Technological and Economic Development of Economy
ISSN: 2029-4913 / eISSN: 2029-4921
2022 Volume 28 Issue 6: 1791–1817
https://doi.org/10.3846/tede.2022.17662

OPTIMAL LICENSING STRATEGY OF GREEN TECHNOLOGY WITH CORPORATE SOCIAL RESPONSIBILITY

Dongdong LI[1*], Chenxuan SHANG[1], Chen JIAO[2]

[1]*School of Public Policy and Administration, Northwestern Polytechnical University, Xi'an, PR China*
[2]*School of Languages, Literacies and Translation, Universiti Sains Malaysia, Penang, Malaysia*

Received 31 October 2021; accepted 01 August 2022; first published online 19 October 2022

Abstract. This study investigates the green technology licensing strategies of firms with corporate social responsibility (CSR) in a duopoly market. The results show that in the absence of CSR, the optimal licensing contract is fixed-fee licensing for a patent holding firm. In the precent of CSR, the optimal licensing contract for a patent holding firm switches from fixed-fee licensing to royalty licensing with increasing level of CSR if the reduction cost of emissions is high. Moreover, we show that the profit goal of firm and the social welfare goal of government are not always mutually exclusive. If the level of CSR is low, a uniform licensing contract would be preferable. If the level of CSR is high, the optimal licensing contract is inconsistent. Finally, we show that CSR is not always beneficial to the social welfare while CSR benefits the environment. Social welfare benefits from increased CSR degree, but vice versa is true when CSR degree decreases. This research may provide valuable insights into licensing and CSR literature.

Keywords: corporate social responsibility, licensing contract, green technology.

JEL Classifications: D42, M14, I13.

Introduction

Firms are now considering green innovation as a way to boost their competitiveness in the market with increasing environmental concern (Porter & Vander Linde, 1995). A good example is Apple, which spent a lot of money on renewable energy projects in 2017. This resulted in a reduction of nearly 2 million tons of carbon emissions compared to last year. Similarly, Gree, a well-known appliance manufacturer in China, abates carbon emissions by about 9064.4 tons per year by investing in next-generation refrigeration technology (Zhang et al., 2020). For technology innovations to spread, the licensing process is of importance. It is estimated that developed countries account for more than 60% of all green technology patents, according to the PTC database. In the meantime, developing nations are steadily obtaining eco-technology from other nations in order to reduce their GHS (Green House Gases).

*Corresponding author. E-mail: *lidd@nwpu.edu.cn*

Firms have three options for licensing its innovation: fixed-fee licensing, royalty licensing, and two-part tariff licensing (Chen et al., 2014; Li, 2021). Although a lot of scholars have studied technology licensing, they consistently stress profit maximization as the primary objective of firms, dismissing the possibility that firms may also prioritize corporate social responsibility (CSR). CSR is a firm's social and environmental responsibility via ethical and transparent behavior (International Organization for Standardization, 2010). Firms are increasingly contemplating CSR initiatives as public awareness of CSR grows. Since 2011, more than 90% of the world's biggest firms, according to KPMG (2016), have issued CSR reports. At the same time, the aforementioned practices are widespread among small, medium, and big firms (Reicher, 2019; Marakova et al., 2021). CSR-compliant firms take into consideration consumer surplus, environmental advantages, and environmentally friendly production in their decisions. According to PricewaterhouseCoopers [PwC] (2010), environmental activities attract the greatest emphasis in CSR reports. KPMG (2015, 2017) tells that 82% of firms globally reported carbon emissions in 2017, up from 58% in 2015. It is therefore true that while designing green technology licensing strategy, firms cannot disregard the influence of CSR effort.

According to previous research, the most effective licensing method for green technology is fixed-fee licensing when the licensor is located in a private market (Li, 2021). Keep in mind that this study only applies to profit-oriented companies. Through CSR, firms develop a corporate self-regulation that prioritizes not only profits but also the protection of the environment (Kliestik et al., 2020; Kovacova & Lazaroiu, 2021; Kovacova & Lewis, 2021; Durana et al., 2021; Lazaroiu et al., 2021). As a result, firms may devote more resources to CSR and make costly improvements to their operations. Under this situation, firms will choose the licensing arrangement that best suits their needs. Therefore, we attempt to understand the most effective way to license green technology from a CSR perspective. Furthermore, we want to know whether the ideal green technology licensing contract differs depending on the presence of CSR. Also, the environmental and social impacts of CSR on green technology licensing contract is an important issue. Based on the above background, the following questions are addressed in this paper:

1) In the CSR case, which of the following licensing arrangements – fixed-fee, royalty, or two-part tariff – is the best option for green technology?
2) Does the absence or presence of CSR make any difference in the ideal licensing contracts for green technology, and if so, what are those differences?
3) In the context of green technology licensing, does CSR lead to a cleaner environment and greater levels of social welfare?

With the aim of investigating the research questions, this paper develops a duopoly model of a patent holding firm's CSR under a time-consistent emissions tax. We first examine the licensing strategies of green innovation, and then compare equilibrium outcomes in the absence and presence of CSR. The main results are as follows. First, fixed-fee licensing with no CSR is always the best option for the patent holder, while the optimal licensing contract with CSR changes from fixed-fee licensing to royalty licensing as the degree of CSR increases. Second, the profit goal of firm and the social welfare goal of government are not always mutually exclusive. If CSR degree is low, there is no contradiction between corporate incentive goal and social welfare; otherwise, there is a contradiction between corporate incentive goal and social welfare. Third, we show that CSR is not always beneficial to the social welfare while CSR benefits the environment, as we've learned through our research. More specifically, a higher level of CSR is good for social welfare, whereas a lower level of CSR is detrimental.

The remainder of the paper is organized as follows. In Section 1, we include a review of the related literature. Section 2 presences our research models and assumptions. In Section 3, we investigate the optimal licensing contract. In Section 4, we conduct numerical experiments to analyze the influence of related parameters. In the last Section, a summary of this paper and the future research directions are presented.

1. Literature review

Our work is related to two research streams technology licensing and CSR. In the subsection, we will explore both streams, and then describe the differences between our findings and earlier studies.

1.1. Technology licensing

One of the related streams concentrates on different approaches to the licensing of technology. A variety of market structures, competitive models, and information structure models have all been used to study technology licensing approaches by scholars (Wang & Yang, 1999; Wang, 2002; Sen, 2005; Sen & Tauman, 2007; Sen & Bhattacharya, 2017; Niu, 2018; Jeon, 2019; Hattori & Tanaka, 2018, 2021). Other factors that influence the optimal licensing contract include product differentiation (Li & Wang, 2010; Ye & Mukhopadhyay, 2013; Rau et al., 2019; Zou & Chen, 2020; Sen et al., 2021; San Martín & Saracho, 2021), the number of participants (Antelo & Sampayo, 2017), and network effects (Zhao et al., 2014; Zhang et al., 2018). However, in the studies mentioned above, licensing of green technologies has received far less consideration than licensing for manufacturing technologies.

Only a few papers have been published in the field of green technology licensing. Innovators' green licensing strategy with an emissions tax and their effects on social welfare are examined by Kim and Lee (2014). In a mixed duopoly, Kim et al. (2018) further investigate the findings of Kim and Lee (2014). Hattori (2017) explored the ideal trade-off between environmental policies and innovations when it comes to technology licensing. Xia et al. (2019) evaluated command-and-control regulation's influence on green technology licensing. Moreover, in a mixed oligopoly, Li (2021) investigated the licensing options for green innovations. Results show that fixed-fee is the optimal contract to license green technology under a private market.

1.2. The effects of CSR on corporate decision making

Another one is the impact of CSR on corporate decision making. In the last several years, CSR has been getting more attention, including both theoretical and empirical studies. Studies have shown that CSR behaviors can influence the actions of profit-maximizing private enterprises competing with those that participate in socially responsible activities in various oligopoly models (Lambertini & Tampieri, 2015; Liu et al., 2015; Ee et al., 2018; Fukuda & Ouchida, 2020). Regarding environmental externalities, Lambertini and Tampieri (2015) studied how CSR impacts firms' profits and social welfare in the Cournot oligopoly market. Findings from their research suggest companies that engage in CSR activities are more profitable and generate more social benefits than their profit-driven competitors if the market is large enough. It was found that the competitive environment influences the motivation of businesses to engage in CSR and how CSR can benefit businesses and consumers in Liu et al. (2015) research. Investing in CSR has an effect on income distribution, as studied by Ee et al. A widening wage gap and increasing capital costs are shown to be caused by CSR investments. CSR model of time-consistent emissions tax by Fukuda and Ouchida (2020) is constructed. According to their findings, CSR is always beneficial to social welfare because it reduces emissions in the monopoly model. Our findings are in contrast to those of Fukuda and Ouchida (2020). According to our model, this abatement effect does not take place; rather, it is dependent on the level of CSR. Many studies have evaluated the impact of CSR activity on the bottom line. CSR has been shown in previous research to boost a firm's reputation, economic efficiency, stock value, employee loyalty, consumer loyalty, and green innovations (Sethi & Sethi, 1990; Noci & Verganti, 1999; Bhattacharya & Sen, 2003; Porter & Kramer, 2006, 2011; Lioui & Sharma, 2012; Flammer, 2013; Chuang & Huang, 2018; Lu et al., 2020; Stojanovic. et al., 2020; Singh et al., 2020; Dimitrova et al., 2021; Malkawi & Khayrullina, 2021; Tijani et al., 2021).

According to the literature survey, no research has coupled green technology licensing with CSR, despite the fact that multiple studies have been reported on both topics separately. In contrast to prior research, this paper will include CSR into the setting of green technology licensing with an emission tax.

2. Methods

Consider a Cournot duopoly where firms produce homogeneous products, with an inverse linear demand function: $p = a - q_1 + q_2$, where $a(a > 0)$ is the parameter of market size. $q_i (i = 1,2)$ and p donate the output and price of product. Firms' production cost is assumed as $C(q_i) = cq_i$, in which $c(a > c)$ is the unit production cost. Firms emits pollutants during their production processes, where unit of output results in one unit of emission. An emission tax τ is imposed by the government on each unit of emissions. Firms have ability to adopt green technology for abatement with the aim of emission tax reduction. Moreover, we assume $\frac{\gamma_i z_i^2}{2}$ is the cost function of firm i, where z_i is abatement effort and $\gamma_i (\gamma_i > 0)$ is the efficiency of abatement cost. After adopting green technology, firm i can abate emissions from a level of q_i to $e_i = q_i - z_i$.

In the absence of new green technologies, the efficiency of abatement cost is $\gamma_i = \gamma$. Moreover, when firms (i.e., patent holders) develops new green technologies and makes them application in order to reduce emission pollution, the efficiency of abatement cost becomes $\gamma_i = \gamma - \varepsilon$, in which ε is the degree of reduction of the abatement innovation cost. In addition, it is patent holder (i.e., firm 1 in this paper) that has the right to determine whether or not to license its green innovations to other firms 2 (i.e., firm 2) and if so, which licensing type to use.

Under fixed-fee licensing, firm 2 has to pay a fixed-fee to firm 1. Under royalty licensing, firm 2 has to pay a royalty rate to firm 1 for each unit of emission. What's more, refer to Wang (1998) and Fauli-Oller and Sandonis (2002), two-part tariff licensing degenerates into fixed-fee licensing in this paper. Therefore, we only study fixed-fee and royalty licensing with the aim of keeping model simple and tractable.

Obviously, without and with licensing, the patent holder's profits can be depicted as following, respectively:

$$\pi_1{}^N = (a - q_1 - q_2 - c)q_1 - \tau(q_1 - z_1) - \frac{(\gamma - \varepsilon)z_1{}^2}{2}; \tag{1}$$

$$\pi_1{}^L = (a - q_1 - q_2 - c)q_1 - \tau(q_1 - z_1) - \frac{(\gamma - \varepsilon)z_1{}^2}{2} + F + rz_2. \tag{2}$$

Superscripts " N " and " L " are used to defined no licensing and licensing. Likewise, the firm 2's profits can be described as Eqs (3)–(4):

$$\pi_2{}^N = (a - q_1 - q_2 - c)q_2 - \tau(q_2 - z_2) - \frac{\gamma z_2{}^2}{2}; \tag{3}$$

$$\pi_2{}^L = (a - c - q_1 - q_2)q_2 - \tau(q_2 - z_2) - \frac{(\gamma - \varepsilon)z_2{}^2}{2} - F - rz_2. \tag{4}$$

Social welfare includes four components: consumer surplus, profits of firms, tax revenues, and environmental damage.

$$SW = CS + \sum_{i=1}^{2} \pi_i + \tau \sum_{i=1}^{2} e_i - ED \tag{5}$$

in which $CS = \frac{1}{2}(q_1 + q_2)^2$ is consumer surplus. Besides, the function of pollution damage can be measured by $ED = dE$, in which $d > 0$ is marginal pollution damage and $E = \sum_{i=1}^{2} e_i$. Such a linear damage function has been widely used in the literature (Petrakis & Xepapadeas, 1998; Tsai et al., 2016; Xing et al., 2021).

Assume that firm 1 engage in CSR activities. In other words, not only firm 1 concerns its profit, but it also concerns its consumers and the damage it causes to the environment (Goering, 2012, 2014; Fanti & Buccella, 2017, 2018, 2019; Ouchida, 2019; Fukuda & Ouchida, 2020). Hence, firm 1's objective function can be described as:

$$\Omega = \pi_1 + \theta(CS - ED) \tag{6}$$

in which $\theta \in [0,1]$ (an exogenous parameter) is the level of CSR.

Based on Ouchida and Goto (2016), Martín-Herrán and Rubio (2018), Yong et al. (2018) and Fukuda and Ouchida (2020), the government do not have the capacity to commit to an emissions tax rate τ in advance. As a result, the government chooses a tax rate with the aim of maximizing social welfare after firms make emission abatement efforts. An important example of this is that Australian carbon tax rates were often adjusted over time. Indeed, we can also understand it in another way that abatement activities time-horizon can be approximately over the long run, and thus the government do not have the capacity to commit to an emission tax rate. The timing of this game is as follows: In the first stage, the patent holder (firm 1) announces how patents will be licensed and decide the types of licensing (fixed-fee licensing or royalty licensing) in order to maximize its own profit. In the second stage, the licensee (firm 2) determines whether to accept the license or not. In the third stage, both firms determine their emission abatement efforts respectively with the aiming of maximizing profit. In the fourth stage, the government chooses an emission tax rate with the aim of maximizing social welfare. In the fifth stage, firms decide their output dependently. We solve the game through backward induction method.

3. Results

Now, we examine the effects of green technology licensing arrangement and then compare equilibrium outcomes without CSR and with CSR.

3.1. Green technology licensing in the absence of CSR

We now investigate a benchmark – no CSR scenario (i.e., $\theta = 0$). Superscripts "*N*", "*LF*", and "*LR*" are assumed with the aiming of denoting no licensing , fixed-fee licensing, and royalty licensing.

3.1.1. No licensing

In this scenario, the fifth-stage game is degraded to a three-stage game. Firm 1 and firm 2 decide their output dependently in the last stage (i.e., the third stage) and the firms' profits can be depicted as Eqs (7)–(8):

$$\pi_1^N = (a - c - q_1 - q_2)q_1 - \tau(q_1 - z_1) - \frac{(\gamma - \varepsilon){z_1}^2}{2}; \tag{7}$$

$$\pi_2^N = (a - c - q_1 - q_2)q_2 - \tau(q_2 - z_2) - \frac{\gamma {z_2}^2}{2}. \tag{8}$$

The resulting equilibrium is as follows:

$$q_1{}^N = q_2{}^N = \frac{a - c - \tau}{3}. \tag{9}$$

In the second stage, an emission tax is chosen by the government with the aim of maximizing social welfare. Then, by directly solving $\frac{\partial SW^N}{\partial \tau} = 0$, we can obtain

$$\tau^N = \frac{1}{2}(c - a + 3d). \tag{10}$$

Both firms determine their emission reduction efforts respective with the aiming of optimizing profit in the first stage. Solving $\frac{\partial \pi_1^N}{\partial z_1} = 0$ and $\frac{\partial \pi_2^N}{\partial z_2} = 0$, we get:

$$z_1^N = \frac{3d + c - a}{2(\gamma - \varepsilon)}; \tag{11}$$

$$z_2^N = \frac{3d + c - a}{2\gamma}. \tag{12}$$

$d+c<a<3d+c$ is assumed to insure positive output and pollution abatement efforts. Note that this assumption differs from cost-reducing innovations scenario. Specifically, when the patent holder acts as a monopolist and has the ability to drive competitors out of business (i.e., ${q_2}^N \le 0$), cost-reducing innovations may drastic. Given that the outputs of both firms are same, the new green innovations in this paper are assumed to be non-drastic.

Substituting Eqs (9)–(12) backward, we can obtain other equilibrium outcomes:

$$\pi_1^N = \frac{(c+\tau^N-a)^2}{9} - \frac{(\gamma-\varepsilon)(z_1^N)^2}{2} + \tau^N z_1^N ; \tag{13}$$

$$\pi_2^N = \frac{(c+\tau^N-a)^2}{9} - \frac{\gamma(z_2^N)^2}{2} + \tau^N z_2^N ; \tag{14}$$

$$CS^N = \frac{2}{9}(a-\tau^N-c)^2 ; \tag{15}$$

$$\begin{aligned} SW^N = dz_1^N + dz_2^N - \frac{(d-\tau^N)(2a-2\tau^N-2c)}{3} - \\ \frac{(\gamma-\varepsilon)[(z_1^N)^2+(z_2^N)^2]}{2} + \frac{(2a-2\tau^N-2c)^2}{9}. \end{aligned} \tag{16}$$

3.1.2. Fixed-fee licensing

In this scenario, firm 1 licenses its green innovations to firm 2 by F and the cost function of firm 2's abatement activities becomes $\frac{(\gamma-\varepsilon)z_2^2}{2}$ after licensing. Similarly, solving stages 3–5 of the game, the equilibrium outcomes are given as:

$$q_1^{LF} = q_2^{LF} = \frac{a-c-d}{2} ; \tag{17}$$

$$\tau^{LF} = \frac{3d+c-a}{2} ; \tag{18}$$

$$z_1^{LF} = z_2^{LF} = \frac{3d+c-a}{2(\gamma-\varepsilon)} . \tag{19}$$

In the second stage, firm 2 decide whether to accept licensing or not. Only when the firm 2's profit after accepting licensing is no less than that before accepting licensing, can firm 2 accepts licensing i.e., $\pi_2^{LF} \ge \pi_2^N$. In the first stage, firm 1 decides to charge a fixed-fee F with the aiming of maximizing its profit. As such, the firm 1's problem can be depicted as follows

$$\max_F \pi_1^{LF} = [\frac{(c+\tau^{LF}-a)^2}{9} - \frac{(\gamma-\varepsilon)(z_1^{LF})^2}{2} + \tau^{LF} z_1^{LF} + F]$$

$$s.t.\ \pi_2^{LF} \ge \pi_2^N.$$

The optimal licensing fee F for firm1 is determined by $\pi_2^{LF} = \pi_2^N$:

$$F = \frac{\varepsilon(a-c-3d)^2}{8\gamma(\gamma-\varepsilon)}. \tag{20}$$

Substituting Eqs (17)–(20) backward, we can obtain other equilibrium outcomes:

$$\pi_1^{LF} = \frac{(c+\tau^{LF}-a)^2}{9} - \frac{(\gamma-\varepsilon)(z_1^{LF})^2}{2} + \tau^{LF} z_1^{LF} + \frac{\varepsilon(a-c-3d)^2}{8\gamma(\gamma-\varepsilon)} ; \tag{21}$$

$$\pi_2^{LF} = \frac{(c+\tau^{LF}-a)^2}{9} - \frac{(\gamma-\varepsilon)(z_2^{LF})^2}{2} + \tau^{LF} z_2^{LF} - \frac{\varepsilon(a-c-3d)^2}{8(\gamma^2-\gamma\varepsilon)}; \tag{22}$$

$$CS^{LF} = \frac{(a-d-c)^2}{2}; \tag{23}$$

$$SW^{LF} = d(z_1^{LF}+z_2^{LF}) - \frac{(\gamma-\varepsilon)[(z_1^{LF})^2+(z_2^{LF})^2]}{2} + \frac{(2a-2\tau^{LF}-2c)^2}{9} + \frac{2[-ad+cd+a\tau^{LF}-c\tau^{LF}+d\tau^{LF}-(\tau^{LF})^2]}{3}. \tag{24}$$

Next, we investigate whether firm 1 can benefit from green inventions licensing by a fixed-fee. Comparing the profits before and after licensing reveals the following Lemma 1.

Lemma 1. *Firm 1 always licenses new green technology to firm 2 under fixed-fee licensing.*

This lemma suggests that fixed-fee licensing benefits firm 1. The intuition lies in the fact that the gap between licensing profit and non-licensing profit is determined by F. In this model, F is positive and the patent holder always chooses to license its green inventions.

3.1.3. Royalty licensing

Under royalty licensing, firm 1 licenses its green inventions to firm 2 at a royalty rate r. The equilibrium outcomes are as follow:

$$q_1^{LR} = q_2^{LR} = \frac{a-c-d}{2}; \tag{25}$$

$$\tau^{LR} = \frac{1}{2}(c-a+3d); \tag{26}$$

$$z_1^{LR} = \frac{1}{2\gamma-2\varepsilon}(c-a+3d), \quad z_2^{LR} = \frac{3d+c-a-2r}{2(\gamma-\varepsilon)}. \tag{27}$$

Then, firm 1 charges the optimal royalty rate r with aim of maximizing profit and the firm 1's problem can be defined as follows,

$$\max_r \pi_1^{LR} = \left[\frac{(c+\tau^{LF}-a)^2}{9} - \frac{(\gamma-\varepsilon)(z_1^{LF})^2}{2} + \tau^{LF} z_1^{LF} + r z_2^{LR}\right]$$

$$s.t.\ \pi_2^{LR} \geq \pi_2^N.$$

Accordingly, firm 1 chooses the following royalty rate to extract the increased profit of firm 2.

$$r = \begin{cases} \frac{1}{2}[(a-c-3d)(\sqrt{1-\frac{\varepsilon}{\gamma}})-1], & 0<\frac{\varepsilon}{\gamma}<\frac{3}{4} \\ \frac{1}{4}(c-a+3d), & \frac{3}{4}<\frac{\varepsilon}{\gamma}<1 \end{cases}. \tag{28}$$

Substituting Eqs (25)–(28) backward, we can obtain other equilibrium outcomes:

$$\pi_1^{LR} = \frac{(c+\tau^{LF}-a)^2}{9} - \frac{(\gamma-\varepsilon)(z_1^{LF})^2}{2} + \tau^{LF} z_1^{LF} + r z_2^{LR}; \tag{29}$$

$$\pi_2^{LR} = \frac{(c+\tau^{LF}-a)^2}{9} - \frac{(\gamma-\varepsilon)(z_2^{LF})^2}{2} + \tau^{LF} z_2^{LF} - r z_2^{LR}; \tag{30}$$

$$CS^{LR} = \frac{(a-d-c)^2}{2}; \tag{31}$$

$$SW^{LR} = \frac{4(a-c-\tau^{LR})^2}{9} + \frac{2(\tau^{LR}-d)(a-c-\tau^{LR})}{3} + d(z_1^{LR}+z_2^{LR}) - \frac{(\gamma-\varepsilon)[(z_1^{LR})^2+(z_2^{LR})^2]}{2}. \tag{32}$$

Lemma 2. *Firm 1 always licenses new green technology to firm 2 under royalty licensing in the scenario of no CSR.*

The reason is the same as Lemma 1.

3.1.4. Fixed-fee licensing versus royalty licensing in the absence of CSR

In this section, we compare the equilibrium outcomes of the two types of licensing.

Proposition 1. *The optimal licensing strategy preferred by the government and the patent holding firm are the same, a fixed-fee licensing.*

For both licensing contracts, the output and abatement efforts remain unchanged. Hence, the gap between fixed-fee licensing profit and royalty licensing profit is determined by license fee. Due to F is larger than rz_2^{LR} (see Proof in Appendix), the patent holder prefers to choose the fixed fee licensing strategy. Similarly, this is true for the government. The intuition behind Proposition 1 is clear. It is fixed-fee licensing that can increase firms' profits and motivate firms to make more abatement effort, resulting in increasing social welfare. According to Proposition 1, there is no contradiction between corporate incentive goals and social welfare. What's more, both types of licensing contracts benefit social welfare.

Proposition 2. *Both types of licensing* contract *are the best options from consumers' standpoint in the absence of CSR.*

The intuition behind this result could be provided by the following argument: the output remains unchanged regardless of the types of licensing that patent holder used. Hence, the consumer surplus determined by the output is equal in both types of licensing. In other words, both types of licensing contracts are the best options from the view of customers.

3.2. Green technology licensing in the presence of CSR

Subscript "C" is assumed with the aiming of denoting the scenario of CSR.

3.2.1. No licensing

Here are the equilibrium outcomes:

$$q_{1C}^N = \frac{a(1+\theta)-c(1+\theta)-d(1+2\theta)}{2},\ q_{2C}^N = \frac{a(1-\theta)-c(1-\theta)-d(1-2\theta)}{2}; \tag{33}$$

$$z_{1C}^N = \frac{3d-a(1-\theta)+c(1-\theta)}{2\gamma-2\varepsilon},\ z_{2C}^N = \frac{d(3-2\theta)+c(1-\theta)-a(1-\theta)}{2\gamma}; \tag{34}$$

$$\tau_C^N = \frac{d(3-2\theta)+c(1-\theta)-a(1-\theta)-c\theta}{2}; \tag{35}$$

$$\Omega_C^N = \frac{(a^2 - 2ac + c^2)(1 + 2\theta - \theta^2) + d^2(1 + 6\theta - 4\theta^2) + 2d(c - a)(1 + 4\theta - 2\theta^2)}{4} + d\theta(z_{1C}^N + z_{2C}^N) + \tau_C^N z_{1C}^N - \frac{(\gamma - \varepsilon)(z_{1C}^N)^2}{2}; \tag{36}$$

$$\pi_{1C}^N = \frac{(a - d - c)^2 - (a - 2d - c)^2\theta^2}{4} - \frac{(\gamma - \varepsilon)(z_{1C}^N)^2}{2} + \tau_C^N z_{1C}^N; \tag{37}$$

$$\pi_{2C}^N = \frac{[a - d - c - (a - 2d - c)\theta]^2}{4} - \frac{\gamma(z_{2C}^N)^2}{2} + \tau_C^N z_{2C}^N; \tag{38}$$

$$CS_C^N = \frac{(a - d - c)^2}{2}; \tag{39}$$

$$SW_C^N = \frac{1}{2}(a - d - c)^2 + d(z_{1C}^N + z_{2C}^N) - \frac{(\gamma - \varepsilon)(z_{1C}^N)^2}{2} - \frac{\gamma(z_{2C}^N)^2}{2}. \tag{40}$$

$2d + c < a < 3d + c$ is assumed to insure positive green innovation investment and output, implying green innovation is non-drastic. The following are some comparative-static effects with respect to θ.

Lemma 3. $\frac{\partial q_{1C}^N}{\partial\theta} > 0$, $\frac{\partial q_{2C}^N}{\partial\theta} < 0$, $\frac{\partial z_{1C}^N}{\partial\theta} > 0$, and $\frac{\partial z_{2C}^N}{\partial\theta} > 0$.

This lemma describes how CSR degree influences firms' decision making. According to these results, an increase in the CSR degree drives firm 1 (the CSR firm) to increase output whereas firm 2 (the non-CSR firm) reduces production. In the meanwhile, an increase in the CSR degree motivates firms to make more abatement efforts. The corresponding intuition is that: the marginal social concern grows as the CSR degree rises, resulting in an increase in firm 1's output and abatement efforts. In contrast, the non-CSR firm's production decrease due to the output substitution impact. With the aim of compensating for the loss of profits, firm 2 shall strengthen its emission reduction efforts.

3.2.2. Fixed-fee licensing

Here are the equilibrium outcomes:

$$q_{1C}^{LF} = \frac{a(1+\theta) - c(1+\theta) - d(1 - 2\theta)}{2}, \quad q_{2C}^{LF} = \frac{a(1-\theta) - d(1 - 2\theta) - c(1 - \theta)}{2}; \tag{41}$$

$$z_{1C}^{LF} = \frac{3d + c(1-\theta) - a(1-\theta)}{2\gamma - 2\varepsilon}, \quad z_{2C}^{LF} = \frac{d(3 - 2\theta) - a(1-\theta) + c(1-\theta)}{2\gamma - 2\varepsilon}; \tag{42}$$

$$\tau_C^{LF} = \frac{d(3 - 2\theta) - a(1 - \theta) + c(1 - \theta)}{2}; \tag{43}$$

$$F_C = \frac{\varepsilon[c - a + \theta(a - c - 2d) + 3d]^2}{8\gamma(\gamma - \varepsilon)}; \tag{44}$$

$$\Omega_C^{LF} = \frac{(a - c)^2(1 + 2\theta - \theta^2) + d^2(1 + 6\theta - 4\theta^2) + 2d(c - a)(1 + 4\theta - 2\theta^2)}{4} + d\theta(z_{1C}^{LF} + z_{2C}^{LF}) + \tau_C^{LF} z_{1C}^{LF} - \frac{(\gamma - \varepsilon)(z_{1C}^{LF})^2}{2} + \frac{\varepsilon[3d + c - a + \theta(a - c - 2d)]^2}{8\gamma(\gamma - \varepsilon)}; \tag{45}$$

$$\pi_{1C}^{LF} = \frac{(a-d-c)^2 - (a-2d-c)^2\theta^2}{4} - \frac{(\gamma-\varepsilon)(z_{1C}^{LF})}{2} + \tau^{LF} z_{1C}^{LF} + \frac{\varepsilon[3d+c-a+\theta(a-c-2d)]^2}{8\gamma(\gamma-\varepsilon)}; \quad (46)$$

$$\pi_{2C}^{LF} = \tau^{LF} z_{2C}^{LF} - \frac{(\gamma-\varepsilon)(z_{2C}^{LF})^2}{2} - \frac{\varepsilon[3d+c-a+(-c-2d+a)\theta]^2}{8\gamma(\gamma-\varepsilon)} + \frac{(c+d-a+a\theta-c\theta-2d\theta)^2}{4}; \quad (47)$$

$$CS_C^{LF} = \frac{(a-d-c)^2}{2}; \quad (48)$$

$$SW_C^{LF} = \frac{1}{2}(d+c-a)^2 + d(z_{1C}^{LF} + z_{2C}^{LF}) - \frac{(\gamma-\varepsilon)[(z_{1C}^{LF})^2 + (z_{2C}^{LF})^2]}{2}. \quad (49)$$

Using Eqs (36) and (45), we can derive the following Lemma 4.

Lemma 4. *Firm 1 always licenses new green technology to firm 2 under fix fee licensing in the presence of CSR.*

The idea underlying this lemma is as follows: the gap between no licensing payoff and fixed-fee licensing payoff is $\Omega_C^{LF} - \Omega_C^{N} = d(z_{2C}^{LF} - z_{2C}^{N}) + F_C$, including $d(z_{2C}^{LF} - z_{2C}^{N})$ and F_C. Since $z_{2C}^{LF} - z_{2C}^{N} > 0$ and $F_C > 0$, it is better for firm 1 to license its inventions with a fix fee. Notably, the result in the presence of CSR is similar to that in the absence of CSR.

3.2.3. Royalty licensing

Here are the equilibrium outcomes:

$$q_{1C}^{LR} = \frac{a-d-c+a\theta-c\theta-2d\theta}{2},\ q_{2C}^{LR} = \frac{a-d-c-a\theta-2d\theta-c\theta}{2}; \quad (50)$$

$$z_{1C}^{LR} = \frac{3d+c-a+a\theta-c\theta}{2\gamma-2\varepsilon},\ z_{2C}^{LR} = \frac{3d+c-a+a\theta-c\theta-2d\theta-2r_C}{2\gamma-2\varepsilon}; \quad (51)$$

$$\tau_C^{LR} = \frac{3d+c-a+a\theta-2d\theta-c\theta}{2}; \quad (52)$$

$$r_C = \begin{cases} \dfrac{[3d+c-a+(a\theta-c\theta-2d\theta)](1-\sqrt{1-\frac{\varepsilon}{\gamma}})}{2}, & 0<\dfrac{\varepsilon}{\gamma}<\dfrac{3}{4}; \\ \dfrac{3d+c-a+\theta(a-c-2d)}{4}, & \dfrac{3}{4}<\dfrac{\varepsilon}{\gamma}<1 \end{cases} \quad (53)$$

$$\Omega_C^{LR} = \frac{(a-c)^2(1+2\theta-\theta^2) + d^2(1+6\theta-4\theta^2) + 2d(c-a)(1+4\theta-2\theta^2)}{4} + d\theta(z_{1C}^{LR} + z_{2C}^{LR}) - \frac{(\gamma-\varepsilon)(z_{1C}^{LR})^2}{2} + \tau_C^{LR} z_{1C}^{LR} + r_C z_{2C}^{LR}; \quad (54)$$

$$\pi_{1C}^{LR} = \frac{(a-d-c)^2 - (a-2d-c)^2\theta^2}{4} - \frac{(\gamma-\varepsilon)(z_{1C}^{LR})^2}{2} + \tau^{LR} z_{1C}^{LR} + r_C z_{2C}^{LR}; \quad (55)$$

$$\pi_{2C}^{LR} = \frac{[-a(1-\theta)+c(1-\theta)+d(1-2\theta)]^2}{4} - \frac{(\gamma-\varepsilon)(z_2^{LR})^2}{2} + z_2^{LR}(\tau_C^{LR} - r_C); \quad (56)$$

$$CS_C^{LR} = \frac{(a-d-c)^2}{2}; \quad (57)$$

$$SW_C^{LR} = \frac{1}{2}(c+d-a)^2 + d(z_{1C}^{LR} + z_{2C}^{LR}) - \frac{(\gamma-\varepsilon)[(z_{1C}^{LR})^2 + (z_{2C}^{LR})^2]}{2}. \quad (58)$$

Lemma 5. *Firm 1 always licenses new green technology to firm 2 under royalty licensing in the precent of CSR.*

The intuition is the same as that of Lemma 4.

3.2.4. Fixed-fee licensing versus royalty licensing in the presence of CSR

In this section, we compare the equilibrium outcomes of the two types of licensing.

Proposition 3. (i) *if* $0 < \frac{\varepsilon}{\gamma} < \frac{3}{4}$, *fixed-fee licensing is the best options for any value of CSR degree from the patent holding firm' s standpoint* (ii) *if* $\frac{3}{4} < \frac{\varepsilon}{\gamma} < 1$, *royalty licensing is the best options if CSR degree is high*; *fixed-fee licensing is optimal if CSR degree is low from patent holding firm' s standpoint.*

A patent holding firm is more likely to license green technologies to other companies for a fixed charge if ε is low. Nevertheless, if ε is large, the optimal licensing arrangement changes from fixed-fee to royalty licensing as CSR degree increases. It is worth noting that the result is different from the scenario in which no firm engages in CSR activities (i.e., CSR degree is 0). The intuition behind this result could be provided by the following argument: without CSR, the licensing fee only depends on firms' emission abatement efforts since both firms have the same level of output. In contrast, the licensing fee relies on firms' emission abatement efforts and output with CSR, resulting in optimal green technology licensing strategy which is determined by CSR degree.

Proposition 4. *Fixed-fee licensing and royalty licensing are both the best options from consumers' standpoint when it comes to CSR.*

Proposition 4 can be analyzed in the same way as Proposition 2.

Proposition 5. *The optimal licensing strategy preferred by the government is fixed-fee licensing when it comes to CSR.*

Different from most existing studies, such as those conducted by Kabiraj (2005), Erutku and Richelle (2007) and Li and Song (2009), Propositions 3 and Propositions 5 suggest that there are never a conflicts or inconsistencies between firms' payoff goals and social welfare. If the CSR degree is high, optimal license contracts may conflict. Specifically, the patent holder prefers a royalty licensing contract while the government prefers fixed-fee licensing contract. However, if CSR degree is low, fixed-fee licensing is always the best option for both. It is worth noting that Propositions 3 and Propositions 5 are not in agreement with Propositions 1.

3.3. Comparison of results

Now, we compare the equilibrium outcomes with and without CSR. After doing some straightforward computations, we are able to get the following lemma and proposition.

Lemma 6. *A comparison of the equilibrium results with and without CSR is as follows:*

i) $q_{1C}^{LF} > q_1^{LF}$, $q_{1C}^{LR} > q_1^{LR}$, $z_{1C}^{LF} > z_1^{LF}$, $z_{1C}^{LR} > z_1^{LR}$, *and* $\pi_{1C}^{LF(R)} > (<) \pi_1^{LF(R)}$ *for* $\theta < (>) \theta^{LF(R)}$, *where* $\theta^{LF(R)}$ *satisfies* $\pi_{1C}^{LF(R)} = \pi_1^{LF(R)}$ *and* $0 < \theta^{LF(R)} \le 1$;

ii) $q_{2C}^{LF} < q_2^{LF}$, $q_{2C}^{LR} < q_2^{LR}$, $z_{2C}^{LF} > z_2^{LF}$, $z_{2C}^{LR} > z_2^{LR}$, *and* $\pi_{2C}^{LF(R)} > (<) \pi_2^{LF(R)}$ *for* $\theta < (>) \theta^{LF(R)}$, *where* $\theta^{LF(R)}$ *satisfies* $\pi_{2C}^{LF(R)} = \pi_2^{LF(R)}$ *and* $0 < \theta^{LF(R)} \le 1$.

From Lemma 6-(i), we can easily indicate that the impact of CSR on firm is twofold: an output enhancing effect and abatement enhancing effect. In addition, the patent holder (i.e., firm 1) is able to increase its profit by engage in CSR activities when CSR degree is low. An abatement cost effect and a tax payment effect make up abatement enhancing effect. In details, if CSR degree is low, the tax payment effect is favorable. In this context, the emission abatement enhancing effect depends on the output enhancing effect.

Lemma 6-(ii) states that with CSR degree increasing, the production substitute effect of CSR firms on non-CSR firms increases. Moreover, the abatement effort of the non-CSR firm increases. This is because if firm 1 takes CSR activities and firm 2 is a profit-oriented firm, firm 1 has ability to draw more consumers from its competitors, leading an increasing output of firm 1 while a decreasing output of firm 2. Further, this lemma demonstrates that the impact of CSR on firm 2 is uncertain. The economic intuition is the same as that in Lemma 6-(i).

Proposition 6. $E_C^{LR(F)} < E^{LR(F)}$; $SW_C^{LR(F)} > (<) SW^{LR(F)}$ *for* $\theta < (>) \theta^{LR(F)}$, *where* $\theta^{LR(F)}$ *satisfies* $SW_C^{LR(F)} = SW^{LR(F)}$ *and* $0 < \theta^{LR(F)} \le 1$.

This proposition implies that the emissions without CSR are more than those with CSR for any value of θ. The reason is as follows: according to Lemma 6, CSR has an influence of abatement-enhancing on both firms, but has no effect on the output level of the entire industry (i.e., $q_{1C}^{LF} + q_{2C}^{LF} = q_1^{LF} + q_2^{LF}$ and $q_{1C}^{LR} + q_{2C}^{LR} = q_1^{LR} + q_2^{LR}$). Hence, total emissions decrease as CSR degree increases. In addition, a higher CSR degree ($\theta > \theta^{LF(R)}$) makes social welfare without CSR greater than social welfare with CSR. Nevertheless, a lower CSR degree ($0 < \theta < \theta^{LF(R)}$) has an opposite effect on social welfare. The economic intuition behind this is as follows. The cost of abatement and total emissions determine the difference between social welfare with CSR and social welfare without CSR. Obviously, the marginal increase in θ increases the cost of abatement while reduce firms' total emissions. Social welfare changes as a result of both factors working together. When θ is low, the effect of the former is greater than that of the latter, leading to increased social welfare. However, when θ is low, the effect of the latter is greater than that of the former, leading to decreased social welfare.

This proposition demonstrates that CSR doesn't always benefit social welfare. In contrast to what was found by Fukuda and Ouchida (2020), who show that, the higher the CSR degree, the higher the social welfare in a monopoly model. Their framework includes two distortions. One is an external diseconomy resulting from a monopolist's emissions, and the other one is a distortion resulting from a monopolist's market power. Emission tax can address the external diseconomy, but it cannot address the market power. Due to the impossibility of correcting the distortion of market power within its framework, emissions are reduced as a result of output reduction resulting from CSR. In sum, social welfare increases as CSR degree increases because of the emission reducing effect caused by CSR. Nevertheless, this effect does not exist in our model, and social welfare is determined by CSR degree. Specifically, when CSR degree is low, CSR benefits social welfare. When CSR degree is high, the effect is reversed.

4. Discussion

We conduct a few suitable numerical experiments in this section. First, a numerical example is used to describe the best way to license green technology in the absence of CSR and in the presence of CSR. Further, we perform a sensitivity analysis of the equilibrium results with respect to ε, γ and *d*. According to Fukuda and Ouchida (2020), we assume $\gamma = 10$, $d = 0.15$, $a = 1, c = 0.6$, $\varepsilon = 7, \theta = 0.5$.

4.1. Optimal licensing strategies from patent holder and government's standpoint

The following figures numerically indicates the best way to license green technology in the absence of CSR and in the presence of CSR. Noting that we assume ε = 7 and ε = 9 to describe the impacts of CSR degree on the licensing strategies. The following are the most significant findings:

Figure 1 shows that fixed-fee licensing is more favorable to firm 1's profit and social welfare for any value of ε if there is no CSR. This finding in agreement with proposition 1. What's more, Figure 2 suggests that fixed-fee licensing is more favorable to firm 1's profit than

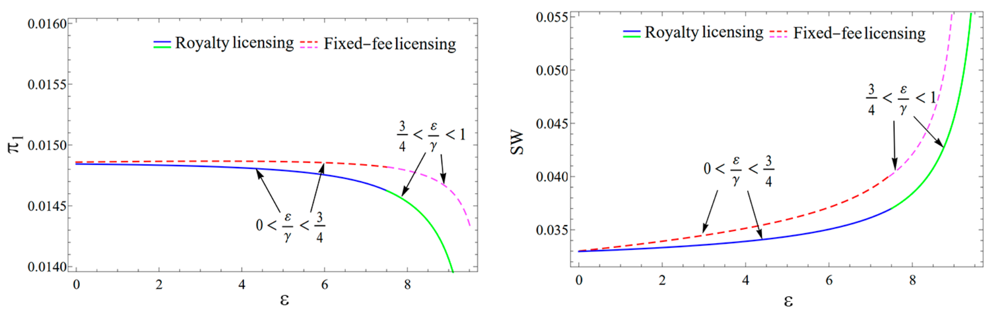


Figure 1. Optimal licensing strategies for patent holder in the absence of CSR

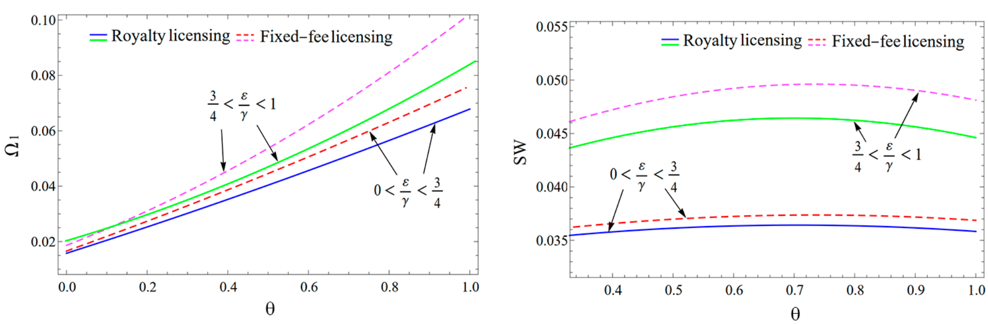


Figure 2. Optimal licensing strategies for patent holder in the precent of CSR

royalty licensing when $0 < \frac{\varepsilon}{\gamma} < \frac{3}{4}$ in the precent of CSR. In contrast, in the precent of CSR, when $\frac{3}{4} < \frac{\varepsilon}{\gamma} < 1$, royalty licensing is more favorable to firm 1's profit than fixed-fee licensing for $0 < \theta < 0.03$, while fixed-fee licensing is more favorable to firm 1's profit than royalty licensing for $0.03 < \theta < 1$. Further, fixed-fee licensing is more favorable to social welfare than royalty licensing for any value of ε in the precent of CSR.

4.2. Analytical sensitivity

In order to gain more managerial insights, the analytical sensitivity of key parameters, ε, γ and *d* is conducted.

4.2.1. Analytical sensitivity on total emissions

Analytical sensitivity of ε, γ, and *d* on total emissions with different θ are depicted in the following figures.

Figure 3 indicates that if θ remains a constant, total emissions decrease with ε regardless the types of licensing contracts. The reason for this is that as ε increases, the cost of green technology innovation decreases and firms make more emission reduction efforts, resulting in more emission reductions. The impact of *d* is similar to total emissions. Nevertheless, total emissions decrease with γ. The rationale is that the firms are motivated to make more abatement efforts if γ is low, which reducing total emissions. Figure 3 also states that if other parameters remain a constant, total emissions decrease with θ.

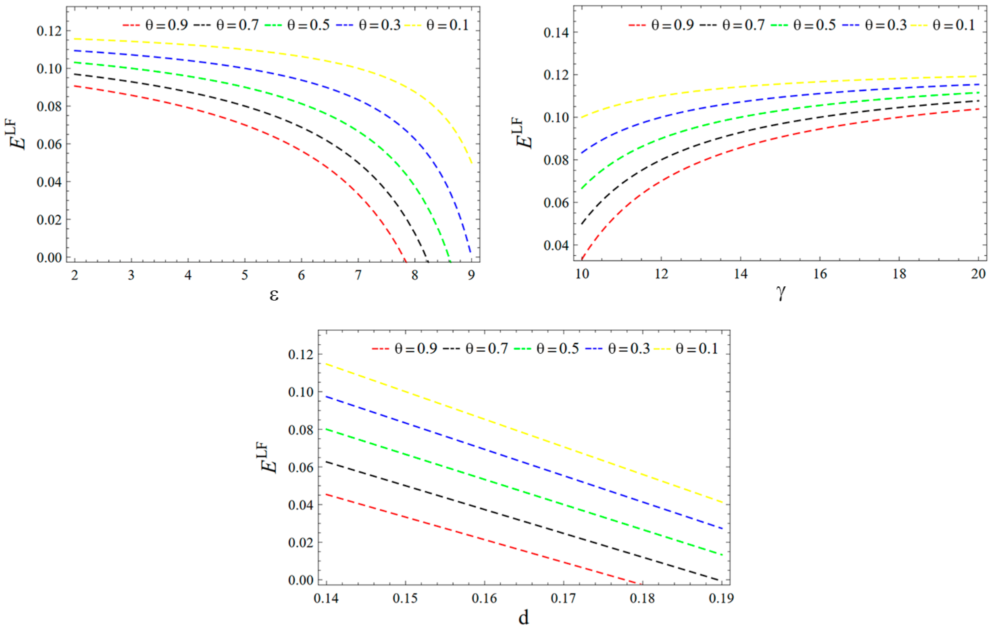
4.2.2. Analytical sensitivity on firm 1 and firm 2's profits

The analytical sensitivity of ε, γ and *d* on firms' profits with different θ are depicted in the following figures.

Figure 4 suggests that the relationship between ε and firm 1's profit is determined by θ regardless the types of licensing contracts. If θ is high, firm 1's profit increases with ε. Nevertheless, if θ is low, firm 1's profit decreases with ε. In addition, Figure 4 shows an inverse relationship between firm 1's profit and γ. Specifically, firm 1's profit decreases as γ increases if θ is sufficiently larger while firm 1's profit increases as γ if θ is sufficiently low. Moreover, Figure 4 indicates firm 1's profit decreases with *d*. What's more, Figure 4 states that if other parameters remain a constant, firms' profits increase with θ. The finding demonstrates that CSR benefits firms' profits because of the abatement-enhancing effect caused by CSR.

Figure 5 suggests that ε has no impact on firm 2's profit, while firm 2's profit decreases as γ regardless the types of licensing contracts. Moreover, Figure 5 shows a non-linear relationship between *d* and firm 2's profit. In other words, if θ is low, firm 2's profit decreases with *d*. If θ is high, firm 2's profit increases with *d*. What's more, Figure 5 indicates that if other parameters remain a constant, firm 1's profit increases with θ.

(i) Fixed-fee licensing



(ii) Royalty licensing

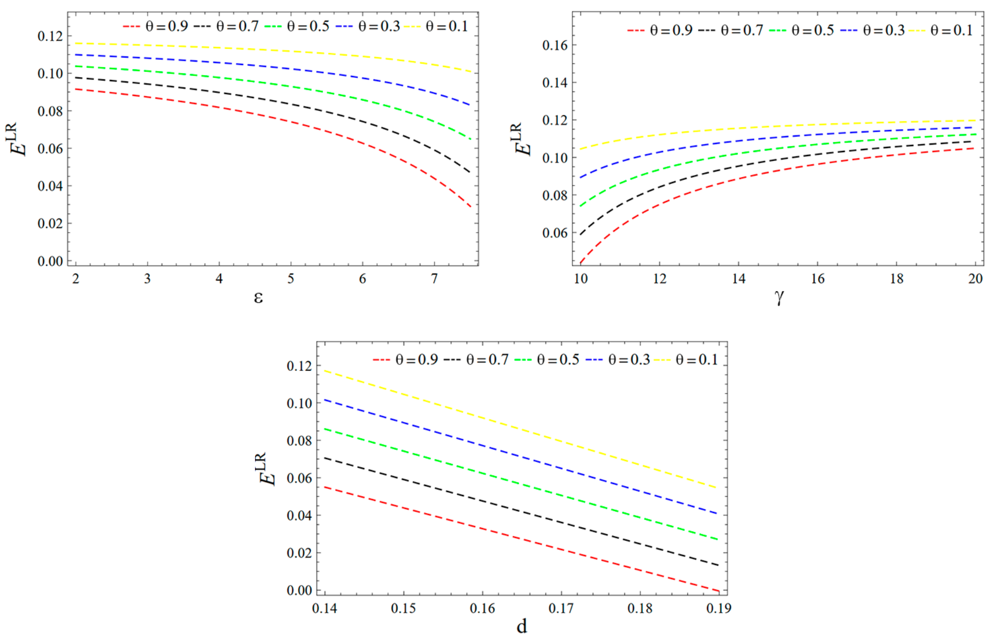
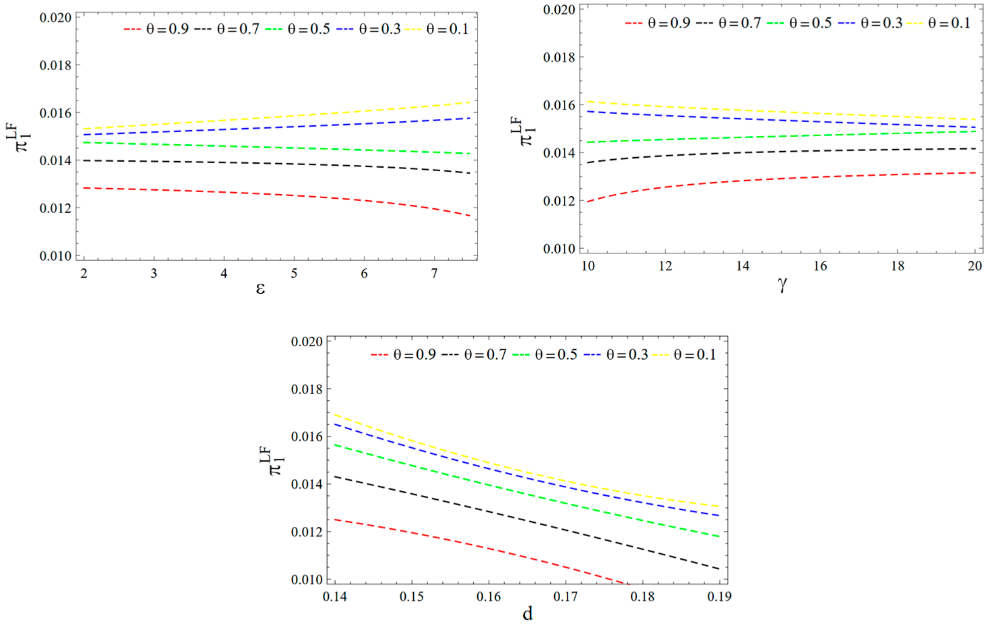


Figure 3. Analytical sensitivity on $E^{LR(F)}$ with different θ

(i) Fixed-fee licensing



(ii) Royalty licensing

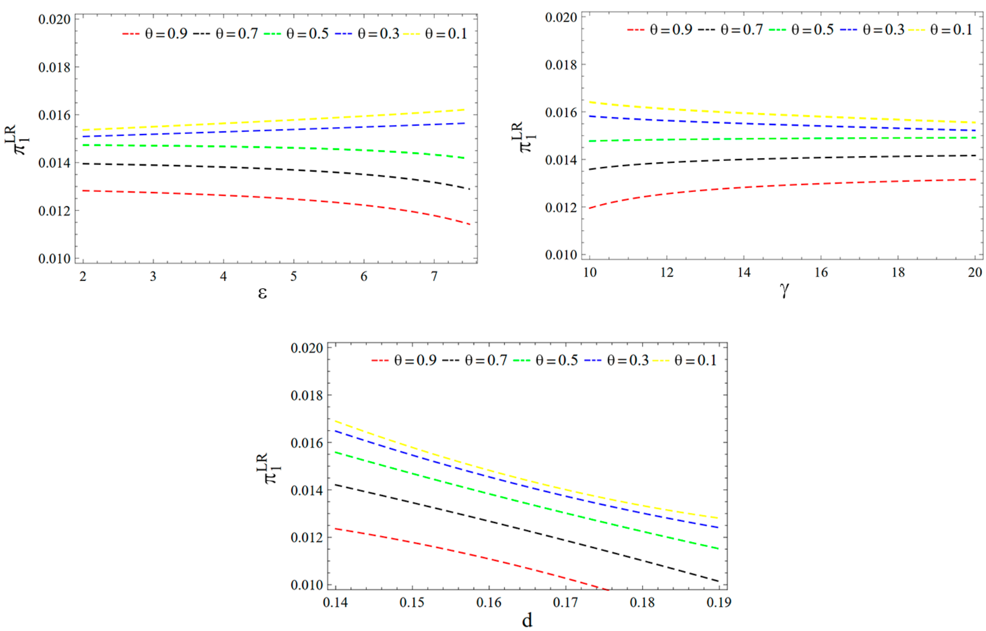
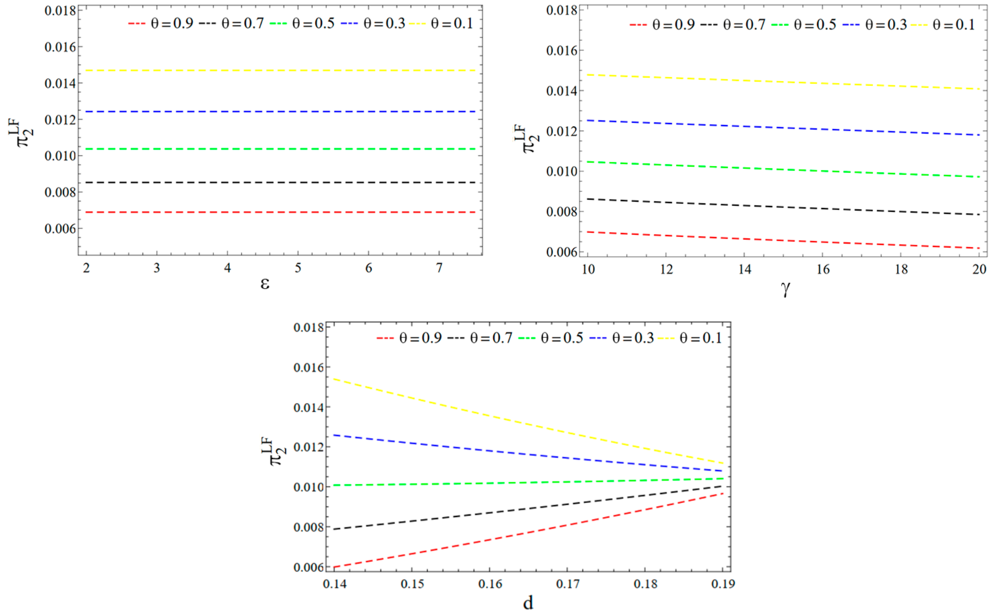


Figure 4. Analytical sensitivity on $\pi_1^{LR(F)}$ with different θ

(i) Fixed-fee licensing



(ii) Royalty licensing

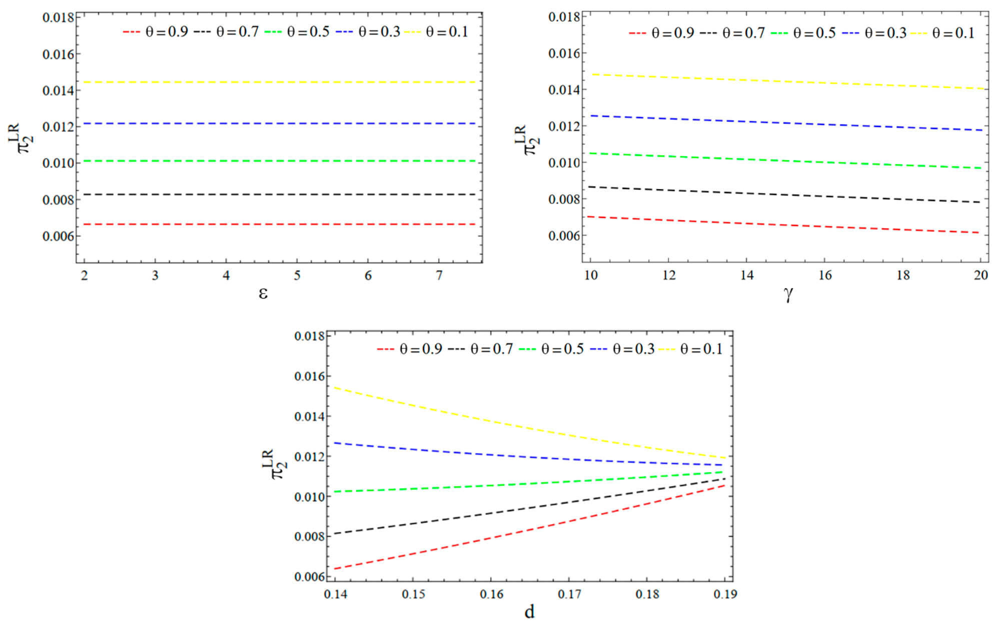
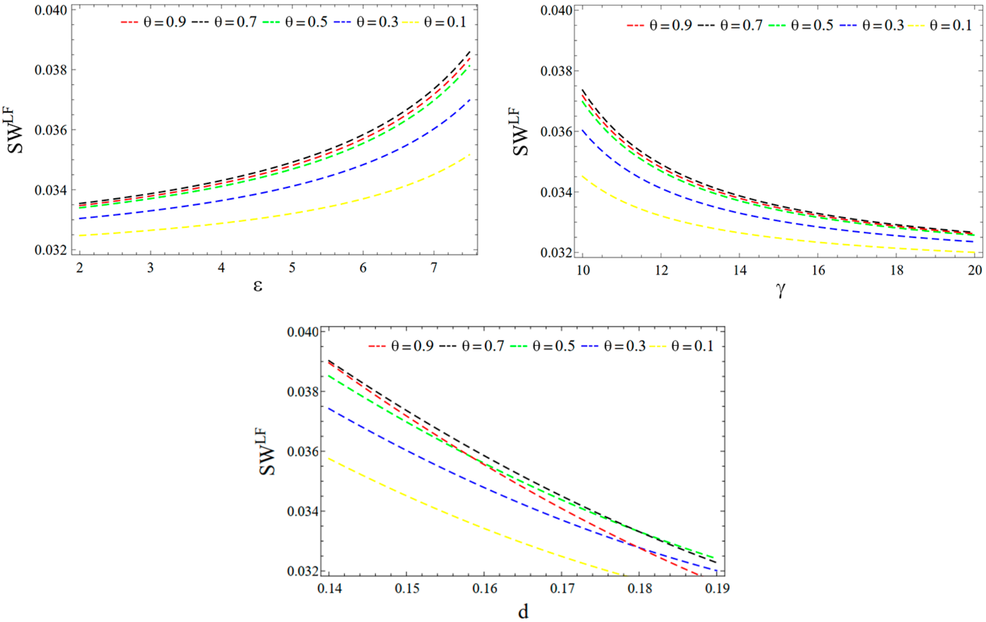


Figure 5. Analytical sensitivity on $\pi_2^{LR(F)}$ with different θ

(i) Fixed-fee licensing



(ii) Royalty licensing

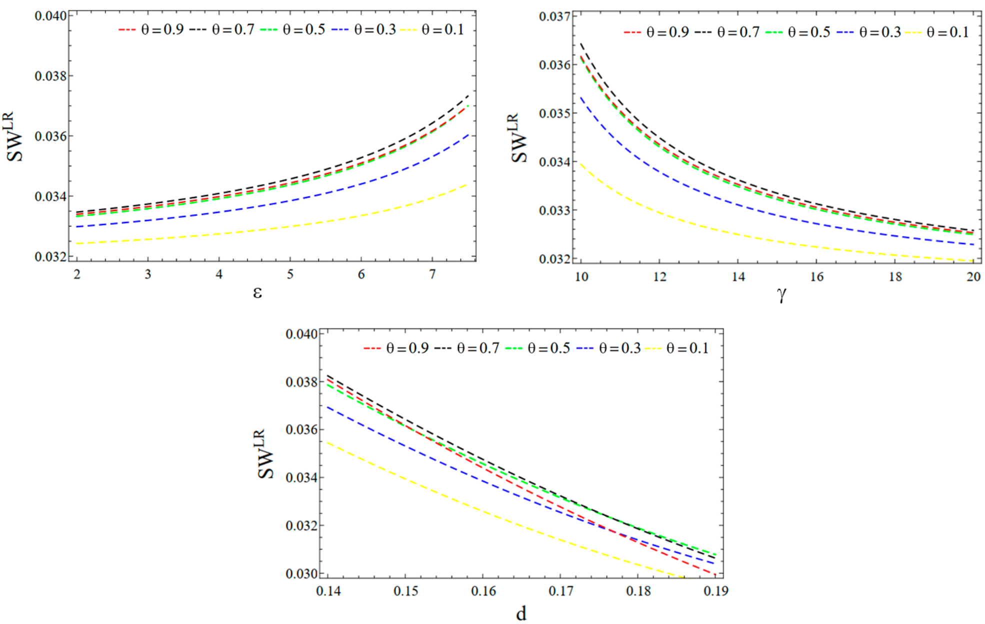


Figure 6. Analytical sensitivity on $SW^{LR(F)}$ with different θ

4.2.3. Analytical sensitivity on social welfare

Finally, the analytical sensitivity of ε, γ, and d on social welfare with different θ are depicted in the following figures.

From Figure 6, we can reveal that for both licensing contracts, if θ remains a constant, social welfare increases with ε. This is because the higher ε, the more the firms' abatement effects, which in turn increases social welfare. However, γ and d have different impacts on social welfare. This is because for a given θ, if γ and d are higher, firms are more motivated to reduce emissions, which increases social welfare. Last but not least, Figure 6 suggests that if other parameters remain a constant, social welfare first increases and the decreases with θ, which is identical to Proposition 6.

Conclusions

Main results

In this paper, we examine the optimal green technology licensing strategy with CSR and study how environment and social welfare are impacted by CSR. Following are the main results: Firstly, fixed-fee licensing with no CSR is always the best option for a patent holder. In contrast, when a patent holding firm engages in CSR, the situation is different. The best option changes from fixed-fee licensing to royalty licensing as the level of CSR increases. In other words, for the patent holder, royalty licensing is the best option if the level of CSR degree is high; fixed-fee licensing is the best option if the level of CSR degree is low.

Secondly, we demonstrate that there are no contradictions or inconsistencies between corporate incentive goal and social welfare in the context of CSR. If the level of CSR is high, there is a contradiction between firm profit goal and government social welfare goal. Specifically, royalty licensing is preferred by the patent holding firm, whereas fixed-fee licensing is preferred by the government. In contrast, there is no contradiction between firm profit goal and government social welfare goal if the level of CSR is low. In other words, fixed-fee licensing contract is optimal for both.

Thirdly, CSR is not always beneficial to the social welfare while CSR benefits the environment. Specifically, social welfare without CSR is higher than social welfare with CSR for a high level of CSR, while social welfare without CSR is lower than social welfare with CSR for a low level of CSR.

Limitations and future research

Our work can be extended in the following ways. Firstly, we only study the optimal green technology licensing strategy with CSR in a duopoly market. Future research may examine that in a mixed oligopoly. Secondly, we only consider a case where only one firm engages in CSR activities and the other doesn't engage in CSR activities. In fact, both firms may adopt the CSR strategy. Further research may extend research in this regard, i.e., both firms engage in CSR activities. Lastly, the CSR degree is an exogenous parameter in this paper. However, firms always maximize their objective function with respect to the level of CSR. Therefore, it is interesting to consider the case in which the level of CSR is endogenously chosen by firm 1.

Acknowledgements

This work was supported by the Social Sciences of Ministry of Education Foundation of PR China (22YJC630098) and the National Natural Science Foundation of PRC (72201207, 72202177, 72202173, 72102182, 71974081).

References

Antelo, M., & Sampayo, A. (2017). On the number of licenses with signalling. *The Manchester School*, *85*(6), 635–660. https://doi.org/10.1111/manc.12157

Bhattacharya, C. B., & Sen, S. (2003). Consumer-company identification: A framework for understanding consumers' relationships with companies. *Journal of Marketing*, *67*(2), 76–88. https://doi.org/10.1509/jmkg.67.2.76.18609

Chen, Y.-W., Yang, Y.-P., Wang, L. F. S., & Wu, S.-J. (2014). Technology licensing in mixed oligopoly. *International Review of Economics and Finance*, *31*, 193–204. https://doi.org/10.1016/j.iref.2014.02.002

Chuang, S. P., & Huang, S. J. (2018). The effect of environmental corporate social responsibility on environmental performance and business competitiveness: The mediation of green information technology capital. *Journal of Business Ethics*, *150*(4), 991–1009. https://doi.org/10.1007/s10551-016-3167-x

Dimitrova, M., Treapat, L. M., & Tulaykova, I. (2021). Value at risk as a tool for economic-managerial decision-making in the process of trading in the financial market. *Ekonomicko-manazerske Spektrum*, *15*(2), 13–26. https://doi.org/10.26552/ems.2021.2.13-26

Durana, P., Perkins, N., & Valaskova, K. (2021). Artificial intelligence data-driven internet of things systems, real-time advanced analytics, and cyber-physical production networks in sustainable smart manufacturing. *Economics*, *Management*, *and Financial Markets*, *16*(1), 20–30. https://doi.org/10.22381/emfm16120212

Ee, M. S., Chao, C. C., Wang, L. F., & Eden, S. H. (2018). Environmental corporate social responsibility, firm dynamics and wage inequality. *International Review of Economics* & *Finance*, *56*, 63–74. https://doi.org/10.1016/j.iref.2018.03.018

Erutku, C., & Richelle, Y. (2007). Optimal licensing contracts and the value of a patent. *Journal of Economics* & *Management Strategy*, *16*(2), 407–436. https://doi.org/10.1111/j.1530-9134.2007.00144.x

Fanti, L., & Buccella, D. (2017). Bargaining agenda in a unionized monopoly with network effects: When corporate social responsibility may be welfare-reducing. *Economia Politica*, *34*(3), 471–489. https://doi.org/10.1007/s40888-016-0051-1

Fanti, L., & Buccella, D. (2018). Corporate social responsibility and the choice of price versus quantities. *Japan and the World Economy*, *48*, 71–78. https://doi.org/10.1016/j.japwor.2018.08.004

Fanti, L., & Buccella, D. (2019). Managerial delegation games and corporate social responsibility. *Managerial and Decision Economics*, *40*(6), 610–622. https://doi.org/10.1002/mde.3031

Fauli-Oller, R., & Sandonis, J. (2002). Welfare reducing licensing. *Games and Economic Behavior*, *41*(2), 192–205. https://doi.org/10.1016/S0899-8256(02)00513-4

Flammer, C. (2013). Corporate social responsibility and shareholder reaction: The environmental awareness of investors. *Academy of Management Journal*, *56*(3), 758–781. https://doi.org/10.5465/amj.2011.0744

Fukuda, K., & Ouchida, Y. (2020). Corporate social responsibility (CSR) and the environment: Does CSR increase emissions? *Energy Economics*, *92*, 104933. https://doi.org/10.1016/j.eneco.2020.104933

Goering, G. E. (2012). Corporate social responsibility and marketing channel coordination. *Research in Economics*, *66*(2), 142–148. https://doi.org/10.1016/j.rie.2011.10.001

Goering, G. E. (2014). The profit-maximizing case for corporate social responsibility in a bilateral monopoly. *Managerial and Decision Economics*, *35*(7), 493–499. https://doi.org/10.1002/mde.2643

Hattori, K. (2017). Optimal combination of innovation and environmental policies under technology licensing. *Economic Modelling*, *64*, 601–609. https://doi.org/10.1016/j.econmod.2017.04.024

Hattori, M., & Tanaka, Y. (2018). License and entry strategies for an outside innovator under duopoly. *Italian Economic Journal*, *4*(1), 135–152. https://doi.org/10.1007/s40797-017-0048-0

Hattori, M., & Tanaka, Y. (2021). License and entry strategies for an outside innovator in Stackelberg duopoly with royalty and fixed-fee under vertical differentiation. *International Journal of Economic Theory*, *17*(3), 234–257. https://doi.org/10.1111/ijet.12219

International Organization for Standardization. (2010). *Guidance on social responsibility* (ISO/FDIS 26000: 2010) (E), Final Draft. Geneva.

Jeon, H. (2019). Licensing and information disclosure under asymmetric information. *European Journal of Operational Research*, *276*(1), 314–330. https://doi.org/10.1016/j.ejor.2019.01.005

Kabiraj, T. (2005). Technology transfer in a Stackelberg structure: Licensing contracts and welfare. *The Manchester School*, *73*(1), 1–28. https://doi.org/10.1111/j.1467-9957.2005.00421.x

Kim, S. L., & Lee, S. H. (2014). Eco-technology licensing under emission tax: Royalty vs. Fixed-fee. *Korean Economic Review*, *30*(2), 273–300.

Kim, S. L., Lee, S. H., & Matsumura, T. (2018). Eco-technology licensing by a foreign innovator and privatization policy in a polluting mixed duopoly. *Asia-Pacific Journal of Accounting* & *Economics*, *25*(3–4), 433–448. https://doi.org/10.1080/16081625.2017.1339617

Kliestik, T., Nica, E., Musa, H., Poliak, M., & Mihai, E. A. (2020). Networked, smart, and responsive devices in Industry 4.0 manufacturing systems. *Economics*, *Management*, *and Financial Markets*, *15*(3), 23–29. https://doi.org/10.22381/EMFM15320203

Kovacova, M., & Lazaroiu, G. (2021). Sustainable organizational performance, cyber-physical production networks, and deep learning-assisted smart process planning in industry 4.0-based manufacturing systems. *Economics*, *Management*, *and Financial Markets*, *16*(3), 41–54.

Kovacova, M., & Lewis, E. (2021). Smart factory performance, cognitive automation, and industrial big data analytics in sustainable manufacturing internet of things. *Journal of Self-Governance and Management Economics*, *9*(3), 9–21. https://doi.org/10.22381/jsme9320211

KPMG. (2015). *The KPMG survey of corporate responsibility reporting 2015*. KPMG International. Zurich, Switzerland.

KPMG. (2016). *The KPMG survey of corporate responsibility reporting 2016*. KPMG International. Zurich, Switzerland.

KPMG. (2017). *The KPMG survey of corporate responsibility reporting 2017*. KPMG International. Zurich, Switzerland.

Lambertini, L., & Tampieri, A. (2015). Incentives, performance and desirability of socially responsible firms in a Cournot oligopoly. *Economic Modelling*, *50*, 40–48. https://doi.org/10.1016/j.econmod.2015.05.016

Lazaroiu, G., Kliestik, T., & Novak, A. (2021). Internet of things smart devices, industrial artificial intelligence, and real-time sensor networks in sustainable cyber-physical production systems. *Journal of Self-Governance and Management Economics*, *9*(1), 20–30. https://doi.org/10.22381/jsme9120212

Li, C., & Song, J. (2009). Technology licensing in a vertically differentiated duopoly. *Japan and the World Economy*, *21*(2), 183–190. https://doi.org/10.1016/j.japwor.2008.04.002

Li, C., & Wang, J. (2010). Licensing a vertical product innovation. *Economic Record*, *86*(275), 517–527. https://doi.org/10.1111/j.1475-4932.2010.00630.x

Li, D. (2021). Optimal licensing strategy of green technology in a mixed oligopoly: Fixed-fee versus royalty. *Managerial and Decision Economics*, *42*(4), 942–951. https://doi.org/10.1002/mde.3283

Lioui, A., & Sharma, Z. (2012). Environmental corporate social responsibility and financial performance: Disentangling direct and indirect effects. *Ecological Economics*, *78*, 100–111. https://doi.org/10.1016/j.ecolecon.2012.04.004

Liu, C. C., Wang, L. F., & Lee, S. H. (2015). Strategic environmental corporate social responsibility in a differentiated duopoly market. *Economics Letters*, *129*, 108–111. https://doi.org/10.1016/j.econlet.2015.02.027

Lu, J., Ren, L., Zhang, Ch., Qiao, J., Kováčová, M., & Streimikis, J. (2020). Assessment of corporate social responsibility and its impacts on corporate reputation of companies in selected Balkan countries former Yugoslavia states. *Technological and Economic Development of Economy*, *26*(2), 504–524. https://doi.org/10.3846/tede.2020.12069

Malkawi, E., & Khayrullina, M. (2021). Digital human skills from the corporate economy and business development. *Ekonomicko-manazerske spektrum*, *15*(1), 64–74. https://doi.org/10.26552/ems.2021.1.64-74

Marakova, V., Wolak-Tuzimek, A., & Tučková, Z. (2021). Corporate social responsibility as a source of competitive advantage in large enterprises. *Journal of Competitiveness*, *13*(1), 113–128. https://doi.org/10.7441/joc.2021.01.07

Martín-Herrán, G., & Rubio, S. J. (2018). Second-best taxation for a polluting monopoly with abatement investment. *Energy Economics*, *73*, 178–193. https://doi.org/10.1016/j.eneco.2018.05.019

Niu, S. (2018). Price and quantity competition in an asymmetric duopoly with licensing. *Journal of Public Economic Theory*, *20*(6), 896–913. https://doi.org/10.1111/jpet.12332

Noci, G., & Verganti, R. (1999). Managing "green" product innovation in small firms. *R&D Management*, *29*(1), 3–15. https://doi.org/10.1111/1467-9310.00112

Ouchida, Y. (2019). Cooperative choice of corporate social responsibility in a bilateral monopoly model. *Applied Economics Letters*, *26*(10), 799–806. https://doi.org/10.1080/13504851.2018.1497843

Ouchida, Y., & Goto, D. (2016). Cournot duopoly and environmental R&D under regulator's precommitment to an emissions tax. *Applied Economics Letters*, *23*(5), 324–331. https://doi.org/10.1080/13504851.2015.1073831

Petrakis, E., & Xepapadeas, A. P. (1998). *Does government precommitment promote environmental innovation*? (Nota di Lavoro, No. 88.1998). Fondazione Eni Enrico Mattei (FEEM), Milano.

Porter, M. E., & Kramer, M. R. (2006). Strategy and society: The link between corporate social responsibility and competitive advantage. *Harvard Business Review*, *84*(12), 78–92.

Porter, M. E., & Kramer, M. R. (2011). Creating shared value. *Harvard Business Review*, *89*, 62–77.

Porter, M. E., & Van der Linde, C. (1995). Toward a new conception of the environment-competitiveness relationship. *Journal of Economic Perspectives*, *9*(4), 97–118. https://doi.org/10.1257/jep.9.4.97

PricewaterhouseCoopers. (2010). *CSR Trends* 2010. http://admin.csrwire.com/system/report_pdfs/1189/original/CSR_TRENDS_2010.pdf

Rau, H., Budiman, S. D., Regencia, R. C., & Salas, A. D. P. (2019). A decision model for competitive remanufacturing systems considering technology licensing and product quality strategies. *Journal of Cleaner Production*, *239*, 118011. https://doi.org/10.1016/j.jclepro.2019.118011

Reicher, Z. R. (2019). Opportunities for small and medium sized enterprises in the field of corporate social responsibility. *Ekonomicko-manazerske Spektrum*, *13*(1), 26–37. https://doi.org/10.26552/ems.2019.1.26-37

San Martín, M., & Saracho, A. I. (2021). Revenue royalties: Comment. *Journal of Economics*, *132*(2), 187–192. https://doi.org/10.1007/s00712-020-00721-9

Sen, D. (2005). Fee versus royalty reconsidered. *Games and Economic Behavior*, *53*(1), 141–147. https://doi.org/10.1016/j.geb.2004.09.005

Sen, D., & Tauman, Y. (2007). General licensing schemes for a cost-reducing innovation. *Games and Economic Behavior*, *59*(1), 163–186. https://doi.org/10.1016/j.geb.2006.07.005

Sen, N., & Bhattacharya, S. (2017). Technology licensing between rival firms in presence of asymmetric information. *The B.E. Journal of Theoretical Economics*, *17*(1), 1–35. https://doi.org/10.1515/bejte-2015-0097

Sen, N., Kaul, S., & Biswas, R. (2021). Technology licensing under product differentiation. *Journal of Economics*, *134*(3), 219–260. https://doi.org/10.1007/s00712-021-00750-y

Sethi, A. K., & Sethi, S. P. (1990). Flexibility in manufacturing: A survey. *International Journal of Flexible Manufacturing Systems*, *2*(4), 289–328. https://doi.org/10.1007/BF00186471

Singh, S. K., Del Giudice, M., Chierici, R., & Graziano, D. (2020). Green innovation and environmental performance: The role of green transformational leadership and green human resource management. *Technological Forecasting and Social Change*, *150*, 119762. https://doi.org/10.1016/j.techfore.2019.119762

Stojanovic, A., Milosevic, I., Arsic, S., Urosevic, S., & Mihaljovic, I. (2020). Corporate social responsibility as a determinant of employee loyalty and business performance. *Journal of Competitiveness*, *12*(2), 149–166. https://doi.org/10.7441/joc.2020.02.09

Tijani, A. A., Osagie, R. O., & Afolabi, B. K. (2021). Effect of strategic alliance and partnership on the survival of MSMEs post COVID-19 pandemic. *Ekonomicko-manazerske spektrum*, *15*(2), 126–137. https://doi.org/10.26552/ems.2021.2.126-137

Tsai, T. H., Wang, C. C., & Chiou, J. R. (2016). Can privatization be a catalyst for environmental R&D and result in a cleaner environment? *Resource and Energy Economics*, *43*, 1–13. https://doi.org/10.1016/j.reseneeco.2015.10.002

Wang, X. H. (1998). Fee versus royalty licensing in a Cournot duopoly model. *Economics Letters*, *60*(1), 55–62. https://doi.org/10.1016/S0165-1765(98)00092-5

Wang, X. H. (2002). Fee versus royalty licensing in a differentiated Cournot duopoly. *Journal of Economics and Business*, *54*(2), 253–266. https://doi.org/10.1016/S0148-6195(01)00065-0

Wang, X. H., & Yang, B. Z. (1999). On licensing under Bertrand competition. *Australian Economic Papers*, *38*(2), 106–119. https://doi.org/10.1111/1467-8454.00045

Xia, H., Fan, T., & Chang, X. (2019). Emission reduction technology licensing and diffusion under command-and-control regulation. *Environmental and Resource Economics*, *72*(2), 477–500. https://doi.org/10.1007/s10640-017-0201-0

Xing, M. Q., Tan, T. T., & Wang, W. (2021). Emission taxes and environmental R&D risk choices in a duopoly market. *Economic Modelling*, *101*, 105530. https://doi.org/10.1016/j.econmod.2021.105530

Ye, G., & Mukhopadhyay, S. K. (2013). Role of demand-side strategy in quality competition. *International Journal of Production Economics*, *145*(2), 696–701. https://doi.org/10.1016/j.ijpe.2013.05.024

Yong, S. K., Friesen, L., & McDonald, S. (2018). Emission taxes, clean technology cooperation, and product market collusion: Experimental evidence. *Economic Inquiry*, *56*(4), 1950–1979. https://doi.org/10.1111/ecin.12573

Zhang, H., Wang, X., Hong, X., & Lu, Q. (2018). Technology licensing in a network product market: Fixed-fee versus royalty licensing. *Economic Record*, *94*(305), 168–185. https://doi.org/10.1111/1475-4932.12385

Zhang, X., Chen, T., & Shen, C. (2020). Green investment choice in a duopoly market with quality competition. *Journal of Cleaner Production*, *276*, 124032. https://doi.org/10.1016/j.jclepro.2020.124032

Zhao, D., Chen, H., Hong, X., & Liu, J. (2014). Technology licensing contracts with network effects. *International Journal of Production Economics*, *158*, 136–144. https://doi.org/10.1016/j.ijpe.2014.07.023

Zou, Y., & Chen, T. L. (2020). Quality differentiation and product innovation licensing. *Economic Modelling*, *87*, 372–382. https://doi.org/10.1016/j.econmod.2019.08.010

APPENDIX

Proof of Lemma 1. Firm 1 can get more profit than it would get in the absence of the license under fixed-fee licensing arrangement because $\pi_1^{LF} - \pi_1^N = F > 0$.

Proof of Lemma 2. Under royalty licensing arrangement, firm 1 gets more profit than it would get in the absence of the license, because $\pi_1^{LR} - \pi_1^N = rz_2^{LR} > 0$.

Proof of Proposition 1. Comparing Eqs (21)–(29) and (24)–(32) yields:

$$\pi_1^{LF} - \pi_1^{LR} = \begin{cases} \dfrac{(a-c-3d)^2}{8(\gamma-\varepsilon)}(1-\sqrt{1-\dfrac{\varepsilon}{\gamma}})^2 > 0, & 0 < \dfrac{\varepsilon}{\gamma} < \dfrac{3}{4} \\ \dfrac{(\dfrac{\varepsilon}{\gamma}-\dfrac{1}{2})(a-c-3d)^2}{8(\gamma-\varepsilon)^2} > 0, & \dfrac{3}{4} < \dfrac{\varepsilon}{\gamma} < 1 \end{cases},$$

$$SW^{LF} - SW^{LR} = \begin{cases} \dfrac{(\dfrac{a-c+d}{2} - \tau^{LR}\sqrt{1-\dfrac{\varepsilon}{\gamma}})}{2(\gamma-\varepsilon)} > 0, & 0 < \dfrac{\varepsilon}{\gamma} < \dfrac{3}{4} \\ \dfrac{ar-cr-dr}{8(\gamma-\varepsilon)} > 0, & \dfrac{3}{4} < \dfrac{\varepsilon}{\gamma} < 1 \end{cases}.$$

Proof of Proposition 2. This is a result of simple mathematics. $CS^{LF} = 0.5(a-c-d)^2 = CS^{LR}$.

Proof of Lemma 3. With respect to θ, the derivative of Eqs (33)–(34) is

$$\frac{\partial q_{1C}^N}{\partial\theta} = -d + 0.5a - 0.5c > 0,\ \frac{\partial q_{2C}^N}{\partial\theta} = d - 0.5a + 0.5c < 0,$$

$$\frac{\partial z_{1C}^N}{\partial\theta} = \frac{1}{\gamma-\varepsilon}(0.5a - 0.5c) > 0,\ \frac{\partial z_{2C}^N}{\partial\theta} = \frac{1}{\gamma}(0.5a - 0.5c - d) > 0.$$

Proof of Lemma 4. Due to $z_{2C}^{LF} > z_{2C}^N$ and $F_C > 0$, a simple calculation yields the following conclusion: $\Omega_C^{LF} - \Omega_C^N = d(z_{2C}^{LF} - z_{2C}^N) + F_C > 0$.

Proof of Lemma 5. A simple calculation yields:

$$\Omega_C^{LR} - \Omega_C^N = \begin{cases} \dfrac{A}{4\gamma-4\varepsilon} > 0, & 0 < \dfrac{\varepsilon}{\gamma} < \dfrac{3}{4} \\ \dfrac{[a-c-3d-(a-2d-c)\theta]^2}{16\gamma-16\varepsilon} > 0, & \dfrac{3}{4} < \dfrac{\varepsilon}{\gamma} < 1 \end{cases},$$

in which $A = (\dfrac{\varepsilon}{\gamma} + \sqrt{1-\dfrac{\varepsilon}{\gamma}} - 1)[-(a-2d-c)\theta + a - 3d - c][a - 5d - c - (a-2d-c)\theta] > 0$.

Proof of Proposition 3. A simple calculation yields that

$$\Omega_C^{LF} - \Omega_C^{LR} = \begin{cases} \dfrac{B}{8(\gamma-\varepsilon)}, & 0 < \dfrac{\varepsilon}{\gamma} < \dfrac{3}{4} \\ D[3d + c - a + \theta(a-c-2d)], & \dfrac{3}{4} < \dfrac{\varepsilon}{\gamma} < 1 \end{cases}.$$

If $0<\frac{\varepsilon}{\gamma}<\frac{3}{4}$, then $\Omega_C^{LF}>\Omega_C^{LR}$; if $\frac{3}{4}<\frac{\varepsilon}{\gamma}<1$, then $\begin{cases}\Omega_C^{LF}<\Omega_C^{LR}, & 0<\theta<\theta^* \\ \Omega_C^{LF}>\Omega_C^{LR}, & \theta^*<\theta\le 1\end{cases}$

in which $\theta^* = \dfrac{(2\varepsilon-\gamma)(3d-a+c)}{-a\gamma+c\gamma+6d\gamma+2a\varepsilon-2c\varepsilon-4d\varepsilon}$,

$$B=(3d-a+c+2d\theta+c\theta-a\theta)\{-\sqrt{\gamma(\gamma-\varepsilon)}[(6d+2c-2a)+(2a-2c)\theta]-2a\gamma+2c\gamma+6d\gamma+ a\varepsilon-c\varepsilon-3d\varepsilon+[4d(\gamma-\varepsilon)\theta+2a\gamma\theta-2c\gamma\theta-4d\gamma\theta-a\varepsilon\theta+c\varepsilon\theta+2d\varepsilon\theta]\}>0,$$

$$D=\frac{(\gamma-2\varepsilon)[a(1-\theta)-d(3-2\theta)-c(1-\theta)]+4\gamma\theta d}{16(\gamma^2-\gamma\varepsilon)}>0.$$

Proof of Proposition 4. This is a result of simple mathematics. $CS_C^{LF}=0.5(a-c-d)^2=CS_C^{LR}$.

Proof of Proposition 5. A simple calculation gives:

$$SW_C^{LF}-SW_C^{LR}=\begin{cases}G[3d+c-a+\theta(a-c-2d)]>0, & 0<\frac{\varepsilon}{\gamma}<\frac{3}{4} \\ H\{3[a-c-\theta(a-c-2d)]-d\}>0, & \frac{3}{4}<\frac{\varepsilon}{\gamma}<1\end{cases},$$

in which $G=\dfrac{1}{8(\gamma-\varepsilon)}[(a\varepsilon-c\varepsilon-2d\varepsilon)(1-\theta)+(4d-\dfrac{\varepsilon d}{\gamma}-4d\sqrt{1-\dfrac{\varepsilon}{\gamma}})]>0,$

$H=\dfrac{1}{32(\gamma-\varepsilon)}[3d+c-a+\theta(a-c-2d)]>0.$

Proof of Lemma 6. Here are outcomes in the absence and presence of CSR when it comes to production and abatement:

(i) $q_{1C}^{LR(F)}-q_1^{LR(F)}=\theta(0.5a-0.5c-d)>0,\ z_{1C}^{LR(F)}-z_1^{LR(F)}=0.5a\theta-0.5c\theta>0,$

$\begin{cases}\pi_{1C}^{LR(F)}>\pi_1^{LR(F)}, & 0<\theta<\theta^{LR(F)} \\ \pi_{1C}^{LR(F)}<\pi_1^{LR(F)}, & \theta^{LR(F)}<\theta\le 1\end{cases}$,

where, $\theta^{LF}=\dfrac{2(3d+c-a)(\gamma+\varepsilon)(a-2d-c)}{\gamma(4d+c-a)a+(-a+2d+c)^2[2\gamma^2-(1+2\gamma)\varepsilon]-(4d+c-a)\gamma c}$

$$\theta^{LR}=\begin{cases}\dfrac{(6d-2a+2c)(a-2d-c)(2\gamma\sqrt{1-\frac{\varepsilon}{\gamma}}+2\gamma\varepsilon-\gamma)}{(a-2d-c)^2(\gamma+2\gamma^2-2\varepsilon-2\gamma\varepsilon-2\gamma\sqrt{1-\frac{\varepsilon}{\gamma}})+4d^2\gamma}, & 0<\frac{\varepsilon}{\gamma}<\frac{3}{4} \\ \dfrac{(18d+6c-6a)(a-2d-c)}{(a-2d-c)^2(4\gamma-4\varepsilon-3)+8d^2}, & \frac{3}{4}<\frac{\varepsilon}{\gamma}<1\end{cases}.$$

(ii) $q_{2C}^{LF}-q_2^{LF}=q_{2C}^{LR}-q_2^{LR}=-\dfrac{\theta(a-c-2d)}{2}<0,\ z_{1C}^{LF}-z_1^{LF}=\dfrac{\theta(a-c)}{2}>0,$

$$z_{2C}^{LF}-z_{2}^{LF}=\begin{cases}\dfrac{\theta\left(a-c-2d\right)\sqrt{1-\dfrac{\varepsilon}{\gamma}}}{2(\gamma-\varepsilon)}>0, & 0<\dfrac{\varepsilon}{\gamma}<\dfrac{3}{4}\\ \dfrac{\theta(a-c-2d)}{4\gamma-4\varepsilon}>0, & \dfrac{3}{4}<\dfrac{\varepsilon}{\gamma}<1\end{cases},\quad \begin{cases}\pi_{2C}^{LF(R)}>\pi_{2}^{LF(R)}, & 0<\theta<\theta^{LF(R)}\\ \pi_{2C}^{LF(R)}<\pi_{2}^{LF(R)}, & \theta^{LF(R)}<\theta\le 1\end{cases}$$

in which, $\theta^{LR}=\begin{cases}\dfrac{2(3d-a+c)}{(a-c-2d)(2\gamma-1)}, & 0<\dfrac{\varepsilon}{\gamma}<\dfrac{3}{4}\\ \dfrac{2(3d-a+c)}{(a-c-2d)(8\gamma-8\varepsilon-1)}, & \dfrac{3}{4}<\dfrac{\varepsilon}{\gamma}<1\end{cases}$ and

$$\theta^{LF}=\frac{(6d+2c-2a)(3-4\dfrac{\varepsilon}{\gamma})}{(8\gamma-8\varepsilon-3+4\dfrac{\varepsilon}{\gamma})(a-c-2d)}.$$

Proof of Proposition 6. Here are outcomes in the absence and presence of CSR when it comes to emissions and social welfare:

$$E_{C}^{LF}-E^{LF}=-\frac{a\theta-d\theta-c\theta}{\gamma-\varepsilon}<0,\ E_{C}^{LR}-E^{LR}=-\left(\frac{r-r_C}{\gamma-\varepsilon}+\frac{a\theta-d\theta-c\theta}{\gamma-\varepsilon}\right)<0,$$

$$\begin{cases}SW_{C}^{LR(F)}>SW^{LR(F)}, & 0<\theta<\theta^{LR(F)}\\ SW_{C}^{LR(F)}<SW^{LR(F)}, & \theta^{LR(F)}<\theta\le 1\end{cases},$$

where $\theta^{LF}=\dfrac{4}{\dfrac{a^2-2ac+c^2}{(a-d-c)^2}+1}$,

$$\theta^{LR}=\begin{cases}\dfrac{\gamma[(4a-4c)(a-3d-c)+12d^2]+I}{-(a-c-2d)^2\varepsilon+[2\gamma(-a+c)^2-2d(2\gamma a-2\gamma d-2\gamma c)]}, & 0<\dfrac{\varepsilon}{\gamma}<\dfrac{3}{4}\\ \dfrac{2[5(a-c)^2+d(5c-5a-2d)]}{5(a^2-2ac+c^2)-4d(a-c-d)}, & \dfrac{3}{4}<\dfrac{\varepsilon}{\gamma}<1\end{cases},$$

$$I=(a-c-2d)[4d\gamma\sqrt{1-\frac{\varepsilon}{\gamma}}-2\varepsilon(a-c-3d)].$$