CPT-TODIM METHOD FOR INTERVAL NEUTROSOPHIC MAGDM AND ITS APPLICATION TO THIRD-PARTY LOGISTICS SERVICE PROVIDERS SELECTION

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Abstract. The multiple attribute group decision making (MAGDM) has always been a concern in the research field. In this article, we establish the interval neutrosophic TODIM method based on cumulative prospect theory (CPT-IN-TODIM) for MAGDM issue. This new developed CPT-IN-TODIM method has markedly superiority in describing decision maker’s psychological states, which utilizes the weight function to adjust weighting attributes distinguishing from the classical TODIM method. Then, this new developed method has been applied to select the third-party logistics service providers and been expound on the disparity with existing methods. Finally, the results of contrastive analysis indicate that this new developed method can lead to the appropriate conclusion and sticks out the differences between alternatives to provide clearer direction. Hence, the new developed CPT-IN-TODIM method is reliable and valid.

Keywords: multiple attribute group decision making (MAGDM), interval neutrosophic sets, TODIM, cumulative prospect theory, third-party logistics service providers.

JEL Classification: C43, C61, D81.

Introduction

Both multiple attribute decision making (MADM) and multiple attribute group decision making (MAGDM) (Kim et al., 1999; Liao et al., 2021; Xu & Chen, 2007; Zhao et al., 2021b) refer to the process of selecting the optimal alternative among several alternatives, and these alternatives involve multiple identical attributes (Chen et al., 2012; Lei et al., 2021; Wei et al., 2021). In our daily life, we have to face many complex and uncertain questions that cannot be answered with full determined or absolute negation, so does the management decision-making process. The uncertainty of environment makes it difficult to describe the attribute

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value accurately with traditional mathematics. Therefore, Zadeh (1965) proposed the fuzzy set for the first time. Then, neutrosophic set created by Smarandache (2002) and consisted of the truth-membership, the indeterminacy-membership and the falsity-membership which were the standard or non-standard real subsets of $[0,1]$. Wang et al. (2010) defined single-valued neutrosophic set (SVNS). Wang et al. (2005) designed the interval neutrosophic set (INS). Many scholars used SVNS or INS to study the problem of MADM or MAGDM (Biswas et al., 2016; Kahraman et al., 2020; Meng et al., 2019; Nafei et al., 2019; Wu et al., 2018; Yang et al., 2020). Wang et al. (2019) researched the choice about strategic supplier by utilizing interval neutrosophic information. A series of aggregation operators about SVNS and INS are put forward by Garg and Nancy (2020) and in their mind, both objective factor and subjective factor are considered concurrently. Wang (2020) proposed a new interval neutrosophic MADM method taking example by the projection model. Awang et al. (2020) proposed the interval neutrosophic Shapley normalized weighted BM operator. Fan et al. (2021) developed interval-valued neutrosophic TODIM method for risk assessment. Pramanik et al. (2017) constructed neutrosophic cubic TODIM method. Pramanik et al. (2018) also considered to establish the VIKOR model under neutrosophic cubic environment. Pramanik et al. (2016) created bipolar neutrosophic TODIM method. Pramanik and Mallick (2019) investigated the feasibility of building TODIM model with trapezoidal neutrosophic number.

On the basis of the prospect theory (Kahneman & Tversky, 1979), Gomes and Lima (1979) first established the TODIM method, which is effective to solve the MADM problems where the DMs’ psychological behaviors are considered. The TODIM method had been widely applied in many fields, such as selection of green supplier (Nie et al., 2019), products selection (Liu & Teng, 2019), Water security evaluation (Zhang et al., 2019). Moreover, many scholars combined TODIM method under different fuzzy sets. For example, Liang et al. (2019) built TODIM method under proportional hesitant fuzzy linguistic and environment. Liu et al. (2019) focused on fermatean fuzzy linguistic information. Ji et al. (2020) selected dual hesitant PFSs to investigate TODIM method.

Flavio and Gonzalez (2012) initially proposed the idea of the combination of TODIM and cumulative prospect theory (CPT). Tian et al. (2019) realized a more detailed improvement of TODIM by using CPT. Huang et al. (2021) defined EDAS method based on Prospect Theory. Zhao et al. (2021c) rebuilt the CPT-TODIM by using intuitionistic fuzzy information. Su et al. (2021) built the PT-TODIM method for probabilistic linguistic MAGDM. Zhao et al. (2021a) defined the Intuitionistic fuzzy MABAC method based on CPT. Zhao et al. (2021d) established interval-valued intuitionistic fuzzy TODIM model on the basis of CPT for studying the urban ecological risk. To a certain extent, increasing the risk weighting moderately is conducive to the enterprise’s risk avoidance and conforms to the enterprise’s decision-making requirements. Therefore, in my opinion, the CPT-TODIM method has obvious advantages in dealing with MAGDM problems. The TODIM method took into account decision makers’ different psychological perceptions of gains and losses, but unfortunately it is only appropriate for certain types of decision makers. But the CPT-TODIM method can depict the difference of psychological perception for different types of decision makers. In addition, compared with other fuzzy sets, interval neutrosophic set is a more flexible and more applicable method for evaluating uncertain information. However, there are few studies
using this method. Therefore, this paper aims to establish the interval neutrosophic TODIM based on cumulative prospect theory (CPT-IN-TODIM) method for selecting the cold chain logistics service providers. In addition, the new developed method also takes a more scientific approach, the entropy weight method, to acquire the original attribute weights avoiding the interference of subjective factors. Although other methods have been proposed to determine the weight of attributes (Keshavarz-Ghorabaee et al., 2019), entropy weight method is still a very effective and straightforward method.

The structure of such paper is as follows. In second part, we introduce the knowledge involved in this paper, such as the definition, the score and accuracy function of INSs, and CPT-TODIM method. After that, we demonstrate the developed interval neutrosophic TODIM method based on cumulative prospect theory more detail in third part. And then, in section 4 and 5, we not only apply this new developed method to select the third-party logistics service providers, but also expound on the disparity with existing methods. Finally, the results of contrastive analysis indicates that this new developed method can lead to the appropriate conclusion and sticks out the differences between alternatives to provide clearer direction.

1. Preliminary knowledge

In this section, we sort out the basic knowledge about INSs.

**Definition 1** (Wang et al., 2010). A NS \( Y \), which consists of the truth-membership \( \rho_Y(n) \), indeterminacy-membership \( \sigma_Y(n) \) and falsity-membership \( \lambda_Y(n) \), can be expressed as follows in a fix set \( N \)

\[
Y = \{ (n, \rho_Y(n), \sigma_Y(n), \lambda_Y(n) | n \in N) \},
\]

where \( \rho_Y(n), \sigma_Y(n), \lambda_Y(n) \) are lying in \([0^-, 1^+]\) and \( 0^- \leq \sup \rho_Y(n) + \sup \sigma_Y(n) + \sup \lambda_Y(n) \leq 3^+ \).

**Definition 2** (Wang et al., 2004). An INS \( \tilde{Y} \) in a fix set \( N \) can be expressed:

\[
\tilde{Y} = \{ (n, \tilde{\rho}_Y(n), \tilde{\sigma}_Y(n), \tilde{\lambda}_Y(n) | n \in N) \},
\]

where \( \tilde{\rho}_Y(n) = [\rho_Y^l(n), \rho_Y^u(n)] \subseteq [0,1], \tilde{\sigma}_Y(n) = [\sigma_Y^l(n), \sigma_Y^u(n)] \subseteq [0,1] \) and \( \tilde{\lambda}_Y(n) = [\lambda_Y^l(n), \lambda_Y^u(n)] \subseteq [0,1] \) represent truth membership, indeterminacy membership and falsity membership respectively, satisfying \( 0 \leq \rho_Y^u(n) + \sigma_Y^u(n) + \lambda_Y^u(n) \leq 3 \).

Generally speaking, interval neutrosophic number (INN) \( \tilde{Y} = (\tilde{\rho}_Y, \tilde{\sigma}_Y, \tilde{\lambda}_Y) = ([\rho_Y^l, \rho_Y^u], [\sigma_Y^l, \sigma_Y^u], [\lambda_Y^l, \lambda_Y^u]) \), the convenient form of INS, is a usual expression in calculating. Moreover, the score function and the accuracy function are created to describe relative precision.

**Definition 3** (Tang, 2016). The score of INN \( \tilde{Y} = ([\rho_Y^l, \rho_Y^u], [\sigma_Y^l, \sigma_Y^u], [\lambda_Y^l, \lambda_Y^u]) \) is

\[
S(\tilde{Y}) = \frac{1}{6} \left[ (2 + \rho_Y^l - \sigma_Y^l - \lambda_Y^l) + (2 + \rho_Y^u - \sigma_Y^u - \lambda_Y^u) \right], \quad S(\tilde{Y}) \in [0,1].
\]

**Definition 4** (Tang, 2016). The accuracy function of INN \( \tilde{Y} = ([\rho_Y^l, \rho_Y^u], [\sigma_Y^l, \sigma_Y^u], [\lambda_Y^l, \lambda_Y^u]) \) is
\[
A(\bar{Y}) = \frac{1}{2} \left[ (\rho^L_Y + \rho^U_Y) - (\lambda^L_Y + \lambda^U_Y) \right], \quad A(\bar{X}) \in [-1, 1].
\]

**Definition 5** (Tang, 2016). There are two INNs \( \bar{Y} = \left[ (\rho^L_Y, \rho^U_Y), [\sigma^L_Y, \sigma^U_Y], [\lambda^L_Y, \lambda^U_Y] \right] \) and \( \bar{X} = \left[ (\rho^L_X, \rho^U_X), [\sigma^L_X, \sigma^U_X], [\lambda^L_X, \lambda^U_X] \right] \).

1. If \( S(\bar{Y}) > S(\bar{X}) \), then \( \bar{Y} > \bar{X} \);
2. If \( S(\bar{Y}) = S(\bar{X}) \), then we need to compare the accuracy function of \( \bar{Y} \) and \( \bar{X} \);
   i. If \( A(\bar{Y}) > A(\bar{X}) \), then \( \bar{Y} > \bar{X} \);
   ii. If \( A(\bar{Y}) = A(\bar{X}) \), then \( \bar{Y} = \bar{X} \).

**Definition 6** (Duong et al., 2020). Let \( \bar{Y} = \left[ (\rho^L_Y, \rho^U_Y), [\sigma^L_Y, \sigma^U_Y], [\lambda^L_Y, \lambda^U_Y] \right] \) and \( \bar{X} = \left[ (\rho^L_X, \rho^U_X), [\sigma^L_X, \sigma^U_X], [\lambda^L_X, \lambda^U_X] \right] \) be two INNs, and the following shows some basic operations of INNs.

1. \( \bar{Y}^c = \left[ (1 - \rho^L_Y, 1 - \rho^U_Y), [1 - \sigma^L_Y, 1 - \sigma^U_Y], [1 - \lambda^L_Y, 1 - \lambda^U_Y] \right] \);
2. \( \beta \bar{Y} = \left[ \left( 1 - (1 - \rho^L_Y)^{\beta}, 1 - (1 - \rho^U_Y)^{\beta} \right), \left( \sigma^L_Y \sigma^U_Y \right)^{\beta}, \left( \lambda^L_Y \lambda^U_Y \right)^{\beta} \right], \beta > 0; \)
3. \( (\bar{Y})^\beta = \left[ \left( \rho^L_Y \rho^U_Y \right)^{\beta}, \left( \sigma^L_Y \sigma^U_Y \right)^{\beta}, \left( \lambda^L_Y \lambda^U_Y \right)^{\beta} \right], \beta > 0; \)
4. \( \bar{Y} \oplus \bar{X} = \left[ \left( \rho^L_Y + \rho^L_X - \rho^U_Y \rho^U_X \right), \left( \sigma^L_Y \sigma^U_Y \sigma^L_X \sigma^U_X \right), \left( \lambda^L_Y \lambda^L_X \lambda^U_Y \lambda^U_X \right) \right]; \)
5. \( \bar{Y} \odot \bar{X} = \left[ \left( \sigma^L_Y + \sigma^L_X - \sigma^U_Y \sigma^U_X \right), \left( \lambda^L_Y \lambda^L_X \lambda^U_Y \lambda^U_X \right) \right]. \)

**Definition 7** (Tang, 2016). Let \( \bar{Y} = \left[ (\rho^L_Y, \rho^U_Y), [\sigma^L_Y, \sigma^U_Y], [\lambda^L_Y, \lambda^U_Y] \right] \) and \( \bar{X} = \left[ (\rho^L_X, \rho^U_X), [\sigma^L_X, \sigma^U_X], [\lambda^L_X, \lambda^U_X] \right] \) be two INNs, the normalized Hamming distance between these two INNs can be acquired by the following Eq. (5):

\[
d(\bar{Y}, \bar{X}) = \frac{\left| \rho^L_Y - \rho^L_X \right| + \left| \rho^U_Y - \rho^U_X \right| + \left| \sigma^L_Y - \sigma^L_X \right| + \left| \sigma^U_Y - \sigma^U_X \right| + \left| \lambda^L_Y - \lambda^L_X \right| + \left| \lambda^U_Y - \lambda^U_X \right|}{6}.
\]

**Definition 8** (Garg & Nancy, 2020). If there is a collection of INNs \( \bar{Y}_l = \left[ (\rho^L_{Y_l}, \rho^U_{Y_l}), [\sigma^L_{Y_l}, \sigma^U_{Y_l}], [\lambda^L_{Y_l}, \lambda^U_{Y_l}] \right] \) \((l = 1, 2, ..., f)\) and the weighting vector of \( \bar{Y}_l \) \((l = 1, 2, ..., f)\) is \( \alpha = (\alpha_1, \alpha_2, ..., \alpha_f)^T \) where \( \alpha_i \geq 0 \) and \( \sum_{l=1}^{f} \alpha_l = 1 \), then the interval neutrosophic weighted averaging (INWA) aggregation operator is

\[
\text{INWA}_\alpha \left( \bar{Y}_1, \bar{Y}_2, ..., \bar{Y}_f \right) = \bigoplus_{l=1}^{f} (\alpha_l \bar{Y}_l)
\]

\[
= \left[ \prod_{l=1}^{f} \left( 1 - \rho^L_{Y_l} \right)^{\alpha_l}, \prod_{l=1}^{f} \left( 1 - \rho^U_{Y_l} \right)^{\alpha_l}, \prod_{l=1}^{f} \left( \sigma^L_{Y_l} \sigma^U_{Y_l} \right)^{\alpha_l}, \prod_{l=1}^{f} \left( \lambda^L_{Y_l} \lambda^U_{Y_l} \right)^{\alpha_l} \right].
\]
Definition 9 (Garg & Nancy, 2020). If there is a collection of INNs \( \tilde{Y}_l = \left[ \begin{array}{c} \rho^L_{Y_l} \\ \lambda^L_{Y_l} \\ \lambda^U_{Y_l} \end{array} \right] \) and the weighting vector of \( \tilde{Y}_l \) \((l=1,2,\ldots,f)\) is \( \alpha = \left( \alpha_1, \alpha_2, \ldots, \alpha_f \right)^T \) where \( \alpha_i \geq 0 \) and \( \sum_{l=1}^{f} \alpha_l = 1 \), then the interval neutrosophic weighted geometric (INWG) aggregation operator is

\[
\text{INWG}_\alpha \left( \tilde{Y}_1, \tilde{Y}_2, \ldots, \tilde{Y}_f \right) = \bigotimes_{l=1}^{f} \left( \alpha_l \tilde{Y}_l \right)
\]

\[
= \left[ \prod_{l=1}^{f} \left( \rho^L_{Y_l} \right)^{\alpha_l}, \prod_{l=1}^{f} \left( \rho^U_{Y_l} \right)^{\alpha_l} \right] \left[ 1 - \prod_{l=1}^{f} \left( 1 - \lambda^L_{Y_l} \right)^{\alpha_l}, 1 - \prod_{l=1}^{f} \left( 1 - \lambda^U_{Y_l} \right)^{\alpha_l} \right] \left[ 1 - \prod_{l=1}^{f} \left( 1 - \lambda^L_{Y_l} \right)^{\alpha_l}, 1 - \prod_{l=1}^{f} \left( 1 - \lambda^U_{Y_l} \right)^{\alpha_l} \right] \right).
\]

2. Interval neutrosophic TODIM based on cumulative prospect theory

In this topic, we introduce a novel MAGDM method named the CPT-IN-TODIM method. Suppose there are three collections \( T = \{T_1, T_2, \ldots, T_h\} \), \( B = \{B_1, B_2, \ldots, B_p\} \) and \( D = \{D_1, D_2, \ldots, D_f\} \), which stand for alternatives, attributes and decision makers respectively. At the same time, the weighting vector of DMs is \( \alpha = \left( \alpha_1, \alpha_2, \ldots, \alpha_f \right)^T \) where \( \alpha_i \geq 0 \) and \( \sum_{l=1}^{f} \alpha_l = 1 \). The weighting vector of attributes is unknown, but it is expressed as \( \iota = (\iota_1, \iota_2, \ldots, \iota_p)^T \) \((\iota_0 \geq 0 \) and \( \sum_{o=1}^{p} \iota_o = 1 \).

**Step 1.** Build the interval neutrosophic decision matrix \( \tilde{Z}^{(l)} = \left( \tilde{z}^{(l)} \right)_{h \times p} = \left( \tilde{z}^{(l)}_{ko}, \tilde{Z}^{(l)} \right)_{h \times p} \)

of the decision maker \( D_l \), where indicate truth-membership degree \( \tilde{\rho}^{(l)}_{ko} = \left[ \rho^L_{ko}^{(l)}, \rho^U_{ko}^{(l)} \right] \), indeterminacy-membership degree \( \tilde{\sigma}^{(l)}_{ko} = \left[ \sigma^L_{ko}^{(l)}, \sigma^U_{ko}^{(l)} \right] \) and falsity-membership degree \( \tilde{\lambda}^{(l)}_{ko} = \left[ \lambda^L_{ko}^{(l)}, \lambda^U_{ko}^{(l)} \right] \) and all of them satisfy \( \tilde{\rho}^{(l)}_{ko}, \tilde{\sigma}^{(l)}_{ko}, \tilde{\lambda}^{(l)}_{ko} \subseteq [0,1] \) and \( 0 \leq \rho^L_{ko} + \sigma^L_{ko} + \lambda^L_{ko} \leq 3 \) \((k = 1, 2, \ldots, h, o = 1, 2, \ldots, p, l = 1, 2, \ldots, f)\).

**Step 2.** Translate negative attributes by using Eq. (8) to ensure consistency of measurement scale and finally make up the standardized interval neutrosophic decision matrix \( \tilde{G}^{(l)} = \left( \tilde{g}^{(l)}_{ko} \right)_{h \times p} \) \((k = 1, 2, \ldots, h, o = 1, 2, \ldots, p, l = 1, 2, \ldots, f)\).
\[
\begin{align*}
\bar{g}^{(l)}_{ko} &= \left\langle \bar{v}^{(l)}_{ko}, \bar{\phi}^{(l)}_{ko}, \bar{e}^{(l)}_{ko} \right\rangle = \left\langle \left[ v^{L}_{ko}, v^{U}_{ko} \right], \left[ \phi^{L}_{ko}, \phi^{U}_{ko} \right], \left[ e^{L}_{ko}, e^{U}_{ko} \right] \right\rangle \\
\bar{z}^{(l)}_{ko} &= \left\langle \bar{\rho}^{(l)}_{ko}, \bar{\sigma}^{(l)}_{ko}, \bar{\lambda}^{(l)}_{ko} \right\rangle, \quad T_k \text{ is a positive attribute} \\
&= \left\langle \left[ \rho^{L}_{ko}, \rho^{U}_{ko} \right], \left[ \sigma^{L}_{ko}, \sigma^{U}_{ko} \right], \left[ \lambda^{L}_{ko}, \lambda^{U}_{ko} \right] \right\rangle. \quad (8)
\end{align*}
\]

Step 3. The INWA operator is used to integrate all decision matrices from different decision makers into one group decision matrix \( \hat{Q} = (\tilde{q}_{ko})_{hx,p} \), shown as the Eq. (9):

\[
\begin{align*}
\tilde{q}_{ko} &= \left\langle \tilde{v}_{ko}, \tilde{\phi}_{ko}, \tilde{e}_{ko} \right\rangle = \left\langle \left[ v^{L}_{ko}, v^{U}_{ko} \right], \left[ \phi^{L}_{ko}, \phi^{U}_{ko} \right], \left[ e^{L}_{ko}, e^{U}_{ko} \right] \right\rangle \\
&= \text{INWA}_{\alpha}(\tilde{g}^{(1)}_{ko}, \tilde{g}^{(2)}_{ko}, \ldots, \tilde{g}^{(f)}_{ko}) = \bigoplus_{i=1}^{f} \alpha_i \tilde{g}^{(i)}_{ko} \\
&= \left\langle \prod_{l=1}^{f} \left( 1 - \prod_{i=1}^{l-1} \left( 1 - v^{L}_{ko} \right) \right)^{\alpha_i}, \prod_{l=1}^{f} \left( 1 - \prod_{i=1}^{l-1} \left( 1 - v^{U}_{ko} \right) \right)^{\alpha_i} \right\rangle.
\end{align*}
\]

Step 4. The entropy weight method, just as equation (10) and (11), helps to achieve the initial weighting vector of attributes \( \iota = (\iota_1, \iota_2, \ldots, \iota_p)^T \) (\( \iota_0 \geq 0 \) and \( \sum_{o=1}^{p} \iota_o = 1 \)) in line with the known information that the decision matrix \( \hat{Q} = (\tilde{q}_{ko})_{hx,p} \) gives us.

\[
\begin{align*}
E_o &= -\frac{1}{\ln h} \sum_{k=1}^{h} S(\tilde{q}_{ko}) \ln(S(\tilde{q}_{ko})); \quad (10) \\
\iota_o &= \frac{1 - E_o}{\sum_{o=1}^{p} (1 - E_o)}, \quad o = 1, 2, \ldots, p. \quad (11)
\end{align*}
\]

Step 5. The weighting function (12) and Eq. (13) are taking advantage of disposing the initial weighting vector of attributes \( \iota = (\iota_1, \iota_2, \ldots, \iota_p)^T \) to obtain the modified weights \( \tilde{\iota}_{ko}^{*} \) (\( \iota_0 = (\iota_0, \iota_1, \ldots, \iota_h, \iota_o) \)).

\[
\begin{align*}
\tilde{\iota}_{ko}^{*} &= \begin{cases} 
\begin{align*}
(\iota_0)^a / \left( (\iota_0)^a + (1 - \iota_0)^a \right)^{\frac{1}{a}}, & \tilde{q}_{ko} \geq \tilde{q}_{lo} \\
(\iota_0)^b / \left( (\iota_0)^b + (1 - \iota_0)^b \right)^{\frac{1}{b}}, & \tilde{q}_{ko} < \tilde{q}_{lo}
\end{align*}
\end{cases} \quad (12)
\end{align*}
\]
\[ \partial_{\text{kio}}^\ast \left( v_o \right) = \frac{\partial_{\text{kio}} \left( v_o \right)}{\max \left\{ \partial_{\text{kit}} \left( v_i \right) \mid t \in p \right\}}, \quad \forall k, i \in h; \quad o = 1, 2, \ldots, p; \tag{13} \]

**Step 6.** Compare the differences under each attribute between any two alternatives based on Eqs (14)–(15), just as \( \delta_o \left( T_k, T_i \right) \) means the relative predominance of alternative \( T_k \) as compared to alternative \( T_i \) under the attribute \( B_o \). And then utilize Eq. (16) to put the holistic preponderance of alternative \( T_k \) compared with alternative \( T_i \) together.

\[
d_o \left( T_k, T_i \right) = \frac{1}{6} \left( \begin{array}{c} \left( \| \gamma^L_{k0} - \gamma^L_{io} \| + \| \gamma^U_{k0} - \gamma^U_{io} \| \right) \\
\left( \| \varphi^L_{k0} - \varphi^L_{io} \| + \| \varphi^U_{k0} - \varphi^U_{io} \| \right) \\
\left( \| \varepsilon^L_{k0} - \varepsilon^L_{io} \| + \| \varepsilon^U_{k0} - \varepsilon^U_{io} \| \right) \end{array} \right), \quad k, i = 1, 2, \ldots, h; \tag{14} \]

\[
\delta_o \left( T_k, T_i \right) = \begin{cases} 0, & \text{if } \tilde{q}_{ko} > \tilde{q}_{ko} \\
\sum_{o=1}^{p} \partial_{\text{kio}}^\ast \left( v_o \right) \cdot \left( d_o \left( T_k, T_i \right) \right)^\eta, & \text{if } \tilde{q}_{ko} = \tilde{q}_{ko} \\
-0. \cdot \frac{\sum_{o=1}^{p} \partial_{\text{kio}}^\ast \left( v_o \right) \cdot \left( d_o \left( T_k, T_i \right) \right)^\mu}{\partial_{\text{kio}}^\ast \left( v_o \right)}, & \text{if } \tilde{q}_{ko} < \tilde{q}_{ko} \end{cases} \tag{15} \]

\[
\varphi \left( T_k, T_i \right) = \sum_{o=1}^{p} \delta_o \left( T_k, T_i \right), \quad k, i = 1, 2, \ldots, p, \tag{16} \]

where \( \theta, \eta \) and \( \mu \) are the parameters. And we can show the relative predominance \( \delta_o \left( T_k, T_i \right) \) and the overall predominance \( \varphi \left( T_k, T_i \right) \) in matrix form, just as the relative predominance matrix \( \delta_o = \left( \delta_o \left( T_k, T_i \right) \right)_{h \times h} \) and the overall predominance matrix \( \varphi = \left( \varphi \left( T_k, T_i \right) \right)_{h \times h} \) \((k, i = 1, 2, \ldots, h)\).

\[
\delta_o = \left( \delta_o \left( T_k, T_i \right) \right)_{h \times h} = \begin{bmatrix} T_1 & T_2 & \cdots & T_h \\
T_1 & 0 & \delta_o \left( T_1, T_2 \right) & \cdots & \delta_o \left( T_1, T_h \right) \\
T_2 & \delta_o \left( T_2, T_1 \right) & 0 & \cdots & \delta_o \left( T_2, T_h \right) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
T_h & \delta_o \left( T_h, T_1 \right) & \delta_o \left( T_h, T_2 \right) & \cdots & 0 \end{bmatrix}, \quad o = 1, 2, \ldots, p \tag{17} \]

\[
\varphi = \left( \varphi \left( T_k, T_i \right) \right)_{h \times h} = \begin{bmatrix} T_1 & T_2 & \cdots & T_h \\
T_1 & 0 & \varphi \left( T_1, T_2 \right) & \cdots & \varphi \left( T_1, T_h \right) \\
T_2 & \varphi \left( T_2, T_1 \right) & 0 & \cdots & \varphi \left( T_2, T_h \right) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
T_h & \varphi \left( T_h, T_1 \right) & \varphi \left( T_h, T_2 \right) & \cdots & 0 \end{bmatrix}, \quad o = 1, 2, \ldots, p \tag{18} \]

**Step 7.** In line with Eq. (19), standardize the overall predominance \( \varphi \left( T_k, T_i \right) \) to acquire the standard overall predominance \( \mathcal{S} \left( T_k \right) \) \((k = 1, 2, \ldots, h)\) for the sake of comparison. The greater the standard overall predominance is, the better the alternative is.
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\[ \mathcal{I}(T_k) = \frac{\sum_{i=1}^{h} \varphi(T_k, T_i) - \min_k \left\{ \sum_{i=1}^{h} \varphi(T_k, T_i) \right\}}{\max_k \left\{ \sum_{i=1}^{h} \varphi(T_k, T_i) \right\} - \min_k \left\{ \sum_{i=1}^{h} \varphi(T_k, T_i) \right\}} \quad k = 1, 2, \ldots, h. \quad (19) \]

### 3. Numerical instance

In this part, we tend to apply this new CPT-IN-TODIM method to give companies a hand assessing third-party cold chain logistics service providers. Now, there are four cold chain logistics service providers \( T_k \) \( (k = 1, 2, 3, 4) \) awaiting evaluation. Three experts \( D_l \) \( (l = 1, 2, 3) \) are invited to analyze five aspects of these providers: (1) \( B_1 \) is the degree of wastage reduction of fresh agricultural products; (2) \( B_2 \) is the degree of advanced service concept; (3) \( B_3 \) is the technical level of cold chain; (4) \( B_4 \) is the intelligence degree of cold chain information system; (5) \( B_5 \) is the distance from logistics point to producing area of fresh agricultural products. Additionally, the weighting vector of experts is \( \alpha = (\alpha_1, \alpha_2, \alpha_3)^T = (0.4, 0.25, 0.35)^T \).

And each expert’s assessment of these systems is depicted in the following Table 1 to Table 3.

#### Table 1. Decision matrix \( \hat{Z}^{(1)} \) given by the expert \( D_1 \)

<table>
<thead>
<tr>
<th></th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_4 )</th>
<th>( B_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 )</td>
<td>( {0.45, 0.55}, {0.25, 0.30}, {0.35, 0.45} )</td>
<td>( {0.70, 0.80}, {0.15, 0.20}, {0.25, 0.35} )</td>
<td>( {0.55, 0.65}, {0.25, 0.30}, {0.35, 0.50} )</td>
<td>( {0.65, 0.70}, {0.30, 0.40}, {0.40, 0.50} )</td>
<td>( {0.30, 0.40}, {0.50, 0.60}, {0.45, 0.50} )</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>( {0.70, 0.85}, {0.10, 0.15}, {0.15, 0.25} )</td>
<td>( {0.65, 0.70}, {0.25, 0.30}, {0.30, 0.40} )</td>
<td>( {0.75, 0.85}, {0.15, 0.20}, {0.20, 0.25} )</td>
<td>( {0.70, 0.80}, {0.25, 0.30}, {0.30, 0.40} )</td>
<td>( {0.20, 0.25}, {0.65, 0.70}, {0.65, 0.70} )</td>
</tr>
<tr>
<td>( T_3 )</td>
<td>( {0.65, 0.70}, {0.20, 0.30}, {0.35, 0.50} )</td>
<td>( {0.60, 0.75}, {0.20, 0.30}, {0.25, 0.35} )</td>
<td>( {0.70, 0.80}, {0.20, 0.25}, {0.30, 0.35} )</td>
<td>( {0.65, 0.75}, {0.30, 0.35}, {0.35, 0.45} )</td>
<td>( {0.15, 0.25}, {0.70, 0.80}, {0.75, 0.80} )</td>
</tr>
<tr>
<td>( T_4 )</td>
<td>( {0.50, 0.55}, {0.25, 0.35}, {0.40, 0.45} )</td>
<td>( {0.65, 0.75}, {0.25, 0.35}, {0.35, 0.40} )</td>
<td>( {0.60, 0.70}, {0.25, 0.35}, {0.35, 0.40} )</td>
<td>( {0.70, 0.75}, {0.25, 0.35}, {0.35, 0.40} )</td>
<td>( {0.35, 0.45}, {0.55, 0.65}, {0.50, 0.60} )</td>
</tr>
</tbody>
</table>
Table 2. Decision matrix $\widetilde{Z}^{(2)}$ given by the expert $D_2$

<table>
<thead>
<tr>
<th></th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>$[0.45,0.55],\frac{1}{3},[0.25,0.30],\frac{1}{3},[0.35,0.45]$</td>
<td>$[0.65,0.75],\frac{1}{3},[0.20,0.25],\frac{1}{3},[0.25,0.35]$</td>
<td>$[0.65,0.70],\frac{1}{3},[0.30,0.35],\frac{1}{3},[0.35,0.45]$</td>
<td>$[0.60,0.70],\frac{1}{3},[0.30,0.35],\frac{1}{3},[0.35,0.45]$</td>
<td>$[0.30,0.35],\frac{1}{3},[0.60,0.65],\frac{1}{3},[0.50,0.65]$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$[0.70,0.85],\frac{1}{5},[0.10,0.15],\frac{1}{5},[0.15,0.25]$</td>
<td>$[0.70,0.75],\frac{1}{5},[0.20,0.25],\frac{1}{5},[0.25,0.35]$</td>
<td>$[0.70,0.80],\frac{1}{5},[0.30,0.35],\frac{1}{5},[0.35,0.45]$</td>
<td>$[0.75,0.85],\frac{1}{5},[0.20,0.25],\frac{1}{5},[0.25,0.35]$</td>
<td>$[0.15,0.20],\frac{1}{5},[0.75,0.80],\frac{1}{5},[0.75,0.85]$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>$[0.65,0.70],\frac{1}{3},[0.20,0.30],\frac{1}{3},[0.35,0.50]$</td>
<td>$[0.65,0.70],\frac{1}{3},[0.20,0.30],\frac{1}{3},[0.35,0.50]$</td>
<td>$[0.70,0.80],\frac{1}{3},[0.30,0.35],\frac{1}{3},[0.35,0.40]$</td>
<td>$[0.75,0.80],\frac{1}{3},[0.25,0.30],\frac{1}{3},[0.30,0.35]$</td>
<td>$[0.20,0.30],\frac{1}{3},[0.65,0.70],\frac{1}{3},[0.65,0.75]$</td>
</tr>
<tr>
<td>$T_4$</td>
<td>$[0.50,0.55],\frac{1}{3},[0.25,0.35],\frac{1}{3},[0.40,0.45]$</td>
<td>$[0.60,0.65],\frac{1}{3},[0.25,0.30],\frac{1}{3},[0.30,0.40]$</td>
<td>$[0.65,0.75],\frac{1}{3},[0.20,0.25],\frac{1}{3},[0.30,0.40]$</td>
<td>$[0.65,0.75],\frac{1}{3},[0.30,0.35],\frac{1}{3},[0.40,0.50]$</td>
<td>$[0.30,0.35],\frac{1}{3},[0.65,0.75],\frac{1}{3},[0.55,0.65]$</td>
</tr>
</tbody>
</table>

Table 3. Decision matrix $\widetilde{Z}^{(3)}$ given by the expert $D_3$

<table>
<thead>
<tr>
<th></th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>$[0.45,0.55],\frac{1}{3},[0.25,0.30],\frac{1}{3},[0.35,0.45]$</td>
<td>$[0.75,0.80],\frac{1}{3},[0.15,0.20],\frac{1}{3},[0.20,0.25]$</td>
<td>$[0.65,0.75],\frac{1}{3},[0.30,0.40],\frac{1}{3},[0.40,0.45]$</td>
<td>$[0.65,0.75],\frac{1}{3},[0.30,0.35],\frac{1}{3},[0.35,0.40]$</td>
<td>$[0.25,0.35],\frac{1}{3},[0.65,0.70],\frac{1}{3},[0.60,0.65]$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$[0.70,0.85],\frac{1}{5},[0.10,0.15],\frac{1}{5},[0.15,0.25]$</td>
<td>$[0.60,0.70],\frac{1}{5},[0.30,0.35],\frac{1}{5},[0.35,0.45]$</td>
<td>$[0.70,0.80],\frac{1}{5},[0.25,0.30],\frac{1}{5},[0.30,0.40]$</td>
<td>$[0.70,0.85],\frac{1}{5},[0.20,0.25],\frac{1}{5},[0.30,0.40]$</td>
<td>$[0.20,0.30],\frac{1}{5},[0.70,0.75],\frac{1}{5},[0.75,0.80]$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>$[0.65,0.70],\frac{1}{3},[0.20,0.30],\frac{1}{3},[0.35,0.50]$</td>
<td>$[0.65,0.75],\frac{1}{3},[0.25,0.30],\frac{1}{3},[0.35,0.45]$</td>
<td>$[0.75,0.85],\frac{1}{3},[0.20,0.25],\frac{1}{3},[0.30,0.40]$</td>
<td>$[0.75,0.80],\frac{1}{3},[0.20,0.25],\frac{1}{3},[0.30,0.40]$</td>
<td>$[0.10,0.20],\frac{1}{3},[0.80,0.90],\frac{1}{3},[0.80,0.85]$</td>
</tr>
<tr>
<td>$T_4$</td>
<td>$[0.50,0.55],\frac{1}{3},[0.25,0.35],\frac{1}{3},[0.40,0.45]$</td>
<td>$[0.70,0.75],\frac{1}{3},[0.20,0.30],\frac{1}{3},[0.35,0.45]$</td>
<td>$[0.60,0.65],\frac{1}{3},[0.35,0.45],\frac{1}{3},[0.45,0.50]$</td>
<td>$[0.60,0.70],\frac{1}{3},[0.35,0.40],\frac{1}{3},[0.40,0.50]$</td>
<td>$[0.25,0.35],\frac{1}{3},[0.55,0.60],\frac{1}{3},[0.55,0.65]$</td>
</tr>
</tbody>
</table>

**Step 1.** Build the interval neutrosophic decision matrix $\widetilde{Z}^{(l)} = \left(\tilde{z}_{k_0}^{(l)}, \sigma_k^{(l)}, \lambda_k^{(l)}\right)_{4 \times 5}$ for $l = 1, 2, 3$ just like the shown in Table 1 to Table 3.

**Step 2.** Translate negative attributes by using Eq. (8) to ensure consistency of measurement scale and finally make up the standardized interval neutrosophic decision matrix $\tilde{G}^{(l)} = \left(\tilde{g}_{k_0}^{(l)}\right)_{4 \times 5}$ for $l = 1, 2, 3$. Please see Tables 4–6.
Table 4. Standardized decision matrix $\tilde{C}^{(1)}$ given by the expert $D_1$

<table>
<thead>
<tr>
<th></th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>$\begin{bmatrix} 0.45,0.55 \ \ 0.25,0.30 \ \ 0.35,0.45 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.70,0.80 \ \ 0.15,0.20 \ \ 0.25,0.35 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.55,0.65 \ \ 0.25,0.30 \ \ 0.35,0.50 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.70 \ \ 0.30,0.40 \ \ 0.40,0.50 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.45,0.50 \ \ 0.40,0.50 \ \ 0.30,0.40 \end{bmatrix}$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$\begin{bmatrix} 0.70,0.85 \ \ 0.10,0.15 \ \ 0.15,0.25 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.70 \ \ 0.25,0.30 \ \ 0.30,0.40 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.75,0.85 \ \ 0.15,0.20 \ \ 0.20,0.25 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.70,0.80 \ \ 0.25,0.30 \ \ 0.30,0.40 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.70 \ \ 0.30,0.35 \ \ 0.20,0.25 \end{bmatrix}$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>$\begin{bmatrix} 0.65,0.70 \ \ 0.20,0.30 \ \ 0.35,0.50 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.60,0.75 \ \ 0.20,0.30 \ \ 0.25,0.35 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.70,0.80 \ \ 0.20,0.25 \ \ 0.30,0.35 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.75 \ \ 0.30,0.35 \ \ 0.35,0.45 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.75,0.80 \ \ 0.20,0.30 \ \ 0.15,0.25 \end{bmatrix}$</td>
</tr>
<tr>
<td>$T_4$</td>
<td>$\begin{bmatrix} 0.50,0.55 \ \ 0.25,0.35 \ \ 0.40,0.45 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.75 \ \ 0.25,0.35 \ \ 0.20,0.25 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.60,0.70 \ \ 0.25,0.35 \ \ 0.35,0.40 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.70,0.75 \ \ 0.25,0.35 \ \ 0.35,0.40 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.50,0.60 \ \ 0.35,0.45 \ \ 0.35,0.45 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

Table 5. Standardized decision matrix $\tilde{C}^{(2)}$ given by the expert $D_2$

<table>
<thead>
<tr>
<th></th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>$\begin{bmatrix} 0.45,0.55 \ \ 0.25,0.30 \ \ 0.35,0.45 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.75 \ \ 0.20,0.25 \ \ 0.25,0.35 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.70 \ \ 0.30,0.35 \ \ 0.35,0.45 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.60,0.70 \ \ 0.25,0.40 \ \ 0.40,0.45 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.50,0.65 \ \ 0.35,0.40 \ \ 0.30,0.35 \end{bmatrix}$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$\begin{bmatrix} 0.70,0.85 \ \ 0.10,0.15 \ \ 0.15,0.25 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.70,0.75 \ \ 0.20,0.25 \ \ 0.25,0.35 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.70,0.80 \ \ 0.20,0.30 \ \ 0.30,0.35 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.75,0.85 \ \ 0.20,0.30 \ \ 0.15,0.20 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.75,0.85 \ \ 0.20,0.25 \ \ 0.15,0.20 \end{bmatrix}$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>$\begin{bmatrix} 0.65,0.70 \ \ 0.20,0.30 \ \ 0.35,0.50 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.70 \ \ 0.20,0.30 \ \ 0.30,0.35 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.70,0.80 \ \ 0.15,0.25 \ \ 0.25,0.30 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.75,0.80 \ \ 0.25,0.30 \ \ 0.35,0.40 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.75 \ \ 0.30,0.35 \ \ 0.20,0.30 \end{bmatrix}$</td>
</tr>
<tr>
<td>$T_4$</td>
<td>$\begin{bmatrix} 0.50,0.55 \ \ 0.25,0.35 \ \ 0.40,0.45 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.60,0.65 \ \ 0.25,0.30 \ \ 0.30,0.40 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.75 \ \ 0.20,0.25 \ \ 0.30,0.35 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.65,0.75 \ \ 0.30,0.35 \ \ 0.40,0.50 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.55,0.65 \ \ 0.25,0.35 \ \ 0.30,0.35 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
Table 6. Standardized decision matrix $\tilde{G}^{(3)}$ given by the expert $D_3$

<table>
<thead>
<tr>
<th></th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>$[0.45,0.55]$, $[0.25,0.30]$, $[0.35,0.45]$</td>
<td>$[0.75,0.80]$, $[0.15,0.20]$, $[0.20,0.25]$</td>
<td>$[0.65,0.75]$, $[0.30,0.40]$, $[0.40,0.45]$</td>
<td>$[0.65,0.75]$, $[0.30,0.35]$, $[0.35,0.40]$</td>
<td>$[0.60,0.65]$, $[0.30,0.35]$, $[0.25,0.35]$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$[0.70,0.85]$, $[0.10,0.15]$, $[0.15,0.25]$</td>
<td>$[0.50,0.55]$, $[0.35,0.45]$, $[0.30,0.35]$</td>
<td>$[0.70,0.80]$, $[0.20,0.30]$, $[0.30,0.35]$</td>
<td>$[0.70,0.85]$, $[0.20,0.30]$, $[0.30,0.35]$</td>
<td>$[0.75,0.80]$, $[0.20,0.30]$, $[0.30,0.35]$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>$[0.65,0.70]$, $[0.20,0.30]$, $[0.35,0.50]$</td>
<td>$[0.65,0.75]$, $[0.25,0.30]$, $[0.30,0.35]$</td>
<td>$[0.75,0.85]$, $[0.20,0.25]$, $[0.25,0.35]$</td>
<td>$[0.75,0.80]$, $[0.30,0.35]$, $[0.35,0.40]$</td>
<td>$[0.25,0.30]$, $[0.20,0.30]$, $[0.25,0.30]$</td>
</tr>
<tr>
<td>$T_4$</td>
<td>$[0.50,0.55]$, $[0.25,0.35]$, $[0.40,0.45]$</td>
<td>$[0.70,0.75]$, $[0.20,0.30]$, $[0.25,0.35]$</td>
<td>$[0.60,0.65]$, $[0.35,0.45]$, $[0.45,0.50]$</td>
<td>$[0.60,0.70]$, $[0.35,0.40]$, $[0.40,0.45]$</td>
<td>$[0.55,0.65]$, $[0.40,0.45]$, $[0.25,0.35]$</td>
</tr>
</tbody>
</table>

**Step 3.** Integrate all decision matrices into the group decision matrix $\tilde{Q}=(\tilde{q}_{ko})_{4 \times 5}$ by taking advantage of the Eq. (9) (see Table 7).

Table 7. Group decision matrix $\tilde{Q}$

<table>
<thead>
<tr>
<th></th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>$[0.4630,0.5945]$, $[0.2660,0.3318]$, $[0.3822,0.4827]$</td>
<td>$[0.7075,0.7885]$, $[0.1612,0.2115]$, $[0.2312,0.3111]$</td>
<td>$[0.6130,0.7006]$, $[0.2789,0.3448]$, $[0.3667,0.4694]$</td>
<td>$[0.6381,0.7185]$, $[0.2866,0.3817]$, $[0.3817,0.4504]$</td>
<td>$[0.5196,0.5963]$, $[0.3498,0.4174]$, $[0.2815,0.3692]$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>$[0.6709,0.7828]$, $[0.1516,0.2038]$, $[0.2038,0.2899]$</td>
<td>$[0.6471,0.7134]$, $[0.2345,0.2861]$, $[0.3025,0.3879]$</td>
<td>$[0.7211,0.8217]$, $[0.1927,0.2551]$, $[0.2551,0.3206]$</td>
<td>$[0.7134,0.8317]$, $[0.2187,0.2689]$, $[0.2815,0.3692]$</td>
<td>$[0.7140,0.7811]$, $[0.2543,0.3049]$, $[0.1861,0.2520]$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>$[0.7006,0.7648]$, $[0.1683,0.2487]$, $[0.2502,0.3516]$</td>
<td>$[0.6308,0.7383]$, $[0.2162,0.3166]$, $[0.2944,0.3667]$</td>
<td>$[0.7192,0.8192]$, $[0.1861,0.2665]$, $[0.2866,0.3368]$</td>
<td>$[0.7140,0.7813]$, $[0.2487,0.2994]$, $[0.3316,0.4193]$</td>
<td>$[0.7485,0.8088]$, $[0.1737,0.2705]$, $[0.1399,0.2420]$</td>
</tr>
<tr>
<td>$T_4$</td>
<td>$[0.5270,0.5945]$, $[0.2420,0.3429]$, $[0.4031,0.4669]$</td>
<td>$[0.6571,0.7281]$, $[0.2312,0.3191]$, $[0.2393,0.3163]$</td>
<td>$[0.6131,0.6975]$, $[0.2660,0.3513]$, $[0.3677,0.4325]$</td>
<td>$[0.6552,0.7335]$, $[0.2944,0.3667]$, $[0.3792,0.4573]$</td>
<td>$[0.5306,0.6308]$, $[0.3372,0.4226]$, $[0.2994,0.3870]$</td>
</tr>
</tbody>
</table>
Step 4. Take advantage of the entropy weight method, the Eqs (10) and (11) to achieve original attribute weights \( ι_1 = 0.1854, ι_2 = 0.2267, ι_3 = 0.2034, ι_4 = 0.1809, ι_5 = 0.2036 \) in line with the known information of the decision matrix \( \tilde{Q} = (\tilde{q}_{ko})_{4×5} \).

Step 5. According to Eqs (12) and (13), compute the modified weights \( \tilde{c}^*_{kio}(t_o) \) \((k, i = 1, 2, \cdots, h; o = 1, 2, \cdots, p)\). The results are depicted in Table 8 to Table 11. \((a = 0.61, b = 0.69, \text{based on the experiment of Kahneman (1992)})\).

Table 8. The modified weights \( \tilde{c}^*_{1io}(t_o) \)

<table>
<thead>
<tr>
<th></th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_4 )</th>
<th>( B_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{c}^*_{12} )</td>
<td>0.8868</td>
<td>1.0000</td>
<td>0.9368</td>
<td>0.8738</td>
<td>0.9373</td>
</tr>
<tr>
<td>( \tilde{c}^*_{13} )</td>
<td>0.8868</td>
<td>1.0000</td>
<td>0.9368</td>
<td>0.8738</td>
<td>0.9373</td>
</tr>
<tr>
<td>( \tilde{c}^*_{14} )</td>
<td>0.8868</td>
<td>1.0000</td>
<td>0.9368</td>
<td>0.8738</td>
<td>0.9373</td>
</tr>
</tbody>
</table>

Table 9. The modified weights \( \tilde{c}^*_{2io}(t_o) \)

<table>
<thead>
<tr>
<th></th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_4 )</th>
<th>( B_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{c}^*_{21} )</td>
<td>0.9079</td>
<td>1.0000</td>
<td>0.9496</td>
<td>0.8971</td>
<td>0.9500</td>
</tr>
<tr>
<td>( \tilde{c}^*_{23} )</td>
<td>0.9079</td>
<td>1.0000</td>
<td>0.9496</td>
<td>0.8971</td>
<td>0.9382</td>
</tr>
<tr>
<td>( \tilde{c}^*_{24} )</td>
<td>0.9079</td>
<td>1.0000</td>
<td>0.9496</td>
<td>0.8971</td>
<td>0.9500</td>
</tr>
</tbody>
</table>

Table 10. The modified weights \( \tilde{c}^*_{3io}(t_o) \)

<table>
<thead>
<tr>
<th></th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_4 )</th>
<th>( B_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{c}^*_{31} )</td>
<td>0.9079</td>
<td>1.0000</td>
<td>0.9496</td>
<td>0.8971</td>
<td>0.9500</td>
</tr>
<tr>
<td>( \tilde{c}^*_{32} )</td>
<td>0.8868</td>
<td>1.0000</td>
<td>0.9368</td>
<td>0.8738</td>
<td>0.9491</td>
</tr>
<tr>
<td>( \tilde{c}^*_{34} )</td>
<td>0.9079</td>
<td>1.0000</td>
<td>0.9496</td>
<td>0.8971</td>
<td>0.9500</td>
</tr>
</tbody>
</table>

Table 11. The modified weights \( \tilde{c}^*_{4io}(t_o) \)

<table>
<thead>
<tr>
<th></th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_4 )</th>
<th>( B_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{c}^*_{41} )</td>
<td>0.9079</td>
<td>1.0000</td>
<td>0.9496</td>
<td>0.8971</td>
<td>0.9500</td>
</tr>
<tr>
<td>( \tilde{c}^*_{42} )</td>
<td>0.8868</td>
<td>1.0000</td>
<td>0.9368</td>
<td>0.8738</td>
<td>0.9491</td>
</tr>
<tr>
<td>( \tilde{c}^*_{43} )</td>
<td>0.8868</td>
<td>1.0000</td>
<td>0.9368</td>
<td>0.8738</td>
<td>0.9491</td>
</tr>
</tbody>
</table>

Step 6. Suppose \( \eta = 0.88, \mu = 0.88 \) and \( \theta = 2.25 \) (Kahneman, 1992), compare the differences under each attribute between any two alternatives based on Eqs (14)–(15) (see Table 12). And then utilize Eq. (16) to put the holistic preponderance of alternative \( T_k \) compared with alternative \( T_i \) together. The relative predominance \( δ_o(T_k, T_i) \) and the overall predominance \( ϕ(T_k, T_i) \) are expressed in the relative predominance matrix \( δ_o = (δ_o(T_k, T_i))_{4×4} \) \((o = 1, 2, 3, 4, 5)\) and the overall predominance matrix \( ϕ = (ϕ(T_k, T_i))_{4×4} \).

Table 12. Distance between each of the two alternatives

<table>
<thead>
<tr>
<th></th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_4 )</th>
<th>( B_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_o(T_1, T_2) )</td>
<td>0.1683</td>
<td>0.0719</td>
<td>0.1109</td>
<td>0.0918</td>
<td>0.1333</td>
</tr>
<tr>
<td>( d_o(T_1, T_3) )</td>
<td>0.1420</td>
<td>0.0676</td>
<td>0.1013</td>
<td>0.0567</td>
<td>0.1722</td>
</tr>
<tr>
<td>( d_o(T_1, T_4) )</td>
<td>0.0227</td>
<td>0.0503</td>
<td>0.0101</td>
<td>0.0107</td>
<td>0.0165</td>
</tr>
<tr>
<td>( d_o(T_2, T_3) )</td>
<td>0.0362</td>
<td>0.0199</td>
<td>0.0118</td>
<td>0.0353</td>
<td>0.0389</td>
</tr>
<tr>
<td>( d_o(T_2, T_4) )</td>
<td>0.1563</td>
<td>0.0326</td>
<td>0.1044</td>
<td>0.0860</td>
<td>0.1304</td>
</tr>
<tr>
<td>( d_o(T_3, T_4) )</td>
<td>0.1300</td>
<td>0.0266</td>
<td>0.0948</td>
<td>0.0509</td>
<td>0.1693</td>
</tr>
</tbody>
</table>
Step 7. In line with Eq. (19), standardize the overall predominance \( \wp(T_k, T_i) \) to acquire the standardized overall predominance \( \wp_k(T_k) \) (\( k = 1, 2, 3, 4 \)) for the sake of comparison. The greater the standard overall predominance is, the better the alternative is.

\[
\wp_k(T_k) = \begin{cases} 
0, & k = 1, 2, 3, 4
\end{cases}
\]

Therefore, the alternative \( T_2 \) is the most excellent one.
4. Sensitivity analysis

As parameters, the value of $a$, $b$, $\theta$, $\eta$, $\mu$ can be adjusted according to the needs of the actual situation and their value will affect the final calculation result to some extent. Therefore, in this section, we discuss the influence of variation of these parameters.

4.1. The sensitivity analysis of parameters from the weighting function

When the parameters $a$ and $b$ from the weighting function vary separately, we can plot the following graphs, Figure 1 and Figure 2, according to the values of the standardized overall predominance $\mathcal{I}(T_k)$ ($k = 1, 2, 3, 4$). We can see clearly from the graphs that the ranking results remain the same in other cases except that the optimal alternative is changed when the value of parameter $b$ is 0.1. However, changes in parameter $b$ have a slightly greater effect than changes in parameter $a$.

4.2. The sensitivity analysis of parameters from the value function

In this section, we also discuss the effects of individual changes in parameters $\theta$, $\eta$, and $\mu$ of the value function. The corresponding results are also presented in the form of graphs, Figure 3, Figure 4 and Figure 5. In general, the individual variation of the three parameters does not affect the ranking result of alternatives. However, the fluctuation of values of the standardized overall predominance $\mathcal{I}(T_k)$ caused by the change of parameter $\mu$ is relatively larger.

![Figure 1](image1.png)  
**Figure 1.** When parameter $a$ changes alone, the results of $\mathcal{I}(T_k)$ in different $a$

![Figure 2](image2.png)  
**Figure 2.** When parameter $b$ changes alone, the results of $\mathcal{I}(T_k)$ in different $b$
5. Comparative analysis

Comparing with other testified methods is indispensable for any new method which is to be proven effective. For verifying this new proposed CPT-IN-TODIM method, in this part, we choose Interval neutrosophic cross-entropy (Sahin, 2017), interval neutrosophic TODIM method (Xu et al., 2017), INWA operator and INWG operator (Garg & Nancy, 2020) to compare with the new method in this paper. Table 13 exhibits the results from different selected methods.
Table 13. The sequence from different methods

<table>
<thead>
<tr>
<th>Method</th>
<th>The sequence</th>
<th>The best alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval Neutrosophic Cross-entropy (Sahin, 2017)</td>
<td>$T_2 &gt; T_3 &gt; T_1 &gt; T_4$</td>
<td>$T_2$</td>
</tr>
<tr>
<td>IN-TODIM method (Xu et al., 2017)</td>
<td>$T_2 &gt; T_3 &gt; T_4 &gt; T_1$</td>
<td>$T_2$</td>
</tr>
<tr>
<td>INWA operator (Garg &amp; Nancy, 2020)</td>
<td>$T_2 &gt; T_3 &gt; T_1 &gt; T_4$</td>
<td>$T_2$</td>
</tr>
<tr>
<td>INWG operator (Garg &amp; Nancy, 2020)</td>
<td>$T_2 &gt; T_3 &gt; T_1 &gt; T_4$</td>
<td>$T_2$</td>
</tr>
<tr>
<td>CPT-IN-TODIM</td>
<td>$T_2 &gt; T_3 &gt; T_4 &gt; T_1$</td>
<td>$T_2$</td>
</tr>
</tbody>
</table>

On the basis of above comparison, there is no doubt that the alternative $T_2$ is the best one. In addition, each method has slightly different in aspects of results and concepts. Obviously, the CPT-IN-TODIM method has markedly superiority in describing decision maker’s psychological states.

Conclusions

In this paper, after absorbing the experiences and deficiencies of previous studies, we establish the developed CPT-IN-TODIM method which is an expansion of MAGDM method with INSs. For one thing, this new developed CPT-IN-TODIM method, utilizing the weighting function to adjust attribute weights and the value function to improve the relative predominance, has marked superiority in describing decision maker’s psychological states. For another, the new developed method not only takes a more scientific approach to acquire the original attribute weights, but also utilizes interval neutrosophic number as the tool to evaluate the uncertain information. Finally, the results of contrastive analysis indicates that this new developed method can lead to the appropriate conclusion and sticks out the differences between alternatives to provide clearer direction. Hence, the new developed CPT-IN-TODIM method is reliable and valid. In the future, we shall continue to explore the application of such method in other fields and put forward more scientific and reasonable new methods to solve MAGDM problems.

References


