Technological and Economic Development of Economy
ISSN: 2029-4913 / eISSN: 2029-4921
2022 Volume 28 Issue 3: 638–654
https://doi.org/10.3846/tede.2022.16483

MODEL FOR NETWORK SECURITY SERVICE PROVIDER SELECTION WITH PROBABILISTIC UNCERTAIN LINGUISTIC TODIM METHOD BASED ON PROSPECT THEORY

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Received 30 March 2021; accepted 19 October 2021; first published online 31 March 2022

Abstract. Choosing the optimal network security service provider (NSSP) is a very important part of enterprise management decision. And the choice of NSSP is a typical multiple attribute group decision making (MAGDM) issue. In order to provide a better decision method for MAGDM problems, this paper integrates the TODIM method and the probabilistic uncertainty linguistic term set (PULTS), so as to propose the probabilistic uncertain linguistic TODIM model based on the prospect theory (PT-PUL-TODIM). The model combines the advantages of the prospect theory and TODIM method. In the end, a case study concerning NSSP selection problem is given to demonstrate the merits of the developed methods.

Keywords: multiple attribute group decision making (MAGDM), probabilistic uncertain linguistic term set, TODIM, prospect theory, Network Security Service Providers Selection.

JEL Classification: C43, C61, D81.

Introduction

The Interactive Multi-Criteria Decision Making (TODIM) method (Gomes & Rangel, 2009; Liao et al., 2021; Su et al., 2021a), the multi-attributive border approximation area comparison (MABAC) method (Pamucar & Cirovic, 2015), VIKOR method (He et al., 2020b; Opricovic & Tzeng, 2004), the Evaluation based on Distance from Average Solution (EDAS) (He et al., 2020a; Keshavarz Ghorabaee et al., 2015; Lei et al., 2022; Jiang et al., 2022; Huang et al., 2021; Su et al., 2021b) method, SWARA method (Hashemkhani Zolfani & Saparauskas, 2013) and CODAS method (Keshavarz Ghorabaee et al., 2016) are very celebrated methods in the field of decision-making. Usually, these methods are used to deal with MADM or MAGDM issues (Tabatabaei et al., 2019; Wei et al., 2021a; Zhao et al., 2021a). Among these

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well-known methods, TODIM method is unique because of the different treatment of gains and losses. Ashofteh et al. (2020) believed that TODIM method could have effective performance in choosing the best river-water transfer strategy. Another application study based on TODIM method about site selection of low-speed wind farms was realized by Wu et al. (2020). Xian et al. (2020) developed TODIM method in interval-valued Pythagorean fuzzy linguistic environment and for avoiding the multicollinearity. Tian et al. (2020) extended the application of traditional TODIM method in green supplier selection on the basis of probabilistic hesitant fuzzy information. Ju et al. (2021) achieved the combination between TODIM method and T-spherical fuzzy environment. Luo and Liang (2021) applied TODIM method to evaluate the property of cleaning products. Li et al. (2021) defined the TODIM method under interval-valued intuitionistic fuzzy set (IVIFS). Wu et al. (2021) constructed linguistic distribution behavioral MCGDM model which incorporated the TODIM method. Arya and Kumar (2020) combined TODIM method and VIKOR method under picture fuzzy set (PFS). In the study of linguistic picture fuzzy set developed from PFS, Liu et al. (2020a) proposed a hybrid model of TOPSIS and TODIM.

In 1965, Zadeh (1965) mentioned fuzzy set (FS) firstly. Novel fuzzy set concepts have been proposed one after another, such as intuitionistic fuzzy set (IFS) (Atanassov, 1986), picture fuzzy set (PFS) (Cuong, 2014), and Pythagorean fuzzy set (PyFS) (Yager, 2013). Liu et al. (2020b) asserted that the normal wiggly hesitant fuzzy linguistic term set (NWHFLTS) was a more effective tool to assist us in further excavating much valuable information. Luo et al. (2020) proposed a new Pearson correlation coefficient for probabilistic linguistic term set (PLTS). Mo (2020) integrated PLTS with the D number theory. Du and Liu (2021) developed dual Muirhead mean operators under PLTS. Wei et al. (2021b) defined the generalized Dice similarity measures under PLTS. A new linguistic term set named probabilistic uncertain linguistic term set (PULTS) defined by Lin et al. (2018) is noteworthy. Wang et al. (2022) defined the probabilistic uncertain linguistic GRP method. Xie et al. (2018) created the preference relation of PULTS. He et al. (2019) established EDAS in PULTS environment. Lei et al. (2020) constructed PULTS-based QUALIFLEX model. Wei et al. (2020b) researched MABAC method under PULTS environment. Bashir et al. (2021) defined new operations, distance measure and operators about PULTS.

From the review of existing studies, it is clear that there has been a wealth of research based on the traditional TODIM method. For example, the TODIM method was extended from the real number to different fuzzy numbers, and the TODIM method was combined with other decision methods. However, there are still insufficient studies on the modification of TODIM method by introducing novel theoretical viewpoints into TODIM model (Tian et al., 2019; Zhao et al., 2021b; Zhang et al., 2022).

PT is a theory proposed by Kahneman and Tversky (1979), which changed the factors affecting decision results from probability and final assets to weight and profit loss. The fusion of PT and TODIM under PULTS and can make a good psychological description and correct subjective information for any type of decision makers. Finally, the application of the PT-PUL-TODIM method in the selection of network security service providers (NSSPs) becomes very valuable for enterprise management decisions. In summary, the main contributions of PT-based TODIM model constructed in the context of PUL to MADM and

MAGDM are as follows: (1) the model is an extension of the traditional TODIM method; (2) the model enriches the way to solve the MAGDM problems; (3) the model creates a unique PUL evaluation model; (4) the model is successfully applied to the selection of network security service providers.

The main structure of this article is as follows. In the Section 1, we introduce the related theoretical knowledge of PULTS. In the Section 2, the fundamental theory of the new proposed method, namely the prospect theory, is expounded. The Section 3 is the key content of this paper, which elaborates the model architecture of this paper. In the example of the fourth part, we focus on the choice of NSSPs. Eventually, in order to ensure the application effect of the proposed method, we chose two existing methods in the probabilistic uncertain linguistic environment and compared them with the PT-PUL-TODIM method in the last Section.

1. Preliminary knowledge

In this section, we tended to introduce some basic knowledge about probabilistic uncertain linguistic term set (PULTS).

Definition 1 (Nie & Wang, 2020). $V=\{\nu_{-3}=\text{extremely bad},\nu_{-2}=\text{very bad},\nu_{-1}=\text{bad},$ $\nu_0=\text{medium},\ \nu_1=\text{great},\nu_2=\text{very great},\nu_3=\text{extremely great}\}$ is an example of the common LTS $V=\{\nu_\vartheta \mid \vartheta=-\varpi,\cdots,-2,-1,0,1,2,\cdots,\varpi\}$ in which each element ν_ϑ stands for a linguistic term. The following function TF can help us transform the linguistic term ν_ϑ into a crisp $\hat{t}$.

$$TF:[\nu_{-\varpi},\nu_{\varpi}]\to[0,1],\ TF(\nu_\vartheta)=\frac{\vartheta+\varpi}{2\varpi}=\hat{t}. \tag{1}$$

On the other hand, there is an opposite function TF^{-1} to restore the crisp $\hat{t}$ back to the linguistic term ν_ϑ.

$$TF^{-1}:[0,1]\to[\nu_{-\varpi},\nu_{\varpi}],TF^{-1}(\hat{t})=\nu_{(2\hat{t}-1)\varpi}=\nu_\vartheta. \tag{2}$$

After hesitant fuzzy term set (HFTS) and probabilistic linguistic term set (PLTS), Lin et al. (2018) further developed the probabilistic uncertain linguistic term set (PULTS).

Definition 2 (Lin et al., 2018). In accordance with the LTS $V=\{\nu_\vartheta \mid \vartheta=-\varpi,\cdots,-2,-1,0,1,2,\cdots,\varpi\}$, the PULTS is defined as:

$$PU(\hat{\pi})=\left\{\left[\hat{L}^{(m)},\hat{U}^{(m)}\right]\left(\hat{\pi}^{(m)}\right)\middle|\hat{\pi}^{(m)}\ge 0;m=1,2,\cdots,\#PU(\hat{\pi});\sum_{m=1}^{\#PU(\pi)}\hat{\pi}^{(m)}\le 1\right\}, \tag{3}$$

where $\hat{L}^{(m)}$ and $\hat{U}^{(m)}$ are the lower and upper limits of uncertain linguistic term (ULT) $\left[\hat{L}^{(m)},\hat{U}^{(m)}\right]$, respectively ($\hat{L}^{(m)},\hat{U}^{(m)}\in V$ as well as $\hat{L}^{(m)}\le\hat{U}^{(m)}$). In addition, $\hat{\pi}^{(m)}$ represents the corresponding probability of ULT $\left[\hat{L}^{(m)},\hat{U}^{(m)}\right]$, and the total amount of ULT in PULTS $PU(\hat{\pi})$ is $\#PU(\hat{\pi})$.

In particular, if there is an inclusion or crossover relationship between two different ULTs in $PU(\hat{\pi})=\left\{\left[\hat{L}^{(m)},\hat{U}^{(m)}\right]\left(\hat{\pi}^{(m)}\right)\middle|m=1,2,\cdots,\#PU(\hat{\pi})\right\}$, it is necessary to reprocess original PULTS. For inclusion, the more extensive ULT is further subdivided, for example, $[\nu_0,\nu_2](0.6)$ is divided into $[\nu_0,\nu_1](0.3)$ and $[\nu_1,\nu_2](0.3)$ in $PU(\hat{\pi})=\{[\nu_0,\nu_2](0.6),[\nu_0,\nu_1](0.4)\}$ so that $PU(\hat{\pi})=\{[\nu_0,\nu_2](0.6),[\nu_0,\nu_1](0.4)\}$. For crossover, the same part is separated out from original PULTSs, for instance, $PU(\hat{\pi})=\{[\nu_{-1},\nu_1](0.6),[\nu_0,\nu_2](0.4)\}$ is turned into $PU(\hat{\pi})=\{[\nu_{-1},\nu_0](0.3),[\nu_0,\nu_1](0.5),[\nu_1,\nu_2](0.2)\}$.

Definition 3 (Lin et al., 2018). Based on $\overline{\pi}^{(m)}=\hat{\pi}^{(m)}\Big/\sum_{m=1}^{\#PU(\hat{\pi})}\hat{\pi}^{(m)}$, the PULTS $PU(\hat{\pi})=\left\{\left[\hat{L}^{(m)},\hat{U}^{(m)}\right]\left(\hat{\pi}^{(m)}\right)\middle|m=1,2,\cdots,\#PU(\hat{\pi})\right\}$ can be easily standardized to $\overline{PU}(\overline{\pi})=\left\{\left[\hat{L}^{(m)},\hat{U}^{(m)}\right]\left(\overline{\pi}^{(m)}\right)\middle|\overline{\pi}^{(m)}\geq 0;m=1,2,\cdots,\#PU(\hat{\pi});\ \sum_{m=1}^{\#PU(\hat{\pi})}\overline{\pi}^{(m)}=1\right\}$.

Definition 4 (Lin et al., 2018). To facilitate the calculation of PULTS, we usually process PULTSs of different lengths to have the same number of ULT. When $\#PU_1(\hat{\pi}_1)>\#PU_2(\hat{\pi}_2)$, $\#PU_1(\hat{\pi}_1)-\#PU_2(\hat{\pi}_2)$ minimal ULTs with zero probability from PULTS $PU_2(\hat{\pi}_2)$ is added in PULTS $PU_2(\hat{\pi}_2)$.

Definition 5 (Lin et al., 2018). The following equations (4) and (5) severally define the expected value $EX(PU(\hat{\pi}))$ and deviation value $DE(PU(\hat{\pi}))$ of PULTS $PU(\hat{\pi})=\left\{\left[\hat{L}^{(m)},\hat{U}^{(m)}\right]\left(\hat{\pi}^{(m)}\right)\middle|m=1,2,\cdots,\#PU(\hat{\pi})\right\}$.

$$EX(PU(\hat{\pi}))=\frac{\sum_{m=1}^{\#PU(\hat{\pi})}\left(\frac{TF\left(\hat{L}^{(m)}\right)\cdot\hat{\pi}^{(m)}+TF\left(\hat{U}^{(m)}\right)\cdot\hat{\pi}^{(m)}}{2}\right)}{\sum_{m=1}^{\#PU(\hat{\pi})}\hat{\pi}^{(t)}}; \tag{4}$$

$$DE(PU(\hat{\pi}))=\frac{\sqrt{\sum_{m=1}^{\#PU(\hat{\pi})}\left(\frac{TF\left(\hat{L}^{(m)}\right)\cdot\hat{\pi}^{(m)}+TF\left(\hat{U}^{(m)}\right)\cdot\hat{\pi}^{(m)}}{2}-EX(PU(\hat{\pi}))\right)^2}}{\sum_{m=1}^{\#PU(\hat{\pi})}\hat{\pi}^{(t)}}. \tag{5}$$

Moreover, there are the following rules for any two PULTSs $PU_1(\hat{\pi}_1)=\left\{\left[\hat{L}_1^{(m)},\hat{U}_1^{(m)}\right]\left(\hat{\pi}_1^{(m)}\right)\middle|m=1,2,\cdots,\#PU_1(\hat{\pi}_1)\right\}$ and $PU_2(\hat{\pi}_2)=\left\{\left[\hat{L}_2^{(m)},\hat{U}_2^{(m)}\right]\left(\hat{\pi}_2^{(m)}\right)\middle|m=1,2,\cdots,\#PU_2(\hat{\pi}_2)\right\}$. Firstly, when $EX(PU_1(\hat{\pi}_1))>EX(PU_2(\hat{\pi}_2))$, we can directly acquire that $PU_1(\hat{\pi}_1)>PU_2(\hat{\pi}_2)$. Secondly, when $EX(PU_1(\hat{\pi}_1))=EX(PU_2(\hat{\pi}_2))$ and $DE(PU_1(\hat{\pi}_1))>DE(PU_2(\hat{\pi}_2))$, we can also get the identical conclusion that $PU_1(\hat{\pi}_1)>PU_2(\hat{\pi}_2)$. Finally, if and only if $EX(PU_1(\hat{\pi}_1))=EX(PU_2(\hat{\pi}_2))$ and $DE(PU_1(\hat{\pi}_1))=DE(PU_2(\hat{\pi}_2))$, $PU_1(\hat{\pi}_1)=PU_2(\hat{\pi}_2)$ appears.

Definition 6 (Wei et al., 2020a). Assume that both $PU_1\left(\hat{\pi}_1\right)=\left\{\left[\hat{L}_1^{(m)},\hat{U}_1^{(m)}\right]\left(\hat{\pi}_1^{(m)}\right)\middle|m=1,2,\cdots,\#PU_1\left(\hat{\pi}_1\right)\right\}$ and $PU_2\left(\hat{\pi}_2\right)=\left\{\left[\hat{L}_2^{(m)},\hat{U}_2^{(m)}\right]\left(\hat{\pi}_2^{(m)}\right)\middle|m=1,2,\cdots,\#PU_2\left(\hat{\pi}_2\right)\right\}$ are PULTS, then the equation (6) is the development of the Hamming distance. ($\#PU_1\left(\hat{\pi}_1\right)=\#PU_2\left(\hat{\pi}_2\right)=\#PU$)

$$d\left(PU_1\left(\hat{\pi}_1\right),PU_2\left(\hat{\pi}_2\right)\right)=\frac{1}{2\#PU}\sum_{m=1}^{\#PU}\left(\begin{array}{l}\left|TF\left(\hat{L}_1^{(m)}\right)\cdot\hat{\pi}_1^{(m)}-TF\left(\hat{L}_2^{(m)}\right)\cdot\hat{\pi}_2^{(m)}\right|\\+\left|TF\left(\hat{U}_1^{(m)}\right)\cdot\hat{\pi}_1^{(m)}-TF\left(\hat{U}_2^{(m)}\right)\cdot\hat{\pi}_2^{(m)}\right|\end{array}\right). \tag{6}$$

2. Prospect theory

Prospect theory (PT) demonstrates the viewpoint that two factors, gains as well as losses and decision weight, are capable of influencing the decision maker's choice. Kahneman and Tversky (1979) expressed this distinctive opinion by utilizing the following three Eqs (7)–(9), the prospect function $\tilde{P}(w)$, the value function $\tilde{C}(w_s)$ and the weighting function $\tilde{G}(h_s)$.

$$\tilde{P}(w)=\sum_{s=1}^{f}\tilde{C}(w_s)\cdot\tilde{G}(h_s); \tag{7}$$

$$\tilde{C}(w_s)=\begin{cases}(w_s-w_0)^{\gamma}, & \text{if } w_s\geq w_0\\ -\kappa(w_0-w_s)^{\xi}, & \text{if } w_s<w_0\end{cases}; \tag{8}$$

$$\tilde{G}(h_s)=\begin{cases}\dfrac{h_s^x}{\left(h_s^x+(1-h_s)^x\right)^{\frac{1}{x}}}, & \text{if } w_s\geq w_0\\[2ex] \dfrac{h_u^z}{\left(h_u^z+(1-h_s)^z\right)^{\frac{1}{z}}}, & \text{if } w_s<w_0\end{cases}. \tag{9}$$

For the value function $\tilde{C}(w_s)$, if the actual value w_s is no less than the selected standard point w_0, the value $w_s–w_0$ means gains for decision makers. Otherwise, the value $w_0–w_s$ means losses for decision makers. Actually, in a real decision making, decision makers with different personality traits have different psychological perception of gain and loss. Hence, the parameters κ, γ and ξ in Eq. (8) are the mathematical embodiment of the decision-maker's psychology. To sum up, the more the decision maker pursues risk, the greater the value of A is than that of B and $\kappa < 1$. However, in more cases, the decision maker is risk averse that corresponds to $\kappa > 1$ as well as $\gamma \leq \xi$.

The weighting function $\tilde{G}(h_s)$ represents a modification of probabilities that have been distorted by the decision makers' psychology. As far as Kahneman and Tversky (1979) are concerned, the decision makers' psychology can affect the decision maker's cognition about the objective probability of the occurrence of the event. Therefore, in order to make a more accurate judgment, it is necessary to modify the subjective probability value according to the weighting function $\tilde{G}(h_s)$. x and z represent the curvature of the weighting function $\tilde{G}(h_s)$.

3. PT-TODIM method for MAGDM under PULTS

The ambition of this paper is to construct a novel PULTS MAGDM model based on the combination of PT as well as TODIM method. In this section, we intend to specify that how this new model actually works. First of all, there is relevant basal information in the following. $\Phi=\{\Phi_1,\Phi_2,\cdots,\Phi_\tau\}$, $\Delta=\{\Delta_1,\Delta_2,\cdots,\Delta_\theta\}$ and $\wp=\{\wp_1,\wp_2,\cdots,\wp_\delta\}$ are the collections about alternatives, attributes and decision-makers respectively, where the information about attribute weights is unknown completely. Moreover, the decision-makers utilize the uncertain linguistic term set (ULTS) to set forth standpoints, which are collected in δ ULTS decision matrices $\tilde{U}^{(e)}=\left(\left[\hat{L}_{\iota\varepsilon}^{(e)},\hat{U}_{\iota\varepsilon}^{(e)}\right]\right)_{\tau\times\theta}$ ($\hat{L}_{\iota\varepsilon}^{(e)},\hat{U}_{\iota\varepsilon}^{(e)}\in V;\hat{L}_{\iota\varepsilon}^{(e)}\le\hat{U}_{\iota\varepsilon}^{(e)};\iota=1,2,\cdots,\tau;\ \varepsilon=1,2,\cdots,\theta;$ $e=1,2,\cdots,\delta$). Immediately, the elaborate process is as follows.

Step 1. In order to ensure the consistent information, transforming the negative attribute into positive one is finely essential. Specifically, if the value of negative attribute is $\left[\nu_a,\nu_b\right]$, then transform it into $\left[\nu_{-b},\nu_{-a}\right]$.

Step 2. Based on the consistent ULTS decision matrices and Definition 3, the pretreated and rearranged PULTS matrix is able to be acquired as $\tilde{K}=\left(PU_{\iota\varepsilon}\left(\hat{\pi}\right)\right)_{\delta\times\tau}=\left(\left\{\left[\hat{L}_{\iota\varepsilon}^{(m)},\hat{U}_{\iota\varepsilon}^{(m)}\right]\left(\hat{\pi}_{\iota\varepsilon}^{(m)}\right)\middle|m=1,2,\cdots,\#PU_{\iota\varepsilon}\left(\hat{\pi}\right)\right\}\right)_{\tau\times\theta}$. Importantly, $\hat{\pi}_{\iota\varepsilon}^{(m)}$ is the possibility of ULTS $\left[\hat{L}_{\iota\varepsilon}^{(m)},\hat{U}_{\iota\varepsilon}^{(m)}\right]$ appearing in alternative Φ_ι under attribute Δ_ε.

Step 3. The CRITIC method (Wei et al., 2020a), just as Eqs (10)–(13), is selected as an approach of determining the objective weighting vector of attributes $y=\left(y_1,y_2,\cdots,y_\theta\right)^T$ ($y_\varepsilon\ge0$ and $\sum_{\varepsilon=1}^{\theta}y_\varepsilon=1$).

$$PCC_{\varepsilon\alpha}=\frac{\sum_{\iota=1}^{\tau}\left(\left(\sum_{m=1}^{\#PU_{\iota\varepsilon}(\hat{\pi})}\frac{\left(TF\left(\hat{L}_{\iota\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\iota\varepsilon}^{(m)}-TF\left(\hat{L}_{\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\varepsilon}^{(m)}\right)+\left(TF\left(\hat{U}_{\iota\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\iota\varepsilon}^{(m)}-TF\left(\hat{U}_{\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\varepsilon}^{(m)}\right)}{2}\right)\cdot\left(\sum_{m=1}^{\#PU_{\iota\varepsilon}(\hat{\pi})}\frac{\left(TF\left(\hat{L}_{\iota\alpha}^{(m)}\right)\cdot\hat{\pi}_{\iota\alpha}^{(m)}-TF\left(\hat{L}_{\alpha}^{(m)}\right)\cdot\hat{\pi}_{\alpha}^{(m)}\right)+\left(TF\left(\hat{U}_{\iota\alpha}^{(m)}\right)\cdot\hat{\pi}_{\iota\alpha}^{(m)}-TF\left(\hat{U}_{\alpha}^{(m)}\right)\cdot\hat{\pi}_{\alpha}^{(m)}\right)}{2}\right)\right)}{\left(\sqrt{\sum_{\iota=1}^{\tau}\left(\sum_{m=1}^{\#PU_{\iota\varepsilon}(\hat{\pi})}\frac{\left(TF\left(\hat{L}_{\iota\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\iota\varepsilon}^{(m)}-TF\left(\hat{L}_{\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\varepsilon}^{(m)}\right)+\left(TF\left(\hat{U}_{\iota\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\iota\varepsilon}^{(m)}-TF\left(\hat{U}_{\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\varepsilon}^{(m)}\right)}{2}\right)}\cdot\sqrt{\sum_{\iota=1}^{\tau}\left(\sum_{m=1}^{\#PU_{\iota\varepsilon}(\hat{\pi})}\frac{\left(TF\left(\hat{L}_{\iota\alpha}^{(m)}\right)\cdot\hat{\pi}_{\iota\alpha}^{(m)}-TF\left(\hat{L}_{\alpha}^{(m)}\right)\cdot\hat{\pi}_{\alpha}^{(m)}\right)+\left(TF\left(\hat{U}_{\iota\alpha}^{(m)}\right)\cdot\hat{\pi}_{\iota\alpha}^{(m)}-TF\left(\hat{U}_{\alpha}^{(m)}\right)\cdot\hat{\pi}_{\alpha}^{(m)}\right)}{2}\right)}\right)}$$

$$\varepsilon,\alpha=1,2,\cdots,\theta, \tag{10}$$

where $\widehat{PU}_{\varepsilon}(\widehat{\pi}) = \left\{\left[\widehat{L}_{\varepsilon}^{(m)}, \widehat{U}_{\varepsilon}^{(m)}\right]\left(\widehat{\pi}_{\varepsilon}^{(m)}\right) = \left[\frac{1}{\tau}\sum_{\iota=1}^{\tau}\hat{L}_{\iota\varepsilon}^{(m)}, \frac{1}{\tau}\sum_{\iota=1}^{\tau}\hat{U}_{\iota\varepsilon}^{(m)}\right]\left(\frac{1}{\tau}\sum_{\iota=1}^{\tau}\hat{\pi}_{\iota\varepsilon}^{(m)}\right)\right\}$ and

$\widehat{PU}_{\alpha}(\widehat{\pi}) = \left\{\left[\widehat{L}_{\alpha}^{(m)}, \widehat{U}_{\alpha}^{(m)}\right]\left(\widehat{\pi}_{\alpha}^{(m)}\right) = \left[\frac{1}{\tau}\sum_{\iota=1}^{\tau}\hat{L}_{\iota\alpha}^{(m)}, \frac{1}{\tau}\sum_{\iota=1}^{\tau}\hat{U}_{\iota\alpha}^{(m)}\right]\left(\frac{1}{\tau}\sum_{\iota=1}^{\tau}\hat{\pi}_{\iota\alpha}^{(m)}\right)\right\}$.

$$PSD_{\varepsilon} = \sqrt{\frac{\sum_{\iota=1}^{\tau}\left(\sum_{m=1}^{\#PU_{\iota\varepsilon}(\hat{\pi})}\frac{1}{2}\left(\begin{array}{l}\left(TF\left(\hat{L}_{\iota\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\iota\varepsilon}^{(m)} - TF\left(\hat{L}_{\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\varepsilon}^{(m)}\right)\\ +\left(TF\left(\hat{U}_{\iota\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\iota\varepsilon}^{(m)} - TF\left(\hat{U}_{\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\varepsilon}^{(m)}\right)\end{array}\right)\right)}{\tau-1}}; \quad \varepsilon = 1,2,\cdots,\theta \tag{11}$$

$$I_{\varepsilon} = PSD_{\varepsilon}\cdot\sum_{\alpha=1}^{\theta}\left(1 - PCC_{\alpha}\right),\ \ \varepsilon = 1,2,\cdots,\theta; \tag{12}$$

$$y_{\varepsilon} = \frac{I_{\varepsilon}}{\sum_{\varepsilon=1}^{\theta}I_{\varepsilon}},\ \ \varepsilon = 1,2,\cdots,\theta. \tag{13}$$

Step 4. As the viewpoint that the perceptual factor in human nature may be cause the deviation of attribute weights in PT, the Eqs (14) as well as (15) blend original weights with the weighting function to correct attribute weights, and finally the corrected relative weight $\ell_{\iota\sigma\varepsilon}^{*}\left(y_{\varepsilon}\right)$ is figured out.

$$\ell_{\iota\sigma\varepsilon}\left(y_{\varepsilon}\right) = \begin{cases}\left(y_{\varepsilon}\right)^{x} / \left(\left(y_{\varepsilon}\right)^{x} + \left(1-y_{\varepsilon}\right)^{x}\right)^{\frac{1}{x}}, & PU_{\iota\varepsilon}\left(\hat{\pi}\right) \geq PU_{\sigma\varepsilon}\left(\hat{\pi}\right)\\ \left(y_{\varepsilon}\right)^{z} / \left(\left(y_{\varepsilon}\right)^{z} + \left(1-y_{\varepsilon}\right)^{z}\right)^{\frac{1}{z}}, & PU_{\iota\varepsilon}\left(\hat{\pi}\right) < PU_{\sigma\varepsilon}\left(\hat{\pi}\right)\end{cases}; \tag{14}$$

$$\ell_{\iota\sigma\varepsilon}^{*}\left(y_{\varepsilon}\right) = \frac{\ell_{\iota\sigma\varepsilon}\left(y_{\varepsilon}\right)}{\max_{\varepsilon}\left\{\ell_{\iota\sigma\varepsilon}\left(y_{\varepsilon}\right)\right\}} \quad \iota,\sigma = 1,2,\cdots,\tau;\ \varepsilon = 1,2,\cdots,\theta. \tag{15}$$

Step 5. Compute the relative dominance degree $\tilde{D}_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right)$ $(\iota,\sigma = 1,2,\cdots,\tau;\ \varepsilon = 1,2,\cdots,\theta)$ according to Eq. (17). In addition to the corrected relative weight $\ell_{\iota\sigma\varepsilon}^{*}\left(y_{\varepsilon}\right)$, the distance $d_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right)$ which is obtained by using Eq. (16) is another prerequisite for the relative dominance degree $\tilde{D}_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right)$. Meanwhile, all values of the relative dominance degree keeping in line with identical attribute can be saved in the same matrix $\tilde{D}_{\varepsilon}$ ($\varepsilon = 1,2,\cdots,\theta$).

$$d_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right) = \frac{1}{2\cdot\#PU}\sum_{m=1}^{\#PU}\left(\begin{array}{l}\left|TF\left(\hat{L}_{\iota\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\iota\varepsilon}^{(m)} - TF\left(\hat{L}_{\sigma\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\sigma\varepsilon}^{(m)}\right|\\ +\left|TF\left(\hat{U}_{\iota\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\iota\varepsilon}^{(m)} - TF\left(\hat{U}_{\sigma\varepsilon}^{(m)}\right)\cdot\hat{\pi}_{\sigma\varepsilon}^{(m)}\right|\end{array}\right); \quad \iota,\sigma = 1,2,\cdots,\tau;\varepsilon = 1,2,\cdots,\theta \tag{16}$$

$$\tilde{D}_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right)=\begin{cases}\dfrac{\ell^{*}_{\iota\sigma\varepsilon}\left(y_{\varepsilon}\right)\cdot\left(d_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right)\right)^{\gamma}}{\sum\limits_{\varepsilon=1}^{\theta}\ell^{*}_{\iota\sigma\varepsilon}\left(y_{\varepsilon}\right)} & \text{, if } PU_{\iota\varepsilon}\left(\hat{\pi}\right)>PU_{\sigma\varepsilon}\left(\hat{\pi}\right)\\ 0 & \text{, if } PU_{\iota\varepsilon}\left(\hat{\pi}\right)=PU_{\sigma\varepsilon}\left(\hat{\pi}\right),\\ -\kappa\cdot\dfrac{\left(\sum\limits_{\varepsilon=1}^{\theta}\ell^{*}_{\iota\sigma\varepsilon}\left(y_{\varepsilon}\right)\right)\cdot\left(d_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right)\right)^{\xi}}{\ell^{*}_{\iota\sigma\varepsilon}\left(y_{\varepsilon}\right)} & \text{, if } PU_{\iota\varepsilon}\left(\hat{\pi}\right)<PU_{\sigma\varepsilon}\left(\hat{\pi}\right)\end{cases} \tag{17}$$

where all κ, γ and ξ are parameters.

$$\tilde{D}_{\varepsilon}=\left(\tilde{D}_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right)\right)_{\tau\times\tau}=\begin{array}{c} \\ \Phi_1\\ \Phi_2\\ \vdots\\ \Phi_\tau\end{array}\begin{array}{c}\begin{array}{cccc}\Phi_1 & \Phi_2 & \cdots & \Phi_\tau\end{array}\\ \begin{pmatrix}0 & \tilde{D}_{\varepsilon}\left(\Phi_1,\Phi_2\right) & \cdots & \tilde{D}_{\varepsilon}\left(\Phi_1,\Phi_\tau\right)\\ \tilde{D}_{\varepsilon}\left(\Phi_2,\Phi_1\right) & 0 & \cdots & \tilde{D}_{\varepsilon}\left(\Phi_2,\Phi_\tau\right)\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{D}_{\varepsilon}\left(\Phi_\tau,\Phi_1\right) & \tilde{D}_{\varepsilon}\left(\Phi_\tau,\Phi_2\right) & \cdots & 0\end{pmatrix}\end{array}. \tag{18}$$

Step 6. Just as Eq. (19), summing the values of the relative dominance degree $\tilde{D}_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right)$ under different attributes is the overall dominance degree $\tilde{D}\left(\Phi_{\iota},\Phi_{\sigma}\right)$ ($\iota,\sigma=1,2,\cdots,\tau$). At the same time, gather the outcomes into matrix $\tilde{D}$.

$$\tilde{D}\left(\Phi_{\iota},\Phi_{\sigma}\right)=\sum_{\varepsilon=1}^{\theta}\tilde{D}_{\varepsilon}\left(\Phi_{\iota},\Phi_{\sigma}\right)\qquad \iota,\sigma=1,2,\cdots,\tau; \tag{19}$$

$$\tilde{D}=\left(\tilde{D}\left(\Phi_{\iota},\Phi_{\sigma}\right)\right)_{\tau\times\tau}=\begin{array}{c} \\ \Phi_1\\ \Phi_2\\ \vdots\\ \Phi_\tau\end{array}\begin{array}{c}\begin{array}{cccc}\Phi_1 & \Phi_2 & \cdots & \Phi_\tau\end{array}\\ \begin{pmatrix}0 & \tilde{D}\left(\Phi_1,\Phi_2\right) & \cdots & \tilde{D}\left(\Phi_1,\Phi_\tau\right)\\ \tilde{D}\left(\Phi_2,\Phi_1\right) & 0 & \cdots & \tilde{D}\left(\Phi_2,\Phi_\tau\right)\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{D}\left(\Phi_\tau,\Phi_1\right) & \tilde{D}\left(\Phi_\tau,\Phi_2\right) & \cdots & 0\end{pmatrix}\end{array}. \tag{20}$$

Step 7. Take advantage of Eq. (21) to acquire the standardized overall dominance degree $\tilde{\mathrm{N}}\left(\Phi_{\iota}\right)$ ($\iota=1,2,\cdots,\tau$) which is the eventual criterion for distinguishing the optimal alternative. Generally speaking, the bigger value of $\tilde{\mathrm{N}}\left(\Phi_{\iota}\right)$ means the better alternative.

$$\tilde{\mathrm{N}}\left(\Phi_{\iota}\right)=\frac{\sum\limits_{\sigma=1}^{\tau}\tilde{D}\left(\Phi_{\iota},\Phi_{\sigma}\right)-\min\limits_{\iota}\left\{\sum\limits_{\sigma=1}^{\tau}\tilde{D}\left(\Phi_{\iota},\Phi_{\sigma}\right)\right\}}{\max\limits_{\iota}\left\{\sum\limits_{\sigma=1}^{\tau}\tilde{D}\left(\Phi_{\iota},\Phi_{\sigma}\right)\right\}-\min\limits_{\iota}\left\{\sum\limits_{\sigma=1}^{\tau}\tilde{D}\left(\Phi_{\iota},\Phi_{\sigma}\right)\right\}}. \qquad \iota=1,2,\cdots,\tau \tag{21}$$

4. Numerical example

Network security problems brought by the Internet have threatened the reputation and development of enterprises. In addition, the arrival of 5G era also means that more and more IOT devices will be connected to the network. Therefore, network security has become more and more essential. For each enterprise, it is important to prevent attackers from jumping into the enterprise network from insecure IOT devices and then penetrating the business system so that causing irreparable damage. Because network security is a very strong professional technology, it is difficult for most enterprises to set up a comprehensive professional security team to ensure the security of enterprise data. According to the principle of risk and return, it is an essential decision-making activity in enterprise management to choose the best one from the numerous network security service providers (NSSPs) in the market. The evaluation of NSSPs usually bases on the following four aspects: (1) Δ_1 is the equipment performance; (2) Δ_2 is the maintenance cost; (3) Δ_3 is the testing capability; (4) Δ_4 is the emergency processing capability. Moreover, five experts $\wp_e$ ($e = 1,2,3,4,5$) have given the following evaluative information, just as Table 1 to Table 5, to five NSSPs Φ_ι ($\iota = 1,2,3,4,5$).

$$V = \left\{ \begin{array}{l} \nu_{-3} = \text{extremely bad}(\underset{\sim}{\text{EB}}), \nu_{-2} = \text{very bad}(\underset{\sim}{\text{VB}}), \nu_{-1} = \text{bad}(\underset{\sim}{\text{B}}), \nu_0 = \text{medium}(\underset{\sim}{\text{M}}), \\ \nu_1 = \text{great}(\underset{\sim}{\text{G}}), \nu_2 = \text{very great}(\underset{\sim}{\text{VG}}), \nu_3 = \text{extremely great}(\underset{\sim}{\text{EG}}) \end{array} \right\}.$$

Table 1. The ULT matrix $\tilde{U}^{(1)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	[VB, M]	[M, G]	[M, G]	[B, G]
Φ_2	[G, VG]	[EB, VB]	[M, G]	[G, EG]
Φ_3	[M, VG]	[VB, M]	[G, VG]	[VB, B]
Φ_4	[M, G]	[VB, B]	[B, M]	[EB, B]
Φ_5	[B, G]	[B, M]	[M, VG]	[G, VG]

Table 2. The ULT matrix $\tilde{U}^{(2)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	[B, G]	[G, VG]	[M, G]	[VB, B]
Φ_2	[G, VG]	[VB, B]	[B, G]	[G, EG]
Φ_3	[G, VG]	[VB, M]	[B, M]	[M, VG]
Φ_4	[VB, M]	[VB, M]	[VB, M]	[B, M]
Φ_5	[M, G]	[M, G]	[B, G]	[VG, EG]

Table 3. The ULT matrix $\tilde{U}^{(3)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	[VB, B]	[M, G]	[G, VG]	[B, M]
Φ_2	[G, EG]	[VB, M]	[G, VG]	[G, VG]
Φ_3	[M, G]	[B, M]	[M, VG]	[M, VG]
Φ_4	[B, M]	[VB, M]	[M, G]	[VB, B]
Φ_5	[M, G]	[M, G]	[M, VG]	[VG, EG]

Table 4. The ULT matrix $\tilde{U}^{(4)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	[VB, M]	[M, G]	[VG, EG]	[B, M]
Φ_2	[G, VG]	[VB, M]	[G, VG]	[G, VG]
Φ_3	[M, G]	[B, M]	[G, VG]	[M, G]
Φ_4	[VB, M]	[VB, B]	[B, M]	[VB, B]
Φ_5	[M, VG]	[VB, M]	[B, G]	[G, VG]

Table 5. The ULT matrix $\tilde{U}^{(5)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	[VB, B]	[G, VG]	[M, G]	[B, M]
Φ_2	[G, VG]	[VB, B]	[G, VG]	[G, EG]
Φ_3	[M, VG]	[EB, B]	[M, VG]	[M, G]
Φ_4	[B, M]	[B, G]	[M, G]	[B, M]
Φ_5	[M, VG]	[VB, M]	[M, G]	[G, VG]

The following clearly demonstrates the application of PT-PUL-TODIM in NSSPs selection.

Step 1. Transform the negative attribute into positive one and take the results in Table 6 to Table 10.

Table 6. The standardized ULT matrix from $\tilde{U}^{(1)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	$[\underset{\sim}{VB},\underset{\sim}{M}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{B},\underset{\sim}{G}]$
Φ_2	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{VG},\underset{\sim}{EG}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{G},\underset{\sim}{EG}]$
Φ_3	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{VB},\underset{\sim}{B}]$
Φ_4	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{EB},\underset{\sim}{B}]$
Φ_5	$[\underset{\sim}{B},\underset{\sim}{G}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$

Table 7. The standardized ULT matrix from $\tilde{U}^{(2)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	$[\underset{\sim}{B},\underset{\sim}{G}]$	$[\underset{\sim}{VB},\underset{\sim}{B}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{VB},\underset{\sim}{B}]$
Φ_2	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{B},\underset{\sim}{G}]$	$[\underset{\sim}{G},\underset{\sim}{EG}]$
Φ_3	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$
Φ_4	$[\underset{\sim}{VB},\underset{\sim}{M}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{VB},\underset{\sim}{M}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$
Φ_5	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{B},\underset{\sim}{G}]$	$[\underset{\sim}{VG},\underset{\sim}{EG}]$

Table 8. The standardized ULT matrix from $\tilde{U}^{(3)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	$[\underset{\sim}{VB},\underset{\sim}{B}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$
Φ_2	$[\underset{\sim}{G},\underset{\sim}{EG}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$
Φ_3	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$
Φ_4	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{VB},\underset{\sim}{B}]$
Φ_5	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{VG},\underset{\sim}{EG}]$

Table 9. The standardized ULT matrix from $\tilde{U}^{(4)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	$[\underset{\sim}{VB},\underset{\sim}{M}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{VG},\underset{\sim}{EG}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$
Φ_2	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$
Φ_3	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$
Φ_4	$[\underset{\sim}{VB},\underset{\sim}{M}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{VB},\underset{\sim}{B}]$
Φ_5	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{B},\underset{\sim}{G}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$

Table 10. The standardized ULT matrix from $\tilde{U}^{(5)}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	$[\underset{\sim}{VB},\underset{\sim}{B}]$	$[\underset{\sim}{VB},\underset{\sim}{B}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$
Φ_2	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{EG}]$
Φ_3	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{G},\underset{\sim}{EG}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$
Φ_4	$[\underset{\sim}{B},\underset{\sim}{M}]$	$[\underset{\sim}{B},\underset{\sim}{G}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{B},\underset{\sim}{M}]$
Φ_5	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{VG}]$	$[\underset{\sim}{M},\underset{\sim}{G}]$	$[\underset{\sim}{G},\underset{\sim}{VG}]$

Step 2. Based on the consistent ULTS decision matrices and Definition 3, the pretreated and rearranged PULTS matrix is able to be acquired as $\tilde{K}=\left(PU_{\iota\varepsilon}\left(\hat{\pi}\right)\right)_{\delta\times\tau}=\left(\left\{\left[\hat{L}_{\iota\varepsilon}^{(m)},\hat{U}_{\iota\varepsilon}^{(m)}\right]\left(\hat{\pi}_{\iota\varepsilon}^{(m)}\right)\middle|m=1,2,\cdots,\#PU_{\iota\varepsilon}\left(\hat{\pi}\right)\right\}\right)_{\tau\times\theta}$, just as Table 11.

Table 11. The PULTS matrix $\tilde{K}$

	Δ_1	Δ_2	Δ_3	Δ_4
Φ_1	$\{[v_{-2},v_{-1}](0.6),[v_{-1},v_0](0.3),[v_0,v_1](0.1)\}$	$\{[v_{-2},v_{-1}](0),[v_{-2},v_{-1}](0.4),[v_{-1},v_0](0.6)\}$	$\{[v_0,v_1](0.6),[v_1,v_2](0.2),[v_2,v_3](0.2)\}$	$\{[v_{-2},v_{-1}](0.2),[v_{-1},v_0](0.7),[v_0,v_1](0.1)\}$
Φ_2	$\{[v_1,v_2](0),[v_1,v_2](0.9),[v_2,v_3](0.1)\}$	$\{[v_0,v_1](0.2),[v_1,v_2](0.6),[v_2,v_3](0.2)\}$	$\{[v_{-1},v_0](0.1),[v_0,v_1](0.3),[v_1,v_2](0.6)\}$	$\{[v_1,v_2](0),[v_1,v_2](0.7),[v_2,v_3](0.3)\}$
Φ_3	$\{[v_0,v_1](0),[v_0,v_1](0.6),[v_1,v_2](0.4)\}$	$\{[v_0,v_1](0.6),[v_1,v_2](0.3),[v_2,v_3](0.1)\}$	$\{[v_{-1},v_0](0.2),[v_0,v_1](0.2),[v_1,v_2](0.6)\}$	$\{[v_{-2},v_{-1}](0.2),[v_0,v_1](0.6),[v_1,v_2](0.2)\}$
Φ_4	$\{[v_{-2},v_{-1}](0.2),[v_{-1},v_0](0.6),[v_0,v_1](0.2)\}$	$\{[v_{-1},v_0](0.1),[v_0,v_1](0.3),[v_1,v_2](0.6)\}$	$\{[v_{-2},v_{-1}](0.1),[v_{-1},v_0](0.5),[v_0,v_1](0.4)\}$	$\{[v_{-3},v_{-2}](0.1),[v_{-2},v_{-1}](0.5),[v_{-1},v_0](0.4)\}$
Φ_5	$\{[v_{-1},v_0](0.1),[v_0,v_1](0.7),[v_1,v_2](0.2)\}$	$\{[v_{-1},v_0](0.4),[v_0,v_1](0.4),[v_1,v_2](0.2)\}$	$\{[v_{-1},v_0](0.2),[v_0,v_1](0.6),[v_1,v_2](0.2)\}$	$\{[v_1,v_2](0),[v_1,v_2](0.6),[v_2,v_3](0.4)\}$

Step 3. The CRITIC method, just as Eqs (10)–(13), is selected as an approach of determining the objective weighting vector of attributes $y=\left(y_1,y_2,y_3,y_4\right)^T$ ($y_\varepsilon\geq 0$ and $\sum_{\varepsilon=1}^{\theta}y_\varepsilon=1$).

$$y=\left(y_1,y_2,y_3,y_4\right)^T=\left(0.1805,0.3064,0.2056,0.3076\right)^T.$$

Step 4. As the viewpoint that the perceptual factor in human nature may be cause the deviation of attribute weights in PT, the Eqs (14) as well as (15) blend original weights with the weighting function to correct attribute weights, and finally the corrected relative weight $\ell_{\iota\sigma\varepsilon}^{*}\left(y_\varepsilon\right)$ is figured out shown in Table 12 (Notes: the values of parameters x = 0.61 and z = 0.69 in Eq. (14) are derived from Kahneman (1992)'s the experimental proof.).

Table 12. The corrected relative weights

	Δ_1	Δ_2	Δ_3	Δ_4		Δ_1	Δ_2	Δ_3	Δ_4
ℓ_{12}^{*}	0.7271	0.9976	0.7946	1	ℓ_{34}^{*}	0.7478	1	0.7965	0.9717
ℓ_{13}^{*}	0.7271	0.9976	0.7946	1	ℓ_{35}^{*}	0.7460	0.9675	0.7946	1
ℓ_{14}^{*}	0.7289	1	0.7965	0.9717	ℓ_{41}^{*}	0.7460	0.9675	0.7856	1
ℓ_{15}^{*}	0.7271	0.9976	0.7946	1	ℓ_{42}^{*}	0.7271	0.9976	0.7856	1
ℓ_{21}^{*}	0.7695	0.9980	0.8104	1	ℓ_{43}^{*}	0.7271	0.9675	0.7856	1
ℓ_{23}^{*}	0.7695	0.9980	0.8197	1	ℓ_{45}^{*}	0.7271	0.9675	0.7856	1
ℓ_{24}^{*}	0.7695	0.9980	0.8197	1	ℓ_{51}^{*}	0.7695	0.9980	0.8104	1
ℓ_{25}^{*}	0.7460	0.9675	0.7946	1	ℓ_{52}^{*}	0.7289	1	0.7875	0.9717
ℓ_{31}^{*}	0.7695	0.9980	0.8104	1	ℓ_{53}^{*}	0.7289	1	0.7875	0.9717
ℓ_{32}^{*}	0.7271	0.9976	0.7856	1	ℓ_{54}^{*}	0.7478	1	0.7965	0.9717

Step 5. Compute the relative dominance degree $\tilde{D}_\varepsilon(\Phi_\iota,\Phi_\sigma)$ $(\iota,\sigma=1,2,3,4,5;\varepsilon=1,2,3,4)$ according to Eq. (17) and the outcomes are listed in Table 8. In addition, the distances $d_\varepsilon(\Phi_\iota,\Phi_\sigma)$ obtained by using Eq. (16) are shown in Table 13. Meanwhile, all values of the relative dominance degree keeping in line with identical attribute can be saved in the same matrix $\tilde{D}_\varepsilon$ $(\varepsilon=1,2,3,4)$ (Notes: the values of parameters γ = 0.88, ξ = 0.88 and κ = 2.25 in Eq. (17) are derived from Kahneman (1992)).

Table 13. Distance between each two alternatives

	Δ_1	Δ_2	Δ_3	Δ_4
$d(\Phi_1,\Phi_2)$	0.2444	0.1778	0.2000	0.1667
$d(\Phi_1,\Phi_3)$	0.2056	0.2111	0.1889	0.0500
$d(\Phi_1,\Phi_4)$	0.0944	0.1056	0.1444	0.1056
$d(\Phi_1,\Phi_5)$	0.1611	0.1333	0.1667	0.1722
$d(\Phi_2,\Phi_3)$	0.1778	0.1833	0.0333	0.1167
$d(\Phi_2,\Phi_4)$	0.1667	0.2056	0.0889	0.1722
$d(\Phi_2,\Phi_5)$	0.1222	0.1000	0.1722	0.0556
$d(\Phi_3,\Phi_4)$	0.1111	0.2389	0.1222	0.0944
$d(\Phi_3,\Phi_5)$	0.0833	0.0833	0.1778	0.1222
$d(\Phi_4,\Phi_5)$	0.0667	0.1611	0.0944	0.1778

$$\tilde{D}_1=\left(\tilde{D}_1(\Phi_\iota,\Phi_\sigma)\right)_{5\times5}=\begin{array}{c} \\ \Phi_1\\ \Phi_2\\ \Phi_3\\ \Phi_4\\ \Phi_5\end{array}\begin{array}{c}\begin{array}{ccccc}\Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 & \Phi_5\end{array}\\ \begin{pmatrix} 0 & -3.1523 & -2.7065 & -1.3533 & -2.1843\\ 0.0623 & 0 & 0.0469 & 0.0443 & 0.0334\\ 0.0535 & -2.3758 & 0 & 0.0308 & 0.0239\\ 0.0267 & -2.2446 & -1.5575 & 0 & -0.9936\\ 0.0431 & -1.6936 & -1.2090 & 0.0196 & 0\end{pmatrix}\end{array};$$

$$\tilde{D}_2=\left(\tilde{D}_2(\Phi_\iota,\Phi_\sigma)\right)_{5\times5}=\begin{array}{c} \\ \Phi_1\\ \Phi_2\\ \Phi_3\\ \Phi_4\\ \Phi_5\end{array}\begin{array}{c}\begin{array}{ccccc}\Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 & \Phi_5\end{array}\\ \begin{pmatrix} 0 & -1.7361 & -2.0196 & -1.0878 & -1.3478\\ 0.0610 & 0 & 0.0625 & 0.0691 & 0.0364\\ 0.0710 & -1.7792 & 0 & -2.2442 & 0.0310\\ 0.0382 & -1.9677 & 0.0789 & 0 & 0.0558\\ 0.0474 & -1.0346 & -0.8812 & -1.5868 & 0\end{pmatrix}\end{array};$$

$$\tilde{D}_3=\left(\tilde{D}_3(\Phi_\iota,\Phi_\sigma)\right)_{5\times5}=\begin{array}{c} \\ \Phi_1\\ \Phi_2\\ \Phi_3\\ \Phi_4\\ \Phi_5\end{array}\begin{array}{c}\begin{array}{ccccc}\Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 & \Phi_5\end{array}\\ \begin{pmatrix} 0 & 0.0548 & 0.0521 & 0.0415 & 0.0467\\ -2.4101 & 0 & 0.0115 & 0.0272 & 0.0482\\ -2.2919 & -0.5040 & 0 & 0.0356 & 0.0495\\ -1.8259 & -1.1949 & -1.5678 & 0 & -1.2495\\ -2.0528 & -2.1198 & -2.1798 & 0.0284 & 0\end{pmatrix}\end{array};$$

$$\tilde{D}_4=\left(\tilde{D}_4\left(\Phi_\iota,\Phi_\sigma\right)\right)_{5\times5}=\begin{array}{c} \\ \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \\ \Phi_5 \end{array}\begin{array}{c} \begin{array}{ccccc} \Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 & \Phi_5 \end{array} \\ \begin{pmatrix} 0 & -1.6363 & -0.5672 & 0.0384 & -1.6843 \\ 0.0578 & 0 & 0.0421 & 0.0593 & 0.6203 \\ 0.0200 & -1.1925 & 0 & 0.0346 & -1.2415 \\ -1.0884 & -1.6799 & -0.9816 & 0 & -1.7127 \\ 0.0594 & 0.0219 & 0.0438 & 0.0604 & 0 \end{pmatrix} \end{array}.$$

Step 6. Just as Eq. (19), summing the values of the relative dominance degree $\tilde{D}_\varepsilon\left(\Phi_\iota,\Phi_\sigma\right)$ under different attributes is the overall dominance degree $\tilde{D}\left(\Phi_\iota,\Phi_\sigma\right)$ ($\iota,\sigma=1,2,3,4,5$). At the same time, gather the outcomes into matrix $\tilde{D}$.

$$\tilde{D}=\left(\tilde{D}\left(\Phi_\iota,\Phi_\sigma\right)\right)_{5\times5}=\begin{array}{c} \\ \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \\ \Phi_5 \end{array}\begin{array}{c} \begin{array}{ccccc} \Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 & \Phi_5 \end{array} \\ \begin{pmatrix} 0 & -6.4700 & -5.2412 & -2.3612 & -5.1697 \\ -2.2291 & 0 & 0.1630 & 0.1999 & -0.5023 \\ -2.1474 & -5.8515 & 0 & -2.1431 & -1.1371 \\ -2.8494 & -7.0871 & -4.0281 & 0 & -3.9001 \\ -1.9029 & -4.8261 & -4.2263 & -1.4783 & 0 \end{pmatrix} \end{array}.$$

Step 7. Take advantage of Eq. (21) to acquire the standardized overall dominance degree $\tilde{\mathrm{N}}\left(\Phi_\iota\right)$ ($\iota=1,2,3,4,5$) which is the eventual criterion for distinguishing the optimal alternative. Generally speaking, the alternative Φ_2 with the biggest value of $\tilde{\mathrm{N}}\left(\Phi_\iota\right)$ is the best choice.

$$\tilde{\mathrm{N}}\left(\Phi_1\right)=0,\tilde{\mathrm{N}}\left(\Phi_2\right)=1,\tilde{\mathrm{N}}\left(\Phi_3\right)=0.4719,\tilde{\mathrm{N}}\left(\Phi_4\right)=0.0816,\tilde{\mathrm{N}}\left(\Phi_5\right)=0.4035,$$

$$\Phi_2>\Phi_3>\Phi_5>\Phi_4>\Phi_1.$$

5. Comparative analysis

In this section, two methods, PUL-EDAS (He et al., 2019) and PUL-GAR (Wei et al., 2020a), are selected and are calculated based on the same initial data. The final calculation results are shown in the Table 14. Both the PT-PUL-TODIM method and the other two methods get the consistent optimal solution, which undoubtedly demonstrates the reliability of the decision-making results of the new PT-PUL-TODIM model. The PT-PUL-TODIM model shows very clearly the degree of variation between different solutions. Moreover, from the perspective of model ideology, the PT-PUL-TODIM method fully considers the influence of DM's psychology on the decision results.

Table 14. Comparison of different methods

	PUL-EDAS	PUL-GAR	PT-PUL-TODIM
Φ_1	0.0478	0.4477	0
Φ_2	1	0.6209	1
Φ_3	0.5894	0.5410	0.4719
Φ_4	0.1476	0.3934	0.0816
Φ_5	0.6801	0.5979	0.4035
The order	$\Phi_2>\Phi_5>\Phi_3>\Phi_4>\Phi_1$	$\Phi_2>\Phi_5>\Phi_3>\Phi_1>\Phi_4$	$\Phi_2>\Phi_3>\Phi_5>\Phi_4>\Phi_1$

Conclusions

In order to provide a better decision method for MAGDM problems, this paper propose a new decision model named the probabilistic uncertainty linguistic TODIM model based on the prospect theory (PT-PUL-TODIM). The model combines the advantages of the PT and the TODIM method, that is, the DM's mentality is fully considered in the whole process of decision information processing. In the end, a case study concerning NSSP selection problem is given to demonstrate the merits of the developed methods. At the end, two existing methods is compared with the PT-PUL-TODIM method in order to ensure the application effect. Although there are slightly different in terms of the overall ranking of the alternatives, these differences are probably due to the diversities in the evaluation criteria of the different methods. However, it is obvious that the PT-PUL-TODIM method has noteworthy advantages over previous methods in the overall design of the model logic structure. In the future, our team will continue to make efforts in the field of MADM and MAGDM. And we hope to make achievements in theory, application and other aspects.

Acknowledgements

The work was supported by the Humanities and Social Sciences Foundation of Ministry of Education of the People's Republic of China (No. 17XJA630003).

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