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EDAS METHOD FOR MULTIPLE CRITERIA GROUP DECISION MAKING WITH PICTURE FUZZY INFORMATION AND ITS APPLICATION TO GREEN SUPPLIERS SELECTIONS

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Abstract. In this paper, we construct picture fuzzy EDAS model based on traditional EDAS (Evaluation based on Distance from Average Solution) model. Firstly, we briefly review the definition of picture fuzzy sets (PFSs) and introduce the score function, accuracy function and operational laws of picture fuzzy numbers (PFNs). Then, we combine traditional EDAS model for MCGDM with PFNs. In our model, it's more accuracy and effective for considering the conflicting attributes. Finally, a numerical example for green supplier selection has been given to illustrate this new model and some comparisons between EDAS model with PFNs and PFWA, PFWG aggregation operators are also conducted to further illustrate advantages of the new method.

Keywords: multiple criteria group decision making (MCGDM) problems, picture fuzzy sets (PFSs), EDAS model, picture fuzzy weighted average (PFWA) operator, picture fuzzy weighted geometric (PFWG) operator, picture fuzzy EDAS model, green supplier selection.

JEL Classification: C43, C61, D81.

Introduction

Keshavarz Ghorabaee, Zavadskas, Olfat, and Turskis (2015) firstly defined the original EDAS (Evaluation based on Distance from Average Solution) method to deal with many multiple criteria decision making (MCDM) problems (S. M. Chen, Yang, Yang, Sheu, & Liau, 2012; Gao, 2018; Keshavarz Ghorabaee, Amiri, Zavadskas, Turskis, & Antucheviciene, 2017a, 2017b; M. Tang et al., 2019; Tang, Wei, & Gao, 2019; Y. Wei, Qin, Li, Zhu, & Wei, 2019; L. P. Wu, Wei, Gao, & Wei, 2018; Xu & Yager, 2009). The EDAS method is very effective especially when the conflicting criteria exist in the MCDM problem. Similar to VIKOR method (Wang, Wei, & Lu, 2018) and TOPSIS method, distances from positive and negative ideal

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solutions are also derived in EDAS method. However, EDAS method is based on the PDA (Positive Distance from Average) and NDA (Negative Distance from Average) from the average solution, not the positive and negative ideal solutions. The best alternative is the one with the biggest value of PDA and the smallest value of NDA (Keshavarz Ghorabaee, Zavadskas, Amiri, & Turskis, 2016; Keshavarz Ghorabaee et al., 2015). In previous work, lots of MCDM models such as the VIKOR method (H. C. Liu, L. Liu, & Wu, 2013; Yang, Pang, Shi, & Wang, 2018; Zhou, Wang, & Zhang, 2018), the ELECTRE method (N. Chen & Xu, 2015; J. J. Peng, Wang, & Wu, 2017; Zhang, Wang, & Chen, 2016), the TOPSIS method (Liao, Si, Xu, & Fujita, 2018; Wan, Li, & Dong, 2018; Zeng & Xiao, 2018), the PROMETHEE method (S. Hajlaoui & N. Halouani, 2013; Liao & Xu, 2014; Muirhead, 1902; Sennaroglu, Yilmazer, Tuzkaya, & Tuzkaya, 2018; Y. N. Wu, Wang, Hu, Ke, & Li, 2018), the GRA method, the MULTIMOORA method (Aydin, 2018; Liu, You, Lu, & Shan, 2014; Zhao, You, & Liu, 2017) and the TODIM method (Huang & Wei, 2018; Ji, Zhang, & Wang, 2018; Qin, Liang, Li, Chen, & Yu, 2017; Ren, Xu, Wang, & Ieee, 2017; G. W. Wei, 2018c) were broadly investigated by a large amount of scholars. Compare with the existed work, the EDAS model owns the merit of only taking average solution (AV) into account with respect to the intangibility of decision maker (DM) and the uncertainty of decision-making environment to obtain more accuracy and effective aggregation results.

K. T. Atanassov (1986) introduced the concept of intuitionistic fuzzy sets (IFSs), which is a generalization of the concept of fuzzy set (Zadeh, 1965). K. Atanassov and Gargov (1989) and K. T. Atanassov (1994) proposed the concept of interval-valued intuitionistic fuzzy sets (IVIFS), which are characterized by a membership function, a non-membership function, and a hesitancy function whose values are intervals. Recently, Cuong and Kreinovich (2013) proposed picture fuzzy sets (PFSs) and investigated the some basic operations and properties of PFSs. The picture fuzzy set is characterized by three functions expressing the degree of membership, the degree of neutral membership and the degree of non-membership. The only constraint is that the sum of the three degrees must not exceed 1. Singh (2015) presented the geometrical interpretation of PFSs and proposed correlation coefficients for PFSs. Son (2015) presented a novel distributed picture fuzzy clustering method on PFSs. N. T. Thong and Son (2015) proposed the model between picture fuzzy clustering and intuitionistic fuzzy recommender systems for medical diagnosis. P. H. Thong and Son (2016) proposed the Automatic Picture Fuzzy Clustering (AFC-PFS) for determining the most suitable number of clusters for FC-PFS. G. W. Wei (2016) proposed the multiple attribute decision making (MADM) method based on the proposed picture fuzzy cross entropy. Son (2017) defined the generalized picture distance measures and picture association measures. Son and Thong (2017) developed some novel hybrid forecast models with picture fuzzy clustering for weather nowcasting from satellite image sequences. G. W. Wei (2017c) gave some cosine similarity measures of PFSs for strategic decision making on the basis of the traditional similarity measures (Hung & Yang, 2004; D. F. Li, 2004; Szmidt & Kacprzyk, 2004; Ye, 2018; Zhai, Xu, & Liao, 2018). G. W. Wei (2017a) proposed some aggregation operators for MCDM based on the PFSs based on the traditional aggregation operators (Deng, Wei, Gao, & Wang, 2018; Gao, Lu, G. W. Wei, & Y. Wei, 2018; Z. X. Li, Wei, & Lu, 2018). G. W. Wei (2018b) defined some similarity measures for PFSs. G. W. Wei (2018c) proposed the TODIM method for picture fuzzy MADM. G. W. Wei and Gao (2018) developed The generalized dice similarity measures for PFSs. G. W. Wei (2018a) proposed some picture fuzzy Hamacher aggregation operators in MADM with traditional Hamacher operations. G. W. Wei, Alsaadi, Hayat, and Alsaedi (2018b) designed the projection models for MADM with picture fuzzy information. G. W. Wei, Alsaadi, Hayat, and Alsaedi (2018a) proposed some picture 2-tuple linguistic operators in MADM. G. W. Wei (2017b) defined some picture uncertain linguistic Bonferroni mean operators for MADM.

Keshavarz Ghorabaee et al. (2015) originally defined the EDAS method and studied it with the problems of multi-criteria inventory classification. Then, Keshavarz Ghorabaee et al. (2016) proposed an extended EDAS method for supplier selection. Kahraman et al. (2017) established a new EDAS model under intuitionistic fuzzy information for solid waste disposal site selection. Keshavarz Ghorabaee, Amiri, Zavadskas, and Turskis (2017) extended EDAS method with interval type-2 fuzzy sets. Keshavarz Ghorabaee et al. (2017a) defined a novel multi-criteria EDAS model with interval type-2 fuzzy sets. Keshavarz Ghorabaee et al. (2017b) proposed the stochastic EDAS method for MCDM with normally distributed data. X. D. Peng and Liu (2017) defined the algorithms for neutrosophic soft decision making based on EDAS and new similarity measure. Ecer (2018) gave third-party logistics (3PLS) provider selection with fuzzy AHP and EDAS integrated method. Feng, Wei, and Liu (2018) developed the EDAS method for extended hesitant fuzzy linguistic MCDM. Ilieva (2018) assigned the group decision models with EDAS for interval fuzzy sets. Karasan and Kahraman (2018b) defined a novel interval-valued neutrosophic EDAS method. Karasan and Kahraman (2018a) constructed the interval-valued neutrosophic EDAS method. Keshavarz-Ghorabaee, Amiri, Zavadskas, Turskis, and Antucheviciene (2018b) developed the dynamic fuzzy approach with the EDAS method for multi-criteria subcontractor evaluation. Stevic, Vasiljevic, Zavadskas, Sremac, and Turskis (2018) gave the selection of carpenter manufacturer using fuzzy EDAS method. Keshavarz-Ghorabaee, Amiri, Zavadskas, Turskis, and Antucheviciene (2018a) gave a comparative analysis of the rank reversal phenomenon with the EDAS and TOPSIS methods.

However, it's clear that the study about the EDAS model for PFSs is not existed. Hence, it's necessary to take picture fuzzy EDAS model into account. The purpose of our work is to establish an extended EDAS model according to the traditional EDAS method and PFNs to study multiple criteria group decision making (MCGDM) problems more effectively. The structure of our paper is organized as follows: the definition, score function, accuracy function and operational formulas of PFNs are briefly introduced in Section 1. Some aggregation operators of PFNs are introduced in Section 2. The traditional EDAS model for MCGDM with PFNs is established and the computing steps are simply depicted in Section 3. A numerical example for green supplier selection has been given to illustrate this new model and some comparisons between EDAS model with PFNs and PFWA, PFWG operators are also conducted to further illustrate advantages of the new method in Section 4. Last Section gives some conclusions of our works.

1. Preliminaries

1.1. Picture fuzzy sets

Definition 1 (Cuong & Kreinovich, 2013)**.** A picture fuzzy set (PFS) on the universe X is an object of the form

$$A=\{\langle x,\mu_A(x),\eta_A(x),\nu_A(x)\rangle | x\in X\}, \tag{1}$$

where $\mu_A(x)\in[0,1]$ is called the "degree of positive membership of A", $\eta_A(x)\in[0,1]$ is called the "degree of neutral membership of A" and $\nu_A(x)\in[0,1]$ is called the "degree of negative membership of A", and $\mu_A(x)$, $\eta_A(x)$, $\nu_A(x)$ satisfy the following condition: $0\le\mu_A(x)+\eta_A(x)+\nu_A(x)\le 1,\ \ \forall x\in X$. Then for $x\in X$, $\pi_A(x)=1-(\mu_A(x)+\eta_A(x)+\nu_A(x))$ could be called the degree of refusal membership of x in A.

Definition 2 (Cuong & Kreinovich, 2013)**.** Let $\alpha=(\mu_\alpha,\eta_\alpha,\nu_\alpha)$ and $\beta=(\mu_\beta,\eta_\beta,\nu_\beta)$ be two picture fuzzy numbers (PFNs), the operation formula of them can be defined:

(1) $\alpha\oplus\beta=(\mu_\alpha+\mu_\beta-\mu_\alpha\mu_\beta,\eta_\alpha\eta_\beta,\nu_\alpha\nu_\beta)$;

(2) $\alpha\otimes\beta=(\mu_\alpha\mu_\beta,\eta_\alpha+\eta_\beta-\eta_\alpha\eta_\beta,\nu_\alpha+\nu_\beta-\nu_\alpha\nu_\beta)$;

(3) $\lambda\alpha=\left(1-(1-\mu_\alpha)^\lambda,\eta_\alpha^\lambda,\nu_\alpha^\lambda\right),\lambda>0$;

(4) $\alpha^\lambda=\left(\mu_\alpha^\lambda,1-(1-\eta_\alpha)^\lambda,1-(1-\nu_\alpha)^\lambda\right),\lambda>0$;

According to the Definition 2, it's clear that the operation laws have the following properties (Cuong & Kreinovich, 2013).

$$(1)\ \alpha\oplus\beta=\beta\oplus\alpha,\alpha\otimes\beta=\beta\otimes\alpha,\left((\alpha)^{\lambda_1}\right)^{\lambda_2}=(\alpha)^{\lambda_1\lambda_2}; \tag{2}$$

$$(2)\ \lambda(\alpha\oplus\beta)=\lambda\alpha\oplus\lambda\beta,(\alpha\otimes\beta)^\lambda=(\alpha)^\lambda\otimes(\beta)^\lambda; \tag{3}$$

$$(3)\ \lambda_1\alpha\oplus\lambda_2\alpha=(\lambda_1+\lambda_2)\alpha,(\alpha)^{\lambda_1}\otimes(\alpha)^{\lambda_2}=(\alpha)^{(\lambda_1+\lambda_2)}. \tag{4}$$

Definition 3 (Cuong & Kreinovich, 2013)**.** Let $\alpha=(\mu_\alpha,\eta_\alpha,\nu_\alpha)$ and $\beta=(\mu_\beta,\eta_\beta,\nu_\beta)$ be PFNs, the score and accuracy functions of α and β can be expressed:

$$S(\alpha)=\mu_\alpha-\nu_\alpha,S(\beta)=\mu_\beta-\nu_\beta; \tag{5}$$

$$H(\alpha)=\mu_\alpha+\eta_\alpha+\nu_\alpha,H(\beta)=\mu_\beta+\eta_\beta+\nu_\beta. \tag{6}$$

For two PFNs α and β, based on the Definition 3, then

(1) if $s(\alpha)\prec s(\beta)$, then $\alpha\prec\beta$;

(2) if $s(\alpha)\succ s(\beta)$, then $\alpha\succ\beta$;

(3) if $s(\alpha)=s(\beta)$, $h(\alpha)\prec h(\beta)$, then $\alpha\prec\beta$;

(4) if $s(\alpha)=s(\beta)$, $h(\alpha)\succ h(\beta)$, then $\alpha\succ\beta$;

(5) if $s(\alpha)=s(\beta)$, $h(\alpha)=h(\beta)$, then $\alpha=\beta$.

2. Picture fuzzy aggregation operators

In this section, we introduced some aggregation operators with PFNs, such as picture fuzzy weighted averaging (PFWA) operator and picture fuzzy weighted geometric (PFWG) operator.

Definition 4 (G. W. Wei, 2017a)**.** Let $\alpha_j = \left(\mu_{\alpha_j}, \eta_{\alpha_j}, \nu_{\alpha_j}\right)(j = 1,2,\cdots,n)$ be a collection of PFNs, the picture fuzzy weighted averaging (PFWA) operator can be defined as:

$$\mathrm{PFWA}_{\omega}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \overset{n}{\underset{j=1}{\oplus}} \left(\omega_j \alpha_j\right), \tag{7}$$

where $\omega = \left(\omega_1, \omega_2, ..., \omega_n\right)^T$ be the weight vector of $\alpha_j \left(j = 1,2,...,n\right)$ and $\omega_j > 0, \sum_{j=1}^{n} \omega_j = 1.$

Based on the Definition 4, we can get the following result:

Theorem 1. The aggregated value by using PFWA operator is also a PFN, where

$$\mathrm{PFWA}_{\omega}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \overset{n}{\underset{j=1}{\oplus}} \left(\omega_j \alpha_j\right) = \left(1 - \prod_{j=1}^{n} \left(1 - \mu_{\alpha_j}\right)^{\omega_j}, \prod_{j=1}^{n} \left(\eta_{\alpha_j}\right)^{\omega_j}, \prod_{j=1}^{n} \left(\nu_{\alpha_j}\right)^{\omega_j}\right), \tag{8}$$

where $\omega = \left(\omega_1, \omega_2, ..., \omega_n\right)^T$ be the weight vector of $\alpha_j \left(j = 1,2,...,n\right)$ and $\omega_j > 0, \sum_{j=1}^{n} \omega_j = 1.$

Definition 5 (G. W. Wei, 2017a). Let $\alpha_j \left(j = 1,2,\cdots,n\right)$ be a collection of PFNs, the picture fuzzy weighted geometric (PFWG) operator can be defined as:

$$\mathrm{PFWG}_{\omega}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \overset{n}{\underset{j=1}{\otimes}} \left(\alpha_j\right)^{\omega_j}, \tag{9}$$

where $\omega = \left(\omega_1, \omega_2, ..., \omega_n\right)^T$ be the weight vector of $\alpha_j \left(j = 1,2,...,n\right)$ and $\omega_j > 0, \sum_{j=1}^{n} \omega_j = 1.$

Based on the Definition 5, we can get the following result:

Theorem 2. The aggregated value by using PFWG operator is also a PFN, where

$$\mathrm{PFWG}_{\omega}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \overset{n}{\underset{j=1}{\otimes}} \left(\alpha_j\right)^{\omega_j} = \left(\prod_{j=1}^{n} \left(\mu_{\alpha_j}\right)^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - \eta_{\alpha_j}\right)^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - \nu_{\alpha_j}\right)^{\omega_j}\right), \tag{10}$$

where $\omega = \left(\omega_1, \omega_2, ..., \omega_n\right)^T$ be the weight vector of $\alpha_j \left(j = 1,2,...,n\right)$ and $\omega_j > 0, \sum_{j=1}^{n} \omega_j = 1.$

3. The EDAS model with picture fuzzy information

The traditional EDAS (Evaluation based on Distance from Average Solution) method (Keshavarz Ghorabaee et al., 2015), which can consider the conflicting attributes, has been studied in many multi-attribute decision making (MCDM) problems. By computing the average solution (AV), this model can describe the difference between all the alternatives and the average solution (AV) based on two distance measures which are namely PDA (Positive Distance from Average) and NDA (Negative Distance from Average), the alternative with higher values of PDA and lower values of PDA is the best choice. To combine the EDAS model with PFNs, we build the picture fuzzy EDAS model which the evaluation values are presented by PFNs. The computing steps of our proposed model can be established as follows.

Suppose there are m alternatives $\{A_1, A_2, \ldots A_m\}$, n attributes $\{G_1, G_2, \ldots G_n\}$ and t experts $\{E_1, E_2, \ldots E_r\}$, let $\{\omega_1, \omega_2, \ldots \omega_n\}$ and $\{\theta_1, \theta_2, \ldots \theta_r\}$ be the attribute's weighting vector and expert's weighting vector which satisfy $\omega_i \in [0,1], \theta_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1, \sum_{r=1}^{t} \theta_r = 1$.

The computing steps are listed as follows:

Step 1. Construct the evaluation matrix $R = \left[\delta_{ij}^r\right]_{m\times n}, i = 1,2,\cdots, m, j = 1,2,\cdots, n, r = 1,2,\cdots, t$, which can be depicted as follows:

$$R = \left[\delta_{ij}^r\right]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & \ldots & G_n \end{array} \\ \begin{bmatrix} \delta_{11}^r & \delta_{12}^r & \ldots & \delta_{1n}^r \\ \delta_{21}^r & \delta_{22}^r & \ldots & \delta_{2n}^r \\ \vdots & \vdots & \vdots & \vdots \\ \delta_{m1}^r & \delta_{m2}^r & \ldots & \delta_{mn}^r \end{bmatrix} \end{array}, \tag{11}$$

where δ_{ij}^r denotes the PFNs of alternative A_i on attribute G_j by expert E^t.

Step 2. Normalize the evaluation matrix $R = \left[\delta_{ij}^r\right]_{m\times n}$ to $R' = \left[\delta_{ij}'^r\right]_{m\times n}$;

For benefit attributes:

$$\delta_{ij}'^r = \delta_{ij}^r = \left(\mu_{\alpha_{ij}}^r, \eta_{\alpha_{ij}}^r, \nu_{\alpha_{ij}}^r\right), i = 1,2,\cdots,m, j = 1,2,\cdots,n, r = 1,2,\cdots,t, \tag{12}$$

For cost attributes:

$$\delta_{ij}'^r = \left(\delta_{ij}^r\right)^c = \left(\nu_{\alpha_{ij}}^r, \eta_{\alpha_{ij}}^r, \mu_{\alpha_{ij}}^r\right), i = 1,2,\cdots,m, j = 1,2,\cdots,n, r = 1,2,\cdots,t, \tag{13}$$

Step 3. According to the decision making matrix $R = \left[\delta_{ij}'^r\right]_{m\times n}$ and expert's weighting vector $\{\theta_1, \theta_2, \ldots \theta_r\}$, we can derive the overall $\delta_{ij}'^r$ to δ_{ij}' by using PFWA operator, the computing results can be presented as follows.

$$R = \left[\delta_{ij}'\right]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & \ldots & G_n \end{array} \\ \begin{bmatrix} \delta_{11}' & \delta_{12}' & \ldots & \delta_{1n}' \\ \delta_{21}' & \delta_{22}' & \ldots & \delta_{2n}' \\ \vdots & \vdots & \vdots & \vdots \\ \delta_{m1}' & \delta_{m2}' & \ldots & \delta_{mn}' \end{bmatrix} \end{array}. \tag{14}$$

Step 4. Compute the value of average solution (AV) based on all proposed attributes:

$$AV = \left[AV_j\right]_{1\times n} = \left[\frac{\sum_{i=1}^{m} \delta_{ij}'}{m}\right]_{1\times n}. \tag{15}$$

Based on **Definition 2, we can get:**

$$\sum_{i=1}^{m} \delta_{ij}' = \left(1 - \prod_{i=1}^{m}\left(1 - \mu_{ij}'\right), \prod_{i=1}^{m} \eta_{ij}', \prod_{i=1}^{m} \nu_{ij}'\right)_{1\times n}; \tag{16}$$

$$AV = \left[AV_j\right]_{1\times n} = \left[\frac{\sum_{i=1}^{m} \delta_{ij}'}{m}\right]_{1\times n} = \left\{1 - \prod_{i=1}^{m}\left(1 - \mu_{ij}'\right)^{\frac{1}{m}}, \prod_{i=1}^{m}\left(\eta_{ij}'\right)^{\frac{1}{m}}, \prod_{i=1}^{m}\left(\nu_{ij}'\right)^{\frac{1}{m}}\right\}_{1\times n}. \tag{17}$$

Step 5. According to the results of average solution (AV), we can compute the positive distance from average (PDA) and negative distance from average (NDA) by using the following formula:

$$PDA_{ij} = \left[PDA_{ij} \right]_{m \times n} = \frac{\max\left(0, \left(\delta'_{ij} - AV_j\right)\right)}{AV_j}; \tag{18}$$

$$NDA_{ij} = \left[NDA_{ij} \right]_{m \times n} = \frac{\max\left(0, \left(AV_j - \delta'_{ij}\right)\right)}{AV_j}. \tag{19}$$

For convenience, we can use the score function of PFNs presented in **Definition 3** to determine the results of PDA and NDA as follows:

$$PDA_{ij} = \left[PDA_{ij} \right]_{m \times n} = \frac{\max\left(0, \left(s\left(\delta'_{ij}\right) - s\left(AV_j\right)\right)\right)}{s\left(AV_j\right)}; \tag{20}$$

$$NDA_{ij} = \left[NDA_{ij} \right]_{m \times n} = \frac{\max\left(0, \left(s\left(AV_j\right) - s\left(\delta'_{ij}\right)\right)\right)}{s\left(AV_j\right)}. \tag{21}$$

Step 6. Calculate the values of SP_i and SN_i which denotes the weighted sum of PDA and NDA, the computing formula are provided as follows:

$$SP_i = \sum_{j=1}^{n} w_j PDA_{ij}, \; NP_i = \sum_{j=1}^{n} w_j NDA_{ij}. \tag{22}$$

Step 7. The results of equation (20) can be normalized as:

$$NSP_i = \frac{SP_i}{\max_i\left(SP_i\right)}, \; NSN_i = 1 - \frac{SN_i}{\max_i\left(SN_i\right)}. \tag{23}$$

Step 8. Compute the values of appraisal score (AS) based on each alternative's NSP_i and NSN_i:

$$AS_i = \frac{1}{2}\left(NSP_i + NSN_i\right). \tag{24}$$

Step 9. According to the calculating results of AS, we can rank all the alternatives, the bigger value of AS is, the best alternative selected will be.

4. The Numerical Example

4.1. Numerical for MCGDM problems with PFNs

With the rapid development of economic globalization and the growing enterprise competition environment, the competition between modern enterprises has become the competition between supply chain and supply chain. The diversity of people consumption concept, the new product life cycles are getting shorter, volatility of demand market and those from external factors drives enterprises for effective supply chain integration and management, and strategic alliance with other enterprises in order to enhance core competitiveness and resist external risk. And the key measure to achieving this goal is the supplier selection. Therefore, supplier selection problem has gain great attention no matter in supply chain management theory or in actual production management problems. In this section, we provide a numerical example for green supplier selection by using EDAS model with PFNs. There are five possible green suppliers $A_i\left(i=1,2,3,4,5\right)$ to be selected and four attributes to assess these green suppliers: ①G_1 is the price factor; ②G_2 is the delivery factor; ③G_3 is the environmental factors; ④G_4 is the product quality factor. The five possible green suppliers $A_i\left(i=1,2,3,4,5\right)$ are to be evaluated with PFNs with the four criteria by three experts a^r (attributes weight $\omega=\left(0.25,0.18,0.35,0.22\right)$, expert's weight $\theta=\left(0.35,0.2,0.45\right)$. The evaluation matrix are listed in Tables 1–3.

Step 1. Construct the evaluation matrix $R=\left[\delta_{ij}^{r}\right]_{m\times n},i=1,2,\ldots,m,j=1,2,\ldots,n$.

Table 1. Picture fuzzy evaluation information by E^1

	G_1	G_2	G_3	G_4
A_1	(0.56,0.12,0.20)	(0.62,0.15,0.23)	(0.47,0.33,0.10)	(0.51,0.34,0.15)
A_2	(0.43,0.25,0.18)	(0.50,0.28,0.22)	(0.54,0.29,0.17)	(0.64,0.17,0.19)
A_3	(0.60,0.32,0.08)	(0.58,0.12,0.30)	(0.62,0.11,0.28)	(0.80,0.15,0.05)
A_4	(0.58,0.22,0.10)	(0.61,0.13,0.26)	(0.55,0.27,0.18)	(0.67,0.26,0.07)
A_5	(0.50,0.13,0.37)	(0.65,0.10,0.25)	(0.76,0.00,0.24)	(0.47,0.35,0.18)

Table 2. Picture fuzzy evaluation information by E^2

	G_1	G_2	G_3	G_4
A_1	(0.48,0.35,0.17)	(0.53,0.27,0.10)	(0.61,0.28,0.19)	(0.80,0.15,0.05)
A_2	(0.53,0.27,0.10)	(0.64,0.17,0.19)	(0.43,0.37,0.20)	(0.23,0.22,0.65)
A_3	(0.66,0.20,0.14)	(0.59,0.21,0.20)	(0.18,0.11,0.77)	(0.73,0.17,0.10)
A_4	(0.41,0.28,0.31)	(0.18,0.32,0.50)	(0.29,0.32,0.39)	(0.49,0.34,0.17)
A_5	(0.07,0.39,0.64)	(0.27,0.28,0.45)	(0.55,0.27,0.08)	(0.68,0.14,0.18)

Table 3. Picture fuzzy evaluation information by E^3

	G_1	G_2	G_3	G_4
A_1	(0.51,0.24,0.25)	(0.70,0.12,0.18)	(0.52,0.25,0.23)	(0.56,0.12,0.20)
A_2	(0.69,0.08,0.23)	(0.59,0.21,0.10)	(0.36,0.00,0.64)	(0.76,0.09,0.15)
A_3	(0.37,0.21,0.42)	(0.45,0.23,0.32)	(0.73,0.14,0.13)	(0.55,0.27,0.08)
A_4	(0.42,0.25,0.33)	(0.66,0.19,0.15)	(0.57,0.26,0.17)	(0.18,0.32,0.50)
A_5	(0.26,0.18,0.56)	(0.47,0.34,0.19)	(0.64,0.17,0.19)	(0.82,0.10,0.08)

Step 2. Normalize the evaluation matrix $R=\left[\delta_{ij}^{r}\right]_{5\times 4}$ to $R'=\left[\delta_{ij}'^{r}\right]_{5\times 4}$, for all the attributes are benefit, the normalization is not needed.

Step 3. According to the decision making matrix $R=\left[\delta_{ij}^{r}\right]_{5\times 4}$ and expert's weighting vector $\left\{v_1,v_2,\ldots v_r\right\}$, we compute the overall $\delta_{ij}'^{r}$ to δ_{ij}' by using PFWA aggregation operator, the computing results can be presented in Table 4.

Table 4. The fused values by using PFWA operator

	G_1	G_2	G_3	G_4
A_1	$(0.5225, 0.2031, 0.2141)$	$(0.6435, 0.1526, 0.1744)$	$(0.5233, 0.2818, 0.1654)$	$(0.6098, 0.1807, 0.1371)$
A_2	$(0.9015, 0.1520, 0.1787)$	$(0.5718, 0.2226, 0.1498)$	$(0.4429, 0.0000, 0.3189)$	$(0.6508, 0.1344, 0.2185)$
A_3	$(0.5250, 0.2410, 0.1887)$	$(0.5281, 0.1799, 0.2848)$	$(0.6200, 0.1226, 0.2427)$	$(0.6941, 0.2004, 0.0710)$
A_4	$(0.4802, 0.2445, 0.3875)$	$(0.5746, 0.1846, 0.2314)$	$(0.5170, 0.2746, 0.2048)$	$(0.4577, 0.3012, 0.2025)$
A_5	$(0.3247, 0.1875, 0.4975)$	$(0.5113, 0.2131, 0.2485)$	$(0.6734, 0.0000, 0.1734)$	$(0.7053, 0.1658, 0.1250)$

Step 4. According to Table 4, we can obtain the value of average solution (AV) based on all proposed attributes by formula (17):

$$AV_1 = \left\{ \begin{array}{l} \left(1 - \left((1-0.5225)\times(1-0.9015)\times(1-0.5250)\times(1-0.4802)\times(1-0.3247) \right)^{\frac{1}{5}} \right), \\ (0.2031\times0.1520\times0.2410\times0.2445\times0.1875)^{\frac{1}{5}} \\ (0.2141\times0.1787\times0.1887\times0.3875\times0.4975)^{\frac{1}{5}} \end{array} \right\} = (0.6208, 0.2026, 0.2683);$$

$$AV_2 = \left\{ \begin{array}{l} 1 - \left((1-0.6435)\times(1-0.5718)\times(1-0.5281)\times(1-0.5746)\times(1-0.5113) \right)^{\frac{1}{5}}, \\ (0.1526\times0.2226\times0.1799\times0.1846\times0.2131)^{\frac{1}{5}}, \\ (0.1744\times0.1498\times0.2848\times0.2314\times0.2485)^{\frac{1}{5}} \end{array} \right\} = (0.5684, 0.1889, 0.2120);$$

$$AV_3 = \left\{ \begin{array}{l} 1 - \left((1-0.5233)\times(1-0.4429)\times(1-0.6200)\times(1-0.5170)\times(1-0.6734) \right)^{\frac{1}{5}}, \\ (0.2818\times0.0000\times0.1226\times0.2746\times0.0000)^{\frac{1}{5}}, \\ (0.1654\times0.3189\times0.2427\times0.2048\times0.1734)^{\frac{1}{5}} \end{array} \right\} = (0.5631, 0.0000, 0.2145);$$

$$AV_4 = \left\{ \begin{array}{l} 1 - \left((1-0.6098)\times(1-0.6508)\times(1-0.6941)\times(1-0.4577)\times(1-0.7053) \right)^{\frac{1}{5}}, \\ (0.1807\times0.1344\times0.2004\times0.3012\times0.1658)^{\frac{1}{5}}, \\ (0.1371\times0.2185\times0.0710\times0.2025\times0.1250)^{\frac{1}{5}} \end{array} \right\} = (0.6330, 0.1893, 0.1400).$$

Then we can get the values of AV_j as:

$$AV_{1\times4} = \left\langle \begin{array}{l} (0.6208, 0.2026, 0.2683), (0.5684, 0.1889, 0.2120), \\ (0.5631, 0.0000, 0.2145), (0.6330, 0.1893, 0.1400) \end{array} \right\rangle.$$

Step 5. According to the results of average solution (AV), we can compute the positive distance from average (PDA) and negative distance from average (NDA) by using the formula (20) and (21) which are listed in Tables 5–7.

Table 5. The score values of δ'_{ij} and AV_j

	G_1	G_2	G_3	G_4
A_1	0.3084	0.4691	0.3579	0.4727
A_2	0.7228	0.4220	0.1240	0.4323
A_3	0.3363	0.2433	0.3773	0.6231
A_4	0.0927	0.3432	0.3122	0.2552
A_5	−0.1728	0.2628	0.4999	0.5803
AV	0.3524	0.3565	0.3485	0.4930

Table 6. The results of PDA_{ij}

	G_1	G_2	G_3	G_4
A_1	0.0000	0.3161	0.0268	0.0000
A_2	1.0509	0.1838	0.0000	0.0000
A_3	0.0000	0.0000	0.0825	0.2641
A_4	0.0000	0.0000	0.0000	0.0000
A_5	0.0000	0.0000	0.4344	0.1772

Table 7. The results of NDA_{ij}

	G_1	G_2	G_3	G_4
A_1	0.1248	0.0000	0.0000	0.0411
A_2	0.0000	0.0000	0.6442	0.1230
A_3	0.0458	0.3174	0.0000	0.0000
A_4	0.7369	0.0371	0.1041	0.4822
A_5	1.4903	0.2627	0.0000	0.0000

Step 6. Calculate the values of SP_i and SN_i by equation (23) and attributes weighting vector $\omega = (0.25, 0.18, 0.35, 0.22)$**,** we can obtain the results as:

$$SP_1 = 0.0663,\ SP_2 = 0.2958,\ SP_3 = 0.0870,\ SP_4 = 0.0000,\ SP_5 = 0.1910;$$

$$SN_1 = 0.0402, SN_2 = 0.2525, SN_3 = 0.0686, SN_4 = 0.3334, SN_5 = 0.4199.$$

Step 7. The results of Step 6 can be normalized by formula (24) and the results are listed as:

$$NSP_1 = 0.2241,\ NSP_2 = 1.0000,\ NSP_3 = 0.2940,\ NSP_4 = 0.0000,\ NSP_5 = 0.6458;$$

$$NSN_1 = 0.9041, NSN_2 = 0.3986, NSN_3 = 0.8366, NSN_4 = 0.2059, NSN_5 = 0.0000.$$

Step 8. Based on each alternative's NSP_i and NSN_i to compute the values of AS;

$$AS_1 = 0.5641, AS_2 = 0.6993, AS_3 = 0.5653, AS_4 = 0.1029, AS_5 = 0.3229.$$

Step 9. According to the calculating results of AS, we can rank all the alternatives, the bigger value of AS is, the best alternative selected will be. Obviously, the rank of all alternatives is $\delta'_2 > \delta'_3 > \delta'_1 > \delta'_5 > \delta'_4$ and δ'_2 is the best green supplier.

4.2. Compare EDAS method with some aggregation operators with PFNs

In this chapter, we compare our proposed picture fuzzy EDAS method with the PFWA operator PFWG operator(G. W. Wei, 2017a). According to the results of Table 4 and attributes weighting vector $\omega = (0.25, 0.18, 0.35, 0.22)$, we can compute the overall δ'_{ij} to δ'_i by PFWA and PFWG operators which is listed in Table 8.

Table 8. The fused values by using the PFWA and PFWG operators

	PFWA	PFWG
A_1	$(0.5669, 0.2108, 0.1709)$	$(0.5615, 0.2183, 0.1912)$
A_2	$(0.6891, 0.0000, 0.2216)$	$(0.6028, 0.1116, 0.2047)$
A_3	$(0.6017, 0.1733, 0.1790)$	$(0.5923, 0.1809, 0.2287)$
A_4	$(0.5068, 0.2535, 0.2449)$	$(0.5036, 0.2578, 0.2438)$
A_5	$(0.5883, 0.0000, 0.2241)$	$(0.5395, 0.1262, 0.2518)$

According to the score function of PFNs, we can obtain the alternative score results which are shown in Table 9.

Table 9. Score results of five alternatives

	PFWA	PFWG
$s(A_1)$	0.3960	0.3703
$s(A_2)$	0.4675	0.3981
$s(A_3)$	0.4227	0.3636
$s(A_4)$	0.2619	0.2598
$s(A_5)$	0.3642	0.2876

The ranking of alternatives by some PFNs aggregation operators are listed in Table 10.

Table 10. Rank of Alternatives by some PFNs aggregation operators

	Order
PFWA operator (G. W. Wei, 2017a)	$A_2 > A_3 > A_1 > A_5 > A_4$
PFWG operator (G. W. Wei, 2017a)	$A_2 > A_1 > A_3 > A_5 > A_4$
PFNs EDAS model	$A_2 > A_3 > A_1 > A_5 > A_4$

Compare the results of the picture fuzzy EDAS model with PFWA and PFWG operators, the aggregation results are slightly different in ranking of alternatives and the best alternative is same. However, picture fuzzy EDAS model has the precious characteristics of considering the conflicting attributes and can be more accuracy and effective in the application of MCDM problems. And compared with other MCDM methods, EDAS method has required fewer computations, although it results in the same best alternative. The evaluations of alternatives in EDAS method are based on the distance measure from the average solution unlike TOPSIS and VIKOR methods.

Conclusions

In this paper, we present the picture fuzzy EDAS model for MCGDM based on the traditional EDAS model. Firstly we briefly review the definition of PFNs and introduce the score function, accuracy function and operational laws of PFNs. Next, to fuse the PFNs, we introduce some aggregation operators of PFNs. Furthermore, we propose the picture fuzzy EDAS model for MCGDM and develop the computing steps for MCGDM problem with PFNs. In our presented model, it's more accuracy and effective for considering the conflicting attributes. Finally, a numerical example for green supplier selection has been given to illustrate this new model and some comparisons between picture fuzzy EDAS model, PFWA and PFWG operators are also conducted to further illustrate advantages of the new method. In the future, the picture fuzzy EDAS model can be applied to other MCDM problems and many other uncertain and fuzzy environments. The picture 2-tuple linguistic and picture uncertain linguistic EDAS methods are the other possible extensions of our proposed method for further researches.

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