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ON ONE EFFECTIVE DIFFERENCE SCHEME FOR THE CONVECTION-DIFFUSION PROBLEM

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ABSTRACT

The given paper is devoted to build-up of the special economic difference schemes for non-stationary one and two-dimensional problems of a convection - diffusion permitting to take into account convective and diffusion terms from the uniform point of view. On the basis of a multicomponent schemes build-up procedure, bound up with region decomposition of the cells of mesh, the economic multicomponent iterative algorithm is constructed. A series of numerical calculations on some test problems solution including Burgers problem is reduced, and the comparison with known, most spread schemes is proceeded.

1. INTRODUCTION

The mathematical models of problems describing viscous fluid flows, in particular modelling of hydrodynamics and heat-mass transfer processes, represent systems of nonlinear differential partial equations of the second order and contain terms reflecting a fluid diffusion and its transfer in a movable. It is possible to consider the equation of a convection - diffusion as the basic equation for such problems. The substance diffusion is described by terms of the second order, and the convection is described by terms of the first order. Though a convection is featured only by terms of the first order, the major attention is given to its approximation. The detailed analysis of the numerous difference schemes for hydrodynamics problems solution is surveyed in [1-3]. In [4] the review of convection - diffusion stationary and non-stationary problems solution difference methods is given, stability estimation is obtained and the sufficient conditions of schemes monotonicity are proved. However

the problem of the best difference scheme solution is still an important task. The given paper is devoted to build-up of the special economic difference schemes permitting to take into account convection and diffusion terms from the uniform point of view. One and two-dimensional non-stationary problems of a convection - diffusion are considered, which enable us to take into account the basic singularities of a problem and are the basis for more complex hydrodynamic equations, heat-mass transfer equations etc.

2. PROBLEM STATEMENT. THE DIFFERENCE SCHEMES. REALIZATION ALGORITHMS

2.1. The one-dimensional nonlinear non-stationary problem of a convection - diffusion

We shall consider the nonlinear non-stationary equation of a convection - diffusion

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(\psi(u)u) = \frac{\partial}{\partial x}(\mu\frac{\partial u}{\partial x}) + f(x,t), \quad \mu > 0, \quad t > 0, \quad x \in \Omega, \qquad (1)$$

$$u(x,0) = \varphi(x), \quad x \in \overline{\Omega}; \quad u(0,t) = u_0, \quad u(L,t) = u_L, \quad t \geq 0. \qquad (2)$$

The domain $\overline{\Omega} = \{0 \leq x \leq L, \quad 0 \leq t \leq T\}$ is covered by the rectangular uniform difference grid $\overline{\Omega}_{h\tau} = \overline{\Omega}_h \times \overline{\Omega}_\tau$, $\overline{\Omega}_h = \Omega_h + \partial\Omega_h$, $\Omega_h = \{x_i = ih, i = 1, \ldots N-1, Nh = L\}$, here $\partial\Omega_h$ are boundary points, and we introduce the time grid: $\Omega_\tau = \{t_j = j\tau, j = 0, 1 \ldots\}$.

The general flux consists from convection and diffusion $P = \psi(u)u - \mu\partial u/\partial x$ specified on boundary $x_{i+0.5}$ of the control volumes $\omega_i = [x_{i-0.5}, x_{i+0.5}]$ and $\omega_{i+1} = [x_{i+0.5}, x_{i+1.5}]$. It is necessarily to require realization of equality condition of fluxes $P^+_{i+0.5} = P^-_{i+0.5}$. This requirement gives us the additional information on behaviour of a solution inside control volume. Since unknowns are defined a grid, and it is not known about behaviour of a solution between nodal points, it is possible, for example to suppose, that the solution between nodal points is stationary and satisfies to the equation $dP/dx = 0$.

Thus, for exact definition of a flux on the boundary of interior control volumes it is necessary to solve additionally the problem inside a cell [3]:

$$d(\psi(u^*)u^* - \mu du^*/dx)/dx = 0, \quad x \in \Omega_i = (x_i, x_{i+1}),$$

with boundary conditions $u^*|_{x_i} = u_i$, $u^*|_{x_{i+1}} = u_{i+1}$.

Assuming, that inside each interval $[x_i, x_{i+1}]$ ψ and μ are constant and are defined by the average value, the solution of the auxiliary problem has a form

$$u^*(x) = u_i + (u_{i+1} - u_i)\frac{\exp(\psi_{i+0.5}(x - x_i)/\mu_{i+0.5}) - 1}{\exp(\psi_{i+0.5}h/\mu_{i+0.5}) - 1},$$

where $\psi_{i+0.5} = 0.5(\psi_{i+1} + \psi_i)$ and similar definition is applied for $\mu_{i+0.5}$.

Integrating equation (1) on control volume $\Omega_\tau \times \omega_i$, we shall receive the equation

$$y_{\bar{t}} + (P_{i+0.5} - P_{i-0.5})/h = f_h.$$

In [1, 2, 4] the solution at an intermediate point was defined by linear interpolation on the nearest next points. Using $u^*(x)$ as a profile of a solution and its derivative for definition in half-integer points ω_i, we come to the difference scheme with nonlinear coefficients

$$y_{\bar{t}} + (\psi_{+0.5} y)_{\bar{x}} = (\tilde{\mu}_{+0.5}\, y_x)_{\bar{x}} + f_h\ , \quad x \in \Omega_h\ , \tag{3}$$

where $\tilde{\mu}_{+0.5} = \mu_{+0.5} F_B(\mathrm{Re}_h^{+0.5})$, $\mathrm{Re}_h^{+0.5} = h\psi_{+0.5}/\mu_{+0.5}$.)

In [6] similar scheme of exponential adjustment with its proper parameter for stationary problem of boundary layer is received.

As the grid Reynolds number depends on a solution, for finding a solution of the difference scheme (3) we shall consider the following iterative process:

$$\overset{s+1}{y}{}_{\bar{t}} + (\overset{s}{\psi}_{+0.5}\ \overset{s+1}{y})_{\bar{x}} = (\overset{s}{\tilde{\mu}}_{+0.5}\ \overset{s+1}{y}{}_x)_{\bar{x}} + f_h\ , \quad x \in \Omega_h, \tag{4}$$

where s is the number of iteration, $s = 0, 1 \ldots,$ $\overset{0}{y} = \breve{y}$.

From (4) we have a linear difference equation for definition $\overset{s+1}{y}{}_i$

$$A_i \overset{s+1}{y}{}_{i-1} - C_i \overset{s+1}{y}{}_i + B_i \overset{s+1}{y}{}_{i+1} = -F_i\ , \quad i = \overline{1, N-1}, \tag{5}$$

with coefficients $A_i = \tau h^{-2} \overset{s}{\mu}_{i-0.5} F_A(\overset{s}{\mathrm{Re}}_{hi-0.5})$, $B_i = \tau h^{-2} \overset{s}{\mu}_{i+0.5} F_B(\overset{s}{\mathrm{Re}}_{hi+0.5})$, $F_i = \breve{y} + \tau f_i$, $C_i = 1 + A_i + B_i + \tau(\overset{s}{\psi}_{i+0.5} - \overset{s}{\psi}_{i-0.5})/h$, where the functions $F_A(\mathrm{Re})$ and $F_B(\mathrm{Re})$ are determined by formula

$$F_B = Re/(\exp(Re) - 1), \quad F_A = (Re\ \exp(Re))/(\exp(Re) - 1). \tag{6}$$

They are positive and have three limiting values, thus the limiting values $\pm\infty$ are indefinitely major values of the same order, as Re at $\mathrm{Re} \to \pm\infty$. The scheme is monotone, as the requirements of a monotonicity are fulfilled [4]:

$$A_i > 0, \quad B_i > 0, \quad D_i = C_i - A_i - B_i \geq 0$$

without any restrictions on steps. A defect of the scheme is multiple calculation of exponential curves. It is not acceptable to the majority of practical problems because of economic reasons. For small Re numbers functions $F_A(Re)$, $F_B(Re)$ can be approximated by a Taylor series.

It is obvious that choosing interpolants well enough approximating $F_A(Re)$, $F_B(Re)$, and taking into account their limiting values, it is possible to construct the various difference schemes.

Let's approximate $F_A(Re)$, $F_B(Re)$ by piecewise linear functions on a segment $[R_{min}, R_{max}]$. We shall choose boundary R_{min}, R_{max} so that the deviation of function values $F_A(Re)$, $F_B(Re)$ in these points from the relevant limiting values was insignificantly. The segment $[R_{\min}, R_{\max}]$ is divided into segments $[R_j, R_{j+1}]$ with a step h_R so that a point of a partition R_0 is equal, as it corresponds to a limiting value Re = 0. In each of these intervals function $F_B(\mathrm{Re})$ is approximated by a symmetric function of known precise values. Thus we present approximating function as

$$\tilde{F}_B(v) = \begin{cases} 0 & \text{for } v > R_{max}, \\ F_B(R_j) + (F_B(R_{j+1}) - F_B(R_j))\frac{(v-R_j)}{h_R} & \text{for } R_j \le v \le R_{j+1}, \\ -v & \text{for } v < R_{\min}. \end{cases} \tag{7}$$

The number of segment partitions depends on an accuracy of approximated function. If $R_{min} = -2$, $R_{max} = 2$ and on all segment $\tilde{F}_B(Re) = 1 - 0.5Re$ we have the combined scheme [3].

Let's note, that a replacement of precise F_B by approximate function $\tilde{F}_B$ introduces in equation (3) the error $O(\varepsilon_F(h_R)y_{\bar{x}x})$, which is the analog of some additional dissipation. It is not difficult to carry out the analysis of dispersion and diffusion properties with the help of differential approximation method[7].

Similarly approximate function $\tilde{F}_A(Re)$ for $F_A(Re)$ can be defined. Difference of this problem is nonlinearity of arguments of functions $F_A(Re)$, $F_B(Re)$, stipulated by nonlinearity of coefficients ψ, μ.

2.1.1. Modifications

Let's stop on a more economic numerical algorithm, which will use field integration decomposition in a combination with a multicomponent method [8].

As subregions of a partition the cells of a grid Ω_h are considered. Indexing will be carried out on the left-hand lower knot of a cell. Unknowns for one rectangular cell $\omega_i = (x_i, x_{i+1})$ will be solution values in its left-hand knots. By solution in a cell we shall understand a grid vector function $\mathbf{Y}_i = (Y_i^{(1)}, Y_i^{(2)})^{\mathrm{T}}$, where $Y_i^{(1)}$ and $Y_i^{(2)}$ are values of a solution in grid knots x_i, x_{i+1}, respectively, i.e $Y_i^{(1)} = Y(x_i)$ and $Y_i^{(2)} = Y(x_{i+1})$.

Let's consider operators

$$D^+v = h^{-1}(\tilde{\mu}_{+0.5}v_x - \psi_{+0.5}v), \quad D^-v = -h^{-1}(\tilde{\mu}_{-0.5}v_{\bar{x}} - \psi_{-0.5}v_{-1}).$$

The difference scheme (3) is given by

$$y_{\bar{t}} = D^+y + D^-y + f_h, \tag{8}$$

its component-wise form is given by

$$Y_i^{(1)}{}_{\bar{t}} = D_i^+\, Y_i^{(1)} + D_i^-\, Y_{i-1}^{(2)} + f_{h,i},$$

$$Y_i^{(2)}{}_{\bar{t}} = D_{i+1}^{+}\, Y_{i+1}^{(1)} + D_i^{-}\, Y_i^{(2)} + f_{h,i+1}. \tag{9}$$

If all components corresponding to the same knot of a grid are equal, equations (8) and (9) coincide. The system of equations (9) is non-linear concerning components for one cell. For component-wise realization we shall consider the following iterative process:

$$\overset{s+1}{Y}{}_i^{(1)}{}_{\bar{t}} = \overset{s+1}{D}{}_i^{+}\, \overset{s+1}{Y}{}_i^{(1)} + \overset{s}{D}{}_{i-1}^{-}\, \overset{s}{Y}{}_{i-1}^{(2)} + f_{h,i}\ ,$$

$$\overset{s+1}{Y}{}_i^{(2)}{}_{\bar{t}} = \overset{s}{D}{}_{i+1}^{+}\, \overset{s}{Y}{}_{i+1}^{(1)} + \overset{s+1}{D}{}_i^{-}\, \overset{s+1}{Y}{}_i^{(2)} + f_{h,i+1}\ , \tag{10}$$

$s = 0, 1, \ldots,\quad i = \overline{1, N-1}$. As a zero approximation for components of a solution we take $\overset{0}{Y}{}_i^{(1)} = Y_i$, $\overset{0}{Y}{}_i^{(2)} = Y_{i+1}$.

Equations (10) in a cell ω_i give a system of linear equations

$$\mathbf{AY} = \mathbf{B} \tag{11}$$

with coefficients:

$$a_{11} = 1 + \tau h^{-2} \overset{s}{\mu}_{i+0.5} F_A(\overset{s}{Re}_h^{+0.5}), \quad a_{12} = -\tau h^{-2} \overset{s}{\mu}_{i+0.5} F_B(\overset{s}{Re}_h^{+0.5}),$$
$$a_{21} = -\tau h^{-2} \overset{s}{\mu}_{i+0.5} F_A(\overset{s}{Re}_h^{+0.5}), \quad a_{22} = 1 + \tau h^{-2} \overset{s}{\mu}_{i+0.5} F_B(\overset{s}{Re}_h^{+0.5}),$$
$$b_1 = \check{y}_i - \tau \overset{s}{D}{}_{i-1}^{-}\, \overset{s}{Y}{}_{i-1}^{(2)} + \tau f_{h,i}, \quad b_2 = \check{y}_{i+1} + \tau \overset{s}{D}{}_{i+1}^{+}\, \overset{s}{Y}{}_{i+1}^{(1)} + \tau f_{h,i+1}.$$

The functions F_B, F_A are defined by (6) or (7). The matrix $\mathbf{A}$ has a diagonal dominance and $\det \mathbf{A} = a_{11}a_{22} - a_{21}a_{12} \neq 0$. The solution is defined in a unique fashion and looks like

$$Y_i^{(1)} = (a_{11}b_2 - a_{21}b_1)/\det \mathbf{A}\ , \qquad Y_i^{(2)} = (a_{22}b_1 - a_{12}b_2)/\det \mathbf{A}\ . \tag{12}$$

The components of a solution are calculated on all interior cells ω_i, $i = \overline{1, N-1}$. On boundary cells, taking into account boundary conditions (2), we determine one component of the solution. Thus, the components of the solution are determined sequentially for each cell. The algorithm is iterated till reaching of some criterion.

As a final solution in knots of a grid x_i we shall accept the average values of the solution components corresponding to the same knot: $Y_i = 0.5(Y_i^{(1)} + Y_{i-1}{}^{(2)})$ for $i = \overline{1, N-1}$.

2.2. Two-dimensional case

The constructed schemes are extended to two-dimensional case. In two-dimensional rectangular field $\overline{\Omega} = \{0 \leq x_\alpha \leq L_\alpha,\ \alpha = 1, 2\}$ we shall consider

the nonlinear non-stationary equation of a convection - diffusion

$$\frac{\partial u}{\partial t}+\sum_{\alpha=1}^{2}\frac{\partial}{\partial x_\alpha}(\psi_\alpha(u)u)=\sum_{\alpha=1}^{2}\frac{\partial}{\partial x_\alpha}\left(\mu_\alpha\frac{\partial u}{\partial x_\alpha}\right)+\sum_{\alpha=1}^{2}f_\alpha(\mathbf{x},t), \qquad (13)$$

$\mu_\alpha > 0, \quad t > 0, \quad \mathbf{x} = (x_1, x_2) \in \Omega$ with boundary conditions of Dirichlet $u(\mathbf{x},t) = \Phi(\mathbf{x}), \quad \mathbf{x} \in \partial\overline{\Omega}, \quad t > 0$, and initial conditions $u(\mathbf{x},0) = \varphi(\mathbf{x}), \quad \mathbf{x} \in \overline{\Omega}$.

Let's define uniform on each direction x_α spatial grid $\overline{\Omega}_h = \Omega_h + \partial\Omega_h$, $\Omega_h = \{(i_1h_1, i_2h_2),\ i_\alpha = \overline{0, N_\alpha},\ N_\alpha h_\alpha = L_\alpha,\ \alpha = 1,2\}$, $\partial\Omega_h$ is a set of the boundary knots, and $\Omega_\tau = \{t_j = j\tau,\ j = 0, 1\ldots\}$ is temporal grid.

Integrating (13) on control volume $\Omega_h \times \Omega_\tau$ and using the technique described above, we obtain

$$y_{\bar{t}}+\sum_{\alpha=1}^{2}(\psi_{\alpha,+0.5}y)_{\bar{x}_\alpha}=\sum_{\alpha=1}^{2}(\tilde{\mu}_{\alpha,+0.5}\ y_{x_\alpha})_{\bar{x}_\alpha}+\sum_{\alpha=1}^{2}f_{h,\alpha}\ , \quad x\in\Omega_h\ , \qquad (14)$$

where

$$\tilde{\mu}_{\alpha,+0.5} = \tilde{\mu}_{\alpha,+0.5}Re_{h_\alpha}^{i_\alpha+0.5}) = \mu_{\alpha,+0.5}F_B(Re_{h_\alpha}^{i_\alpha+0.5}),$$
$$Re_{h_\alpha}^{i_\alpha+0.5} = h_\alpha\psi_{\alpha,+0.5}/\mu_{\alpha,+0.5}.$$

For linearization of (14) we shell use the method of simple iterations:

$$A_{i_1i_2}\overset{s+1}{y}_{i_1-1i_2}-C_{i_1i_2}\overset{s+1}{y}_{i_1i_2}+B_{i_1i_2}\overset{s+1}{y}_{i_1+1i_2}+E_{i_1i_2}\overset{s+1}{y}_{i_1i_2-1}+ \qquad (15)$$

$$+G_{i_1i_2}\overset{s+1}{y}_{i_1i_2+1}=-F_{i_1i_2},$$

where

$$A_{i_1i_2} = \tau h_1^{-2}\ \overset{s}{\mu}_{1,i_1-0.5}\ F_A(\overset{s}{\mathrm{Re}}_{h_1}^{i_1-0.5})\ ,$$
$$B_{i_1i_2} = \tau h_1^{-2}\ \overset{s}{\mu}_{1,i_1+0.5}\ F_B(\overset{s}{\mathrm{Re}}_{h_1}^{i_1+0.5}),$$
$$E_{i_1i_2} = \tau h_2^{-2}\ \overset{s}{\mu}_{2,i_2-0.5}\ F_A(\overset{s}{\mathrm{Re}}_{h_2}^{i_2-0.5}),$$
$$G_{i_1i_2} = \tau h_2^{-2}\ \overset{s}{\mu}_{2,i_2+0.5}\ F_B(\overset{s}{\mathrm{Re}}_{h_2}^{i_2+0.5})\ ,$$
$$C_{i_1i_2} = 1 + A_{i_1i_2} + +B_{i_1i_2} + E_{i_1i_2} + G_{i_1i_2}$$
$$+\tau\sum_{\alpha=1}^{2}\frac{(\overset{s}{\psi}_{\alpha,i_\alpha+0.5}-\overset{s}{\psi}_{\alpha,i_\alpha-0.5})}{h_\alpha},$$

$i_\alpha = \overline{1, N_\alpha - 1}$, $\alpha = 1, 2$, the functions $F_A(\mathrm{Re}_{h_\alpha}^{s\ i_\alpha - 0.5})$ and $F_B(\mathrm{Re}_{h_\alpha}^{s\ i_\alpha + 0.5})$ are defined according to (6).

The solution of a system (15) together with boundary conditions can be found by matrix factorization method or one of the economic schemes [4], e.g., the iterative method of alternate directions. However, the algorithm obtained with application of a multicomponent method for grid cells is more preferable.

We shall consider a grid vector function $\mathbf{Y}_{i_1 i_2} = (Y^{(1)}_{i_1 i_2}, Y^{(2)}_{i_1 i_2}, Y^{(3)}_{i_1 i_2}, Y^{(4)}_{i_1 i_2})^{\mathrm{T}}$, which we shall determine as a solution, where $Y^{(1)}_{i_1 i_2}$, $Y^{(2)}_{i_1 i_2}$, $Y^{(3)}_{i_1 i_2}$ Í $Y^{(4)}_{i_1 i_2}$ are components of a solution for a cell $(i_1 i_2)$ in grid knots (x_{i_1}, x_{i_2}), (x_{i_1}, x_{i_2+1}), (x_{i_1+1}, x_{i_2+1}), (x_{i_1+1}, x_{i_2}) respectively, i.e., $Y^{(1)}_{i_1 i_2} = Y(x_{i_1}, x_{i_2})$ etc. We shall carry out cells indexing on the left-hand lower angle of a cell, and inside a cell for a label of components of a solution - from the left-hand lower angle clockwise. Unknowns for each rectangular cell (x_{i_1}, x_{i_2}) will be values of the solution in its knots, i.e. four components of the solution.

Similarly to Sect. 2.1.1 for each spatial direction we shall define operators $D^+_\alpha v = {h_\alpha}^{-1}(\tilde{\mu}_{\alpha,+0.5} v_{x_\alpha} - \psi_{\alpha,+0.5} v)$ and $D^-_\alpha v = -{h_\alpha}^{-1}(\tilde{\mu}_{\alpha,-0.5} v_{x^-_\alpha} - v_{-1_\alpha})$. Taking into account association between ψ_α, μ_α, Re_{h_α}, operators are conversed to more convenient aspect $D^+_\alpha v = = h_\alpha^{-2} \mu_\alpha^{+0.5}(F^{i_\alpha+0.5}_{\alpha,B} v_{i_\alpha+1} - F^{i_\alpha+0.5}_{\alpha,A} v)$, $D^-_\alpha v = -h_\alpha^{-2} \mu_\alpha^{-0.5}(F^{i_\alpha-0.5}_{\alpha,B} v - F^{i_\alpha-0.5}_{\alpha,A} v_{i_\alpha-1})$.

Noting the difference schemes for each knots of a cell $\omega_{i_1 i_2}$ in new labels, we derive a system of four (11) equations concerning four unknowns.

$F^{i_\alpha \pm 0.5}_{\alpha,A} = F_A(Re^{i_\alpha \pm 0.5}_{h_\alpha})$, $F^{i_\alpha \pm 0.5}_{\alpha,B} = F_B(Re^{i_\alpha \pm 0.5}_{h_\alpha})$ are defined (6). Due to the properties of functions $F_{\alpha,B}, F_{\alpha,A}$, $\alpha = 1, 2$, the determinant of matrix $\mathbf{A}$ is distinct from zero. In interior cells of a grid the order of the system is defined by the number of unknowns in the cell knots, in boundary cells the order of the system decreases up to two. The solution is determined in unique way. The algorithm is solved sequentially from one cell to the other.

As each interior knot of a two-dimensional grid belongs simultaneously to four cells, in this knot there will be also four various components of the solution of the appropriate adjacent cells. Therefore as a solution in a knot of a grid we shall use average of appropriate components. Such procedure will be carried out for all interior cells and for each time layer.

Thus the difference schemes is constructed.The basic parameter of this scheme is a grid Reynolds number describing relation between convection and diffusion. Depending on a Reynolds number, in particular on behaviour of functions F_A, F_B, the coefficients of the difference scheme are selected which define behaviour of the numerical scheme. Their limiting values approximate a pure diffusive or convection problems. A series of examples has shown effectiveness of the similar approach.

3. RESULTS AND DISCUSSION

We illustrate the efficiency of proposed schemes on the example of one-dimensional nonlinear Burgers equation

$$\partial u/\partial t + u\partial u/\partial x = \mu\partial^2 u/\partial x^2, \quad \mu > 0, \quad x \in \Omega, \quad t > 0,$$
$$u(x,0) = \varphi(x), \quad 0 \le x \le 1;$$
$$u(0,t) = u_0, \quad u(1,t) = u_1, \ t \ge 0.$$

We present results, obtained by using algorithm (4) (here we take linear function $\tilde{F}_B(R) = \{-R$ at $R > -4$; $0.54 - 0.885\,R$ at $-4 \le R < -2; \ldots;$ 0 at $R > 4\}$) and by some widely used difference schemes, such as the monotonic difference scheme with directional differences [3], the second order accuracy monotonic difference scheme with a perturbed coefficient of diffusion [4], the central differences scheme [5].

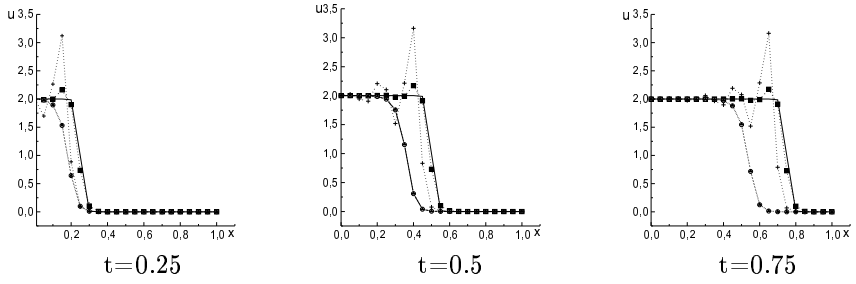

Figure 1. Comparison of the schemes on a test problem: $\mu = 0.01$, $h = 0.05$, $\tau = 0.005$. A continuous line - a precise solution, ■ — a solution by the constructed difference scheme, ∘ — a solution by the monotonic difference scheme with directional differences, ▲ — a solution by the second order monotonic difference scheme with a perturbed diffusion coefficient, + — a solution by the scheme with central differences.

3.1. Example 1

Let take boundary conditions $u_0 = 2/(1 + \exp(-t/\mu))$, $u_1 = 2/(1 + \exp((L - t)/\mu))$ and the initial condition $\varphi(x) = 2/(1 + \exp(x/\mu))$. Then the problem has an exact solution $u(x,t) = 2/(1 + \exp((x - t)/\mu))$.

In Figure 1 results for different time moments are presented. In the region of large gradients the proposed scheme (4) is most accurate. If spatial step reduced 2 times, these differences become less appreciable (see Fig. 1, t=0.25 and Fig. 2a).

The Figure 3 shows change of the flow character depending on a modification of a diffusion coefficient.

3.2. Example 2

Let take boundary conditions $u_0 = f(-\nu t - \beta)$, $u_1 = f(1 - \beta - \nu t)$ and the initial conditions $\varphi(x) = f(x - \beta)$. Then problem has solution $u(x,t) = f(\xi)$,

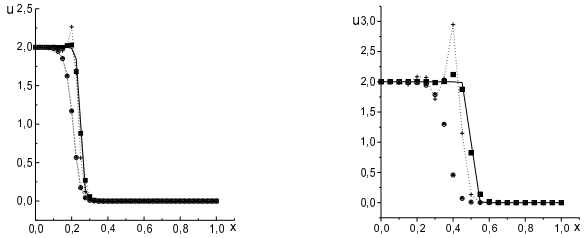

Figure 2. Influence steps grid on nature solution: $\mu = 0.01$. a)-$h = 0.025$, $\tau = 0.005$ $t = 0.25$,b)- $h = 0.025$, $\tau = 0.01$, $t = 0.5$. Of a label same, as in Figure 1.

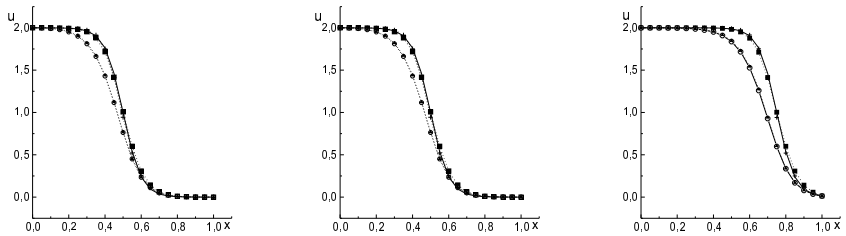

Figure 3. Comparison of the schemes on a test problem: $\mu = 0.05$, $h = 0.05$, $\tau = 0.01$. The labels are same, as in Figure 1.

$\xi = x - \nu t - \beta$, $f(\xi) = (\nu + \alpha + (\nu - \alpha)\exp(\alpha\xi/\mu))/(1 + \exp(\alpha\xi/\mu))$. We use parameters: $\nu = 3,5$, $\beta = 1$, $\alpha = 0.5$. In Fig. 4 the interval is zoomed, in order to show the differences of the considered schemes. Again scheme (4) is best in the field of large gradients.

Thus, as it follows from the presented graphics and from a series of calculations, the approximate solutions can be considerably non accurate. For large steps the constructed scheme is mostly accurate, despite of small oscillations in the neighborhood of maximum growth of the solution. The proposed scheme is sufficiently effective for rough steps of a grid.

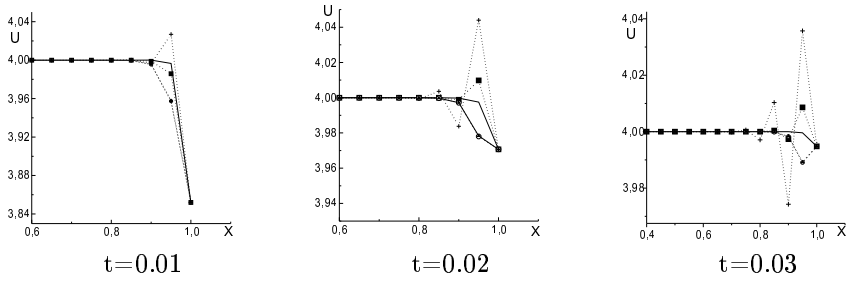

Figure 4. Comparison of the schemes on a test problem: $\mu = 0.05$, $h = 0.05$, $\tau = 0.001$.

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Konvekcijos – difuzijos uždavinių efektyvios baigtinių skirtumų schemos

G. Gromyko

Darbe pasiūlytas naujo tipo baigtinių skirtumų schemos, aproksimuojančios konvekcijos ir difuzijos procesus. Pateikta bendra metodika, kuri apibendrina eksponentinio tipo schemas. Gerai žinoma, kad tokio tipo schemos atskirais atvejais yra tikslios. Sudaryti ekonomiški schemos realizavimo algoritmai, kuriuose panaudota daugiakomponentinių iteracinių metodų idėja. Pateikti apibendrinimai dvimačiams uždaviniams. Teoriniai rezultatai iliustruojami skaitinio eksperimento rezultatais.