PDHL-EDAS METHOD FOR MULTIPLE ATTRIBUTE GROUP DECISION MAKING AND ITS APPLICATION TO 3D PRINTER SELECTION

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Abstract. With the rapid development of 3D printing technology, 3D printers are manufactured based on the principle of 3D printing technology are more and more widely used in the manufacturing industry. Choosing high quality 3D printers for industrial production is of great significance to the economic growth of enterprises. In fact, it is difficult to select the most optimal 3D printers under a single and simple standard. Therefore, this paper establishes the probabilistic double hierarchy linguistic EDAS (PDHL-EDAS) method for the multiple attribute group decision making (MAGDM). Then the CRITIC model is introduced to derive objective weight and the cumulative prospect theory is leaded into obtain the cumulative weight of PDHLTS. In addition, what's more, the PDHL-EDAS method is built and applied to the choice of high-quality 3D printer. Finally, compared with the available MAGDM methods under PDHLTS, the built method is proved to be scientific and effective.

Keywords: multiple attribute group decision making (MAGDM), probabilistic double hierarchy linguistic term set (PDHLTS), EDAS method, CRITIC method, 3D printer selection.

JEL Classification: C43, C61, D81.

Introduction

3D printing is a rapid prototyping information technology based on digital model files, which uses powder metal, plastic and other bonding materials to build objects by stacking and accumulating layer by layer (i.e. "layer by layer molding method"). In the past, it is often employed in mold manufacturing, industrial design and other fields to make models, but now it is gradually used directly to manufacture some products. In particular, it is used for the

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production of some high-value products (such as animal hip joints or teeth, or some parts of automobiles and aircraft). At present, there are many researches on the improvement of 3D printing information technology and the application of 3D printer (Cheng et al., 2020; Chrispin et al., 2020; Liu et al., 2019; Pavan et al., 2021; Yi et al., 2019), but there are few researches on the selection and evaluation of 3D printer. Such as, Prabhu and Ilangkumaran (2019a) used VIKOR method to rank and evaluate 3D printers in fuzzy environment; Prabhu and Ilangkumaran (2019b) combined the GRA Method with TOPSIS method, and put forward the GRA-TOPSIS method to rank 3D printers in real environment.

Through the above literature, we can know that the research on 3D printer evaluation model is very few, and the evaluation information is given in the form of real number. But in the actual production and life, decision makers are more accustomed to using linguistic to describe the evaluation information. Pang, Wang, and Xu (2016) built the probabilistic linguistic term sets (PLTSs). Wei, Lin, Lu, Wu, and Wei (2021b) built the generalized dice similarity measures for PUL-MAGDM. Wei, Wu, Guo, and Wei (2021a) built the CODAS in probabilistic uncertain linguistic setting. But some complex linguistic evaluation information can't describe. for example, the frequency or probabilistic of "just right poor" is 0.3, and "only a little good" is 0.7 is can not represent by those sets. Recently, to solve this issue the probabilistic double hierarchy linguistic term set (PDHLTS) proposed by Gou, Xu, Liao, and Herrera (2021). and Lei, Wei, and Chen (2021) defined the probabilistic double hierarchy linguistic CODAS method. So in PDHLTS, we can use $\left\{ z_{0<h_{-1}>}(0.3), z_{1<h_{-2}>}(0.7) \right\}$ to represent The frequency or probabilistic of "just right poor" is 0.3, and "only a little good" is 0.7, where,

$$z = \left\{ z_{-3} = extremely\ poor,\ z_{-2} = very\ poor,\ z_{-1} = poor, z_0 = medium,\ z_1 = good, \right.$$
$$\left. z_2 = very\ good,\ z_3 = extremely\ good \right\},$$
$$h = \left\{ h_{-3} = far\ form,\ h_{-2} = only\ a\ little,\ h_{-1} = a\ little,\ h_0 = just\ right, \right.$$
$$\left. h_1 = much,\ h_2 = very\ much,\ h_3 = extirely \right\}.$$

Besides, in order to better select the 3D printers that meet the actual production needs of enterprises, we will use EDAS model to evaluate the 3D printers. The EDAS is developed by Ghorabaee, Zavadskas, Olfat, and Turskis (2015), which is a novel and efficient method to solve MAGDM problems. Karunanithi et al. (2015) expanded the EDAS method to evaluate treatment outcome in Moyamoya disease. Stanujkic, Zavadskas, Keshavarz Ghorabaee, and Turskis (2017) proposed the EDAS model under interval grey numbers. Gundogdu, Kahraman, and Civan (2018) defined the HF-EDAS for hospital selection. Liang, Zhao, and Luo (2018) proposed that the cleanliness of gold production can be evaluated by a combination method of the EDAs method and ELECTRE model. Feng, Wei, and Liu (2018) extended hesitant fuzzy linguistic EDAS model to solve a company's investment issue. Kundakci (2019) developed the MACBETH and EDAS for selecting the steam boiler. Wang, Wang, and Wei (2019) employed EDAS to cope with the 2-tuple linguistic neutrosophic MADM. Darko and Liang (2020) combined EDAS method with Hamacher operators to select mobile payment platform. Wei, Wei, and Guo (2021c) defined the EDAS method for PL-MAGDM. He et al. (2019) built the EDAS algorithm for MAGDM with PULTSs.

From the above description, we can know that EDAS method is an efficient method to deal with MAGDM issue, but there is no research on EDAS method under PDHLTSs in the

existing literatures, therefore, in order to effectively solve the problem of 3D printer selection, this paper will construct an PDHL-EDAS method to deal with this problem. The main work of this paper could be showed: (1) the PDHL-EDAS method for MAGDM is established; (2) the CRITIC (Diakoulaki et al., 1995) method is used to calculate the objective weight of attributes under PDHLTSs; (3) the steps of solving MAGDM problem with PDHL-EDAS model are given; (4) the PDHL-EDAS model is used to select the best 3D printer; (5) the model constructed in this paper is compared with the existing models.

1. Preliminaries

The notion of PDHLTS is reviewed in this section.

Definition 1 (Gou et al., 2017). Let $DHL = \left\{ z_{\phi < h_\psi >} \middle| \phi = -\chi, \cdots, -1, 0, 1, \cdots \chi; \psi = -\eta, \cdots, -1, 0, 1, \cdots \eta \right\}$ be a DHLTS, the definition of the DHHFLTS is built as follows:

$$DHHFL = \left\{ z_{\phi < h_\psi >}^k \middle| \phi = -\chi, \cdots, -1, 0, 1, \cdots \chi; \psi = -\eta, \cdots, -1, 0, 1, \cdots \eta \right\}, \tag{1}$$

where $k = 1, 2, \ldots, \#DHL$, the *k-th* double hierarchy linguistic element (DHLE) is denoted as $z_{\phi < h_\psi >}^k$, the numbers of all DHLEs in DHL is expressed as $\#DHL$, and each DHLE in a DHHFLTS is arranged in ascending order.

Definition 2 (Gou et al., 2017). Let $DHL = \left\{ z_{\phi < h_\psi >} \middle| \phi = -\chi, \cdots, -1, 0, 1, \cdots \chi; \psi = -\eta, \cdots, -1, 0, 1, \cdots \eta \right\}$ be a DHLTS, the PDHLTS is built as follows:

$$PDHL(p) = \left\{ z_{\phi < h_\psi >}^k (p^k) \middle| z_{\phi < h_\psi >}^k \in DHL, p^k \geq 0, \sum_{k=1}^{\#PDHL(P)} p^k \leq 1 \right\}, \tag{2}$$

where $k = 1, 2, \ldots, \#PDHL(P); \phi = -\chi, \cdots, -1, 0, 1, \cdots \chi; \psi = -\eta, \cdots, -1, 0, 1, \cdots \eta$, the *k-th* probabilistic double hierarchy linguistic element (PDHLE) is denoted as $z_{\phi < h_\psi >}^k (p^k)$, the numbers of all PDHLEs in $PDHL(P)$ are expressed as $\#PDHL(P)$, and According to $f\left(l_{\varphi < S_\psi >}^k \right)$, each PDHLE in a PDHLTS is arranged in ascending order, where the transformation function f is defined by Eq. (3).

Definition 3 (Gou et al., 2021). Let $DHL = \left\{ z_{\phi < h_\psi >} \middle| \phi = -\chi, \cdots, -1, 0, 1, \cdots \chi; \psi = -\eta, \cdots, -1, 0, 1, \cdots \eta \right\}$ be a DHLTS, and $PDHL(p) = \left\{ z_{\phi < h_\psi >}^k (p^k) \middle| z_{\phi < h_\psi >}^k \in DHL, p^k \geq 0, \sum_{k=1}^{\#PDHL(P)} p^k \leq 1 \right\}$ be a PDHLTS, the equivalent information between the subscript (φ, ψ) of $z_{\phi < h_\psi >}^k$ and the numerical γ_n is converted by transformation functions f and f^{-1}:

$$f : [-\chi, \chi] \times [-\eta, \eta] \to [0, 1], f(\phi, \psi) = \frac{\psi + (\chi + \phi)\eta}{2\chi\eta} = \gamma_n; \tag{3}$$

$$f^{-1} : [0, 1] \to [-\chi, \chi] \times [-\eta, \eta],$$
$$f^{-1}(\chi_n) = [2\chi\gamma_n - \chi]_{< h_{\eta((2\chi\gamma_n - \chi) - [2\chi\gamma_n - \chi])} >} \text{ or } [2\chi\gamma_n - \chi] + 1_{< h_{\eta((2\chi\gamma_n - \chi) - [2\chi\gamma_n - \chi]) - \eta} >}. \tag{4}$$

Considering that the probability sum of all PDHLEs in PDHLTS may not be 1, the following normalization formula is proposed.

$$PDHL(p) = \left\{ z^k_{\phi<h_\psi>}\left(p^k\right) \middle| z^k_{\phi<h_\psi>} \in DHL, p^k \geq 0, \sum_{k=1}^{\#PDHL(P)} p^k < 1 \right\}, \qquad (5)$$

where $\tilde{p}^k = p^k \Big/ \sum_{k=1}^{\#PDHL(P)} p^k$; $\phi \in [-\chi, \chi]$; $\psi \in [-\eta, \eta]$; ϕ, ψ are integer numbers.

Definition 4 (Lei et al., 2021). Let $DHL = \left\{ z_{\phi<h_\psi>} \middle| \phi = -\chi, \cdots, -1, 0, 1, \cdots \chi; \psi = -\eta, \cdots, -1, 0, 1, \cdots \eta \right\}$ be a DHLTS, $PDH\tilde{L}_1(\tilde{p}) = \left\{ z^k_{1\phi<h_\psi>}\left(\tilde{p}^k_1\right) \middle| z^k_{1\phi<h_\psi>} \in DHL; k = 1,2,...,\#PDH\tilde{L}_1(\tilde{p}) \right\}$ and $PDH\tilde{L}_2(\tilde{p}) = \left\{ z^k_{2\phi<h_\psi>}\left(\tilde{p}^k_2\right) \middle| z^k_{2\phi<h_\psi>} \in DHL; k = 1,2,...,\#PDH\tilde{L}_2(\tilde{p}) \right\}$ be two PDHLTSs, where $\#PDH\tilde{L}_1(\tilde{p})$, $\#PDH\tilde{L}_2(\tilde{p})$ are the numbers of all PDHLEs in $PDH\tilde{L}_1(\tilde{p})$ and $PDH\tilde{L}_2(\tilde{p})$. Especially, if $\#PDH\tilde{L}_1(\tilde{p}) > \#PDH\tilde{L}_2(\tilde{p})$, then the numbers of $\#PDH\tilde{L}_1(\tilde{p}) - \#PDH\tilde{L}_2(\tilde{p})$ DHLEs are added to $PDH\tilde{L}_2(\tilde{p})$. What's more, the newly added DHLEs need be the smallest DHLEs in $PDH\tilde{L}_2(\tilde{p})$ and the corresponding probabilities of newly added DHLTS should be zero.

Definition 5 (Lei et al., 2021). Let $PDH\tilde{L}(\tilde{p}) = \left\{ z^k_{\phi<h_\psi>}\left(\tilde{p}^k\right) \middle| z^k_{\phi<h_\psi>} \in DH\tilde{L}; k = 1,2,..., \#PDH\tilde{L}(\tilde{p}) \right\}$ be a PDHLTS, established the expected values $E\left(PDH\tilde{L}(\tilde{p})\right)$ and deviation degree $\sigma\left(PDH\tilde{L}(\tilde{p})\right)$ of $PDH\tilde{L}(\tilde{p})$ as follows:

$$E\left(PDH\tilde{L}(\tilde{p})\right) = \sum_{k=1}^{\#PDH\tilde{L}(\tilde{p})} f\left(PDH\tilde{L}(\tilde{p})\right)\tilde{p}^k \Big/ \sum_{k=1}^{\#PDH\tilde{L}(\tilde{p})} \tilde{p}^k; \qquad (6)$$

$$\sigma\left(PDH\tilde{L}(\tilde{p})\right) = \sqrt{\sum_{k=1}^{\#PDH\tilde{L}(\tilde{p})} \left(f\left(PDH\tilde{L}(\tilde{p})\right)\tilde{p}^k - E\left(PDH\tilde{L}(\tilde{p})\right)\right)^2 \Big/ \sum_{k=1}^{\#PDH\tilde{L}(\tilde{p})} \tilde{p}^k}. \qquad (7)$$

The order relationship between the two PDHLTSs can be further determined by Eqs (6)–(7).

(1) if $E\left(PDH\tilde{L}_1(\tilde{p})\right) > E\left(PDH\tilde{L}_2(\tilde{p})\right)$, then $PDH\tilde{L}_1(\tilde{p}) > PDH\tilde{L}_2(\tilde{p})$;

(2) if $E\left(PDH\tilde{L}_1(\tilde{p})\right) = E\left(PDH\tilde{L}_2(\tilde{p})\right)$, then if $\sigma\left(PDH\tilde{L}_1(\tilde{p})\right) = \sigma\left(PDH\tilde{L}_2(\tilde{p})\right)$, then $PDH\tilde{L}_1(\tilde{p}) = PDH\tilde{L}_2(\tilde{p})$; then if $\sigma\left(PDH\tilde{L}_1(\tilde{p})\right) < \sigma\left(PDH\tilde{L}_2(\tilde{p})\right)$, then, $PDH\tilde{L}_1(\tilde{p}) > PDH\tilde{L}_2(\tilde{p})$.

Definition 6 (Lei et al., 2021). Let $DHL = \left\{ z_{\phi<h_\psi>} \middle| \phi = -\chi, \cdots, -1, 0, 1, \cdots \chi; \psi = -\eta, \cdots, -1, 0, 1, \cdots \eta \right\}$ be a DHLTS, $PDH\tilde{L}_1(\tilde{p}) = \left\{ z^k_{1\phi<h_\psi>}\left(\tilde{p}^k_1\right) \middle| z^k_{1\phi<h_\psi>} \in DHL; k = 1,2,...,\#PDH\tilde{L}_1(\tilde{p}) \right\}$ and $PDH\tilde{L}_2(\tilde{p}) = \left\{ z^k_{2\phi<h_\psi>}\left(\tilde{p}^k_2\right) \middle| z^k_{2\phi<h_\psi>} \in DHL; k = 1,2,...,\#PDH\tilde{L}_2(\tilde{p}) \right\}$ are two PDHLTSs, where $\#PDH\tilde{L}_1(\tilde{p}) = \#PDH\tilde{L}_2(\tilde{p}) = \#PDH\tilde{L}(\tilde{p})$, then Euclidean distance $ED\left(PDH\tilde{L}_1(\tilde{p}), PDH\tilde{L}_2(\tilde{p})\right)$ and Hamming distance $HD\left(PDH\tilde{L}_1(\tilde{p}), PDH\tilde{L}_2(\tilde{p})\right)$ between $PDH\tilde{L}_1(\tilde{p})$ and $PDH\tilde{L}_2(\tilde{p})$ are established.

$$ED\left(PDH\tilde{L}_1(\tilde{p}), PDH\tilde{L}_2(\tilde{p})\right) = \sqrt{\frac{\sum_{k=1}^{\#PDH\tilde{L}(\tilde{p})} \left(f\left(z^k_{1\phi<h_\psi>}\right)\tilde{p}^k_1 - f\left(z^k_{2\phi<h_\psi>}\right)\tilde{p}^k_2 \right)^2}{\#PDH\tilde{L}(\tilde{p})}}; \qquad (8)$$

$$HD\left(PDH\tilde{L}_1(\tilde{p}),PDH\tilde{L}_2(\tilde{p})\right)=\frac{\sum_{k=1}^{\#PDH\tilde{L}(\tilde{p})}\left|f\left(z_{1\phi<h_\psi>}^k\right)\tilde{p}_1^k-f\left(z_{2\phi<h_\psi>}^k\right)\tilde{p}_2^k\right|}{\#PDH\tilde{L}(\tilde{p})}.\tag{9}$$

In order to facilitate readers to understand these two distance formulas, the following example is built.

Let $DHL=\left\{z_{\phi<h_\psi>}\mid\phi=-3,\cdots,-1,0,1,\cdots3;\psi=-3,\cdots,-1,0,1,\cdots3\right\}$ is a DHLTS, and $PDH\tilde{L}_1(\tilde{p})=\left\{z_{0<h_1>}(0.3),z_{0<h_2>}(0.4),z_{2<h_1>}(0.3)\right\}$, $PDH\tilde{L}_2(\tilde{p})=\left\{z_{1<h_2>}(0.5),z_{2<h_0>}(0.2),z_{2<h_2>}(0.3)\right\}$ are two PDLTSs, the Euclidean distance $ED\left(PDH\tilde{L}_1(\tilde{p}),PDH\tilde{L}_2(\tilde{p})\right)$ and Hamming distance $HD\left(PDH\tilde{L}_1(\tilde{p}),PDH\tilde{L}_2(\tilde{p})\right)$ can be obtained as follows:

According to Eq. (3)

$$f(z_{0<h_1>})=\frac{1+(3+0)\times3}{2\times3\times3}=\frac{10}{18},\ f(z_{0<h_2>})=\frac{2+(3+0)\times3}{2\times3\times3}=\frac{11}{18},\ f(z_{2<h_1>})=\frac{1+(3+2)\times3}{2\times3\times3}=\frac{16}{18},$$

$$f(z_{1<h_2>})=\frac{2+(3+1)\times3}{2\times3\times3}=\frac{14}{18},\ f(z_{2<h_0>})=\frac{0+(3+2)\times3}{2\times3\times3}=\frac{15}{18},\ f(z_{2<h_2>})=\frac{2+(3+2)\times3}{2\times3\times3}=\frac{17}{18}.$$

$$ED\left(PDH\tilde{L}_1(\tilde{p}),PDH\tilde{L}_2(\tilde{p})\right)=\sqrt{\frac{\left(\frac{10}{18}\times0.3-\frac{14}{18}\times0.5\right)^2+\left(\frac{11}{18}\times0.4-\frac{15}{18}\times0.2\right)^2+\left(\frac{16}{18}\times0.3-\frac{17}{18}\times0.3\right)^2}{3}}=0.1363;$$

$$HD\left(PDH\tilde{L}_1(\tilde{p}),PDH\tilde{L}_2(\tilde{p})\right)=\frac{\left(\left|\frac{10}{18}\times0.3-\frac{14}{18}\times0.5\right|+\left|\frac{11}{18}\times0.4-\frac{15}{18}\times0.2\right|+\left|\frac{16}{18}\times0.3-\frac{17}{18}\times0.3\right|\right)}{3}=0.1055.$$

2. PDHL-EDAS method for MAGDM with combined weight

Now, EDAS method for PDHLTSs is proposed to tackle MAGDM problems. And the MAGDM issues are represented by the following mathematical symbols. All alternatives is denoted as $AL=\left\{AL_1,AL_2,\cdots,AL_m\right\}$, the $AT=\left\{AT_1,AT_2,\cdots,AT_n\right\}$ is denoted the set of attributes, and combined weight vector is $cw=\left(cw_1,cw_2,\cdots,cw_n\right)$, where $cw_j\in[0,1]$, $j=1,2,\cdots,n$, $\sum_{j=1}^n cw_j=1$, and $E=\left\{EX_1,EX_2,\cdots,EX_g\right\}$ be a set of experts.

Suppose that t-th expert EX_t is evaluated i-th alternative CA_i under j-th attribute AT_j as

$$PDHL_{ij}^{(t)}(p)=\left\{z_{ij\phi<h_\psi>}^{k}{}^{(t)}\left(p_{ij}^{k(t)}\right)\mid z_{ij\phi<h_\psi>}^{k}{}^{(t)}\in DHL,p_{ij}^{k(t)}\geq0,\sum_{k=1}^{\#PDHL(P)}p_{ij}^{k(t)}\leq1\right\},$$

$$\left(i=1,2,\cdots,m,j=1,2,\cdots,n,t=1,2,\cdots,g\right).$$

Then, PDHL-EDAS method is built to tackle MAGDM issue with combined attribute weight.

Step 1. Established all decision makers' decision matrixes PDHLTS $Q^{(t)} = \left(PDHL^{(t)} (P) \right)_{m \times n}$.

Step 2. Converted cost index into benefit index. Let $PDH\tilde{L}(\tilde{p}) = \left\{ z_{\phi < h_\psi >}^k (\tilde{p}^k) \middle| z_{\phi < h_\psi >}^k \in DH\tilde{L}; k = 1, 2, ..., PDH\tilde{L}(\tilde{p}) \right\}$ be a PDHLTS, if $z_{\phi < h_\psi >}^k (\tilde{p}^k)$ is a cost linguistic evaluation, convert it into the corresponding benefit evaluation $z_{-\phi < -h_\psi >}^k (\tilde{p}^k)$.

Step 3. computed the normalized decision matrix $\tilde{Q}^{(t)} = \left(PDH\tilde{L}_{ij}^{(t)} (\tilde{p}) \right)_{m \times n}$.

Step 4. Calculated the PDHLTS prospect weight of each alternative, and obtain The PDHLTSs decision making information with prospects weight of each alternative based on each expert.

Firstly, the probabilistic double hierarchy linguistic prospect values $PW_{ij}^{(t)} (\tilde{p})$ of each alternative is obtained.

$$\text{if: } \frac{\sum_{k=1}^{\# PDH\tilde{L}(\tilde{p})} f\left(z_{ij\phi < h_\psi >}^{k}{}^{(t)} \right) \tilde{p}_{ij}^{k(t)} - f\left(\overline{z_{j\phi < h_\psi >}^{k}{}^{(t)}} \right) \overline{p_j^{k(t)}}}{\# PDH\tilde{L}(\tilde{p})} \geq 0, \tag{10}$$

$$\text{then, } PW_{ij}^{(t)} (\tilde{p}) = \left(HD\left(PDH\tilde{L}_{ij}^{(t)} (\tilde{p}), \overline{PDH\tilde{L}_j}^{(t)} \left(\tilde{p}_j^{(t)} \right) \right) \right)^\alpha;$$

$$\text{if: } \frac{\sum_{k=1}^{\# PDH\tilde{L}(\tilde{p})} f\left(z_{ij\phi < h_\psi >}^{k}{}^{(t)} \right) \tilde{p}_{ij}^{k(t)} - f\left(\overline{z_{j\phi < h_\psi >}^{k}{}^{(t)}} \right) \overline{p_j^{k(t)}}}{\# PDH\tilde{L}(\tilde{p})} < 0, \tag{11}$$

$$\text{then, } PW_{ij}^{(t)} (\tilde{p}) = -\lambda \left(HD\left(PDH\tilde{L}_{ij}^{(t)} (\tilde{p}), \overline{PDH\tilde{L}_j}^{(t)} \left(\tilde{p}_j^{(t)} \right) \right) \right)^\beta,$$

where

$$HD\left(PDH\tilde{L}_{ij}^{(t)} (\tilde{p}), \overline{PDH\tilde{L}_j}^{(t)} (\tilde{p}) \right) = \frac{\sum_{k=1}^{\# PDH\tilde{L}(\tilde{p})} \left| f\left(z_{ij\phi < h_\psi >}^{k}{}^{(t)} \right) \tilde{p}_{ij}^{k(t)} - f\left(\overline{z_{j\phi < h_\psi >}^{k}{}^{(t)}} \right) \overline{p_j^{k(t)}} \right|}{\# PDH\tilde{L}(\tilde{p})}, \tag{12}$$

particularly $\overline{PDH\tilde{L}_j}^{(t)} \left(\overline{\tilde{p}_j^{(t)}} \right) = \left\{ z_{j\phi < h_\psi >}^{k}{}^{(t)} \left(\overline{p_j^{k(t)}} \right) \middle| k = 1, 2, ..., PDH\tilde{L}^{(t)} (\tilde{p}^{(t)}) \right\};$ \tag{13}

$$\left\{ \overline{z_{j\phi < h_\psi >}^{k}{}^{(t)}} \left(\overline{p_j^{k(t)}} \right) \right\} = \left(f^{-1} \left(\frac{\sum_{i=1}^{m} f\left(z_{ij\phi < h_\psi >}^{k}{}^{(t)} \right)}{m} \right), \left(\frac{\sum_{i=1}^{m} \tilde{p}_{ij}^{k(t)}}{m} \right) \right), \tag{14}$$

with $\alpha = \beta = 0.88, \lambda = 2.25$.

Then, the normalized probabilistic double hierarchy linguistic prospect weights $NPW_{ij}^{(t)} (\tilde{p})$ are calculated by Eqs (15)–(16):

$$\omega_{ij}^{(t)} = \frac{NPW_{ij}^{(t)} (\tilde{p})}{\sum_{i=1}^{m} NPW_{ij}^{(t)} (\tilde{p})}, \tag{15}$$

where $NPW_{ij}^{(t)}(\tilde{p}) = \dfrac{PW_{ij}^{(t)}(\tilde{p}) - (-2.25)}{1-(-2.25)}$, $\omega_{ij}^{(t)} \in [0,1], \sum\limits_{i=1}^{m} \omega_{ij}^{(t)} = 1$. (16)

In addition, we can obtain The PDHLTSs decision matrix $PW\tilde{Q}^{(t)} = \left(PWPDH\tilde{L}_{ij}^{(t)}(\tilde{p}) \right)_{m \times n}$ with prospects weight of each alternative based on each expert by Eq. (17). And we can obtain the overall prospect weight PDHLTSs decision matrix $PW\tilde{Q} = \left(PWPDH\tilde{L}_{ij}(\tilde{p}) \right)_{m \times n}$.

$$\left(PWPDH\tilde{L}_{ij}^{(t)}(\tilde{p}) \right)_{m \times n} = \omega_{ij}^{(t)} \left(PDH\tilde{L}_{ij}^{(t)}(\tilde{p}) \right) = \left(f^{-1} \left(\sum_{k=1}^{\#PDH\tilde{L}_{ij}^{(t)}} \omega_{ij}^{(t)} f\left(z_{ij\phi<h_\psi>}^{k}{}^{(t)} \right) \right), \omega_{ij}^{(t)} \sum_{k=1}^{\#PDH\tilde{L}_{ij}^{(t)}} \tilde{p}_{ij}^{k(t)} \right).$$ (17)

Finally, we can calculate the normalized overall prospect weight PDHLTSs decision matrix $NPW\tilde{Q} = \left(NPWPDH\tilde{L}_{ij}(\tilde{p}) \right)_{m \times n}$.

Step 5. Calculated the combined weight.

The CRITIC method is proposed by Diakoulaki et al. (1995), which is a comprehensive measure of the objective weight of attributes. Now, the CRITIC method is extended in PDHLTSs to calculate the objective weight of attributes, and detailed calculation steps are as follows:

(1) Calculated between attributes' coefficient of correlation to establish the probabilistic double hierarchy linguistic coefficient of correlation matrix $PWPDHLCCM = \left(PWPDHLCC_{jr} \right)_{n \times n}$ as follows:

$$NPWPDH\tilde{L}CC_{jr} = \dfrac{\sum\limits_{i=1}^{m} \left(\left(\sum\limits_{k=1}^{\#NPWPDH\tilde{L}_{ij}(p)} \left(f\left(z_{ij\phi<h_\psi>}^{k}\right)p_{ij}^{k} - f\left(z_{j\phi<h_\psi>}^{k}\right)p_{j}^{k} \right) \right) \cdot \left(\sum\limits_{k=1}^{\#NPWPDH\tilde{L}_{ij}(p)} \left(f\left(z_{ir\phi<h_\psi>}^{k}\right)p_{ir}^{k} - f\left(z_{j\phi<h_\psi>}^{k}\right)p_{r}^{k} \right) \right) \right)}{\sqrt{\sum\limits_{i=1}^{m}\left(\sum\limits_{\varphi=1}^{\#NPWPDH\tilde{L}_{ij}(p)} \left(f\left(z_{ij\phi<h_\psi>}^{k}\right)p_{ij}^{k} - f\left(z_{j\phi<h_\psi>}^{k}\right)p_{j}^{k} \right) \right)^2} \cdot \sqrt{\sum\limits_{i=1}^{m}\left(\sum\limits_{k=1}^{\#NPWPDH\tilde{L}_{ij}(p)} \left(f\left(z_{ir\phi<h_\psi>}^{k}\right)p_{ir}^{k} - f\left(z_{r\phi<h_\psi>}^{k}\right)p_{r}^{k} \right) \right)^2}}, j,r=1,2,\dots,n,$$ (18)

where $z_{j\phi<h_\psi>}^{k}\left(p_{j}^{k}\right) = \left(f^{-1}\left(\dfrac{\sum\limits_{i=1}^{m} f\left(z_{ij\phi<h_\psi>}^{k}\right)}{m} \right) \right) \left(\dfrac{\sum\limits_{i=1}^{m} p_{ij}^{k}}{m} \right)$,

$$z_{r\phi<h_\psi>}^{k}\left(p_{r}^{k}\right) = f^{-1}\left(\left(\dfrac{\sum\limits_{i=1}^{m} f\left(z_{ir\phi<h_\psi>}^{k}\right)}{m} \right) \right) \left(\dfrac{\sum\limits_{i=1}^{m} p_{ir}^{k}}{m} \right).$$ (19)

(2) Probabilistic double hierarchy linguistic standard deviation (NPWPDHLSD) of each attribute could be got as follows:

$$NPWPDHL\tilde{S}D_j = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(\sum_{k=1}^{\#NPWPDHL\tilde{}_{ij}(p)}\left(f\left(z_{ij\phi<h_\psi>}^{k}\right)p_{ij}^{k} - f\left(z_{j\phi<h_\psi>}^{k}\right)p_j^{k}\right)\right)^2}, \quad (20)$$

(3) The objective weights of all attributes can be built as follows:

$$ow_j = \frac{NPWPDHL\tilde{S}D_j\sum_{r=1}^{n}\left(1-NPWPDHL\tilde{C}C_{jr}\right)}{\sum_{j=1}^{n}\left(NPWPDHL\tilde{S}D_j\sum_{r=1}^{n}\left(1-NPWPDHL\tilde{C}C_{jr}\right)\right)}, j=1,2\ldots,n, \quad (21)$$

where $o\omega_j \in [0,1], \sum_{j=1}^{n}o\omega_j = 1$.

Then, assumed $s\omega_j \in [0,1], \sum_{j=1}^{n}s\omega_j = 1, j=1,2,\ldots,n$ is the subjective weight provided by expert. Therefore, the comprehensive weight is built:

$$c\omega_j = \frac{\sqrt{o\omega_j s\omega_j}}{\sum_{j=1}^{n}\sqrt{o\omega_j s\omega_j}}, \quad (22)$$

where $c\omega_j \in [0,1], \sum_{j=1}^{n}c\omega_j = 1, j=1,2,\ldots,n$.

Step 6. The probabilistic double hierarchy linguistic average values are obtained by Eqs (23)–(25):

$$NPWAVPDH\tilde{L}\left(\tilde{p}\right) = \left(NPWAVPDH\tilde{L}_j\left(\tilde{p}\right)\right)_{1\times n}; \quad (23)$$

$$NPWAVPDH\tilde{L}_j\left(\tilde{p}\right) = \left\{z_{j\phi<h_\psi>}^{k}\left(p_j^k\right)\Big| k=1,2,\ldots,\#NPWAPDH\tilde{L}\left(\tilde{p}\right)\right\}; \quad (24)$$

$$z_{j\phi<h_\psi>}^{k}\left(p_j^k\right) = \left\{\frac{f^{-1}\left(\sum_{i=1}^{m}f\left(z_{ij\phi<h_\psi>}^{k}{}^{(t)}\right)\right)}{m}, \left(\frac{\sum_{i=1}^{m}\tilde{p}_{ij}^{k(t)}}{m}\right)\right\}. \quad (25)$$

Step 7. The positive distance of probabilistic double hierarchy linguistic from average (NPW-PDHLPDA) can be calculated by Eq. (26), and the positive distance of probabilistic double hierarchy linguistic from average (NPWPDHLNDA) can be obtained by Eq. (27):

$$NPWPDH\tilde{L}^{PDA}\left(\tilde{p}\right) = \left[NPWPDH\tilde{L}_{ij}^{PDA}\left(\tilde{p}\right)\right]_{m\times n}; \quad (26)$$

$$NPWPDH\tilde{L}^{NDA}\left(\tilde{p}\right) = \left[NPWPDH\tilde{L}_{ij}^{NDA}\left(\tilde{p}\right)\right]_{m\times n}. \quad (27)$$

Respectively, beneficial attributes are computed by Eqs (28)–(29):

$$NPWPDH\tilde{L}_{ij}^{PDA}(\tilde{p}) = \frac{\max\left(d\left(NPWPDH\tilde{L}_{ij}(\tilde{p}), NPWAVPDH\tilde{L}_j(\tilde{p})\right), 0\right)}{E\left(NPWAVPDH\tilde{L}_j(\tilde{p})\right)}; \qquad (28)$$

$$NPWPDH\tilde{L}_{ij}^{NDA}(\tilde{p}) = \frac{\max\left(d\left(NPWAVPDH\tilde{L}_j(\tilde{p}), NPWPDH\tilde{L}_{ij}(\tilde{p})\right), 0\right)}{E\left(NPWAVPDH\tilde{L}_j(\tilde{p})\right)}; \qquad (29)$$

Then, cost attributes are computed by Eqs (30)–(31):

$$NPWPDH\tilde{L}_{ij}^{PDA}(\tilde{p}) = \frac{\max\left(d\left(NPWAVPDH\tilde{L}_j(\tilde{p}), NPWPDH\tilde{L}_{ij}(\tilde{p})\right), 0\right)}{E\left(NPWAVPDH\tilde{L}_j(\tilde{p})\right)}; \qquad (30)$$

$$NPWPDH\tilde{L}_{ij}^{NDA}(\tilde{p}) = \frac{\max\left(d\left(NPWPDH\tilde{L}_{ij}(\tilde{p}), NPWAVPDH\tilde{L}_j(\tilde{p})\right), 0\right)}{E\left(NPWAVPDH\tilde{L}_j(\tilde{p})\right)}. \qquad (31)$$

Step 8. Computing the weight sum of NPWPDHLPDA and weight sum of NPWPDHLNDA for all alternatives as:

$$NPWPDH\tilde{L}_i^{SP}(\tilde{p}) = \sum_{j=1}^{n} c\omega_j NPWPDH\tilde{L}_{ij}^{PDA}(\tilde{p}); \qquad (32)$$

$$NPWPDH\tilde{L}_i^{SN}(\tilde{p}) = \sum_{j=1}^{n} c\omega_j NPWPDH\tilde{L}_{ij}^{NDA}(\tilde{p}). \qquad (33)$$

Step 9. The normalized values of $NPWPDH\tilde{L}_i^{SP}(\tilde{p})$ and $NPWPDH\tilde{L}_i^{NP}(\tilde{p})$ can be obtained by Eqs (34)–(35):

$$NPWPDH\tilde{L}_i^{NSP}(\tilde{p}) = \frac{NPWPDH\tilde{L}_i^{SP}(\tilde{p})}{\max\limits_{i=1}^{m} NPWPDH\tilde{L}_i^{SP}(\tilde{p})}; \qquad (34)$$

$$NPWPDH\tilde{L}_i^{NSN}(\tilde{p}) = 1 - \frac{NPWPDH\tilde{L}_i^{SN}(\tilde{p})}{\max\limits_{i=1}^{m} NPWPDH\tilde{L}_i^{SN}(\tilde{p})}. \qquad (35)$$

Step 10. Computed the assessment value $NPWPDH\tilde{L}_i^{AS}(\tilde{p})$ of every alternative:

$$NPWPDH\tilde{L}_i^{AS}(\tilde{p}) = \frac{NPWPDH\tilde{L}_i^{NSP}(\tilde{p}) + NPWPDH\tilde{L}_i^{NSN}(\tilde{p})}{2}. \qquad (36)$$

Step 11. Used assessment value $NPWPDH\tilde{L}_i^{AS}(\tilde{p})$ to rank the alternatives, and the larger $NPWPDH\tilde{L}_i^{AS}(\tilde{p})$, the better the alternative is.

3. Numerical example and comparative analysis

3.1. Numerical example

With the continuous development of 3D printing technology, high-efficiency and low-cost 3D printers have been widely used in manufacturing. A high-end car manufacturer of a certain brand has researched and developed a new high-end car and plans to use a 3D printer to produce certain parts of the new car. Choosing high-quality 3D printer is closely

related to the economic interests of the manufacturer. The high-quality 3D printer selection is also a classical MAGDM issue. Therefore, the PDHL-EDAS method is used to select the optimal 3D printer. Now, there are five 3D printers $DP = \{DP_1, DP_2, DP_3, DP_4, DP_5\}$ that meet the production conditions. Three invited experts $E = \{EX_1, EX_2, EX_3\}$ have selected four attributes $AT = \{AT_1, AT_2, AT_3, AT_4\}$ to evaluate the five given alternatives, And the $s\omega = [0.28, 0.20, 0.21, 0.31]$ is the subjective weight given by the experts. In addition, AT_1: Endurance of production materials. AT_2: Cost of production. AT_3: speed of Production AT_4: After-sales service. Evidently, AT_2 is cost attribute, on the contrary, AT_1, AT_3 and AT_4 are benefit attributes, and $cw = (cw_1, cw_2, cw_3, cw_4)$ is the combined weight of four attributes where $cw_j \in [0,1]$, $j = 1,2,3,4$, $\sum_{j=1}^{4} cw_j = 1$. Supposed that t-th expert EX_t Evaluated i-th alternative CA_i under j-th attribute AT_j as

$$PDHL_{ij}^{(t)}(p) = \left\{ z_{ij\phi<h_\psi>}^{k}{}^{(t)}\left(p_{ij}^{k(t)}\right) \middle| z_{ij\phi<h_\psi>}^{k}{}^{(t)} \in DHL, p_{ij}^{k(t)} \geq 0, \sum_{k=1}^{\#PDHL(P)} p_{ij}^{k(t)} \leq 1 \right\},$$

$(i = 1,2,\cdots,5, j = 1,2,\cdots,4, t = 1,2,3)$ where, the double linguistic hierarchy evaluation information tables are given as follows:

$Z = \{z_{-3} = extremely\ poor, z_{-2} = very\ poor,\ z_{-1} = poor, z_0 = medium,$
$z_1 = good, z_2 = very\ good, z_3 = extremely\ good\},$
$H = \{h_{-3} = far\ form, h_{-2} = only\ a\ little, h_{-1} = a\ little, h_0 = just\ right,$
$h_1 = much, h_2 = very\ much, h_2 = very\ much, h_3 = extirely\}.$

Then, the decision matrixes of each invited expert are expressed in Tables 1–3.

Table 1. The PDHLTSs evaluation of all alternatives is provided by e_1

	AT_1	AT_2
DP_1	$\{z_{-3<h_0>}(0.6), z_{-2<h_1>}(0.3), z_{-2<h_2>}(0.1)\}$	$\{z_{1<h_{-2}>}(0.3), z_{2<h_0>}(0.5), z_{3<h_{-2}>}(0.2)\}$
DP_2	$\{z_{3<h_{-1}>}(0.0), z_{3<h_{-1}>}(0.1), z_{3<h_0>}(0.9)\}$	$\{z_{-3<h_0>}(1.0)\}$
DP_3	$\{z_{0<h_1>}(0.3), z_{1<h_0>}(0.4), z_{2<h_1>}(0.3)\}$	$\{z_{-3<h_0>}(0.2), z_{-2<h_0>}(0.3), z_{-1<h_{-2}>}(0.5)\}$
DP_4	$\{z_{-3<h_2>}(0.4), z_{-2<h_1>}(0.3), z_{-1<h_2>}(0.3)\}$	$\{z_{1<h_0>}(0.3), z_{2<h_{-1}>}(0.2), z_{3<h_0>}(0.5)\}$
DP_5	$\{z_{-2<h_0>}(0.6), z_{-1<h_0>}(0.2), z_{0<h_0>}(0.2)\}$	$\{z_{-2<h_{-1}>}(0.2), z_{-1<h_{-2}>}(0.5), z_{-1<h_0>}(0.3)\}$

	AT_3	AT_4
DP_1	$\{z_{-2<h_0>}(0.3), z_{-2<h_1>}(0.3), z_{-1<h_2>}(0.4)\}$	$\{z_{-3<h_0>}(0.1), z_{-3<h_2>}(0.5), z_{-2<h_2>}(0.4)\}$
DP_2	$\{z_{2<h_0>}(0.1), z_{3<h_{-1}>}(0.5), z_{3<h_0>}(0.4)\}$	$\{z_{2<hs_0>}(0.1), z_{3<h_{-1}>}(0.2), z_{3<h_0>}(0.7)\}$
DP_3	$\{z_{3<h_{-1}>}(0.0), z_{3<h_{-1}>}(0.6), z_{3<h_0>}(0.4)\}$	$\{z_{1<h_2>}(0.7), z_{2<h_0>}(0.1), z_{3<h_0>}(0.2)\}$

	AT_3	AT_4
DP_4	$\left\{z_{-3<h_0>}(0.3), z_{-2<h_1>}(0.5), z_{-2<h_2>}(0.2)\right\}$	$\left\{z_{-2<hs_1>}(0.3), z_{-1<h_1>}(0.3), z_{0<h_1>}(0.4)\right\}$
DP_5	$\left\{z_{-1<h_1>}(0.5), z_{1<h_0>}(0.2), z_{2<h_{-2}>}(0.3)\right\}$	$\left\{z_{3<h_{-2}>}(0.0), z_{3<h_{-2}>}(0.4), z_{3<h_0>}(0.6)\right\}$

Table 2. The PDHLTSs evaluation of all alternatives is provided by e_2

	AT_1	AT_2
DP_1	$\left\{z_{-3<h_0>}(0.2), z_{-3<h_2>}(0.5), z_{-2<h_1>}(0.3)\right\}$	$\left\{z_{2<h_{-1}>}(0.4), z_{3<h_{-2}>}(0.2), z_{3<h_{-1}>}(0.4)\right\}$
DP_2	$\left\{z_{3<h_0>}(0.0), z_{3<h_0>}(0.0), z_{3<h_0>}(1.0)\right\}$	$\left\{l_{-3<s_0>}(0.8), l_{-3<s_1>}(0.1), l_{-3<s_2>}(0.1),\right\}$
DP_3	$\left\{z_{1<h_1>}(0.5), z_{2<h_0>}(0.3), z_{3<h_{-1}>}(0.2)\right\}$	$\left\{z_{-3<h_2>}(0.4), z_{-2<h_1>}(0.2), z_{-2<h_0>}(0.4)\right\}$
DP_4	$\left\{z_{-3<h_2>}(0.3), z_{-2<h_2>}(0.4), z_{-1<h_1>}(0.3)\right\}$	$\left\{z_{2<h_{-2}>}(0.2), z_{2<h_{-1}>}(0.2), z_{3<h_{-2}>}(0.6)\right\}$
DP_5	$\left\{z_{1<h_0>}(0.5), z_{1<h_2>}(0.3), z_{2<h_1>}(0.2)\right\}$	$\left\{z_{-1<h_{-1}>}(0.2), z_{0<h_{-2}>}(0.1), z_{0<h_{-1}>}(0.7)\right\}$

	AT_3	AT_4
DP_1	$\left\{z_{-2<h_1>}(0.2), z_{-2<h_2>}(0.5), z_{-1<h_2>}(0.3)\right\}$	$\left\{z_{-3<h_0>}(0.7), z_{-2<h_0>}(0.2), z_{-2<h_1>}(0.1)\right\}$
DP_2	$\left\{z_{3<h_{-1}>}(0.0), z_{3<h_{-1}>}(0.1), z_{3<h_0>}(0.9)\right\}$	$\left\{z_{2<h_0>}(0.2), z_{3<h_{-2}>}(0.3), z_{3<h_0>}(0.5)\right\}$
DP_3	$\left\{z_{2<h_0>}(0.3), z_{3<h_{-2}>}(0.4), z_{3<h_0>}(0.3)\right\}$	$\left\{z_{1<h_1>}(0.5), z_{2<h_0>}(0.3), z_{2<h_1>}(0.2)\right\}$
DP_4	$\left\{z_{-3<h_1>}(0.3), z_{-2<h_0>}(0.5), z_{-2<h_1>}(0.2)\right\}$	$\left\{z_{-2<h_1>}(0.2), z_{-1<h_0>}(0.2), z_{-1<h_2>}(0.6)\right\}$
DP_5	$\left\{z_{0<h_1>}(0.7), z_{1<h_0>}(0.2), z_{1<h_2>}(0.1)\right\}$	$\left\{z_{2<h_0>}(0.1), z_{3<h_{-1}>}(0.3), z_{3<h_0>}(0.6)\right\}$

Table 3. The PDHLTSs evaluation of all alternatives is provided by e_3

	AT_1	AT_2
DP_1	$\left\{z_{-3<h_2>}(0.5), z_{-2<h_0>}(0.3), z_{-2<h_1>}(0.2)\right\}$	$\left\{z_{2<h_0>}(0.2), z_{3<h_{-1}>}(0.4), z_{3<h_0>}(0.4)\right\}$
DP_2	$\left\{z_{3<h_0>}(1.0)\right\}$	$\left\{z_{-3<h_0>}(0.8), z_{-3<h_1>}(0.2)\right\}$
DP_3	$\left\{z_{1<h_2>}(0.5), z_{2<h_0>}(0.3), z_{3<h_{-2}>}(0.2)\right\}$	$\left\{z_{-3<h_2>}(0.5), z_{-2<h_0>}(0.1), z_{-2<h_1>}(0.4)\right\}$
DP_4	$\left\{z_{-3<h_2>}(0.1), z_{-2<h_1>}(0.6), z_{-1<h_1>}(0.3)\right\}$	$\left\{z_{2<h_{-2}>}(0.1), z_{2<h_{-1}>}(0.2), z_{3<h_{-2}>}(0.7)\right\}$
DP_5	$\left\{z_{-1<h_0>}(0.4), z_{0<h_2>}(0.4), z_{2<h_0>}(0.2)\right\}$	$\left\{z_{-2<h_1>}(0.1), z_{0<h_{-2}>}(0.3), z_{0<h_{-1}>}(0.6)\right\}$

	AT_3	AT_4
DP_1	$\{z_{-3<h_2>}(0.2), z_{-2<h_0>}(0.3), z_{-1<h_2>}(0.5)\}$	$\{z_{-3<h_0>}(0.4), z_{-3<h_1>}(0.6)\}$
DP_2	$\{z_{3<h_{-2}>}(0.2), z_{3<h_{-1}>}(0.4), z_{3<h_0>}(0.4)\}$	$\{z_{3<h_{-2}>}(0.2), z_{3<h_{-1}>}(0.4), z_{3<h_0>}(0.4)\}$
DP_3	$\{z_{3<h_{-1}>}(0.4), z_{3<h_0>}(0.6)\}$	$\{z_{2<h_{-2}>}(0.3), z_{2<h_1>}(0.5), z_{3<h_{-1}>}(0.2)\}$
DP_4	$\{z_{-3<h_1>}(0.5), z_{-3<h_2>}(0.3), z_{-2<h_1>}(0.2)\}$	$\{z_{-2<h_0>}(0.5), z_{-1<h_1>}(0.2), z_{-1<h_2>}(0.3)\}$
DP_5	$\{z_{-2<h_1>}(0.4), z_{0<h_0>}(0.4), z_{2<h_{-2}>}(0.2)\}$	$\{z_{3<h_{-2}>}(0.2), z_{3<h_{-1}>}(0.2), z_{3<h_0>}(0.6)\}$

Now, the built PDHL-EDAS method is used to select the optimal 3D printer.

Step 1. Transform the cost indicators into benefit indicators and derive the normalized decision matrix which are shown in Tables 4–6.

Table 4. The normalized PDHLTSs evaluation of each alternative is provided by e_1

	AT_1	AT_2
DP_1	$\{z_{-3<h_0>}(0.6), z_{-2<h_1>}(0.3), z_{-2<h_2>}(0.1)\}$	$\{z_{-3<h_2>}(0.2), z_{-2<h_0>}(0.5), z_{-1<h_2>}(0.3)\}$
DP_2	$\{z_{3<h_{-1}>}(0.0), z_{3<h_{-1}>}(0.1), z_{3<h_0>}(0.9)\}$	$\{z_{3<h_0>}(0.0), z_{3<h_0>}(0.0), z_{3<h_0>}(1.0)\}$
DP_3	$\{z_{0<h_1>}(0.3), z_{1<h_0>}(0.4), z_{2<h_1>}(0.3)\}$	$\{z_{1<h_2>}(0.5), z_{2<h_0>}(0.3), z_{3<h_0>}(0.2)\}$
DP_4	$\{z_{-3<h_2>}(0.4), z_{-2<h_1>}(0.3), z_{-1<h_2>}(0.3)\}$	$\{z_{-3<h_0>}(0.5), z_{-2<h_1>}(0.2), z_{-1<h_0>}(0.3)\}$
DP_5	$\{z_{-2<h_0>}(0.6), z_{-1<h_0>}(0.2), z_{0<h_0>}(0.2)\}$	$\{z_{1<h_0>}(0.3), z_{1<h_2>}(0.5), z_{2<h_1>}(0.2)\}$

	AT_3	AT_4
DP_1	$\{z_{-2<h_0>}(0.3), z_{-2<h_1>}(0.3), z_{-1<h_2>}(0.4)\}$	$\{z_{-3<h_0>}(0.1), z_{-3<h_2>}(0.5), z_{-2<h_2>}(0.4)\}$
DP_2	$\{z_{2<h_0>}(0.1), z_{3<h_{-1}>}(0.5), z_{3<h_0>}(0.4)\}$	$\{z_{2<hs_0>}(0.1), z_{3<h_{-1}>}(0.2), z_{3<h_0>}(0.7)\}$
DP_3	$\{z_{3<h_{-1}>}(0.0), z_{3<h_{-1}>}(0.6), z_{3<h_0>}(0.4)\}$	$\{z_{1<h_2>}(0.7), z_{2<h_0>}(0.1), z_{3<h_0>}(0.2)\}$
DP_4	$\{z_{-3<h_0>}(0.3), z_{-2<h_1>}(0.5), z_{-2<h_2>}(0.2)\}$	$\{z_{-2<hs_1>}(0.3), z_{-1<h_1>}(0.3), z_{0<h_1>}(0.4)\}$
DP_5	$\{z_{-1<h_1>}(0.5), z_{1<h_0>}(0.2), z_{2<h_{-2}>}(0.3)\}$	$\{z_{3<h_{-2}>}(0.0), z_{3<h_{-2}>}(0.4), z_{3<h_0>}(0.6)\}$

Table 5. The normalized PDHLTSs evaluation of each alternative is provided by e_2

	AT_1	AT_2
DP_1	$\left\{z_{-3<h_0>}(0.2), z_{-3<h_2>}(0.5), z_{-2<h_1>}(0.3)\right\}$	$\left\{z_{-3<h_1>}(0.4), z_{-3<h_2>}(0.2), z_{-2<h_1>}(0.4)\right\}$
DP_2	$\left\{z_{3<h_0>}(0.0), z_{3<h_0>}(0.0), z_{3<h_0>}(1.0)\right\}$	$\left\{l_{3<s_{-2}>}(0.1), l_{3<s_{-1}>}(0.1), l_{3<s_0>}(0.8)\right\}$
DP_3	$\left\{z_{1<h_1>}(0.5), z_{2<h_0>}(0.3), z_{3<h_{-1}>}(0.2)\right\}$	$\left\{z_{2<h_0>}(0.4), z_{2<h_{-1}>}(0.2), z_{3<h_{-2}>}(0.4)\right\}$
DP_4	$\left\{z_{-3<h_2>}(0.3), z_{-2<h_2>}(0.4), z_{-1<h_1>}(0.3)\right\}$	$\left\{z_{-3<h_2>}(0.6), z_{-2<h_1>}(0.2), z_{-2<h_2>}(0.2)\right\}$
DP_5	$\left\{z_{1<h_0>}(0.5), z_{1<h_2>}(0.3), z_{2<h_1>}(0.2)\right\}$	$\left\{z_{0<h_1>}(0.7), z_{0<h_2>}(0.1), z_{1<h_1>}(0.2)\right\}$

	AT_3	AT_4
DP_1	$\left\{z_{-2<h_1>}(0.2), z_{-2<h_2>}(0.5), z_{-1<h_2>}(0.3)\right\}$	$\left\{z_{-3<h_0>}(0.7), z_{-2<h_0>}(0.2), z_{-2<h_1>}(0.1)\right\}$
DP_2	$\left\{z_{3<h_{-1}>}(0.0), z_{3<h_{-1}>}(0.1), z_{3<h_0>}(0.9)\right\}$	$\left\{z_{2<h_0>}(0.2), z_{3<h_{-2}>}(0.3), z_{3<h_0>}(0.5)\right\}$
DP_3	$\left\{z_{2<h_0>}(0.3), z_{3<h_{-2}>}(0.4), z_{3<h_0>}(0.3)\right\}$	$\left\{z_{1<h_1>}(0.5), z_{2<h_0>}(0.3), z_{2<h_1>}(0.2)\right\}$
DP_4	$\left\{z_{-3<h_1>}(0.3), z_{-2<h_0>}(0.5), z_{-2<h_1>}(0.2)\right\}$	$\left\{z_{-2<h_1>}(0.2), z_{-1<h_0>}(0.2), z_{-1<h_2>}(0.6)\right\}$
DP_5	$\left\{z_{0<h_1>}(0.7), z_{1<h_0>}(0.2), z_{1<h_2>}(0.1)\right\}$	$\left\{z_{2<h_0>}(0.1), z_{3<h_{-1}>}(0.3), z_{3<h_0>}(0.6)\right\}$

Table 6. The normalized PDHLTSs evaluation of each alternative is provided by e_3

	AT_1	AT_2
DP_1	$\left\{z_{-3<h_2>}(0.5), z_{-2<h_0>}(0.3), z_{-2<h_1>}(0.2)\right\}$	$\left\{z_{-3<h_0>}(0.4), z_{-3<h_1>}(0.4), z_{-2<h_0>}(0.2)\right\}$
DP_2	$\left\{z_{3<h_0>}(0.0), z_{3<h_0>}(0.0), z_{3<h_0>}(1.0)\right\}$	$\left\{z_{3<h_{-1}>}(0.0), z_{3<h_{-1}>}(0.2), z_{3<h_0>}(0.8)\right\}$
DP_3	$\left\{z_{1<h_2>}(0.5), z_{2<h_0>}(0.3), z_{3<h_{-2}>}(0.2)\right\}$	$\left\{z_{2<h_{-1}>}(0.4), z_{2<h_0>}(0.1), z_{3<h_{-2}>}(0.5)\right\}$
DP_4	$\left\{z_{-3<h_2>}(0.1), z_{-2<h_1>}(0.6), z_{-1<h_1>}(0.3)\right\}$	$\left\{z_{-3<h_2>}(0.7), z_{-2<h_1>}(0.2), z_{-2<h_2>}(0.1)\right\}$
DP_5	$\left\{z_{-1<h_0>}(0.4), z_{0<h_2>}(0.4), z_{2<h_0>}(0.2)\right\}$	$\left\{z_{0<h_1>}(0.6), z_{0<h_2>}(0.3), z_{2<h_{-1}>}(0.1)\right\}$

	AT_3	AT_4
DP_1	$\left\{z_{-3<h_2>}(0.2), z_{-2<h_0>}(0.3), z_{-1<h_2>}(0.5)\right\}$	$\left\{z_{-3<h_0>}(0.0), z_{-3<h_0>}(0.4), z_{-3<h_1>}(0.6)\right\}$
DP_2	$\left\{z_{3<h_{-2}>}(0.2), z_{3<h_{-1}>}(0.4), z_{3<h_0>}(0.4)\right\}$	$\left\{z_{3<h_{-2}>}(0.2), z_{3<h_{-1}>}(0.4), z_{3<h_0>}(0.4)\right\}$
DP_3	$\left\{z_{3<h_{-1}>}(0.0), z_{3<h_{-1}>}(0.4), z_{3<h_0>}(0.6)\right\}$	$\left\{z_{2<h_{-2}>}(0.3), z_{2<h_1>}(0.5), z_{3<h_{-1}>}(0.2)\right\}$

	AT_3	AT_4
DP_4	$\left\{z_{-3<h_1>}(0.5), z_{-3<h_2>}(0.3), z_{-2<h_1>}(0.2)\right\}$	$\left\{z_{-2<h_0>}(0.5), z_{-1<h_1>}(0.2), z_{-1<h_2>}(0.3)\right\}$
DP_5	$\left\{z_{-2<h_1>}(0.4), z_{0<h_0>}(0.4), z_{2<h_{-2}>}(0.2)\right\}$	$\left\{z_{3<h_{-2}>}(0.2), z_{3<h_{-1}>}(0.2), z_{3<h_0>}(0.6)\right\}$

Step 2. Calculating reference points for all alternatives of each attribute by Eqs (12)–(14), which are shown in Table 7.

Table 7. The Reference points

	AT_1	AT_2
EX_1	$\left\{z_{-1<h_{0.13}>}(0.38), z_{-1<h_{0.87}>}(0.26), z_{0<h_{0.73}>}(0.36)\right\}$	$\left\{z_{0<h_{0.07}>}(0.30), z_{0<h_{0.60}>}(0.30), z_{1<h_{0.40}>}(0.40)\right\}$
EX_2	$\left\{z_{0<h_{0.00}>}(0.30), z_{0<h_{0.60}>}(0.30), z_{1<h_{0.13}>}(0.40)\right\}$	$\left\{z_{-1<h_{0.87}>}(0.46), z_{0<h_{0.27}>}(0.14), z_{0<h_{0.73}>}(0.40)\right\}$
EX_3	$\left\{z_{-1<h_{0.80}>}(0.30), z_{0<h_{0.40}>}(0.32), z_{1<h_{0.06}>}(0.38)\right\}$	$\left\{z_{-1<h_{0.87}>}(0.42), z_{0<h_{0.20}>}(0.22), z_{0<h_{0.73}>}(0.36)\right\}$

	AT_3	AT_4
EX_1	$\left\{z_{-1<h_{0.80}>}(0.24), z_{0<h_{0.67}>}(0.42), z_{1<h_{0.20}>}(0.34)\right\}$	$\left\{z_{0<h_{0.27}>}(0.24), z_{0<h_{0.80}>}(0.30), z_{1<h_{0.47}>}(0.46)\right\}$
EX_2	$\left\{z_{0<h_{0.13}>}(0.30), z_{0<h_{0.53}>}(0.34), z_{1<h_{0.13}>}(0.36)\right\}$	$\left\{z_{0<h_{0.13}>}(0.34), z_{0<h_{0.80}>}(0.26), z_{1<h_{0.27}>}(0.38)\right\}$
EX_3	$\left\{z_{-1<h_{0.67}>}(0.26), z_{0<h_{0.20}>}(0.36), z_{1<h_{0.07}>}(0.38)\right\}$	$\left\{z_{0<h_{0.20}>}(0.24), z_{0<h_{0.80}>}(0.34), z_{1<h_{0.13}>}(0.42)\right\}$

Step 3. Calculating the PDHLTS prospect weight of each alternative by Eqs (15)–(16) which are shown in Tables 8–10.

Table 8. The prospect weight of each alternative based on e_1

	AT_1	AT_2	AT_3	AT_4
DP_1	0.1732	0.1685	0.1812	0.1596
DP_2	0.2377	0.2371	0.2259	0.2163
DP_3	0.2168	0.2170	0.2319	0.2252
DP_4	0.1852	0.1614	0.1719	0.1777
DP_5	0.1871	0.2161	0.1891	0.2211

Table 9. The prospect weight of each alternative based on e_2

	AT_1	AT_2	AT_3	AT_4
DP_1	0.1611	0.1685	0.1857	0.1583
DP_2	0.2372	0.2293	0.2460	0.2183
DP_3	0.2156	0.2154	0.2253	0.2189
DP_4	0.1715	0.1711	0.1719	0.1795
DP_5	0.2145	0.2156	0.1712	0.2250

Table 10. The prospect weight of each alternative based on e_3

	AT_1	AT_2	AT_3	AT_4
DP_1	0.1685	0.1622	0.1830	0.1570
DP_2	0.2423	0.2345	0.2235	0.2202
DP_3	0.2210	0.2173	0.2320	0.2235
DP_4	0.1784	0.1693	0.1682	0.1769
DP_5	0.1899	0.2167	0.1934	0.2224

Step 4. Calculating The PDHLTSs decision information with prospects weight of each alternative based on each expert by Eq. (17). And assume that the decision information given by each expert is of the same significance, then got overall decision matrix which are shown in Tables 11–14.

Table 11. The PDHLTSs decision information with prospects weight based on e_1

	AT_1	AT_2
DP_1	$\left\{ z_{-2.4803 < h_{1.5590} >}(0.1732) \right\}$	$\left\{ z_{-2.2700 < h_{2.1899} >}(0.1685) \right\}$
DP_2	$\left\{ z_{1.1209 < h_{0.3627} >}(0.2377) \right\}$	$\left\{ z_{1.2678 < h_{0.8035} >}(0.2371) \right\}$
DP_3	$\left\{ z_{-0.2545 < h_{2.2365} >}(0.2168) \right\}$	$\left\{ z_{0.3995 < h_{1.1984} >}(0.2170) \right\}$
DP_4	$\left\{ z_{-2.1356 < h_{2.5931} >}(0.1852) \right\}$	$\left\{ z_{-2.4620 < h_{1.6140} >}(0.1614) \right\}$
DP_5	$\left\{ z_{-1.8776 < h_{0.3671} >}(0.1871) \right\}$	$\left\{ z_{0.0247 < h_{0.0742} >}(0.2161) \right\}$

	AT_3	AT_4
DP_1	$\left\{ z_{-2.0941 < h_{2.7176} >}(0.1812) \right\}$	$\left\{ z_{-2.6275 < h_{1.1175} >}(0.1596) \right\}$
DP_2	$\left\{ z_{0.7645 < h_{2.2936} >}(0.2259) \right\}$	$\left\{ z_{0.6044 < h_{1.8133} >}(0.2163) \right\}$
DP_3	$\left\{ z_{1.0198 < h_{0.0595} >}(0.2319) \right\}$	$\left\{ z_{0.3787 < h_{1.1362} >}(0.2252) \right\}$
DP_4	$\left\{ z_{-2.4269 < h_{1.7194} >}(0.1719) \right\}$	$\left\{ z_{-1.7558 < h_{0.7327} >}(0.1777) \right\}$
DP_5	$\left\{ z_{-0.9198 < h_{0.2405} >}(0.1891) \right\}$	$\left\{ z_{0.6849 < h_{2.0546} >}(0.2211) \right\}$

Table 12. The PDHLTSs decision information with prospects weight based on e_2

	AT_1	AT_2
DP_1	$\left\{z_{-2.6778<h_{0.9666}>}(0.1611)\right\}$	$\left\{z_{-2.6067<h_{1.1798}>}(0.1685)\right\}$
DP_2	$\left\{z_{1.2698<h_{0.8094}>}(0.2372)\right\}$	$\left\{z_{0.8979<h_{2.6937}>}(0.2293)\right\}$
DP_3	$\left\{z_{0.2347<h_{0.7041}>}(0.2156)\right\}$	$\left\{z_{0.2315<h_{0.6945}>}(0.2154)\right\}$
DP_4	$\left\{z_{-2.1996<h_{2.4013}>}(0.1715)\right\}$	$\left\{z_{-2.3727<h_{1.8820}>}(0.1711)\right\}$
DP_5	$\left\{z_{0.0032<h_{0.0096}>}(0.2145)\right\}$	$\left\{z_{-0.5561<h_{1.3317}>}(0.2156)\right\}$

	AT_3	AT_4
DP_1	$\left\{z_{-1.9478<h_{0.1567}>}(0.1857)\right\}$	$\left\{z_{-2.6307<h_{1.1079}>}(0.1583)\right\}$
DP_2	$\left\{z_{1.2641<h_{0.7923}>}(0.2460)\right\}$	$\left\{z_{0.5655<h_{1.6965}>}(0.2183)\right\}$
DP_3	$\left\{z_{0.6792<h_{2.0375}>}(0.2253)\right\}$	$\left\{z_{0.2111<h_{0.6334}>}(0.2189)\right\}$
DP_4	$\left\{z_{-2.5417<h_{1.3750}>}(0.1719)\right\}$	$\left\{z_{-1.9232<h_{0.2304}>}(0.1615)\right\}$
DP_5	$\left\{z_{-0.9458<h_{0.1625}>}(0.1712)\right\}$	$\left\{z_{0.7504<h_{2.2511}>}(0.2250)\right\}$

Table 13. The PDHLTSs decision information with prospects weight based on e_3

	AT_1	AT_2
DP_1	$\left\{z_{-2.4946<h_{1.5161}>}(0.1685)\right\}$	$\left\{z_{-2.7837<h_{0.6488}>}(0.1622)\right\}$
DP_2	$\left\{z_{1.3606<h_{1.0818}>}(0.2423)\right\}$	$\left\{z_{1.0645<h_{0.1934}>}(0.2345)\right\}$
DP_3	$\left\{z_{0.3148<h_{0.9445}>}(0.2210)\right\}$	$\left\{z_{0.2601<h_{0.7804}>}(0.2173)\right\}$
DP_4	$\left\{z_{-2.1676<h_{2.4973}>}(0.1784)\right\}$	$\left\{z_{-2.3792<h_{1.8623}>}(0.1693)\right\}$
DP_5	$\left\{z_{-0.9741<h_{0.0776}>}(0.1899)\right\}$	$\left\{z_{-0.4723<h_{1.5830}>}(0.2167)\right\}$

	AT_3	AT_4
DP_1	$\left\{z_{-2.069<h_{2.3794}>}(0.1830)\right\}$	$\left\{z_{-2.9477<h_{0.1570}>}(0.1570)\right\}$
DP_2	$\left\{z_{0.7989<h_{2.3966}>}(0.2235)\right\}$	$\left\{z_{0.7432<h_{2.2297}>}(0.2202)\right\}$

	AT_3	AT_4
DP_3	$\left\{ z_{1.0207<h_{0.0620}>}\left(0.2320\right)\right\}$	$\left\{ z_{0.4270<h_{1.2811}>}\left(0.2235\right)\right\}$
DP_4	$\left\{ z_{-2.6076<h_{1.1772}>}\left(0.1682\right)\right\}$	$\left\{ z_{-1.9387<h_{0.1838}>}\left(0.1769\right)\right\}$
DP_5	$\left\{ z_{-1.3241<h_{2.0278}>}\left(0.1934\right)\right\}$	$\left\{ z_{0.7812<h_{2.3436}>}\left(0.2224\right)\right\}$

Table 14. The overall decision matrix

	AT_1	AT_2
DP_1	$\left\{\begin{array}{l}\left\{z_{-2.6778<h_{0.9666}>}\left(0.1611\right)\right\},\left\{z_{-2.4946<h_{1.5161}>}\left(0.1685\right)\right\},\\ \left\{z_{-2.4803<h_{1.5590}>}\left(0.1732\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-2.7837<h_{0.6488}>}\left(0.1622\right)\right\},\left\{z_{-2.6067<h_{1.1798}>}\left(0.1685\right)\right\},\\ \left\{z_{-2.2700<h_{2.1899}>}\left(0.1685\right)\right\}\end{array}\right\}$
DP_2	$\left\{\begin{array}{l}\left\{z_{1.1209<h_{0.3627}>}\left(0.2377\right)\right\},\left\{z_{1.2698<h_{0.8094}>}\left(0.2372\right)\right\},\\ \left\{z_{1.3606<h_{1.0818}>}\left(0.2423\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.8979<h_{2.6937}>}\left(0.2293\right)\right\},\left\{z_{1.0645<h_{0.1934}>}\left(0.2345\right)\right\},\\ \left\{z_{1.2678<h_{0.8035}>}\left(0.2371\right)\right\}\end{array}\right\}$
DP_3	$\left\{\begin{array}{l}\left\{z_{-0.2545<h_{2.2365}>}\left(0.2168\right)\right\},\left\{z_{0.2347<h_{0.7041}>}\left(0.2156\right)\right\},\\ \left\{z_{0.3148<h_{0.9445}>}\left(0.2210\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.2315<h_{0.6945}>}\left(0.2154\right)\right\},\left\{z_{0.2601<h_{0.7804}>}\left(0.2173\right)\right\},\\ \left\{z_{0.3995<h_{1.1984}>}\left(0.2170\right)\right\}\end{array}\right\}$
DP_4	$\left\{\begin{array}{l}\left\{z_{-2.1996<h_{2.4013}>}\left(0.1715\right)\right\},\left\{z_{-2.1676<h_{2.4973}>}\left(0.1784\right)\right\},\\ \left\{z_{-2.1356<h_{2.5931}>}\left(0.1852\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-2.4620<h_{1.6140}>}\left(0.1614\right)\right\},\left\{z_{-2.3792<h_{1.8623}>}\left(0.1693\right)\right\},\\ \left\{z_{-2.3727<h_{1.8820}>}\left(0.1711\right)\right\},\end{array}\right\}$
DP_5	$\left\{\begin{array}{l}\left\{z_{-1.8776<h_{0.3671}>}\left(0.1871\right)\right\},\left\{z_{-0.9741<h_{0.0776}>}\left(0.1899\right)\right\},\\ \left\{z_{0.0032<h_{0.0096}>}\left(0.2145\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-0.5561<h_{1.3317}>}\left(0.2156\right)\right\},\left\{z_{-0.4723<h_{1.5830}>}\left(0.2167\right)\right\},\\ \left\{z_{0.0247<h_{0.0742}>}\left(0.2161\right)\right\}\end{array}\right\}$

	AT_3	AT_4
DP_1	$\left\{\begin{array}{l}\left\{z_{-2.0941<h_{2.7176}>}\left(0.1812\right)\right\},\left\{z_{-2.069<h_{2.3794}>}\left(0.1830\right)\right\},\\ \left\{z_{-1.9478<h_{0.1567}>}\left(0.1857\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-2.9477<h_{0.1570}>}\left(0.1570\right)\right\},\left\{z_{-2.6307<h_{1.1079}>}\left(0.1583\right)\right\},\\ \left\{z_{-2.6275<h_{1.1175}>}\left(0.1596\right)\right\}\end{array}\right\}$
DP_2	$\left\{\begin{array}{l}\left\{z_{0.7645<h_{2.2936}>}\left(0.2259\right)\right\},\left\{z_{0.7989<h_{2.3966}>}\left(0.2235\right)\right\},\\ \left\{z_{1.2641<h_{0.7923}>}\left(0.2460\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.5655<h_{1.6965}>}\left(0.2183\right)\right\},\left\{z_{0.6044<h_{1.8133}>}\left(0.2163\right)\right\},\\ \left\{z_{0.7432<h_{2.2297}>}\left(0.2202\right)\right\}\end{array}\right\}$
DP_3	$\left\{\begin{array}{l}\left\{z_{0.6792<h_{2.0375}>}\left(0.2253\right)\right\},\left\{z_{1.0198<h_{0.0595}>}\left(0.2319\right)\right\},\\ \left\{z_{1.0207<h_{0.0620}>}\left(0.2320\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.2111<h_{0.6334}>}\left(0.2189\right)\right\}\left\{z_{0.3787<h_{1.1362}>}\left(0.2252\right)\right\},\\ \left\{z_{0.4270<h_{1.2811}>}\left(0.2235\right)\right\}\end{array}\right\}$
DP_4	$\left\{\begin{array}{l}\left\{z_{-2.6076<h_{1.1772}>}\left(0.1682\right)\right\},\left\{z_{-2.5417<h_{1.3750}>}\left(0.1719\right)\right\},\\ \left\{z_{-2.4269<h_{1.7194}>}\left(0.1719\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-1.9387<h_{0.1838}>}\left(0.1769\right)\right\},\left\{z_{-1.9232<h_{0.2304}>}\left(0.1615\right)\right\},\\ \left\{z_{-1.7558<h_{0.7327}>}\left(0.1777\right)\right\}\end{array}\right\}$
DP_5	$\left\{\begin{array}{l}\left\{z_{-1.3241<h_{2.0278}>}\left(0.1934\right)\right\},\left\{z_{-0.9458<h_{0.1625}>}\left(0.1712\right)\right\},\\ \left\{z_{-0.9198<h_{0.2405}>}\left(0.1891\right)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.6849<h_{2.0546}>}\left(0.2211\right)\right\},\left\{z_{0.7504<h_{2.2511}>}\left(0.2250\right)\right\},\\ \left\{z_{0.7812<h_{2.3436}>}\left(0.2224\right)\right\}\end{array}\right\}$

Step 5. The normalized overall decision matrix could be obtained with Eq. (5) and shown in Table 15.

Table 15. The normalized overall decision matrix

	AT_1	AT_2
DP_1	$\left\{\begin{array}{l}\left\{z_{-2.6778<h_{0.9666}>}(0.3204)\right\},\left\{z_{-2.4946<h_{1.5161}>}(0.3350)\right\}, \\ \left\{z_{-2.4803<h_{1.5590}>}(0.3445)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-2.7837<h_{0.6488}>}(0.3249)\right\},\left\{z_{-2.6067<h_{1.1798}>}(0.3376)\right\}, \\ \left\{z_{-2.2700<h_{2.1899}>}(0.3374)\right\}\end{array}\right\}$
DP_2	$\left\{\begin{array}{l}\left\{z_{1.1209<h_{0.3627}>}(0.3315)\right\},\left\{z_{1.2698<h_{0.8094}>}(0.3307)\right\}, \\ \left\{z_{1.3606<h_{1.0818}>}(0.3378)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.8979<h_{2.6937}>}(0.3271)\right\},\left\{z_{1.0645<h_{0.1934}>}(0.3346)\right\}, \\ \left\{z_{1.2678<h_{0.8035}>}(0.3383)\right\}\end{array}\right\}$
DP_3	$\left\{\begin{array}{l}\left\{z_{-0.2545<h_{2.2365}>}(0.3317)\right\},\left\{z_{0.2347<h_{0.7041}>}(0.3300)\right\}, \\ \left\{z_{0.3148<h_{0.9445}>}(0.3382)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.2315<h_{0.6945}>}(0.3316)\right\},\left\{z_{0.2601<h_{0.7804}>}(0.3345)\right\}, \\ \left\{z_{0.3995<h_{1.1984}>}(0.3339)\right\}\end{array}\right\}$
DP_4	$\left\{\begin{array}{l}\left\{z_{-2.1996<h_{2.4013}>}(0.3205)\right\},\left\{z_{-2.1676<h_{2.4973}>}(0.3333)\right\}, \\ \left\{z_{-2.1356<h_{2.5931}>}(0.3461)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-2.4620<h_{1.6140}>}(0.3217)\right\},\left\{z_{-2.3792<h_{1.8623}>}(0.3374)\right\}, \\ \left\{z_{-2.3727<h_{1.8820}>}(0.3410)\right\},\end{array}\right\}$
DP_5	$\left\{\begin{array}{l}\left\{z_{-1.8776<h_{0.3671}>}(0.3162)\right\},\left\{z_{-0.9741<h_{0.0776}>}(0.3211)\right\}, \\ \left\{z_{0.0032<h_{0.0096}>}(0.3627)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-0.5561<h_{1.3317}>}(0.3326)\right\},\left\{z_{-0.4723<h_{1.5830}>}(0.3342)\right\}, \\ \left\{z_{0.0247<h_{0.0742}>}(0.3332)\right\}\end{array}\right\}$

	AT_3	AT_4
DP_1	$\left\{\begin{array}{l}\left\{z_{-2.0941<h_{2.7176}>}(0.3314)\right\},\left\{z_{-2.069<h_{2.3794}>}(0.3303)\right\}, \\ \left\{z_{-1.9478<h_{0.1567}>}(0.3383)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-2.9477<h_{0.1570}>}(0.3306)\right\},\left\{z_{-2.6307<h_{1.1079}>}(0.3333)\right\}, \\ \left\{z_{-2.6275<h_{1.1175}>}(0.3362)\right\}\end{array}\right\}$
DP_2	$\left\{\begin{array}{l}\left\{z_{0.7645<h_{2.2936}>}(0.3248)\right\},\left\{z_{0.7989<h_{2.3966}>}(0.3214)\right\}, \\ \left\{z_{1.2641<h_{0.7923}>}(0.3538)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.5655<h_{1.6965}>}(0.3334)\right\},\left\{z_{0.6044<h_{1.8133}>}(0.3303)\right\}, \\ \left\{z_{0.7432<h_{2.2297}>}(0.3363)\right\}\end{array}\right\}$
DP_3	$\left\{\begin{array}{l}\left\{z_{0.6792<h_{2.0375}>}(0.3269)\right\},\left\{z_{1.0198<h_{0.0595}>}(0.3365)\right\}, \\ \left\{z_{1.0207<h_{0.0620}>}(0.3366)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.2111<h_{0.6334}>}(0.3279)\right\}\left\{z_{0.3787<h_{1.1362}>}(0.3374)\right\}, \\ \left\{z_{0.4270<h_{1.2811}>}(0.3347)\right\}\end{array}\right\}$
DP_4	$\left\{\begin{array}{l}\left\{z_{-2.6076<h_{1.1772}>}(0.3285)\right\},\left\{z_{-2.5417<h_{1.3750}>}(0.3357)\right\}, \\ \left\{z_{-2.4269<h_{1.7194}>}(0.3358)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{-1.9387<h_{0.1838}>}(0.3427)\right\},\left\{z_{-1.9232<h_{0.2304}>}(0.3129)\right\}, \\ \left\{z_{-1.7558<h_{0.7327}>}(0.3444)\right\}\end{array}\right\}$
DP_5	$\left\{\begin{array}{l}\left\{z_{-1.3241<h_{2.0278}>}(0.3493)\right\},\left\{z_{-0.9458<h_{0.1625}>}(0.3092)\right\}, \\ \left\{z_{-0.9198<h_{0.2405}>}(0.3416)\right\}\end{array}\right\}$	$\left\{\begin{array}{l}\left\{z_{0.6849<h_{2.0546}>}(0.3307)\right\},\left\{z_{0.7504<h_{2.2511}>}(0.3366)\right\}, \\ \left\{z_{0.7812<h_{2.3436}>}(0.3327)\right\}\end{array}\right\}$

Step 6. Through CRITIC method get the objective weight of attributes is $o\omega = (0.1988, 0.1248, 0.2518, 0.4246)$. Therefore, the combine weight of attribute is $c\omega_j = (0.2391, 0.1601, 0.2330, 0.3677)$.

Step 7. The probabilistic double hierarchy linguistic average values are competed by Eqs (23)–(25) shown in Table 16.

Table 16. NPWAVPDHL for all attributes

	PDHLBBA
AT_1	$\left\{ z_{-0.7554<h_{0.2446}>}\left(0.3241\right), z_{-0.4527<h_{0.5473}>}\left(0.3301\right), z_{-0.1749<h_{0.8251}>}\left(0.3459\right)\right\}$
AT_2	$\left\{ z_{-0.4690<h_{0.5310}>}\left(0.3276\right), z_{-0.4535<h_{0.5465}>}\left(0.3356\right), z_{-0.1803<h_{0.8197}>}\left(0.3368\right)\right\}$
AT_3	$\left\{ z_{-0.2799<h_{0.7201}>}\left(0.3322\right), z_{-0.3050<h_{0.6950}>}\left(0.3266\right), z_{-0.4039<h_{0.5961}>}\left(0.3412\right)\right\}$
AT_4	$\left\{ z_{-0.3700<h_{0.6300}>}\left(0.3331\right), z_{-0.1281<h_{0.8719}>}\left(0.3301\right), z_{0.0273<h_{0.0273}>}\left(0.3369\right)\right\}$

Step 8. The NPWPDHLPDA can be calculated by Eq. (26), and the NPWPDHLNDA can be calculated by Eq. (27) which are shown in Tables 17–18.

Table 17. The NPWPDHLPDA matrix

	AT_1	AT_2	AT_3	AT_4
DP_1	0.0000	0.0000	0.0000	0.0000
DP_2	0.7689	0.7030	0.7047	0.5038
DP_3	0.3872	0.3649	0.5524	0.2941
DP_4	0.0000	0.0000	0.0000	0.0000
DP_5	0.0000	0.1383	0.0000	0.5747

Table 18. NPWPDHLNDA matrix

	AT_1	AT_2	AT_3	AT_4
DP_1	0.6451	0.6587	0.4401	0.8132
DP_2	0.0000	0.0000	0.0000	0.0000
DP_3	0.0000	0.0000	0.0000	0.0000
DP_4	0.3453	0.5471	0.6440	0.5575
DP_5	0.1581	0.0000	0.1717	0.0000

Step 9. Calculating the weight sum of NPWPDHLPDA and NPWPDHLNDA of all given alternatives. And their normalized values by Eqs (32)–(35) shown in Table 19.

Table 19. NPWPDHL(SP/NSP) and NPWPDHL(SN/NSN) matrix

	NPWPDHL(SP)	NPWPDHL(NSP)	NPWPDHL(SN)	NPWPDHL(NSN)
DP_1	0.0000	0.0000	0.6613	0.0000
DP_2	0.6459	1.0000	0.0000	1.0000
DP_3	0.3879	0.6005	0.0000	1.0000
DP_4	0.0000	0.0000	0.5252	0.2058
DP_5	0.2335	0.3615	0.0778	0.8823

Step 10. Calculated the assessment value matrix by Eq. (36) shown in Table 20.

Table 20. NPWPDHL(AS) matrix

	DP$_1$	DP$_2$	DP$_3$	DP$_4$	DP$_5$
$NPWPDH\tilde{L}_i^{AS}(\tilde{p})$	0.0000	1.0000	0.8003	0.1029	0.6219

Step 11. According to the $NPWPDH\tilde{L}_i^{AS}(\tilde{p})$ $(i=1,2,3,4,5)$, the order of the given alternatives could be obtained. Evidently, the order is DP$_2$ > DP$_3$ > DP$_5$ > DP$_4$ > DP$_1$ and the most optimal 3D printer is DP$_2$.

3.2. Comparative analysis

Then, The built method is compared with PDHL-VIKOR method (Gou et al., 2020) shown in Table 21.

Table 21. The numerical results and rank derived by the PDHL-VIKOR

	PDHLGU	rank	PDHLIK	rank	PDHLC	rank
DP$_1$	0.9218	5	0.3645	5	1.0000	5
DP$_2$	−1.045	1	−0.1693	1	0.0000	1
DP$_3$	−0.6588	2	−0.0869	2	0.1754	2
DP$_4$	0.7200	4	0.2325	4	0.8251	4
DP$_5$	−0.3125	3	0.0674	3	0.4080	3

From Table 21, the ranking of the five alternatives can be obtained as: DP$_2$ > DP$_3$ > DP$_5$ > DP$_4$ > DP$_1$ and the most optimal 3D printer is DP$_2$. Therefore, the PDHL-EDAS method in this paper is scientific and effective.

Conclusions

The selection of optimal 3D printers is a great significance in the production and sales process of enterprises. Therefore, enterprises urgently need a set of effective decision-making model based on the optimal 3D printer selection problem. Now, a new PDHL-EDAS method for MAGDM is established. Then the feasibility of this method is illustrated by choosing high quality 3D printing as an example. In addition, the validity and rationality of the developed model is verified, and a comparative analysis is carried out. The contributions of the PDHL-EDAS method established in this paper are as follow: (1) the establishment of this method enriches the decision-making method based on PDHLTS; (2) enrich the 3D printer evaluation model.

Acknowledgements

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