(2r\sigma_x Z\sqrt{1-r^2}\sigma_x E_1, \left(\sqrt{1-r^2}\sigma_y E_2\right)^2\right)+Cov\left(\left(\sqrt{1-r^2}\sigma_x E_1\right)^2, \mu_Y^2\right)+$$
$$Cov\left(\left(\sqrt{1-r^2}\sigma_x E_1\right)^2, 2\mu_y r\sigma_y Z\right)+$$
$$Cov\left(\left(\sqrt{1-r^2}\sigma_x E_1\right)^2, 2\mu_y\sqrt{1-r^2}\sigma_y E_2\right)+$$
$$Cov\left(\left(\sqrt{1-r^2}\sigma_x E_1\right)^2, \left(r\sigma_y Z\right)^2\right)+$$
$$Cov\left(\left(\sqrt{1-r^2}\sigma_x E_1\right)^2, 2r\sigma_y Z\sqrt{1-r^2}\sigma_y E_2\right)+$$
$$Cov\left(\left(\sqrt{1-r^2}\sigma_x E_1\right)^2, \left(\sqrt{1-r^2}\sigma_y E_2\right)^2\right). \tag{35}$$

After decomposition, Eq. (36) can be obtained as follows:

$$Cov\left(X^2, Y^2\right)=Cov\left(2\mu_x r\sigma_x Z, 2\mu_y r\sigma_y Z\right)+Cov\left(\left(r\sigma_x Z\right)^2, \left(r\sigma_y Z\right)^2\right)=$$
$$4r^2\mu_x\mu_y\sigma_x\sigma_y+2r^4\sigma_x^2\sigma_y^2=4Cov\left(X,Y\right)\mu_x\mu_y+2Cov\left(X,Y\right)^2. \tag{36}$$

Therefore, by substituting the derived covariance, the variance structure of the new data distribution can be obtained, as follows:

$$Var\left(XY\right)=\left(Var\left(X\right)+E\left(X\right)^2\right)\left(Var\left(Y\right)+E\left(Y\right)^2\right)+$$
$$4Cov\left(X,Y\right)E\left(X\right)E\left(Y\right)+2Cov\left(X,Y\right)^2-\left(E\left(X\right)E\left(Y\right)+Cov\left(X,Y\right)\right)^2=$$
$$Var\left(X\right)Var\left(Y\right)+Var\left(X\right)E\left(Y\right)^2+E\left(X\right)^2Var\left(Y\right)+E\left(X\right)^2E\left(Y\right)^2+$$
$$4Cov\left(X,Y\right)E\left(X\right)E\left(Y\right)+2Cov\left(X,Y\right)^2-E\left(X\right)^2E\left(Y\right)^2-$$
$$2Cov\left(X,Y\right)E\left(X\right)E\left(Y\right)-Cov\left(X,Y\right)^2=$$
$$Var\left(X\right)Var\left(Y\right)+Var\left(X\right)E\left(Y\right)^2+E\left(X\right)^2Var\left(Y\right)+E\left(X\right)^2E\left(Y\right)^2+$$
$$4Cov\left(X,Y\right)E\left(X\right)E\left(Y\right)+2Cov\left(X,Y\right)^2-E\left(X\right)^2E\left(Y\right)^2-$$
$$2Cov\left(X,Y\right)E\left(X\right)E\left(Y\right)-Cov\left(X,Y\right)^2=$$
$$Var\left(X\right)Var\left(Y\right)+Var\left(X\right)E\left(Y\right)^2+$$
$$E\left(X\right)^2Var\left(Y\right)+2Cov\left(X,Y\right)E\left(X\right)E\left(Y\right)+Cov\left(X,Y\right)^2, \tag{37}$$

then

$$\sqrt{Var\left(XY\right)}=\begin{pmatrix} Var\left(\mathrm{X}\right)Var\left(Y\right)+Var\left(X\right)E\left(Y\right)^2+E\left(X\right)^2Var\left(Y\right)+ \\ 2Cov\left(X,Y\right)E\left(X\right)E\left(Y\right)+Cov\left(X,Y\right)^2 \end{pmatrix}^{1/2}, \tag{38}$$

where $\sqrt{Var(XY)}$ is the standard deviation after the multiplication of two uncertain parameters and can be substituted into $\widehat{ab_{ij}}$ in Eq. (14). In this model, the parameters of the new data distribution can be obtained only with the mean and variance of original data and the correlation coefficient of two uncertainties.

4. Mathematical foundation and solving model

4.1. Scenario description

In a supply chain, the competitive advantages of the manufacturer to complete the product is determined by the conditions of suppliers to supply components. Many companies now focus on selecting quality suppliers for cooperation, so as to produce and deliver high-quality products to customers in a short period of time. In addition, due to increasingly fierce competition, companies seek to build long-term relationships with high-quality supply chain partners. Therefore, selecting suppliers is challenging for companies (Chatterjee & Kar, 2018; Li et al., 2009). Che (2017) and Che et al. (2021) also pointed out that firms must integrate suppliers into the production process to increase their production efficiency and competitiveness in the overall supply chain. Therefore, the supplier selection problem as an analysis case is used to illustrate the applicability of the proposed two-uncertainty robust optimization model.

This study uses a single product supplier selection problem to explain and discuss the applicability of the proposed RO model with two uncertainties. When a factory receives a demand order and must separately purchase the components required, factors relating to both cost and time must be considered. For each component, multiple suppliers are available. Moreover, the two uncertainties of component production, reliability and transportation reliability, must also be considered. In terms of cost, the suppliers' purchase costs and the transportation costs are taken into consideration, as well as component production time and transportation time. The basic assumptions of this study are as follows:

1) A single product and a single-period supplier selection model are the scope.
2) The main criteria for supplier selection are cost and time, with equal weight.
3) Each component can be purchased from multiple suppliers and must not exceed the supplier's capacity limitations, and the component purchasing process must not encounter the out-of-stock phenomenon.
4) Components are delivered in single-item deliveries, not batch deliveries.

4.2. Multi-objective optimization mathematical model for supplier selection

Due to time and cost considerations, this study constructs a multi-objective optimization mathematical model for the selection of suppliers. Definitions of mathematical symbols are shown in Table 1.

Table 1. Mathematical symbols

Symbol	Description
I	Total number of components required for the product
i, j	Component index number, $i = 1,2,3,\ldots,I$; $j = 1,2,3,\ldots,I$
V_i	Total number of suppliers for component i
V	Supplier index number for component i, $v = 1,2,3,\ldots,V_i$
OD	Order quantity
PA_{iv}	Number of components i purchased from supplier v
$TC_{i,v}$	Transportation cost for supplier v and component i
$PC_{i,v}$	Procurement cost for supplier v and component i
$TT_{i,v}$	Transportation time for supplier v and component i
$MT_{i,v}$	Production completing time for supplier v and component i
$MR_{i,v}$	Production reliability for supplier v and component i
$TR_{i,v}$	Transportation reliability for supplier v and component i
$UCP_{i,v}$	Maximum capacity for supplier v and component i
$LCP_{i,v}$	Minimum capacity for supplier v and component i
$P_{i,v}$	$\begin{cases} 1 & \text{Select supplier } v \text{ for the purchase of component } i \\ 0 & \text{otherwise} \end{cases}$
TPC	Total purchase cost
TTC	Total transportation cost
FC	Total cost of order completion
TTT	Total transportation time
TMT	Total production time of components
FT	Total order completing time
$MR_{i,v}$	Production reliability of supplier v for component i
$TR_{i,v}$	Transportation reliability of supplier v for component i
U	Total reliability set of all suppliers
m	Variable coefficient set of the worst-case scenario
t	Variable coefficient set of the Non-worst-case scenario
Γ	Budget parameter
$\beta\left(PA_{i,v},\Gamma\right)$	Budget function

The objectives used are divided into cost and time when suppliers are evaluated; the constraints account for supply, demand, and capacity limitations. A multi-objective mathematical model is constructed as follows:

Objective functions:

1) Minimize total ordering cost: Minimize the supplier's cost of component purchase and cost of transportation.

$$Minimize \quad FC = TPC + TTC; \tag{39}$$

$$TPC = \sum_{i=1}^{I}\sum_{v=1}^{V_i} (P_{i,v} PA_{iv} PC_{i,v}); \tag{40}$$

$$TTC = \sum_{i=1}^{I}\sum_{v=1}^{V_i} (P_{i,v} PA_{iv} TC_{i,v}). \tag{41}$$

2) Minimize order completion time: Minimize the supplier's component production time and transportation time.

$$Minimize \quad FT = TMT + TTT. \tag{42}$$

Production time is the time for components to be manufactured by the supplier, and the transportation time is the maximum transportation time for all selected component suppliers.

$$TMT = \sum_{i=1}^{I}\sum_{v=1}^{V_i} \left(MT_{i,v} P_{i,v} PA_{iv} \right); \tag{43}$$

$$TTT = \max\{TT_{iv} P_{iv} \mid \forall i, v\}. \tag{44}$$

Constraints:

Supply–demand balance: The remaining favorable products after each supplier's purchase is multiplied by reliability and must satisfy the total demand of component orders.

$$\sum_{v=1}^{V_i} PA_{iv} MR_{i,v} TR_{i,v} = OD_i, \quad \forall i. \tag{45}$$

Supplier capacity limitation: Must comply with the supplier's capacity limitation for the ordering of components from suppliers.

$$LCP_{i,v} \le PA_{i,v} \le UCP_{i,v}, \quad \forall i, v. \tag{46}$$

Limited purchase quantity: The purchase quantity of components is a nonnegative integer.

$$PA_{i,v} \ge 0 \text{ and } PA_{i,v} \in \text{Integer}, \ \forall i, v. \tag{47}$$

4.3. Multi-objective optimization mathematical model for supplier selection

Using the RO model derived in Section 3, this study constructs an RO model with two uncertainties, as follows:

Objective functions:

(39)–(44)

Constraints:
(46)–(47)

$$\sum_{v=1}^{V_i} PA_{iv} MR_{i,v} TR_{i,v} - \beta\left(PA_{i,v},\Gamma\right) = OD_i, \ \forall i; \tag{48}$$

$$\beta_i\left(x,\Gamma_i\right) = \max_{\left\{m\cup t \mid m\subseteq U, |m|=\Gamma, t\in U\setminus m\right\}} \left\{ \sum\nolimits_{v\in m} \begin{matrix} \widehat{MR_{i,v}TR_{i,v}}PA_{i,V} + \\ \left(\Gamma - \Gamma \widehat{MR_{i,v}TR_{i,v}}PA_{i,V}\right) \end{matrix} \right\}, \ \forall i. \tag{49}$$

Eq. (48) is a RO model with two uncertainties formed by adding a budget function β to Eq. (45). In addition, by controlling the magnitude of the budget parameter Γ to determine the magnitude of risk to be taken can reduce the effect of uncertainties, the budget function is as shown in Eq. (49), where $\sum\nolimits_{v\in m} \widehat{MR_{i,v}TR_{i,v}}PA_{i,V}$ is the coefficient for the worst-case scenario, and $\Gamma - \Gamma\widehat{MR_{i,v}TR_{i,v}}PA_{i,V}$ is the coefficient for the non-worst-case scenario. The addition of these two values yields the maximum value, which is the RO budget function.

4.4. Nondominated sorting genetic algorithm for solving the optimization mathematical model

NSGA-II is here used to obtain a solution for the supplier's model of two uncertainties. The steps are as follows:

Step 1: Encoding solutions. The solution structure is expressed in the form of chromosomes, and real numbers are used for encoding. The genetic value represents the quantity purchased by each supplier and must satisfy the aforementioned constraints. The chromosome structure is shown in Figure 1.

Step 2: Generation of initial population. According to the order demand and subject to the mathematical constraints of this study, the population of the initial supplier order quantity is generated. The order purchase quantity must take into account the production reliability and transportation reliability of suppliers. If the purchase quantity exceeds the capacity of its supplier, then any supplier should be randomly selected to compensate for the insufficient components.

Step 3: Calculation of objective function value. Substitute chromosomes in the original population into the mathematical objective equation of this study to calculate the objective function values of all chromosomes. The objective function of this study is the total cost and time.

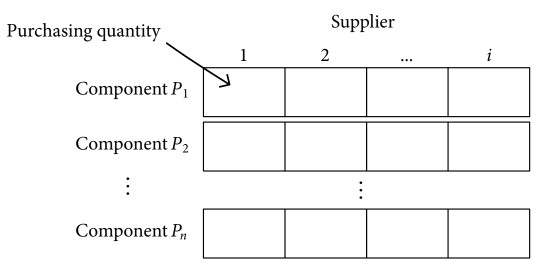

Figure 1. Chromosome structure

Step 4: Nondominated sorting. According to the objective function value of each chromosome, the nondominated solutions for various levels are sorted to judge the quality of chromosomes in the subsequent reproduction stage. The sorting step is to initially identity the chromosomes that are not dominated by other chromosomes, defined as Level 1 and called nondominated solutions; the remaining solutions are called dominated solutions. Again, the process is repeated for the remaining chromosomes and defined as Levels 2, 3, … until all chromosomes are sorted.

Step 5: Calculation of crowding distance. Crowding distance is used to calculate the chromosome scores of the sorted chromosomes according to their levels, thus indicating the density between a particular chromosome and other surrounding chromosomes. Smaller crowding distance implies that the chromosome distribution is denser, and the chance of reproduction is lower. By contrast, greater crowding distance signifies looser chromosome distribution and higher probability of selection for reproduction. The calculation is as Eq. (50):

$$CD_i = \frac{1}{r}\sum_{k=1}^{r}\left|f_k^{i+1} - f_k^{i-1}\right|, \quad i = 1,2,3,\ldots, j-1, \tag{50}$$

where CD_i is the crowding distance of the i^{th} chromosome, r is the number of objective functions, k is one of the objective function values, i is the number of chromosomes, j is the last chromosome in the set, and f_k^{i+1} and f_k^{i-1} are the objective function values of chromosome $(i + 1)$ and chromosome $(i - 1)$ for the k^{th} objective, respectively. In each level, the chromosomes distributed at the end points on both sides set the crowding distance to infinity.

Step 6: Reproduction. After all chromosomes are sorted and the two data for the attributes of crowding distance are obtained, both data are used to determine whether reproduction occurs. The criterion used in this study is the total crowding distance ratio for all chromosomes (i.e., probability of the chromosome being reproduced).

Step 7: Crossover. Genetic codes for chromosome reproduced in the previous step are exchanged so the offspring inherit some of the characteristics of each parent. This step is referred to as crossover, and the algorithm in this study uses a single-point crossover mechanism. First, a group of component suppliers is randomly selected from the parent chromosome as a crossover segment. Subsequently, the supplier combination and purchase quantity corresponding to the segment of the two chromosomes are exchanged with each other, thereby generating two new offspring chromosomes.

Step 8: Mutation. The mutation mechanism expects offspring to evolve characteristics absent in the parents to prevent the chromosomes from converging to a regional optimal solution. The method used in this study is a single-point mutation mechanism, which groups chromosomes by components (i.e., mutations are performed in units of one component). However, tangent points are randomly generated on the grouped chromosomes, and the genes after the tangent points are used as mutant genes. The mutation results must comply with the constraints listed in the previous section, otherwise this step should be repeated.

Step 9: Elitist selection. The purpose of the elitist selection is to combine the original parent chromosomes and the offspring chromosomes produced after the processes of reproduction, crossover, and mutation. All chromosomes in a nondominated solution are sorted, and the crowding distance is calculated. They are sorted according to the levels, and the first 50% of the desirable chromosomes are transferred to the next generation. To prevent the process from falling into a regional optimal solution, chromosomes with the same gene encodings are removed and replaced by the top-ranked chromosomes of the next 50% of chromosomes.

Step 10: Satisfy termination conditions. A maximum number of generations is the termination condition, at which point the iterations stop. If the termination condition is not achieved, the process returns to Step 3 to calculate the objective function value until the termination condition is fulfilled.

Step 11: Optimal supplier partner combination. When the termination condition is satisfied, the optimal supplier combination and relative purchase quantity can be obtained. The optimal objective function value is calculated from these two, and the objective function is the cost and time.

5. Case application and analysis

5.1. Supply selection and component purchase planning

To verify fitness of the proposed methodology, this study uses a real case where factory A, a hemadynamometer manufacturer, has eight major suppliers. For this case, the proposed supplier selection model both evaluates the most efficient and feasible suppliers and allocates the optimal component purchase quantities. After a customer order is received, factory A must produce 260 units' worth of products. A product requires 22 different components, and each component has four suppliers to choose from. Each supplier supplies only one component, and partial component supply information is shown in Table 2.

Because this study uses NSGA-II, the control parameters include population (N), generation (G), crossover rate (Cr), and mutation rate (Mr). For the decision-making support system to quickly achieve the optimal solution, each parameter must be experimentally designed to identify a set of optimal parameter combinations that optimize the efficiency of the algorithm. Wu and Cao (1997) set the P for a stochastically optimized genetic algorithm to 30 groups, with Cr between 0.6 and 0.98 and Mr between 0.01 and 0.2. Rojas et al. (2002) noted that G and P have different sizes, according to the scale and type of the problem, setting the Cr values to 0.4, 0.5, 0.6, 0.7, and 0.8 and Mr values to 0.05, 0.1, 0.15, 0.2, and 0.25. Veldhuizen and Lamont (1999) proposed that error ratio (ER) can be used to identify the algorithm convergence degree to the Pareto front. The ER exhibits the smaller the better characteristic, and its formula is shown in Eq. (51):

$$ER = \frac{\sum_{i=1}^{n} e_i}{n}, \tag{51}$$

where n is the number of dominant solutions found in the algorithm; and e_i is a binary variable. When the dominant solution i is a Pareto solution, then $e_i = 0$, but it is 1 otherwise. When the ER value is closer to 1, fewer nondominated solutions converge on the Pareto front.

Table 2. Partial data for components

Component	Supplier	Minimum capacity	Maximum capacity	Purchase cost	Transportation cost	Transportation time	Production reliability	Transportation reliability
P_1	1	7	142	12.3	1.3	41.2	0.99	0.90
	2	7	115	13.6	2.7	26.5	0.97	0.98
	3	12	136	11.7	2.2	31	0.94	0.90
	4	15	131	12	1.6	35.3	0.96	0.86
P_2	5	5	113	11.3	1.5	27.1	0.99	0.98
	6	9	101	13.5	2.4	39.4	0.94	0.97
	7	14	141	12.2	1.2	31.6	0.97	0.99
	8	9	96	11.9	2	34	0.95	0.85
P_3	9	5	124	13.2	2.6	27.1	0.99	0.93
	10	15	105	11.2	2.4	30	0.94	0.95
	11	5	107	13.5	1.8	42.7	0.95	0.88
	12	13	142	12.1	2.1	36	0.99	0.94
P_4	13	15	146	13.5	1	32.3	0.98	0.97
	14	7	139	11.7	2.3	40.2	0.96	0.89
	15	5	104	12.4	1.3	28	0.99	0.94
	16	8	138	13.1	2.8	38.4	0.98	0.99

Table 3. Taguchi orthogonal array for the L_9 experimental design

Experimental orthogonal array	Population	Generation	Crossover	Mutation
1	150	100	0.4	0.1
2	150	200	0.6	0.2
3	150	300	0.8	0.4
4	200	100	0.6	0.4
5	200	200	0.8	0.1
6	200	300	0.4	0.2
7	250	100	0.8	0.2
8	250	200	0.4	0.4
9	250	300	0.6	0.1

Table 3 sets N at 150, 200, and 250; G at 100, 200, and 300; Cr at 0.4, 0.6, and 0.8; and Mr at 0.1, 0.2, and 0.4 and incorporates the Taguchi experimental design to form an L_9 orthogonal array, as shown in Table 3. The nine combinations in the orthogonal array are tested 10 times each, and the number of Level 1 solutions among various parameter combinations is established; that is, the number of nondominated solutions and the *ER* value of each parameter combination is calculated. Subsequently, the signal-to-noise ratio of the two indicators is as shown in Figure 2. According to the signal-to-noise ratio in Figure 2, when N is 250 groups, G is 300 generations, Cr is 0.4, and Mr is 0.4, the best performance is achieved. Therefore, this parameter level combination is used for the experimental parameters of this study.

On the basis of the Pareto-optimal solution sets obtained from the NSGA-II, case data calculations are performed on the basis of the parameters obtained from the previous experimental design. Figure 3 lists the remaining 250 solution set distribution after iteration. According to Figure 3, the solutions obtained from iteration of the algorithm are all distributed on the Pareto front, and the solutions of Level 1 are not dominated by any other solutions. To save space, 20 reference solutions are obtained. Table 4 provides the objective function values from the intercepting results for supplier selection in 20 Pareto solution sets. From this table, the third Pareto solution set shows that the total ordering cost of production planning is 81953.1, total order completion time is 1044.36. The value of cost and time are smaller the better.

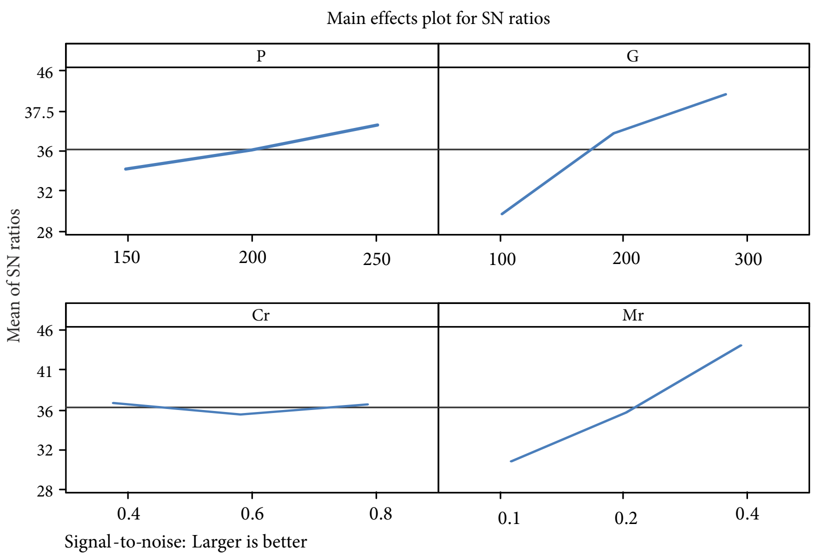

Figure 2. Signal-to-noise (SN) ratios

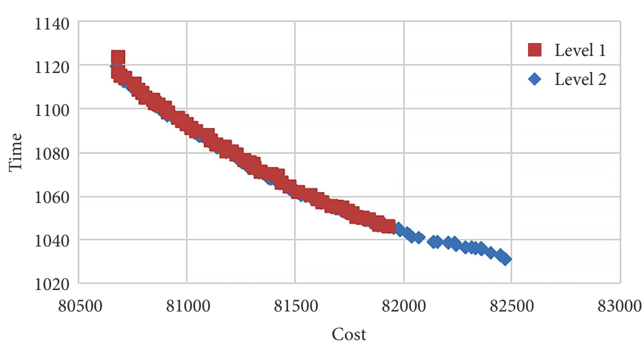

Figure 3. Pareto-optimal solutions

Table 5 presents the third group of Pareto supplier selection combinations and purchase quantities from Table 4. For component P_1, for example, to satisfying the total demand of 260 units suppliers I, III, and IV are selected, who supply 110, 105, and 45 components, respectively. Supplier II is not selected to supply component P_1. For component P_2, suppliers I, II, and III are selected, who supply 89, 63, and 108 components, respectively. Supplier IV is not selected to supply component P_2.

Table 4. Pareto solution sets

Nondominated solution	Total cost	Total time	Nondominated solution	Total cost	Total time
1	81981.7	1044.36	11	81036.6	1089.18
2	82026.6	1042.65	12	80710.8	1112.42
3	81953.1	1045.38	13	81370.4	1069.85
4	80678.3	1119.44	14	80911.6	1097.06
5	81932.1	1045.80	15	81254.3	1075.73
6	80683.4	1116.36	16	81098.7	1073.97
7	81882.7	1046.94	17	80450.7	1123.71
8	81089.8	1086.39	18	81507.6	1054.14
9	81430.5	1065.91	19	80943.5	1082.00
10	81230.3	1077.65	20	81543.0	1052.37

Table 5. Purchase planning of supplier selection

	Supplier			
Component	I	II	III	IV
P_1	110	0	105	45
P_2	89	63	108	0
P_3	69	82	0	109
P_4	0	111	80	69
P_5	102	0	63	95
P_6	0	105	49	106
P_7	94	46	0	120
P_8	58	108	94	0
P_9	0	115	97	48
P_{10}	113	37	110	0
P_{11}	99	0	62	99
P_{12}	80	91	0	89
P_{13}	0	113	81	66
P_{14}	85	81	0	94
P_{15}	119	0	103	38
P_{16}	80	0	108	72

End of Table 5

	Supplier			
Component	I	II	III	IV
P_{17}	94	0	91	75
P_{18}	101	81	0	78
P_{19}	109	105	0	46
P_{20}	0	70	83	107
P_{21}	102	0	80	78
P_{22}	41	103	0	116

5.2. Discussion of the robust planning

RO is used in this study to control the effect of uncertainties. To better represent real life scenarios, this study also considers two uncertainties, the reliability of suppliers' component production and transportation to the factory by suppliers. This study improves the RO model proposed by Bertsimas and Sim (2004) by simultaneously considering the two aforementioned variables and uses scenario analysis to find if a difference exists between two-uncertainty and one-uncertainty situations. According to Bertsimas and Sim (2004), the range of the budget parameter Γ is between 0 and the number of uncertainties, so Γ is set as a continuous function between 0 and 4. The value of Γ is increased by 0.5 for Level 1 and the NSGA-II is used to calculate each budget parameter 10 times and also to calculate the mean, standard deviation, and coefficient of variation (CV). Results are shown in Table 6. The small CV value indicates little difference between the solutions in each budget parameter. In this case, the range of CV values is between [0.05, 0.20], that is, the model constructed by this study and the solution method used have stable planning capabilities. Table 6 shows, when the budget parameter increases, the cost and time also increase. When budget parameter Γ increases from 0 to 4, the mean of total cost increases from 81312.93 to 102732.72 and the mean of total time increase from 1072.90 to 1385.84.

Table 6. Effect of budget parameter on objective value

Γ		0	0.5	1	1.5	2	2.5	3	3.5	4
Cost	max	81452.61	84179.02	87289.94	89835.42	92741.67	95398.63	97975.13	100409.83	102807.49
	min	81078.53	83700.57	86891.88	89240.26	92394.21	94986.74	97573.72	100035.53	102650.29
	mean	81312.93	83951.16	87016.03	89613.63	92540.41	95129.80	97793.07	100314.97	102732.72
	std.	110.91	161.87	116.28	182.32	126.01	143.90	138.28	103.51	53.29
	CV	0.14	0.19	0.13	0.20	0.14	0.15	0.14	0.10	0.05
Time	max	1081.08	1111.46	1154.32	1196.21	1231.79	1269.02	1319.53	1351.29	1389.87
	min	1067.36	1102.11	1141.70	1162.06	1218.57	1255.20	1301.14	1338.73	1382.12
	mean	1072.90	1107.33	1148.12	1183.51	1227.52	1263.84	1308.76	1343.19	1385.84
	std.	4.03	3.05	3.91	9.31	4.17	4.62	5.51	3.43	2.63
	CV	0.38	0.28	0.34	0.79	0.34	0.37	0.42	0.26	0.19

This study evaluates the robust solution $x^*(\Gamma)$ by changing the budget parameter. First, $C\left(x^*\left(\Gamma\right)\right)$ is defined as the objective function value corresponding to the changing state of $x^*(\Gamma)$. For the performance evaluation of robust planning, the price of robustness $\eta(\Gamma)$ is applied to represent the objective value set of the decision-maker under budget parameters and is compared with the objective value generated when robustness is not considered. The definition is shown in Eq. (52):

$$\eta\left(\Gamma\right)=\overline{C}\left(x^*\left(\Gamma\right)\right)-\overline{C}\left(x^*\left(\Gamma=0\right)\right), \tag{52}$$

where $\overline{C}\left(x^*\left(\Gamma\right)\right)$ is the objective function value corresponding to $x^*(\Gamma)$, and $\overline{C}\left(x^*\left(\Gamma=0\right)\right)$ is the expected objective function value produced without robustness considered. According to the aforementioned definition, $\eta(\Gamma)$ is the price that the decision maker must pay to avoid excessive risks. Table 7 shows the effect of budget parameter on the price of robustness. For example, if budget parameter Γ is 0.5, the decision maker will face higher risks and will pay 2638.24 additional cost and 34.43 additional time. To avoid excessive risk, the budget parameter Γ should have a higher value. The extra cost paid and time spent on $\Gamma = 4$ relative to $\Gamma = 0$ are 21419.79 and 312.95. Therefore, higher cost and more time are needed for an enterprise to reduce risks of production activities.

The $\eta(\Gamma)$ value increases with increased budget parameter, as shown in Figure 4. The figure illustrates that, when the budget parameter increases by one level, the total cost and total time both increase significantly. When Γ increases by 1, the total cost and total time increases by approximately 7.5%. The rate of increase in the total cost is higher than the rate of increase in the total time as budget parameter increasing continues.

This study further discusses production uncertainty and transportation uncertainty in a robust model with one uncertainty and compares the price of robustness results with those for the model with two uncertainties proposed in this study. The study first discusses production uncertainty. Transportation uncertainty is fixed and allows changes only in production uncertainty. Prices of robustness are shown in Table 8 and Figure 5 is a line chart for the price of robustness. In production uncertainty, when budget parameter Γ is 0.5, there is less protection and additional cost and additional time are 2173.81 and 36.29. The decision maker can set higher budget parameter values to reduce production risk, and the extra cost paid and time spent on $\Gamma = 4$ over $\Gamma = 0$ are 17870.91 and 256.04. The rate of increase in the total cost without transportation uncertainty is also higher than the rate of increase in the total time as budget parameter increasing continues.

Similarly, Table 9 depicts the variation range for the price of robustness when only the uncertainty of transportation changes. Figure 6 is a line chart for the price of robustness. For transportation uncertainty, when budget parameter Γ is lower and set at 0.5, the additional cost and additional time are 2296.37 and 40.00. A higher budget parameter value is set to reduce the risk, the extra cost paid and time spent on $\Gamma = 4$ over $\Gamma = 0$ are 15104.33 and 215.21. The rate of increase in the total cost without production uncertainty is also higher than the rate of increase in the total time as budget parameters continue increasing. As Figure 6 shows, both slopes of increase in the total cost and time decrease, that is, the budget parameter continue to increase from 0 to 4, there is some moderation of the rates of increase in the total time and cost.

According to Figures 5 and 6, regardless of production uncertainty or transportation uncertainty in a single-uncertainty model, the total cost and time increase with an increase in budget parameter. When Γ increases by 1, η(Γ) increases by approximately 4–5%. Compared with the model with two uncertainties, whose increase rate is approximately 7.5%, the model with one uncertainty increases more slowly. Therefore, when two uncertain parameters exist in a scenario, using a model with only one uncertainty to evaluate the relationship between risk and cost may underestimate the cost of risks, causing decision makers to suffer larger losses.

Table 7. Effect of budget parameter on price of robustness

Γ		0	0.5	1	1.5	2	2.5	3	3.5	4
η(Γ)	cost	0	2638.24	5703.10	8300.71	11227.48	13816.87	16480.14	19002.04	21419.79
	time	0	34.43	75.22	110.61	154.62	190.95	235.86	270.29	312.95

Table 8. Price of robustness for changes in production uncertainty

Γ		0	0.5	1	1.5	2	2.5	3	3.5	4
η(Γ)	cost	0	2173.81	4693.44	6958.28	9234.08	11480.17	13754.68	15869.97	17870.91
	time	0	36.29	67.18	100.28	132.32	157.24	185.97	216.52	256.04

Table 9. Price of robustness for changes in transportation uncertainty

Γ		0	0.5	1	1.5	2	2.5	3	3.5	4
η(Γ)	cost	0	2296.37	4905.33	6891.45	9063.59	10609.70	12458.73	13661.62	15104.33
	time	0	40.00	71.16	103.83	130.43	149.94	172.85	194.63	215.21

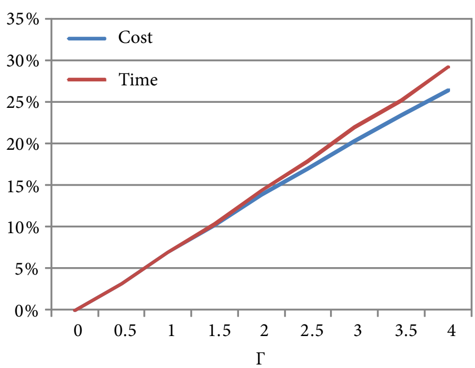

Figure 4. Trend for price paid by the robust model

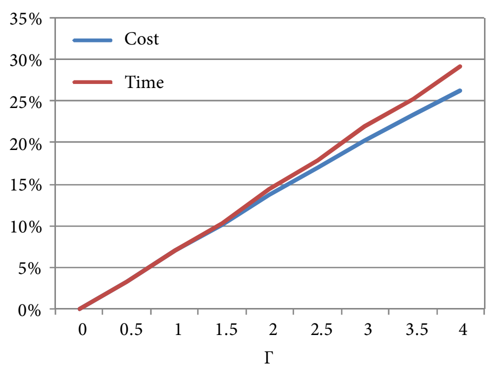

Figure 5. Trend for price of robustness in production uncertainty

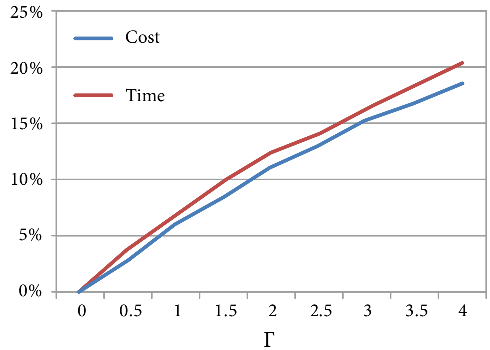

Figure 6. Trend for price of robustness in transportation uncertainty

Conclusions and suggestions

Referring to the robust model proposed by Bertsimas and Sim (2004), this study uses statistical to construct a RO model that can simultaneously consider two uncertainties and test the supplier selection problem. The analysis results indicate that when there are two uncertainties, decision makers encounter both greater risks and greater variation effect. The rate of increase is higher than when only one uncertainty is considered, indicating that more care is required when two uncertainties are involved to identify the correct decision. Several areas deserving of further research are proposed: 1) Expand the scale of the applied problems and consider more scenarios (e.g., the delayed cost of supplier purchasing, production efficiency, and details of factory processing) to improve representation of real life scenarios. 2) Consider a greater number and variety of criteria (e.g., quality level and supplier delivery times) for a more thorough evaluation.

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