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PARTIAL BACKORDERING INVENTORY MODEL WITH LIMITED STORAGE CAPACITY UNDER ORDER-SIZE DEPENDENT TRADE CREDIT

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Abstract. This study formulates an inventory model with limited storage capacity under the condition of order-size dependent trade credit. Shortages are allowed and partially backlogged. The objective of this study is to determine the optimal replenishment cycle length, the optimal fraction of no shortage, and whether retailers should choose to rent an extra warehouse to store more items, such that retailers' total annual profit is maximized. We prove the global optimally of objective functions and derive the closed-form optimal solution. Some numerical examples are presented to illustrate the applicability of the proposed model. Sensitivity analysis is carried out and managerial insights are obtained. We find that if retailers' own warehouse capacity is relatively small, they always benefit from enlarging order quantity and renting an extra warehouse; meanwhile, suppliers further prolong the credit period is beneficial for both parties. On the contrary, as retailers' own warehouse capacity increases and exceeds the optimal order quantity under that of without capacity constraints, adopting the same replenishment strategy as that without capacity constraints is profitable for retailers. Our results also reveal that other model parameters (e.g., ordering cost, inventory holding cost, shortages cost, backordering rate, etc.) have a significant impact on retailers' optimal decisions.

Keywords: inventory, order-size dependent trade credit, limited storage capacity, partial backordering.

JEL Classification: C44, D24.

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Introduction

Providing a reasonable and efficient way to manage inventory is one of the most challenging activities that business organizations face; this provision serves as a significant role in the success of organizations in the current competitive market (Taleizadeh et al., 2013; Diabat et al., 2017; Lashgari et al., 2018; Tiwari et al., 2020). In the traditional economic order quantity (EOQ) model, it is generally assumed that retailers must pay for the entire purchase costs immediately after goods are received. However, as market trends change and competition intensifies in today's business world, suppliers usually grant trade credit to retailers to increase sales and reduce inventory (Ouyang et al., 2008; Teng et al., 2012; Jaggi et al., 2017; Wu et al., 2018; Li et al., 2021). Under this policy, suppliers agree to offer retailers a certain grace period to pay off their purchase costs. During this grace period, retailers can start to accumulate sales revenue and earn interest, but suppliers charge interest beyond this period. Clearly, paying later indirectly reduces inventory holding costs, retailers thus are motivated to enlarge their order quantity. In real business, trade credit has rapidly grown and gained popularity among many industries. It reported that over 80% of B2B transactions in the United Kingdom are underpinned by trade credit. Moreover, in the United States, approximately 80% of companies sell their goods through trade credit (Seifert et al., 2013). Additionally, some larger companies, such as Ford Motor, Gree Electric, IBM, HP, etc., are also willing to offer trade credit to their dealers or retailers (Feng & Chan, 2019; Yao et al., 2021). Seifert et al. (2017) also point out that enterprise profitability is positively related to payment delay after investigating a sample of 3,383 groups of public US enterprises.

In the existing inventory models involving trade credit, it is usually assumed that the length of the trade credit period is constant and regardless of retailers' order quantities. However, in practice, suppliers may provide the trade credit period is linked to the order quantity to encourage retailers to place larger quantities (Chen et al., 2014; Chang et al., 2015; Tiwari et al., 2020). A common form of this trade credit policy is called conditional trade credit, i.e., suppliers preset an order quantity threshold and below which delay in payment is not permitted and retailers must pay for purchase costs with cash. For order quantities above this threshold, trade credit is available (Huang, 2007; Liang & Zhou, 2011; Chung et al., 2013; Tiwari et al., 2020; Taleizadeh et al., 2021). Undeniably, the above conditional trade credit policy has two obvious disadvantages: (1) suppliers must fully grasp all kinds of information about retailers (including market demand information, warehouse capacity information, and cost structures information, etc.) to set an accurate order quantity threshold. Doing so is difficult, even impossible to achieve in today's highly competitive market environment, especially in the buyers' market. (2) Trade credit with a single order quantity threshold forces retailers to make two extreme choices: to enjoy delay in payment by making the order quantity exceeds the predetermined quantity or to pay for the full purchase amount immediately when the order quantity is less than the quantity threshold. Therefore, a flexible order-size dependent trade credit policy (i.e., offer different trade credit periods with different predetermined order quantity thresholds) should be proposed to reduce the difficulty of suppliers' decision-making and increase the choice of retailers. However, as far as we know, only very limited attention has been paid to this kind of trade credit policy, despite it has various advantages and applicability in the real business world.

When retailers are faced with an order-size dependent trade credit policy, they usually prefer to order higher quantities to obtain a longer trade credit period. From retailers' perspectives, holding a high stock level means that they need additional warehouse space. However, retailers' own warehouse (OW) storage capacity is always limited in practice, implying retailers need an extra rented warehouse (RW) to keep the exceeded part (if any) (Ghiami et al., 2013; Tiwari et al., 2016, 2018; Jonas, 2019). Obviously, the above consideration raises a problem for retailers to have to address, i.e., whether retailers should rent an extra warehouse to hold more items and thus obtain a longer trade credit period? In addition, shortages are a natural phenomenon that occurs in many retail industries; meanwhile, in real business, to reduce the inventory holding cost and avoid physical items losses, retailers are always motivated to adopt the planning shortage strategy to manage their inventory (Taleizadeh, 2016; Tiwari et al., 2018; Lashgari et al., 2018). In reality, employing the planning shortage strategy is also an effective approach to alleviate the dilemma of OW capacity constraints. Nevertheless, considering that shortages may not be fully backlogged in a practical scenario and extra costs caused by shortages, retailers must fully account for various costs (i.e., the shortage cost, lost sale cost, and inventory holding costs in RW and OW) in determining whether they should operate an inventory system with shortages. Especially, if adopted, How retailers determine the optimal shortage level?

Motivated by the facts aforementioned, this paper aims to complement the existing works by examining the following questions:

(1) Under the conditions of limited storage capacity, partial backordering, and order-size dependent trade credit, how should retailers decide the optimal replenishment schedule to optimize their annual profit?

(2) Is it always optimal for retailers to rent an additional warehouse? Under what situations in which retailers rent an additional warehouse may not be optimal? That is, how do retailers balance the profit gained from a longer credit period and loss due to a rented warehouse?

(3) How do different operational costs affect retailers' replenishment policy and rented warehouse choice?

To address the above concerns, this study develops an inventory model with limited storage capacity under the condition of order-size dependent trade credit. Shortages are allowed and partially backlogged. We first formulate four possible mathematical models that may arise owing to different parameter values. Then, we establish the condition that objective functions have an interior optimal value, derive the closed-form optimal solution, and design a solution procedure to find the global optimal solution in an integrated manner. Finally, some numerical examples are presented and the sensitivity analysis is performed to illustrate the applicability of the proposed model and obtain some managerial insights. Specific results include: (1) when retailers' OW capacity is relatively small, they always benefit from enlarging order quantity and renting an additional warehouse, thereby enjoying a longer credit period. In this situation, suppliers further prolong the credit period is beneficial for both parties because retailers will place a larger order quantity that is directly favorable for suppliers' business. (2) When retailers' OW capacity becomes larger enough and exceeds the optimal order quantity under that of without capacity constraints, they neither increase the order quantity nor rent an additional warehouse, even though suppliers grant a longer credit period. This result indicates that retailers should adopt the same replenishment strategy as that without capacity constraints. At the same time, the further extension of the credit period is not profitable for suppliers, as such extension only increases suppliers' capital cost. (3) Compared with the fixed credit period policy and the conditional trade credit policy, the order-size dependent trade credit policy has notable advantages in encouraging retailers to order more, reducing the difficulty of suppliers' decision-making, and increasing the choice of retailers. (4) Other model parameters (e.g., ordering cost, inventory holding cost, shortages cost, backordering rate, the interest rate earned, etc.) also play a significant role in influencing retailers' optimal decisions. For example, an increase in backordering rate or interest earned leads to retailers' annual profit, optimal order quantity, and replenishment cycle length all increase, but the optimal fraction of no shortage decreases. When the shortage cost, opportunity cost, or selling price increase (or the backordering rate decreases), retailers' optimal replenishment cycle length and order quantity decrease (or increase). At the same time, retailers tend to operate an inventory system without shortages (or with shortages) and transfer the decision of renting an additional warehouse from "Yes" to "No" (or "No" to "Yes"). Retailers prefer to rent an extra warehouse if the fixed ordering cost, demand rate, backordering rate, interest rate earned, or purchase cost is high.

The rest of this paper is organized as follows. Section 1 presents the literature review. Section 2 introduces notations and assumptions of the problem. Section 3 formulates the mathematical model. In Section 4, we analyze the model and derive the closed-form optimal solution. We also design a solution procedure to search for the global optimal solution in an integrated manner. In Section 5, numerical studies are presented and sensitivity analysis is conducted concerning major parameters. Section 6 discusses some important managerial insights. Finally, we conclude this paper by summarizing conclusions and possible directions for further research.

1. Literature review

Our work is mainly related to two streams of literature: (1) inventory models with trade credit, and (2) inventory models with limited storage capacity. In this part, we only review the related literature in these two areas and clarify the differences between our paper and the extant literature in highlighting our main contributions.

1.1. Inventory models with trade credit

The inventory models under the condition of trade credit is an important and popular topic because it characterizes the real situation in the market, and has received a lot of attention from researchers in recent years.

Goyal (1985) was the first to investigate an EOQ model under the condition of trade credit. Aggarwal and Jaggi (1995) extended Goyal's (1985) study to consider a constant deterioration rate. Jamal et al. (1997) further extended Aggarwal and Jaggi's (1995) work by considering shortages, thereby making the inventory model more applicable in practice than before. Huang (2006) and Teng and Goyal (2007) studied an EOQ model in which suppliers offer retailers upstream credit period M, whereas retailers grant downstream credit period N to customers. Teng et al. (2012) discussed an inventory model with permissible delay in payments under a non-decreasing demand pattern. Chen and Teng (2015) obtained the optimal ordering and credit period decisions for time-varying perishable items under two-level trade credit. Khanna et al. (2017) formulated an inventory model for deteriorating imperfect quality items under trade credit. They assumed demand rate depends on the selling price and shortages are partially backlogged. Wu et al. (2018) addressed the optimal inventory policy for perishable products with two-level trade credit under trapezoidal-type demand patterns. Lin et al. (2019) studied an integrated inventory model for items with in-transit and retail deterioration under trade credit. Yao et al. (2021) explored the optimal replenishment strategy in a three-echelon supply chain under two-level trade credit. Many similar studies can be found in Jain and Aggarwal (2012), Taleizadeh et al. (2013), Taleizadeh (2016), Diabat et al. (2017), Lashgari et al. (2018), Jaggi et al. (2018a, 2018b), Li et al. (2019, 2021), Feng and Chan (2019), Feng et al. (2021), and their references. All the inventory models above assume that the length of the trade credit period is a fixed value and independent of retailers' order quantities.

In today's business transactions, suppliers usually provide the trade credit is linked to the order quantity to encourage retailers to place a larger order. In this regard, the conditional trade credit policy is a common form and considered by several scholars. For example, Chang et al. (2003) formulated an inventory model for perishable goods where suppliers allow retailers to delay in payments only if their order quantity exceeds a specified threshold. Huang (2007) further enriched Chang et al. (2003)'s study to consider partial trade credit if the lot size is lower than a specific threshold. Ouyang et al. (2009a) proposed an inventory model for deteriorating goods under partial trade credit that related to order quantity. Other related studies for references include Chung and Liao (2009), Chung et al. (2013), Chen et al. (2014), Ting (2015), Shah and Cárdenas-Barrón (2015), Zia and Taleizadeh (2015), Tiwari et al. (2020), and Taleizadeh et al. (2021). The above-mentioned studies consider the conditional trade credit based on only one order quantity threshold. As we stated earlier, this form of trade credit policy has two obvious disadvantages. To this end, this study considers a flexible order-size dependent trade credit in formulating a specific inventory model to further adapt to the real business world. To the best of our knowledge, only Ouyang et al. (2008, 2009b) and Chang et al. (2015) incorporated order-size dependent trade credit into inventory models. But their works ignored two significant facts, that is, warehouse capacity constraints and shortages. Intuitively, when the length of the trade credit period is linked to order quantity instead of a given parameter, retailers are motivated to order more items that may exceed their OW can be accommodated. Faced with such a dilemma, retailers can rent an additional warehouse or adopt the planning shortage strategy to cope with OW capacity constraints. As a result, retailers have to reconsider their replenishment policy under the aforementioned conditions.

1.2. Inventory models with limited storage capacity

In practice, there exist many reasons that force retailers to order more items than that can be stored in their own warehouse. For example, to obtain attractive trade credit terms, avoid frequent transportation inconvenience, guard against commodity scarcity, etc. Therefore, capturing limited storage capacity in formulating the inventory model becomes more consistent with the current business environment, that is, extending the traditional single-warehouse inventory model to the two-warehouse inventory model.

Hartley (1976) was the first to establish a two-warehouse inventory model where shortages are not allowed. Subsequently, Sarma (1987) extended Hartley's (1976) work by considering perishable products in which shortages are allowed. Zhou and Yang (2005) addressed the optimal inventory policy for a two warehouses inventory system under the condition of inventory-level-dependent demand rate. Banerjee and Agrawal (2008) investigated a two warehouses inventory system for deterioration items under time-varying demand rate. Agrawal et al. (2013) developed a two warehouses inventory model for deterioration items under ramp-type demand and partially backlogged. Shaikh et al. (2019) considered partial backlogged shortages and interval-valued inventory costs to analyze an inventory model with two storage facilities. Ghiami and Beullens (2020) considered a two-warehouse supply chain to address the optimal integrated inventory policy. Other researchers who have investigated this topic include Lee and Hsu (2009), Ghiami et al. (2013), Xu et al. (2017), Tiwari et al. (2018), and Khan et al. (2019a). All the above studies ignore the impact of trade credit on the optimal inventory strategy.

Huang (2006) first established an inventory model with two warehouses under the condition of two-level trade credit. Liang and Zhou (2011) studied an inventory model with two warehouses and trade credit. They assumed that the OW has a lower deterioration rate than RW. Yang and Chang (2013) described a two warehouses partial backlogged inventory model for perishable goods under trade credit. Tiwari et al. (2016) worked on a two warehouses inventory system for perishable goods with trade credit and inflation. Chakraborty et al. (2018) formulated a partial backordering inventory model for perishable goods with two storage facilities under trade credit, inflation, and ramp-type demand pattern. Panda et al. (2019) considered price- stock- and advertisement frequency-dependent demand rate to investigate a two-warehouse inventory model with trade credit. Gupta et al. (2020) developed a two-warehouse inventory model by capturing partial backlogging, time-varying deterioration rate, and trade credit. Other related papers include Liao et al. (2012, 2013, 2014), Jaggi et al. (2014), Bhunia et al. (2014), Jonas (2019), Mashud et al. (2021), Khan et al. (2020), and their references.

Our study also formulates a two warehouse inventory model under the condition of trade credit, but we consider a flexible order-size dependent trade credit policy. Importantly, in our study, we do not strictly limit retailers to adopt a two-inventory system that allows us to discuss retailers whether or not to rent an extra warehouse and what conditions should be satisfied.

1.3. Contributions to the literature

Table 1 presents a brief comparison between the previous models and ours. Along with Table 1, we clarify the contributions of the current inventory model from two aspects. First, regarding research content, this paper is the first to study an inventory model under the condition of order-size dependent trade credit, limited storage capacity, and partially backlogged. As we stated previously, these practical conditions are closely interrelated and impact each other.

Table 1. A brief comparison between earlier published literature and our paper

References	Trade credit policy	Shortages	Limited storage capacity	Objective function	Solution	
					Non-closed form	Closed-form
Huang (2006)	Two-level	No	×	Convex		√
Huang (2007)	Conditional	No	×	Convex		√
Ouyang et al. (2008)	Order-size dependent	No	×	Concave	√	
Banerjee and Agrawal (2008)	No	Fully backordering	√	Convex	√	
Ouyang et al. (2009b)	Order-size dependent	No	×	Concave	√	
Liao et al. (2012)	Conditional	No	√	Convex		√
Teng et al. (2012)	Fixed	No	×	Concave		√
Taleizadeh et al. (2013)	Fixed	Partial backordering	×	No-convex		√
Agrawal et al. (2013)	No	Partial backordering	√	Convex	√	
Chen et al. (2014)	Conditional	No	×	Convex		√
Zia and Taleizadeh (2015)	Conditional	Full backordering	×	No-concave		√
Chang et al. (2015)	Order-size dependent	No	×	Concave		√
Taleizadeh (2016)	Fixed	Partial backordering	×	No-convex		√
Xu et al. (2017)	No	No	√	Convex	√	
Khanna et al. (2017)	Fixed	Full backordering	×	No-concave	√	
Diabat et al. (2017)	Fixed	Full & Partial backordering	×	No-convex		√
Jaggi et al. (2017)	Fixed	Full backordering	√	Concave	√	
Tiwari et al. (2018)	No	Full backordering	√	No-concave	√	
Lashgari et al. (2018)	Fixed	Partial backordering	×	No-convex		√
Jonas (2019)	Fixed	Full backordering	√	Concave	√	
Feng and Chan (2019)	Two-level	No	×	Concave		√
Ghiami and Beullens (2020)	No	Partial backordering	√	Concave	√	
Gupta et al. (2020)	Fixed	Partial backordering	√	Convex	√	
Tiwari et al. (2020)	Conditional	Full backordering	×	Convex	√	
Li et al. (2021)	Two-level	No	×	Concave		√
This paper	Order-size dependent	Partial backordering	√	No-Concave		√

At the same time, introducing these conditions can serve the need of the current business environment and provide practitioners a more pragmatic inventory model which can be applied to many industries (e.g., retail, wholesale trade, construction, etc.). In addition, another prominent advantage of our model is that it can generate various models with no trade credit, fixed trade credit, conditional trade credit, single warehouse, no shortage, full backordering, partial backordering, and so on. And even any combination of all states has made the method flexible enough to capture various real-life cases. Second, regarding solution methodology, our model involves two decision variables and objective functions are no-concave. Therefore, setting two first-order partial derivatives to zero is evidently not feasible to derive the global optimal solution. To effectively prove global optimality and derive the closed-form solution, we develop an effective methodology to establish the condition under which objective functions have interior optimal values, thereby closed-form optimal solutions can be found. Moreover, we also design an effective solution procedure to find the global optimal solutions in an integrated manner.

2. Notations and assumptions

For simplicity, the below notations and assumptions are introduced to formulate the inventory model.

2.1. Notations

Parameter	Description
A	Fixed ordering cost per order
c	Retailers' purchase cost per unit
p	Retailers' selling price per unit
D	Demand rate per year
c_b	Backordering cost per unit backordered items per unit time
c_g	Cost of goodwill loss per unit lost sale
p	Lost sales cost per unit, including the lost profit and the goodwill loss
h_r	Inventory holding cost per unit per year (excluding interest charged) in RW
h_o	Inventory holding cost per unit per year (excluding interest charged) in OW
W	Maximum storage space of OW
M	The length of trade credit period provided by suppliers
I_e	Interest rate earned per dollar per year
I_c	Interest rate charged per dollar in stocks per year
b	Fraction of shortage that will be backlogged ($0 \leq \beta \leq 1$)
T_w	Length of depletion time for the maximum warehouse capacity of OW, that is, $T_w = W/D$
Q	Retailers' order quantity (decision variable)
F	Fraction of the duration of time with inventory level is positive, $0 \leq F \leq 1$ (decision variable)
T	Retailers' replenishment cycle (decision variable)
$ATP(F, T)$	Retailers' annual profit
$ATC(F, T)$	Retailers' annual cost

2.2. Assumptions

(1) The inventory system involves only one product, the lead time is zero, and the replenishment rate is infinite.

(2) The demand rate is known and keeps constant over time.

(3) Shortages are allowed and partial backordering, and the backordering rate β is constant.

(4) OW has a limited storage capacity of W units. When $Q > W$, retailers have to rent an additional warehouse to stock the excess items. We assume that RW has an unlimited capacity. Moreover, in practice, RW usually offers better preserving facilities than OW. Thus, this study uses the relationship $h_r \geq h_o$ to reflect this real situation. Given that the inventory holding cost in OW is less than that in RW, consuming RW first is cost-effective.

(5) Suppliers offer credit period M_j, $j = 1, 2, \ldots, k$, which is related to retailers' order quantity, and the relationship can be expressed as

$$M = \begin{cases} M_1 & q_1 \leq Q < q_2 \\ M_2 & q_2 \leq Q < q_3 \\ \vdots & \vdots \\ M_k & q_k \leq Q < q_{k+1} \end{cases},$$

where $1 = q_1 < q_2 < \cdots < q_k < q_{k+1} = \infty$, each of which is a boundary value at which a specific credit period is granted. M_j represents the credit period applicable to orders whose order quantity Q falls in the interval q_j to q_{j+1} with $M_1 < M_2 < \cdots < M_k$.

(6) During the credit period, retailers can accumulate sales revenue and earn interest at a rate of I_e. However, retailers need to settle the account at the end of the credit period and pay for the interest charged on the items remaining in stock with the rate I_c.

3. Model formulation

Based on the above notations and assumptions, we know that retailers' replenishment cycle time is T, and the duration of the positive stock level is FT; unsatisfied demand is partially backordered at a rate β. At the time $t = 0$, retailers receive Q units to meet the total accumulated backlogged demand and consumer demand during the time interval $\left[0, FT\right]$. Without loss of generality, we assume that retailers' order quantity satisfies $q_j \leq Q < q_{j+1}$, and they obtain trade credit period is M_j.

Retailers' annual profit comprises sales revenue, fixed ordering cost, purchasing cost, inventory holding cost, backordering cost, opportunity cost, interest earned, and interest charged. These elements are computed as follows:

(i) Annual sales revenue: $pD\left[F + (1-F)\beta\right]$.

(ii) Annual fixed ordering cost: A/T.

(iii) Annual purchasing cost: $cD\left[F + (1-F)\beta\right]$.

(iv) Annual inventory holding cost: if $FT \leq T_w$, then an RW is no longer necessary.

Otherwise, retailers need an extra RW to stock excess units. Therefore, the annual inventory holding cost, excluding the interest charged, can be calculated as follows:

$$\begin{cases} \dfrac{h_o DF^2 T}{2} & FT \le T_w \quad (1a) \\ \dfrac{h_r\left(FDT-W\right)^2}{2DT}+\dfrac{h_o\left(2DFT-W\right)W}{2DT} & FT \ge T_w \quad (1b) \end{cases}. \tag{1}$$

(v) Annual backordering cost: $\dfrac{c_b \beta D\left(1-F\right)^2 T}{2}$.

(vi) Annual opportunity cost caused by lost sales: $c_g D\left(1-F\right)\left(1-\beta\right)$.

(vii) Annual interest earned and interest charged: two situations, which are depicted in Figure 1, are possible based on FT and M_j values. Both situations are separately discussed.

Situation 1: $FT \le M_j\left(j=1,2,\cdots k\right)$

As shown in Figure 1(a), retailers' trade credit period M_j is longer than or equal to positive inventory level length FT. It indicates that retailers have sold all stocks at the time M_j. Therefore, no interest is charged. Meanwhile, retailers' interest earned per cycle can be divided into two parts: (1) during the period from 0 to FT, retailers gain the interest earned on the sales revenue received (including sales revenue from backlogged), and (2) during the period $\left[FT, M_j\right]$, retailers earn interest on full sales revenue. Therefore, in this situation, the annual total interest earned is

$$\left[pI_e\int_0^{FT} Dtdt + pI_e DFT\left(M_j-FT\right)+pI_e\left(1-F\right)\beta DTM_j\right]\Big/T$$
$$= pI_e D\left[F\left(M_j-\frac{FT}{2}\right)+\left(1-F\right)\beta M_j\right]. \tag{2}$$

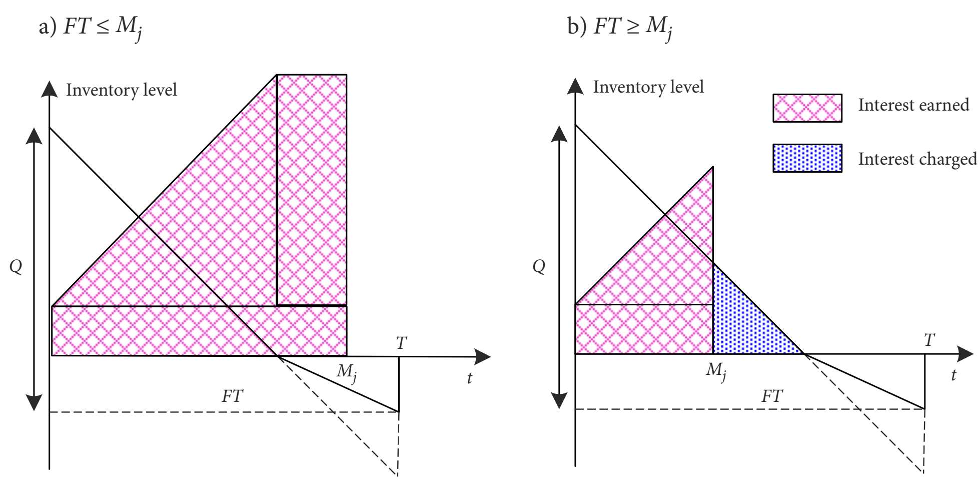

Figure 1. The interest earned and charged for two situations

Situation 2: $FT \geq M_j \left(j = 1,2,\cdots k \right)$

In this situation, retailers' trade credit period M_j is less than or equal to positive inventory level length FT (see Figure 1(b)), indicating that retailers have some inventory available after the due date M_j. Thus, during the period $\left[M_j, FT \right]$, retailers must pay the interest charged on unsold items. The annual total interest charged is

$$\left[cI_c \int_{M_j}^{FT} D\left(FT - t\right) dt \right] \Big/ T = \frac{cI_c D\left(FT - M_j\right)^2}{2T}. \tag{3}$$

During the period $\left[0, M_j \right]$, retailers employ the sales revenue to earn interest. Hence the annual interest earned is

$$\left[pI_e \int_0^{M_j} Dtdt + pI_e \left(1 - F\right) \beta DTM_j \right] \Big/ T = pI_e D \left[\frac{M_j^2}{2T} + \left(1 - F\right) \beta M_j \right]. \tag{4}$$

Combining the above results, given that M_j, $j = 1, 2, \ldots, k$ and based on FT and T_w lengths, retailers' annual profit function under various situations can be expressed as:

$ATP_i^{(j)}\left(F,T\right)\left(i = 1,2\right)$ = annual sales revenue – annual fixed ordering cost – annual purchasing cost – annual inventory holding cost – annual backordering cost – annual opportunity cost – annual interest charged + annual interest earned.

Specifically,

$$ATP_1^{(j)}\left(F,T\right) = \begin{cases} ATP_{11}^{(j)}\left(F,T\right) & M_j \leq FT \leq T_w \\ ATP_{12}^{(j)}\left(F,T\right) & 0 < FT \leq T_w \leq M_j \text{ or } 0 < FT \leq M_j \leq T_w \end{cases}; \tag{5}$$

$$ATP_2^{(j)}\left(F,T\right) = \begin{cases} ATP_{21}^{(j)}\left(F,T\right) & T_w \leq M_j \leq FT \text{ or } M_j \leq T_w \leq FT \\ ATP_{22}^{(j)}\left(F,T\right) & T_w \leq FT \leq M_j \end{cases}, \tag{6}$$

where

$$\begin{aligned} ATP_{11}^{(j)}\left(F,T\right) = {} & pD\left[F + \left(1-F\right)\beta\right] - \frac{A}{T} - cD\left[F + \left(1-F\right)\beta\right] - \frac{c_b \beta D\left(1-F\right)^2 T}{2} - c_g D\left(1-F\right)\left(1-\beta\right) \\ & - \frac{h_o DF^2 T}{2} + \frac{pI_e DM_j^2}{2T} + pI_e\left(1-F\right)D\beta M_j - \frac{cI_c D\left(FT - M_j\right)^2}{2T} \end{aligned}; \tag{7}$$

$$\begin{aligned} ATP_{12}^{(j)}\left(F,T\right) = {} & pD\left[F + \left(1-F\right)\beta\right] - \frac{A}{T} - cD\left[F + \left(1-F\right)\beta\right] - \frac{c_b \beta D\left(1-F\right)^2 T}{2} - c_g D\left(1-F\right)\left(1-\beta\right) \\ & - \frac{h_o DF^2 T}{2} + pI_e D\left[F\left(M_j - FT/2\right) + \left(1-F\right)\beta M_j\right] \end{aligned}; \tag{8}$$

$$\begin{aligned} ATP_{21}^{(j)}\left(F,T\right) = {} & pD\left[F + \left(1-F\right)\beta\right] - \frac{A}{T} - cD\left[F + \left(1-F\right)\beta\right] - \frac{c_b \beta D\left(1-F\right)^2 T}{2} - c_g D\left(1-F\right)\left(1-\beta\right) \\ & - \frac{h_r\left(FDT - W\right)^2}{2DT} - \frac{h_o\left(2DFT - W\right)W}{2DT} + pI_e D\left[\frac{M_j^2}{2T} + \left(1-F\right)\beta M_j\right] - \frac{cI_c D\left(FT - M_j\right)^2}{2T} \end{aligned}; \tag{9}$$

$$\begin{aligned} ATP_{22}^{(j)}\left(F,T\right) = {} & pD\left[F + \left(1-F\right)\beta\right] - \frac{A}{T} - cD\left[F + \left(1-F\right)\beta\right] - \frac{c_b \beta D\left(1-F\right)^2 T}{2} - c_g D\left(1-F\right)\left(1-\beta\right) \\ & - \frac{h_r\left(FDT - W\right)^2}{2DT} - \frac{h_o\left(2DFT - W\right)W}{2DT} + pI_e D\left[F\left(M_j - FT/2\right) + \left(1-F\right)\beta M_j\right] \end{aligned}. \tag{10}$$

Here, Eq. (5) represents retailers' annual profit function when they do not need to rent an additional warehouse (i.e., $FT \le T_w$). More specifically, $M_j \le FT \le T_w$ indicates that retailers must use Eqs. (3) and (4) to calculate interest charged and earned, and the inventory holding cost is referred to Eq. (1a). In this case, retailers' annual profit function can be described as Eq. (7). Meanwhile, $0 < FT \le T_w \le M_j$ or $0 < FT \le M_j \le T_w$ suggests that no interest is charged, retailers must use Eq. (2) to calculate interest earned, and the inventory holding cost is referred to Eq. (1a). In this case, retailers' annual profit function can be described as Eq. (8).

Similarly, Eq. (6) represents retailers' annual profit function when they need to rent an additional warehouse (i.e., $FT \ge T_w$). Note that $T_w \le M_j \le FT$ or $M_j \le T_w \le FT$ indicates that retailers should use Eqs (3) and (4) to calculate interest paid and earned, and the holding cost is referred to Eq. (1b). In this case, retailers' annual profit function can be described as Eq. (9). Moreover, $T_w \le FT \le M_j$ indicates that no interest is charged, retailers must use Eq. (2) to calculate interest earned, and the holding cost is referred to Eq. (1b). In this case, retailers' annual profit function can be described as Eq. (10).

4. Solution methodology

4.1. Deriving closed-form optimal solution

The purpose of this study is to find the optimal solutions of F^{**} and T^{**}, such that retailers' annual profit function $ATP(F, T)$ is maximized. Due to our objective functions are not concave, resulting in it is not feasible to find optimal solutions by employing the common methodology of setting two first-order partial derivatives to zero. To effectively solve the proposed model, this section adopts an effective approach to prove global optimality and derive the closed-form solution. we present this approach for all cases to derive the closed-form optimal solution as follows:

4.1.1. Case 1-1: $M_j \le FT \le T_w$

In this case, retailers don't need to rent an additional warehouse. As illustrated in Supplement A, Eqs. (A1)–(A3), maximizing Eq. (7) is equivalent to minimizing the following function:

$$ATC_{11}^{(j)}\left(F,T\right)=\psi_{111}F^2T-\psi_{112}FT-\psi_{113}F+\psi_{114}T+\frac{\psi_{115}}{T}+\psi_{116}, \tag{11}$$

where

$$\psi_{111}=\frac{D}{2}\left(c_b\beta+h_o+cI_c\right); \tag{12}$$

$$\psi_{112}=c_b\beta D; \tag{13}$$

$$\psi_{113}=\pi D\left(1-\beta\right)+\left(cI_c-\beta pI_e\right)M_jD; \tag{14}$$

$$\psi_{114}={c_b\beta D}\big/{2}; \tag{15}$$

$$\psi_{115}=A+\left(cI_c-pI_e\right)DM_j^2\big/2; \tag{16}$$

$$\psi_{116}=cD+\pi D\left(1-\beta\right)-pI_e\beta DM_j. \tag{17}$$

First, for fixed *F*, calculating the derivatives of $ATC_{11}^{(j)}(F,T)$ with respect to *T*, leading to Eqs (18) and (19), respectively.

$$ATC_{11}^{\prime(j)}(F,T) = \psi_{111}F^2 - \psi_{112}F + \psi_{114} - \frac{\psi_{115}}{T^2}; \tag{18}$$

$$ATC_{11}^{\prime\prime(j)}(F,T) = \frac{2\psi_{115}}{T^3}. \tag{19}$$

From Eq. (19), if $\psi_{115} > 0$ (If not, then see the solution methodology given in Supplement B), then $ATC_{11}^{\prime\prime(j)}(F,T) > 0$, that is, $ATC_{11}^{(j)}(F,T)$ is strictly convex in *T*. Setting $ATC_{11}^{\prime(j)}(F,T) = 0$ yields

$$T = \sqrt{\frac{\psi_{115}}{\theta_{11}(F)}}, \tag{20}$$

where $\theta_{11}(F) = \psi_{111}F^2 - \psi_{112}F + \psi_{114}$.

The discriminant of $\theta_{11}(F)$, $\Delta = \psi_{112}^2 - 4\psi_{111}\psi_{114} = -(h_o + cI_c)c_b\beta D^2 < 0$, is always negative. Thus, $\theta_{11}(F)$ has no roots. Moreover, given that $\theta_{11}(0) = c_b\beta D/2 > 0$, we can conclude that $\theta_{11}(F)$ is strictly positive in $[0,1]$. Therefore, Eq. (20) is feasible, and for $\forall F \in [0,1]$, a unique $T = \sqrt{\frac{\psi_{115}}{\theta_{11}(F)}}$ always exists, such that $ATC_{11}^{(j)}(F,T)$ is minimized.

Substituting Eq. (20) into Eq. (11) (that is, $T = \sqrt{\frac{\psi_{115}}{\theta_{11}(F)}}$ into $ATC_{11}^{(j)}(F,T)$) leads to

$$ATC_{11}^{(j)}(F) = 2\sqrt{\psi_{115}\theta_{11}(F)} - \psi_{113}F + \psi_{116}. \tag{21}$$

From Eq. (21), the derivatives of $ATC_{11}^{(j)}(F)$ with respect to *F* are

$$ATC_{11}^{\prime(j)}(F) = \sqrt{\psi_{115}}\left(\frac{\theta_{11}'(F)}{\sqrt{\theta_{11}(F)}}\right) - \psi_{113}; \tag{22}$$

$$\begin{aligned} ATC_{11}^{\prime\prime(j)}(F) &= \sqrt{\psi_{115}}\left(\frac{2\theta_{11}''(F)\theta_{11}(F) - \left(\theta_{11}'(F)\right)^2}{2\left(\theta_{11}(F)\right)^{3/2}}\right) \\ &= \sqrt{\psi_{115}}\left(\frac{4\psi_{111}\left(\psi_{111}F^2 - \psi_{112}F + \psi_{114}\right) - \left(2\psi_{111}F - \psi_{112}\right)^2}{2\left(\theta_{11}(F)\right)^{3/2}}\right). \\ &= \sqrt{\psi_{115}}\left(\frac{4\psi_{111}\psi_{114} - \psi_{112}^2}{2\left(\theta_{11}(F)\right)^{3/2}}\right) = \sqrt{\psi_{115}}\left(\frac{c_b\beta D^2\left(h_o + cI_c\right)}{2\left(\theta_{11}(F)\right)^{3/2}}\right) > 0 \end{aligned} \tag{23}$$

From Eq. (23), $ATC_{11}^{(j)}(F)$ is a strictly convex function of *F*. We check

$$ATC_{11}^{\prime(j)}(0) = -\psi_{112}\sqrt{\frac{\psi_{115}}{\psi_{114}}} - \psi_{113}. \tag{24}$$

Note that if $ATC_{11}'^{(j)}(0) \geq 0$, then $ATC_{11}^{(j)}(F)$ is increasing in $[0,1]$, that is, $ATC_{11}^{(j)}(F)$ reaches the global minimum at $F = 0$; it indicates that the best choice is that retailers do not build inventory. Therefore, we only need to consider the situation of $ATC_{11}'^{(j)}(0) < 0$. We further investigate

$$\begin{aligned} ATC_{11}'^{(j)}(1) &= \sqrt{\psi_{115}}\left(\frac{2\psi_{111}-\psi_{112}}{\sqrt{\psi_{111}-\psi_{112}+\psi_{114}}}\right)-\psi_{113} \\ &= \sqrt{A+\frac{(cI_c-pI_e)DM_j^2}{2}}\sqrt{2D(h_o+cI_c)}-\pi D(1-\beta)-(cI_c-\beta pI_e)M_jD \end{aligned} . \quad (25)$$

From Eq. (25), $ATC_{11}'^{(j)}(1) > 0$ holds if and only if

$$\beta > \frac{\pi D + cI_cDM - \sqrt{2A+(cI_c-pI_e)DM_j^2}\sqrt{D(h_o+cI_c)}}{\pi D + pI_eDM_j} = \beta_{11}. \quad (26)$$

Thus, if the inequality in Eq. (26) is established, then $ATC_{11}^{(j)}(F)$ has a unique minimizer in (0, 1); and the optimal solutions of T_{11} and F_{11} can be derived by employing Eqs (27) and (28), respectively (see Supplement C, Eqs. (C3) and (C4)). Otherwise, the optimal solutions lie on the boundary point $F_{11} = 1$ (see Supplement D).

$$T_{11} = \sqrt{\frac{4\psi_{111}\psi_{115}-\psi_{113}^2}{4\psi_{111}\psi_{114}-\psi_{112}^2}}; \quad (27)$$

$$F_{11} = \frac{\psi_{112}}{2\psi_{111}} + \frac{\psi_{113}}{2\psi_{111}}\sqrt{\frac{4\psi_{111}\psi_{114}-\psi_{112}^2}{4\psi_{111}\psi_{115}-\psi_{113}^2}}. \quad (28)$$

Here, for the discriminant term β_{11}, note that: (1) if $0 \leq \beta_{11} \leq \beta$, then the optimal is that retailers use partial backordering; and the optimal solutions of T_{11} and F_{11} can be derived using Eqs (27) and (28). (2) If $0 \leq \beta \leq \beta_{11}$, then the optimal is that retailers employ inventory policy without shortages (e.g., $F_{11} = 1$). (3) If $\beta_{11} < 0$, then retailers must compare the cases of no stocking (e.g., $F_{11} = 0$) and partial backlogging to determine which is optimal.

For the solutions T_{11} and F_{11} found using Eqs (27) and (28), if the condition $M_j \leq F_{11}T_{11} \leq T_w$ is not satisfied, then $ATC_{11}^{(j)}(F,T)$ obtains the optimal solution at the boundary. A logical solution is to set $T = \frac{T_w}{F_{11}}$ or $T = \frac{M_j}{F_{11}}$ (refer to the detailed solution process given in Supplement E and optimal solutions are summarized in Table 2).

To sum up, given the analysis above, the order quantity based on trade credit period M_j and purchase cost c_j can be computed using Eq. (29), that is,

$$Q_j = DT\left[(1-F)\beta + F\right]. \quad (29)$$

From Eq. (29), if the optimal order quantity (Q_j) satisfies $q_j \leq Q_j < q_{j+1}$, the solution obtained from the analysis above is feasible. Otherwise, we must use the solution procedure described in Supplement F to derive the optimal solutions of T and F.

4.1.2. Case 1-2: $FT \le M_j \le T_w$ or $FT \le T_w \le M_j$

In this case, an additional RW is not necessary, and credit period length M_j is longer than or equal to the positive inventory level length FT. Analogously, as shown in Supplement A, Eqs (A4)–(A5), maximizing Eq. (8) is equivalent to minimizing the following function:

$$ATC_{12}^{(j)}(F,T) = \psi_{121}F^2T - \psi_{122}FT - \psi_{123}F + \psi_{124}T + \frac{\psi_{125}}{T} + \psi_{126}, \tag{30}$$

where

$$\psi_{121} = \frac{D}{2}(c_b\beta + h_o + pI_e); \tag{31}$$

$$\psi_{122} = c_b\beta D; \tag{32}$$

$$\psi_{123} = \pi D(1-\beta) + (1-\beta)pI_eM_jD; \tag{33}$$

$$\psi_{124} = {c_b\beta D}/{2}; \tag{34}$$

$$\psi_{125} = A; \tag{35}$$

$$\psi_{126} = cD + \pi D(1-\beta) - pI_e\beta DM_j. \tag{36}$$

Note that Eqs (30) and (11) have similar function structures (i.e., ψ_{121} through ψ_{126} instead of ψ_{111} through ψ_{116}). Therefore, the analysis and discussion provided for Eqs (18)–(24) of Case 1-1 are also developed for those of Case 1-2. As a result, the equivalent analysis for Eq. (25) of Cases 1-2 and 1-1 is

$$\begin{aligned} ATC_{12}^{\prime(j)}(1) &= \sqrt{\psi_{125}}\left(\frac{2\psi_{121} - \psi_{122}}{\sqrt{\psi_{121} - \psi_{122} + \psi_{124}}}\right) - \psi_{123} \\ &= \sqrt{2A}\sqrt{D(h_o + pI_e)} - \pi D(1-\beta) - pI_eM_jD(1-\beta) \end{aligned} \tag{37}$$

From Eq. (37), $ATC_{12}^{\prime(j)}(1) > 0$ holds if and only if

$$\beta > \frac{\pi D + pI_eDM_j - \sqrt{2A}\sqrt{D(h_o + pI_e)}}{\pi D + pI_eDM_j} = \beta_{12}. \tag{38}$$

Consequently, if the inequality in Eq. (38) is established, then $ATC_{12}^{(j)}(F)$ has a unique minimizer in (0, 1), and the optimum solutions of T_{12} and F_{12} can be found using Eqs (39) and (40) (see Supplement C). Otherwise, we set $F_{12} = 1$ (see Supplement D).

$$T_{12} = \sqrt{\frac{4\psi_{121}\psi_{125} - \psi_{123}^2}{4\psi_{121}\psi_{124} - \psi_{122}^2}}; \tag{39}$$

$$F_{12} = \frac{\psi_{122}}{2\psi_{121}} + \frac{\psi_{123}}{2\psi_{121}}\sqrt{\frac{4\psi_{121}\psi_{124} - \psi_{122}^2}{4\psi_{121}\psi_{125} - \psi_{123}^2}}. \tag{40}$$

Similar to Case 1-1, we still need to perform the following two steps to ensure the feasibility of the solution: (1) For solutions F_{12} and T_{12} found using Eqs (39) and (40), check whether they satisfy $F_{12}T_{12} \le \min\{M_j, T_w\}$; if not, then refer to Supplement E to derive the optimal values of T and F. (2) Check whether order quantity Q_j satisfies $q_j \le Q_j < q_{j+1}$; if not, then refer to Supplement F to find the optimal values of T and F.

4.1.3. Case 2-1: $T_w \le M_j \le FT$ or $M_j \le T_w \le FT$

In this case, retailers must rent an additional warehouse. Similarly, as presented in Supplement A, Eqs (A6) and (A7), maximizing Eq. (9) is equivalent to minimizing the following function:

$$ATC_{21}^{(j)}\left(F,T\right)=\psi_{211}F^2T-\psi_{212}FT-\psi_{213}F+\psi_{214}T+\frac{\psi_{215}}{T}+\psi_{216}\,, \tag{41}$$

where

$$\psi_{211}=\frac{D}{2}\left(c_b\beta+h_r+cI_c\right); \tag{42}$$

$$\psi_{212}=c_b\beta D; \tag{43}$$

$$\psi_{213}=\pi D\left(1-\beta\right)+\left(h_r-h_o\right)W+\left(cI_c-\beta pI_e\right)M_jD; \tag{44}$$

$$\psi_{214}={c_b\beta D}\big/{2}; \tag{45}$$

$$\psi_{215}=A+\frac{\left(h_r-h_o\right)W^2}{2D}+\frac{\left(cI_c-pI_e\right)DM_j^2}{2}; \tag{46}$$

$$\psi_{216}=cD+\pi D\left(1-\beta\right)-pI_e\beta DM_j\,. \tag{47}$$

Similar to previous cases,

$$\begin{aligned}ATC_{21}^{\prime(j)}\left(1\right)&=\sqrt{\psi_{215}}\left(\frac{2\psi_{211}-\psi_{212}}{\sqrt{\psi_{211}-\psi_{212}+\psi_{214}}}\right)-\psi_{213}\\&=\sqrt{2A+\left(h_r-h_o\right)W^2/D+\left(cI_c-pI_e\right)DM_j^2}\,\sqrt{D\left(h_r+cI_c\right)}-\pi D\left(1-\beta\right).\\&\quad-\left(h_r-h_o\right)W-\left(cI_c-\beta pI_e\right)M_jD\end{aligned} \tag{48}$$

From Eq. (48), $ATC_{21}^{\prime(j)}\left(1\right)>0$ holds if and only if

$$\beta>\frac{\pi D+\left(h_r-h_o\right)W+cI_cM_jD-\sqrt{2A+\left(h_r-h_o\right)W^2/D+\left(cI_c-pI_e\right)DM_j^2}\,\sqrt{D\left(h_r+cI_c\right)}}{\pi D+pI_eM_jD}=\beta_{21}. \tag{49}$$

Consequently, if the inequality in Eq. (49) is established, then $ATC_{21}^{(j)}\left(F\right)$ has a unique minimizer in (0, 1), and the optimum solutions of T_{21} and F_{21} can be determined by employing Eqs (50) and (51) (see Supplement C). Otherwise, we set $F_{21}=1$ (see Supplement D).

$$T_{21}=\sqrt{\frac{4\psi_{211}\psi_{215}-\psi_{213}^2}{4\psi_{211}\psi_{214}-\psi_{212}^2}}\,; \tag{50}$$

$$F_{21}=\frac{\psi_{212}}{2\psi_{211}}+\frac{\psi_{213}}{2\psi_{211}}\sqrt{\frac{4\psi_{211}\psi_{214}-\psi_{212}^2}{4\psi_{211}\psi_{215}-\psi_{213}^2}}\,. \tag{51}$$

Again, similar to previous cases, the feasibility of the solution obtained must be checked. If the solution is not feasible, then refer to the detailed solution process given in Supplement E and F.

4.1.4. Case 2-2: $T_w \le FT \le M_j$

Similarly, as presented in Supplement A, Eqs (A8)–(A9), maximizing Eq. (10) is equivalent to minimizing the following function:

$$ATC_{22}^{(j)}(F,T) = \psi_{221}F^2T - \psi_{222}FT - \psi_{223}F + \psi_{224}T + \frac{\psi_{225}}{T} + \psi_{226}, \tag{52}$$

where

$$\psi_{221} = \frac{D}{2}\left(c_b\beta + h_r + pI_e\right); \tag{53}$$

$$\psi_{222} = c_b\beta D; \tag{54}$$

$$\psi_{223} = \pi D(1-\beta) + \left(h_r - h_o\right)W + (1-\beta)pI_eM_jD; \tag{55}$$

$$\psi_{224} = {c_b\beta D}/{2}; \tag{56}$$

$$\psi_{225} = A + \frac{\left(h_r - h_o\right)W^2}{2D}; \tag{57}$$

$$\psi_{226} = cD + \pi D(1-\beta) - pI_e\beta DM_j. \tag{58}$$

Similarly,

$$\begin{aligned} ATC_{22}^{\prime(j)}(1) &= \sqrt{\psi_{225}}\left(\frac{2\psi_{221} - \psi_{222}}{\sqrt{\psi_{221} - \psi_{222} + \psi_{224}}}\right) - \psi_{223} \\ &= \sqrt{2A + \left(h_r - h_o\right)W^2/D}\sqrt{D\left(h_r + cI_c\right)} - \pi D(1-\beta). \\ &\quad -\left(h_r - h_o\right)W - (1-\beta)pI_eM_jD \end{aligned} \tag{59}$$

From Eq. (59), $ATC_{22}^{\prime(j)}(1) > 0$ holds if and only if

$$\beta > \frac{\pi D + \left(h_r - h_o\right)W + pI_eM_jD - \sqrt{2A + \left(h_r - h_o\right)W^2/D}\sqrt{D\left(h_r + pI_e\right)}}{\pi D + pI_eM_jD} = \beta_{22}. \tag{60}$$

Consequently, if the inequality in Eq. (60) is established, then $ATC_{22}^{(j)}(F)$ has a unique minimizer in (0, 1), and the optimum solutions of T_{22} and F_{22} can be obtained using Eqs (61) and (62) (see Supplement C). Otherwise, the optimal solution lies on the boundary point $F_{21} = 1$ (see Supplement D).

$$T_{22} = \sqrt{\frac{4\psi_{221}\psi_{225} - \psi_{223}^2}{4\psi_{221}\psi_{224} - \psi_{222}^2}}; \tag{61}$$

$$F_{22} = \frac{\psi_{222}}{2\psi_{221}} + \frac{\psi_{223}}{2\psi_{221}}\sqrt{\frac{4\psi_{221}\psi_{224} - \psi_{222}^2}{4\psi_{221}\psi_{225} - \psi_{223}^2}}. \tag{62}$$

In the same way, Appendices E and F are used to derive the optimal solutions of T and F if the solution is not feasible.

Table 2. The optimal solution if T and F don't meet the condition

Cases	Possibility	Optimal solution	
Case 1-1	$\frac{M_j}{F_{11}} > T_{11}$	$T'_{11} = \frac{M_j}{F'_{11}}$	$F'_{11} = \sqrt{\frac{\psi_{114} M_j^2}{\psi_{111} M_j^2 - \psi_{113} M_j + \psi_{115}}}$
	$\frac{T_w}{F_{11}} < T_{11}$	$T'_{11} = \frac{T_w}{F'_{11}}$	$F'_{11} = \sqrt{\frac{\psi_{114} T_w^2}{\psi_{111} T_w^2 - \psi_{113} T_w + \psi_{115}}}$
Case 1-2	$\frac{\min\{T_w, M_j\}}{F_{12}} < T_{12}$	$T'_{12} = \frac{\min\{T_w, M_j\}}{F'_{12}}$	$F'_{12} = \sqrt{\frac{\psi_{124}\left(\min\{T_w, M_j\}\right)^2}{\psi_{121}\left(\min\{T_w, M_j\}\right)^2 - \psi_{123}\min\{T_w, M_j\} + \psi_{125}}}$
Case 2-1	$\frac{\max\{T_w, M_j\}}{F_{21}} > T_{21}$	$T'_{21} = \frac{\max\{T_w, M_j\}}{F'_{21}}$	$F'_{21} = \sqrt{\frac{\psi_{214}\left(\max\{T_w, M_j\}\right)^2}{\psi_{211}\left(\max\{T_w, M_j\}\right)^2 - \psi_{213}\max\{T_w, M_j\} + \psi_{215}}}$
Case 2-2	$\frac{M_j}{F_{22}} < T_{22}$	$T'_{22} = \frac{M_j}{F'_{22}}$	$F'_{22} = \sqrt{\frac{\psi_{224} M_j^2}{\psi_{221} M_j^2 - \psi_{223} M_j + \psi_{225}}}$
	$\frac{T_w}{F_{22}} > T_{22}$	$T'_{22} = \frac{T_w}{F'_{22}}$	$F'_{22} = \sqrt{\frac{\psi_{224} T_w^2}{\psi_{221} T_w^2 - \psi_{223} T_w + \psi_{225}}}$

4.2. Solution procedure

Summarizing the results above, we provide the following solution procedure to determine the global optimal solution of the problem.

Step 1: For each j, $j = 1, 2, \ldots, k-1, k$, perform Steps 2–4.

Step 2: Compare M_j and T_w, if $M_j < T_w$, then proceed to Step 3; if not, then proceed to Step 4.

Step 3: Execute Steps 3.1–3.4 and determine $\left(F_{(j)}^{**}, T_{(j)}^{**}\right)$ and $ATP^{(j)}\left(F_{(j)}^{**}, T_{(j)}^{**}\right)$.

Step 3.1: Determine $\left(F_{11}^{**}, T_{11}^{**}\right)$ and $ATP_{11}{}^{(j)}\left(F_{11}^{**}, T_{11}^{**}\right)$ by using **sub-procedure A**, and then proceed to Step 3.2.

Step 3.2: Determine $\left(F_{12}^{**}, T_{12}^{**}\right)$ and $ATP_{12}{}^{(j)}\left(F_{12}^{**}, T_{12}^{**}\right)$ by using **sub-procedure B**, and then proceed to Step 3.3.

Step 3.3: Determine $\left(F_{21}^{**}, T_{21}^{**}\right)$ and $ATP_{21}{}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right)$ by using **sub-procedure C**, and then proceed to Step 3.4.

Step 3.4: Set $ATP^{(j)}\left(F_{(j)}^{**}, T_{(j)}^{**}\right) = \max\left\{ATP_{11}{}^{(j)}\left(F_{11}^{**}, T_{11}^{**}\right), ATP_{12}{}^{(j)}\left(F_{12}^{**}, T_{12}^{**}\right), ATP_{21}{}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right)\right\}$.

Step 4: Execute Steps 4.1–4.4 and determine $\left(F_{(j)}^{**}, T_{(j)}^{**}\right)$ and $ATP^{(j)}\left(F_{(j)}^{**}, T_{(j)}^{**}\right)$.

Step 4.1: Determine $\left(F_{12}^{**}, T_{12}^{**}\right)$ and $ATP_{12}{}^{(j)}\left(F_{12}^{**}, T_{12}^{**}\right)$ by using **sub-procedure B**, and then proceed to Step 4.2.

Step 4.2: Determine $\left(F_{21}^{**}, T_{21}^{**}\right)$ and $ATP_{21}{}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right)$ by using **sub-procedure C**, and then proceed to Step 4.3.

Step 4.3: Determine $\left(F_{22}^{**}, T_{22}^{**}\right)$ and $ATP_{22}{}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right)$ by using **sub-procedure D**, and then proceed to Step 4.4.

Step 4.4: Set $ATP^{(j)}\left(F_{(j)}^{**}, T_{(j)}^{**}\right) = \max\left\{ATP_{12}{}^{(j)}\left(F_{12}^{**}, T_{12}^{**}\right), ATP_{21}{}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right), ATP_{22}{}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right)\right\}$.

Step 5: Set $ATP\left(F^{**}, T^{**}\right) = \max_{j=1,2,\cdots,k}\left\{ATP^{(j)}\left(F_{(j)}^{**}, T_{(j)}^{**}\right)\right\}$ and $\left(F^{**}, T^{**}\right) = \left(F_d^{**}, T_d^{**}\right)$.

Step 6: Using the optimal solutions F^{**} and T^{**}, determine the optimal order quantity $Q^{**} = DT^{**}\left[\left(1 - F^{**}\right)\beta + F^{**}\right]$.

Note: The specific steps of sub-procedures A, B, C, and D can be found in Supplement G.

5. Numerical examples and sensitivity analysis

This section presents several examples to illustrate the applicability of the model and also the solution procedure proposed in this study. We also perform sensitivity analysis of major parameters to derive additional managerial insights.

Example 1. In this example, we shed light on the validity of the proposed solution procedure. We assume the following numerical data: A = \$100/order, D = 300 units/year, W = 100 units, p = \$15/unit, c = \$10/unit, c_b = \$4/unit/year, c_g = \$2/unit, h_r = \$2/unit/year, h_o = \$1.6/unit/year, I_c = 0.15/\$/year, I_e = 0.12/\$/year, and β = 0.85. In addition, the trade credit schedule offered by suppliers is $M=\left(M_1,M_2,M_3\right)=\left(0.20,0.40,0.60\right)$ years and $q=\left(q_1,q_2,q_3\right)=\left(1,350,500\right)$ units.

Employing the solution procedure proposed in section 5,2, we present the specific solution process as follows:

Set $j=1,2,3$, $T_w=W/D=0.3333$, $q=\left(q_1,q_2,q_3\right)=\left(1,150,350\right)$ units, and $M=\left(M_1,M_2,M_3\right)=\left(0.20,0.40,0.60\right)$ years.

For j = 1, given that $M_1=0.20<0.3333=T_w$, we proceed to **Step 3** in the presented solution procedure. Then, we execute **Steps 3.1–3.3** to determine the solutions of $\left(F_{11}^{**},T_{11}^{**}\right)$, $\left(F_{12}^{**},T_{12}^{**}\right)$, and $\left(F_{21}^{**},T_{21}^{**}\right)$ as below:

- Execute **sub-procedure A** to determine $\left(F_{11}^{**},T_{11}^{**}\right)$ and $ATP_{11}^{(1)}\left(F_{11}^{**},T_{11}^{**}\right)$, then we obtain $\left(F_{11}^{**},T_{11}^{**}\right)=\left(0.7449,0.4475\right)$ and $ATP_{11}^{(1)}\left(F_{11}^{**},T_{11}^{**}\right)=1160.34$;
- Execute **sub-procedure B** to determine $\left(F_{12}^{**},T_{12}^{**}\right)$ and $ATP_{12}^{(1)}\left(F_{12}^{**},T_{12}^{**}\right)$, then we obtain $\left(F_{12}^{**},T_{12}^{**}\right)=\left(0.2929,0.6829\right)$ and $ATP_{12}^{(1)}\left(F_{12}^{**},T_{12}^{**}\right)=1023.38$;
- Execute **sub-procedure C** to determine $\left(F_{21}^{**},T_{21}^{**}\right)$ and $ATP_{21}^{(1)}\left(F_{21}^{**},T_{21}^{**}\right)$, then we obtain $\left(F_{21}^{**},T_{21}^{**}\right)=\left(0.8102,0.5375\right)$ and $ATP_{21}^{(1)}\left(F_{21}^{**},T_{21}^{**}\right)=1172.75$;

Set $ATP^{(1)}\left(F_{(1)}^{**},T_{(1)}^{**}\right)=\max\left\{ATP_{11}^{(1)}\left(F_{11}^{**},T_{11}^{**}\right),ATP_{12}^{(1)}\left(F_{12}^{**},T_{12}^{**}\right),ATP_{21}^{(1)}\left(F_{21}^{**},T_{21}^{**}\right)\right\}$ which yields $\left(F_{(1)}^{**},T_{(1)}^{**}\right)=\left(0.8102,0.5375\right)$ and $ATP^{(1)}\left(F_{(1)}^{**},T_{(1)}^{**}\right)=1172.75$.

For j = 2, given that $M_2=0.40>0.3333=T_w$, we proceed to **Step 4** in the presented solution procedure. Subsequently, we execute **Steps 4.1–4.3** to determine the solutions of $\left(F_{12}^{**},T_{12}^{**}\right)$, $\left(F_{21}^{**},T_{21}^{**}\right)$, and $\left(F_{22}^{**},T_{22}^{**}\right)$ as below:

- Execute **sub-procedure B** to determine $\left(F_{12}^{**},T_{12}^{**}\right)$ and $ATP_{12}^{(2)}\left(F_{12}^{**},T_{12}^{**}\right)$, then we obtain $\left(F_{12}^{**},T_{12}^{**}\right)$, which is not a feasible solution, and $ATP_{12}^{(2)}\left(F_{12}^{**},T_{12}^{**}\right)=-\inf$;
- Execute **sub-procedure C** to determine $\left(F_{21}^{**},T_{21}^{**}\right)$ and $ATP_{21}^{(2)}\left(F_{21}^{**},T_{21}^{**}\right)$, then we obtain $\left(F_{21}^{**},T_{21}^{**}\right)=\left(0.6472,1.2319\right)$ and $ATP_{21}^{(2)}\left(F_{21}^{**},T_{21}^{**}\right)=1166.19$;
- Execute **sub-procedure D** to determine $\left(F_{22}^{**},T_{22}^{**}\right)$ and $ATP_{22}^{(2)}\left(F_{22}^{**},T_{22}^{**}\right)$, then we obtain $\left(F_{21}^{**},T_{21}^{**}\right)$, which is not a feasible solution, and $ATP_{21}^{(2)}\left(F_{21}^{**},T_{21}^{**}\right)=-\inf$.

Set $ATP^{(2)}\left(F_{(2)}^{**},T_{(2)}^{**}\right)=\max\left\{ATP_{12}^{(2)}\left(F_{12}^{**},T_{12}^{**}\right),ATP_{21}^{(2)}\left(F_{21}^{**},T_{21}^{**}\right),ATP_{22}^{(2)}\left(F_{22}^{**},T_{22}^{**}\right)\right\}$ which yields $\left(F_{(2)}^{**},T_{(2)}^{**}\right)=(0.6472,1.2319)$ and $ATP^{(2)}\left(F_{(2)}^{**},T_{(2)}^{**}\right)=1166.19$.

For j = 3, given that $M_3=0.60>0.3333=T_w$, we proceed to **Step 4** in the presented solution procedure. Then, we execute **Steps 4.1–4.3** to determine the solutions of $\left(F_{12}^{**},T_{12}^{**}\right)$, $\left(F_{21}^{**},T_{21}^{**}\right)$, and $\left(F_{22}^{**},T_{22}^{**}\right)$ as below:

– Execute **sub-procedure B** to determine $\left(F_{12}^{**},T_{12}^{**}\right)$ and $ATP_{12}^{(3)}\left(F_{12}^{**},T_{12}^{**}\right)$, then we obtain $\left(F_{12}^{**},T_{12}^{**}\right)$, which is not a feasible solution, and $ATP_{12}^{(3)}\left(F_{12}^{**},T_{12}^{**}\right)=-\inf$;

– Execute **sub-procedure C** to determine $\left(F_{21}^{**},T_{21}^{**}\right)$ and $ATP_{21}^{(3)}\left(F_{21}^{**},T_{21}^{**}\right)$, then we obtain $\left(F_{21}^{**},T_{21}^{**}\right)=(0.6068,1.7711)$ and $ATP_{21}^{(3)}\left(F_{21}^{**},T_{21}^{**}\right)=1139.43$;

– Execute **sub-procedure D** to determine $\left(F_{22}^{**},T_{22}^{**}\right)$ and $ATP_{22}^{(3)}\left(F_{22}^{**},T_{22}^{**}\right)$, then we obtain $\left(F_{21}^{**},T_{21}^{**}\right)$, which is not a feasible solution, and $ATP_{21}^{(3)}\left(F_{21}^{**},T_{21}^{**}\right)=-\inf$.

Set $ATP^{(3)}\left(F_{(3)}^{**},T_{(3)}^{**}\right)=\max\left\{ATP_{12}^{(3)}\left(F_{12}^{**},T_{12}^{**}\right),ATP_{21}^{(3)}\left(F_{21}^{**},T_{21}^{**}\right),ATP_{22}^{(3)}\left(F_{22}^{**},T_{22}^{**}\right)\right\}$ which yields to $\left(F_{(3)}^{**},T_{(3)}^{**}\right)=(0.6068,1.7711)$ and $ATP^{(3)}\left(F_{(3)}^{**},T_{(3)}^{**}\right)=1139.43$.
Set $ATP\left(F^{**},T^{**}\right)=\max\limits_{j=1,2,3}\left\{ATP^{(j)}\left(F_{(j)}^{**},T_{(j)}^{**}\right)\right\}$ which yields $\left(F^{**},T^{**}\right)=(0.8102,0.5375)$ and $ATP\left(F^{**},T^{**}\right)=1172.75$, $Q^{**}=156.65$.

The above results are summarized in Table 3. When j = 1, retailers' annual profit is the largest, that is, $ATP=ATP_{21}^{(1)}=\$1172.75$. Moreover, retailers' optimal inventory policies are $F^{**}=F_{21}=0.8102$ and $T^{**}=T_{21}=0.5375$ years, and the optimal order quantity is $Q^{**}=Q_{21}=156.65$ units. Meanwhile, the trade credit chosen by retailers is M_1 = 0.2. We also notice that $W=100<130.64=0.8102\times0.5375\times300=F^{**}T^{**}D$, which means OW capacity is insufficient to stock the purchased items. Therefore, an additional RW is necessary.

Table 3. Solution procedure of Example 1

j	Case	F^*	T^*	Q^*	ATP^*
1	Case 1-1	$F_{11}=0.7449$	$T_{11}=0.4475$	$Q_{11}=129.11$	$ATP_{11}^{(1)}=1160.34$
	Case 1-2	$F_{12}=0.2929$	$T_{21}=0.5375$	$Q_{12}=183.11$	$ATP_{12}^{(1)}=1023.38$
	Case 2-1	$F_{21}=0.8102$	$T_{21}=0.5375$	$Q_{21}=156.65$	$ATP_{21}^{(2)}=1172.75\leftarrow^{a}$
2	Case 1-2	×	×	×	×
	Case 2-1	$F_{21}=0.6472$	$T_{21}=1.2319$	$Q_{21}=350$	$ATP_{21}^{(3)}=1166.19\leftarrow$
	Case 2-2	×	×	×	×
3	Case 1-2	×	×	×	×
	Case 2-1	$F_{21}=0.6068$	$T_{21}=1.7711$	$Q_{21}=500$	$ATP_{21}^{(3)}=1139.43\leftarrow$
	Case 2-2	×	×	×	×

Note: "×" represents the problem is not feasible in this case. "←" represents the optimal solution for given j. "←^{a}" represents the global optimal solution.

Example 2. In this example, we study the influence of the trade credit period and OW storage space on the retailers' optimal decisions. All parameter values keep the same as those in Example 1, apart from W and (M_1, M_2, M_3). Table 4 presents the optimal solutions for each value of $W \in \{100, 200, 300, 400, 500\}$ units and $(M_1, M_2, M_3) \in \{(0.15, 0.30, 0.45), (0.20, 0.40, 0.60), (0.30, 0.55, 0.80)\}$ years.

As presented in Table 4, for a fixed trade credit value (M_1, M_2, M_3), retailers' optimal profit and order quantity first increase, and then remain unchanged as OW capacity enlarges. At the same time, when retailers' OW storage space raise, the decision of renting the additional warehouse changes from "Yes" to "No." This observation is consistent with our intuition. Since when retailers' OW capacity is relatively small, they have clear motivations to rent an additional warehouse and store enough goods to satisfy the market demand. As retailers' OW capacity increases and exceeds the optimal order quantity under that of without capacity constraint, the further increases of order quantity is unprofitable for retailers, as it may bring large inventory costs caused by an additional RW. In this situation, retailers neither increase their order quantity nor rent an additional warehouse, that is, retailers' ordering behavior as that without capacity constraint. Compared with the case in which retailers' OW capacity is unlimited (i.e., W = 500 units in this example), the storage space-constrained retailers tend to order less because a small OW capacity forces retailers to reduce order quantity such that avoiding paying additional inventory holding costs caused by an extra RW.

Moreover, for a fixed OW capacity W, we observe that when the credit period is relatively long (i.e., $(M_1, M_2, M_3) = (0.20, 0.40, 0.60)$ years or $(0.30, 0.55, 0.80)$ years), retailers always order more goods to benefit from a longer credit period. An RW is thus necessary. Clearly, a longer trade credit period helps retailers prolong payments to suppliers without any penalty, indirectly reducing retailer's inventory costs that can fully offset the increased expenses incurred by renting a warehouse.

Table 4. Result of sensitivity analysis on W and (M_1, M_2, M_3)

(M_1, M_2, M_3)	W	T^{**}	F^{**}	Q^{**}	ATP^{**}	Rented warehouse?	Credit period
(0.15, 0.30, 0.45)	100	0.5400	0.8091	157.37	1148.70	Yes	M_1
	200	0.5507	0.8152	160.63	1150.22	No	M_1
	300	0.5507	0.8152	160.63	1150.22	No	M_1
	400	0.5507	0.8152	160.63	1150.22	No	M_1
	500	0.5507	0.8152	160.63	1150.22	No	M_1
(0.20, 0.40, 0.60)	100	0.5375	0.8102	156.65	1172.75	Yes	M_1
	200	1.2288	0.6626	350.00	1176.12	Yes	M_2
	300	1.2275	0.6696	350.00	1177.25	No	M_2
	400	1.2275	0.6696	350.00	1177.25	No	M_2
	500	1.2275	0.6696	350.00	1177.25	No	M_2
(0.30, 0.55, 0.80)	100	1.2319	0.6470	350.00	1239.37	Yes	M_2
	200	1.7681	0.6174	500.00	1250.43	Yes	M_3
	300	1.7651	0.6279	500.00	1256.47	Yes	M_3
	400	1.7642	0.6314	500.00	1256.89	No	M_3
	500	1.7642	0.6314	500.00	1256.89	No	M_3

Finally, along with the above results, we also find that if retailers' OW capacity is relatively small, the strategy in which suppliers prolong trade credit length to motivate retailers to order additional goods is beneficial for both members, as retailers will place huge orders to suppliers, and such orders are favorable for their business. On the contrary, if retailers' OW capacity is larger enough, the further extension of the credit period is only in favor of retailers. Because such an extension does not enlarge retailers' order quantity but increases suppliers' capital costs.

Example 3. In this example, we aim to investigate the value of order-size dependent trade credit policy. To this end, we consider two benchmark cases, namely, fixed credit period policy (i.e, retailers always obtain a constant trade credit period independent of the order quantity) and conditional trade credit policy (i.e., retailers obtain a constant trade credit period only when their order quantities above a specific threshold). We assume the input parameters of three trade credit policies as follows: (a) under the fixed credit period policy, we set the credit period as $M = 0.3$ years; (b) under the conditional trade credit policy, the credit period is $M = 0.3$ years and the order quantity threshold is 350 units; and (c) under the order-size dependent trade credit policy, we set $\left(M_1, M_2, M_3\right) = \left(0.30, 0.55, 0.80\right)$ years and $q = \left(q_1, q_2, q_3\right) = \left(1, 350, 500\right)$ units. Other parameters adopted are identical to example 1 and the computational results as presented in Table 5.

As shown in Table 5, we find that under the fixed credit period policy, retailers' ordering behavior is extremely conservative. In this situation, retailers always receive a certain grace period to pay off their purchase costs, thereby having no motivation to enlarge the order quantity, especially when the OW capacity space is relatively small. However, under the conditional trade credit policy, retailers are forced to order more such that obtaining a grace period.

Table 5. The value of order-size dependent trade credit policy

Trade credit policy	W	T^{**}	F^{**}	Q^{**}	ATP^{**}	Rented warehouse?	Credit period
Fixed credit period	100	0.5299	0.8139	154.53	1222.14	Yes	–
	200	0.5400	0.8196	157.64	1223.34	No	–
	300	0.5400	0.8196	157.64	1223.34	No	–
	400	0.5400	0.8196	157.64	1223.34	No	–
	500	0.5400	0.8196	157.64	1223.34	No	–
Conditional trade credit	100	1.2318	0.6474	350.00	1118.32	Yes	–
	200	1.2288	0.6628	350.00	1128.27	No	–
	300	1.2274	0.6698	350.00	1129.39	No	–
	400	1.2274	0.6698	350.00	1129.39	No	–
	500	1.2274	0.6698	350.00	1129.39	No	–
Order-size dependent trade credit	100	1.2319	0.6470	350.00	1239.37	Yes	M_2
	200	1.7681	0.6174	500.00	1250.43	Yes	M_3
	300	1.7651	0.6279	500.00	1256.47	Yes	M_3
	400	1.7642	0.6314	500.00	1256.89	No	M_3
	500	1.7642	0.6314	500.00	1256.89	No	M_3

Under this policy, a rational retailer usually chooses the order quantity equal to the predetermined quantity threshold, independent of the OW capacity space. Compared with the fixed credit period policy, although retailers actively increase the order quantity under the conditional trade credit policy, this policy seriously restricts the retailer's choice of order quantity. Especially, if the order quantity threshold is set as 500 units, then we easily show that retailers always pay for the full purchase amount immediately. Obviously, the conditional trade credit policy often forces retailers to make two extreme choices as we have emphasized in the introduction.

By comparison, under the order-size dependent trade credit policy, we observe that retailers' order decisions become more flexible. At the same time, retailers will further enlarge the order quantity and earn more profit under this policy, thereby achieving a win-win situation for both supply parties. Moreover, from the perspective of suppliers, it is normally easier to select several appropriate order quantity thresholds than to select an accurate one. Therefore, we strongly believe that the order-size dependent trade credit has notable applicable value in reducing the difficulty of suppliers' decision-making and increasing the order quantity choice of retailers.

Example 4. Using the same data set in Example 1, except for W = 200 unit, this example outlines the impact of the changes of major parameters A, D, h_r, h_o, c_b, c_g, p, β, I_e, I_c, and c on the optimal solution. The results are summarized in Table 6.

The below conclusions can be observed from Table 6:

(1) Retailers' annual profit ATP^{**} increases as the value of D, p, β, or I_e increases, whereas it decreases as the value of A, h_r, h_o, c_b, c_g, I_c, or c increases. An increases in D, p, β, or I_e can bring additional market demand or interest earned, which results in additional profits for retailers. Note that A, h_r, h_o, c_b, c_g, I_c, or c are all cost structure parameters of the inventory system and have negative effects on retailers' profits. Therefore, the increase in their values must reduce retailers' profits. We also observe that annual profit ATP^{**} is highly sensitive to the changes in D, p, or c.

(2) Optimal order quantity Q^{**} increases if we increase the value of A, D, β, I_e, or c, but it decreases as the value of h_r, h_o, c_b, c_g, I_c, or p increases. This result suggests that if ordering cost (A) increases, then retailers reduce order frequency by increasing order quantity. Moreover, if the demand rate (D) or the backlogged rate (β) increases, the market demand also increases. Therefore, retailers should place a larger order. In addition, the increase of I_e or c motivates retailers to order additional goods to enjoy a longer credit period. If inventory holding costs (h_r / h_o) or the interest rate charged (I_c) increases, retailers reduce order quantity to maintain a low average inventory level. If the backlogging cost (c_b), lost sale cost (c_g), or unit selling price (p) increases, then retailers shorten the replenishment cycle and shortage period to reduce the shortage cost and lost sales cost. As a result, retailers' order quantity decreases.

(3) The optimal fraction of no shortage F^{**} increases as the value of c_b, c_g, or p increases, whereas it decreases as the value of A, β, I_e, or c increases. An increase in c_b, c_g, or p implies that retailers must pay more costs for shortages. Therefore, retailers shorten the shortage period by increasing the fraction of no shortage. However, if A, β, I_e, or c increases, then retailers are encouraged to place huge orders. They also lengthen

the shortage period (i.e., reduce the fraction of no shortage) to avoid paying excessive inventory holding costs. In addition, we observe that when retailers face order-size dependent trade credit policy, no specific monotonic relationship exists between F^{**} and the value of D, h_r, h_o, or I_c.

(4) The optimal replenishment cycle length T^{**} increases if we increase the value of A, I_e, or c, but it decreases if we increase the value of c_b, c_g, or p. Obviously, if A, I_e, or c increases, retailers place large orders, thereby T^{**} increases. However, if c_b, c_g, or p increases, retailers reduce their order quantity and T^{**} eventually decreases. Similarly, no specific monotonic relationship exists between T^{**} and the value of D, h_r, h_o, β, or I_c. Moreover, the optimal replenishment cycle T^{**} is very sensitive to the changes in A, D, h_o, c_b, c_g, p, β, I_e, or c.

(5) As the value of A, D, β, I_e, or c increases, retailers benefit from renting an extra warehouse. By contrast, as the value of h_r, h_o, c_b, c_g, p, or I_c increases, retailers tend to choose not to rent an extra warehouse. In reality, when retailers encounter attractive trade credit terms, high product demands, or relatively high ordering costs, they usually order more items. Therefore, an additional RW is necessary.

Table 6. Result of sensitivity analysis on major parameters

		T^{**}	F^{**}	Q^{**}	ATP^{*}	Rented warehouse?	Credit period
A	80	0.4670	0.8670	137.31	1213.45	No	M_1
	90	0.5091	0.8385	149.04	1192.96	No	M_1
	100	1.2289	0.6626	350.00	1176.13	Yes	M_2
	110	1.2289	0.6621	350.00	1167.99	Yes	M_2
	120	1.2290	0.6615	350.00	1159.86	Yes	M_2
D	100	1.0595	0.6747	100.78	308.40	No	M_1
	200	0.7112	0.7489	136.89	729.75	No	M_1
	300	1.2289	0.6626	350.00	1176.13	Yes	M_2
	400	1.3200	0.6464	500.00	1692.89	Yes	M_3
	500	1.0511	0.6758	500.00	2238.15	Yes	M_3
h_r	2.0	1.2289	0.6626	350.00	1176.13	Yes	M_2
	2.2	1.2295	0.6594	350.00	1175.61	Yes	M_2
	2.4	1.2301	0.6563	350.00	1175.12	Yes	M_2
	2.6	1.2306	0.6533	350.00	1174.65	Yes	M_2
	2.8	0.5480	0.8161	159.87	1174.04	No	M_1
h_o	1.5	1.2273	0.6705	350.00	1184.03	Yes	M_2
	1.6	1.2289	0.6626	350.00	1176.13	Yes	M_2
	1.7	0.5449	0.8054	158.71	1168.65	No	M_1
	1.8	0.5420	0.7949	157.61	1163.43	No	M_1
	1.9	0.5393	0.7847	156.56	1158.37	No	M_1

End of Table 6

		T^{**}	F^{**}	Q^{**}	ATP^{*}	Rented warehouse?	Credit period
c_b	2.0	1.8072	0.4814	500.00	1243.00	Yes	M_3
	3.0	1.2390	0.6110	350.00	1238.96	Yes	M_2
	4.0	1.2289	0.6626	350.00	1176.13	Yes	M_2
	5.0	0.5315	0.8455	155.75	1172.08	No	M_1
	6.0	0.5202	0.8667	152.94	1170.70	No	M_1
c_g	1.0	1.2325	0.6441	350.00	1191.73	Yes	M_2
	2.0	1.2289	0.6626	350.00	1176.13	Yes	M_2
	3.0	0.5181	0.8776	152.59	1167.11	No	M_1
	4.0	0.4820	0.9520	143.57	1163.22	No	M_1
	5.0	0.4595	1.0000	137.86	1162.62	No	M_1
p	14	1.2315	0.6490	350.00	885.07	Yes	M_2
	15	1.2289	0.6626	350.00	1176.13	Yes	M_2
	16	0.5197	0.8705	152.88	1469.27	No	M_1
	17	0.4865	0.9352	144.53	1766.88	No	M_1
	18	0.4545	1.0000	136.34	2067.34	No	M_1
β	0.75	0.4595	1.0000	137.86	1162.62	No	M_1
	0.80	0.4634	0.9915	138.80	1162.64	No	M_1
	0.85	1.2289	0.6626	350.00	1176.13	Yes	M_2
	0.90	1.2123	0.6235	350.00	1217.47	Yes	M_2
	0.95	1.1913	0.5863	350.00	1262.40	Yes	M_2
I_e	0.08	0.5421	0.8483	158.94	1162.30	No	M_1
	0.10	0.5453	0.8320	159.47	1168.05	No	M_1
	0.12	1.2289	0.6626	350.00	1176.13	Yes	M_2
	0.14	1.2312	0.6504	350.00	1192.50	Yes	M_2
	0.16	1.2336	0.6383	350.00	1209.22	Yes	M_2
I_c	0.10	1.2240	0.6879	350.00	1187.33	Yes	M_2
	0.12	1.2260	0.6774	350.00	1182.67	Yes	M_2
	0.14	1.2279	0.6674	350.00	1178.26	Yes	M_2
	0.16	1.2297	0.6579	350.00	1174.06	Yes	M_2
	0.18	0.5281	0.7952	153.57	1166.46	No	M_1
c	9	0.5250	0.8846	154.76	1470.01	No	M_1
	9.5	0.5372	0.8490	157.51	1321.58	No	M_1
	10	1.2289	0.6626	350.00	1176.13	Yes	M_2
	10.5	1.2313	0.6499	350.00	1032.35	Yes	M_2
	11	1.2337	0.6376	350.00	888.95	Yes	M_2

6. Managerial insights

In this section, we present significant managerial insights that can help decision-makers in the industry to make the best action under a certain situation.

- When retailers' OW capacity is relatively small, they should increase the order quantity and rent an extra warehouse. At the same time, suppliers are advised to prolong the trade credit period to motivate retailers to order more because a large order quantity brings additional revenue for suppliers.
- When retailers' OW capacity is large enough, we advise suppliers not to prolong the trade credit period because the further extension of the credit period does not enlarge retailers' order quantity. Such an extension only increases suppliers' capital costs. Moreover, selecting a replenishment policy equal to that without capacity constraint is profitable for retailers.
- Compared with the fixed credit period policy and the conditional trade credit policy, the order-size dependent trade credit policy has various advantages (e.g., encourage retailers to order more, reduce the difficulty of suppliers' decision-making, and increase the choice of retailers), thus we strongly suggest suppliers adopt this policy in the specific business environment.
- Trade credit and OW capacity have significant effects on retailers' profit. We thus recommend retailers to persuade suppliers to provide a longer credit period or to choose suppliers with a long credit period. Retailers may also increase their profits by appropriately expanding OW capacity.
- With the raises of shortage cost (c_b) or opportunity cost (c_g), the annual profit and the optimal replenishment cycle length decrease, whereas the optimal fraction of no shortage increases, suggesting to order less but more frequently to pay less for shortages.
- As the backlogging rate (β) or interest rate earned (I_e) increases, the annual profit and the order quantity increase. Thus, retailers must order more goods to meet increased backlogged demand. At the same time, a larger order quantity leads to a longer trade credit period. As a result, retailers can obtain additional interest revenue.
- High holding costs in RW and OW or the interest rate charged (I_c) leads to low annual profit and order quantity. Therefore, reducing the order quantity when these parameters' values are high is profitable for retailers.
- When the shortage cost (c_b), opportunity cost (c_g), or selling price (p) increases (or backordering rate (β) decreases), operating an inventory system without shortages (or with shortages) is profitable for retailers.
- As the fixed ordering cost (A), the demand rate (D), the backordering rate (β), the interest rate earned (I_e), or the purchase cost (c) increases, renting an additional warehouse is favorable to retailers.

All these insights are essential in the decision process and provide significant managerial guidelines for decision-makers in the industry.

Conclusions and future research directions

Conclusions

In this study, we investigate a deterministic inventory model with limited warehouse storage space under an order-size dependent trade credit policy. Shortages are allowed and partially backlogged. The purpose of this study is to find optimal ordering and backlogging policies for retailers who seek to maximize their annual profit. We also discuss how retailers determine whether to rent an extra warehouse to stock more goods and thus gain a longer credit period. First, four possible inventory models are formulated based on different parameter values. Then, the condition of the objective function to have an interior optimal value is established, and the closed-form solution is derived. Meanwhile, an efficient solution procedure is proposed to find the global optimal solution in an integrated manner. Finally, several numerical examples are provided to shed light on the applicability of the presented model and solution procedure. Sensitivity analysis of major parameters is performed and meaningful managerial insights are obtained.

The results show that if retailers' OW capacity is relatively small, then they have a clear motivation to place huge orders and rent an extra warehouse to obtain good credit terms. Importantly, under this situation, the strategy in which suppliers prolong trade credit length to motivate retailers to order additional goods is beneficial for both members, as retailers place huge orders to suppliers that are favorable for their business. If retailers' OW capacity exceeds the optimal order quantity under that without capacity constraint, then retailers neither increase their order quantity nor rent an additional warehouse, that is, retailers' ordering behavior as that without capacity constraint. In this situation, the further extension of the credit period only in favor of retailers. Because such an extension does not enlarge retailers' order quantity but increases suppliers' capital costs. We also demonstrate that the order-size dependent trade credit has obvious advantages in enlarging retailers' order quantity, increasing retailers' decisions on order quantity choice, and reducing the difficulty of suppliers' decision-making.

Moreover, by further sensitive analysis concerning major parameters, we find that: (1) a high backordering rate (β) or interest rate earned (I_e) increases retailers' annual profit, optimal order quantity, and replenishment cycle length, but decreases the optimal fraction of no shortages; (2) if the shortage cost (c_b) or opportunity cost (c_g) increases, then retailers' annual profit, optimal order quantity, and replenishment cycle length decrease, but the optimal fraction of no shortage increases; (3) high inventory holding costs in OW and RW or interest rate charged (I_c) leads to a low annual profit and optimal order quantity; (4) when the shortage cost (c_b), opportunity cost (c_g), or selling price (p) increases (or the backordering rate (β) decreases), retailers benefit from operating an inventory system without shortages (or with shortages), and (5) as the fixed ordering cost (A), demand rate (D), backordering rate (β), interest earned (I_e), or purchase cost (c) increases, retailers prefer to rent an additional warehouse.

Future research directions

There are many opportunities for future research. First, the current study only involves a single player (i.e., the retailer) to optimize the replenishment schedule. Thus, we can extend our model to consider a general supply chain setting and find the integrated cooperative solution for multiple participants (i.e., the supplier and the retailer), as considered by Tiwari et al. (2018) and Jonas (2019). Second, in our study, we assume that the demand rate is constant. As stated by Feng et al. (2021) and Li et al. (2021), the market demand rate may be influenced by various factors in practice, such as the selling price, stock level, time, credit period, etc. By capturing these factors to extend our current model is a meaningful topic. Finally, our model can be extended to consider the deteriorating items or imperfect imperfect quality items, as described by Jaggi et al. (2017), Khanna et al. (2017), and Gupta et al. (2020).

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