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CALCULATION OF ELECTROMAGNETIC FIELDS, FORCES AND TEMPERATURE IN A FINITE CYLINDER

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ABSTRACT

The distribution of electromagnetic fields, forces and temperatures induced by a three-phase axially-symmetric system of electric current in a conducting cylinder of a finite length has been calculated. An original method was used to calculate the radial and axial components of the magnetic fields and the mean values of electromagnetic forces, as well as the azimuthal component of the electric field and of the mean curl of the electromagnetic forces. For finding the source term in the temperature equation we applied an approximation of the heat transport problem based on the finite-difference method. Such a procedure allows one to calculate the distribution of temperatures inside the cylinder depending on that of currents in the wires.

1. THE MATHEMATICAL MODEL

Let the cylindrical domain $\Omega = \{(r,z)|0 < r < a, 0 < z < Z\}$ contains conducting material, where a, Z are radius and length of the cylinder. On the internal surface of the cylinder $r = a$ there are arranged N discrete circular wires (conductors) $L_i = \{(r,z)|r = a, z = z_i\}$ through which alternating current flows with density

$$j_i = j_0 \cos(\omega t + (i-1)\theta),\ i = \overline{1,N},$$

where j_0 is the amplitude, $\omega = 2\pi f$ is the angular frequency, f is the frequency of alternating current, θ is the phase (usually $\theta = 60^0, f = 50Hz$) and t is the time. For the calculation of the electromagnetic fields the averaging method over the time interval $\frac{2\pi}{\omega} = \frac{1}{f}$ was used.

Most of works devoted to investigation of the influence of alternating current on the distribution of electromagnetic forces deal with the infinity long inductor. Quasi-stationary magnetic field is calculated using the form of Bessel's functions and complex number with the factor $\exp(\mathrm{i}\,\omega t)$, where $\mathrm{i} = \sqrt{-1}$ [1] – [7]. In the present work which employs elliptical integrals, the calculations are performed using only the real form of numbers and assuming the time dependence as $\cos(\omega t)$. This makes it possible to consider alternating current connections of various type, with phase shifts θ and various arrangements of the conductors.

The azimuthal components A_ϕ, E_ϕ of the vector potential A and of the electric field $\boldsymbol{E}$ are determined from Maxwell's equations

$$\mathrm{div}\boldsymbol{B} = 0; \quad \mathrm{rot}\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}; \quad \mathrm{rot}\boldsymbol{H} = \boldsymbol{j}; \quad \boldsymbol{B} = \mu \boldsymbol{H}$$

in the following form:

$$E_\phi = -\frac{\partial A_\phi}{\partial t}; \quad \Delta A_\phi = -\mu j^e_\phi,$$

where $\boldsymbol{B} = \mathrm{rot}\boldsymbol{A}$ and $\boldsymbol{H}, \boldsymbol{B}, \boldsymbol{j}^e, \mu$ are the vectors of magnetic field intensity, induction, the vector of external current density and the coefficient of magnetic permeability of the medium; Δ is the Laplace operator.

Applying the Biot-Savar law we arrive at the following form of the azimuthal component of the vector potential created by the alternating current in each conductor L_i:

$$A_{\phi,i}(t, r, z) = \frac{\mu j_i}{4\pi} \int_{L_i} \frac{dl}{\sqrt{((z - z_i)^2 + (r - a)^2)}},$$

where dl is an element of the circular line. Evaluating line integrals in closed form we have

$$A_{\phi,i}(t, r, z) = \frac{\mu j_0}{2\pi} A_i(r, z) \cos(\omega t + (i - 1)\theta), \tag{1.1}$$

where

$$A_i(r, z) = \sqrt{\frac{a}{r}} \Big[\Big(\frac{2}{k_i} - k_i\Big) K(k_i) - \frac{2}{k_i} E(k_i)\Big],$$
$$k_i = 2\sqrt{ar}/c_i, \ c_i = \sqrt{(a + r)^2 + (z - z_i)^2},$$

$K(k) = \int_0^{\pi/2} \dfrac{d\xi}{\sqrt{1-k^2\sin^2\xi}}$ is the total elliptical integral of first kind, $E(k) = \int_0^{\pi/2} \sqrt{1-k^2\sin^2\xi}d\xi$ is the total elliptical integral of second kind. This integrals can be calculated with the accuracy of four decimal places in the following form [1]:

$$K(k) = 1.3863 + 0.1120\tilde{k} + 0.0725\tilde{k}^2 - (0.5 + 0.1213\tilde{k} + 0.0289\tilde{k}^2)\ln\tilde{k},$$
$$E(k) = 1 + 0.4630\tilde{k} + 0.1078\tilde{k}^2 - (0.2453\tilde{k} + 0.0412\tilde{k}^2)\ln\tilde{k},$$

where $\tilde{k} = 1 - k^2$. Assuming the vectorial components of magnetic field induction as

$$B_r = -\frac{\partial A_\phi}{\partial z}, \quad B_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)$$

we arrive at the following form of such components created by the alternating current in conductor L_i :

$$\begin{aligned} B_{r,i}(t,r,z) &= \frac{\mu j_0}{2\pi} B_i^r(r,z)\cos(\omega t + (i-1)\theta), \\ B_{z,i}(t,r,z) &= \frac{\mu j_0}{2\pi} B_i^z(r,z)\cos(\omega t + (i-1)\theta), \end{aligned} \tag{1.2}$$

where

$$B_i^r(r,z) = \sqrt{\frac{a}{r}}\frac{z-z_i}{c_i}\Big[E(k_i)\frac{a^2+r^2+(z-z_i)^2}{(a-r)^2+(z-z_i)^2} - K(k_i)\Big],$$
$$B_i^z(r,z) = \frac{1}{c_i}\Big[K(k_i) + \frac{a^2-r^2-(z-z_i)^2}{(a-r)^2+(z-z_i)^2}E(k_i)\Big].$$

It follows from Ohm's law for the electricity-conducting media that the azimuthal components j_ϕ of the vector of induced current density are given as $\boldsymbol{j} = \sigma\boldsymbol{E}$, where σ is the electrical conductivity. Therefore $j_\phi = \sigma E_\phi = -\dfrac{\sigma\partial A_\phi}{\partial t,}$ and hence from every conductor L_i electric current with the density current with density

$$j_{\phi,i}(t,r,z) = \frac{\omega\mu\sigma j_0}{2\pi} A_i(r,z)\sin(\omega t + (i-1)\theta) \tag{1.3}$$

is induced. From (1.1) – (1.3) we obtain the total magnetic field, the vector potential and the induced current density

$$B_r = \sum_{i=1}^N B_{r,i}, B_z = \sum_{i=1}^N B_{z,i}, A_\phi = \sum_{i=1}^N A_{\phi,i}, j_\phi = \sum_{i=1}^N j_{\phi,i}. \tag{1.4}$$

If $t = 0$ then from (1.4) one can evaluate the momentary values of the above quantities. Having calculated the vector of electromagnetic force (Lorenz's force) $\boldsymbol{F} = \boldsymbol{j} \times \boldsymbol{B}$ we can write the momentary value of the radial and axial components as

$$F_r = B_z j_\phi, \qquad F_z = -B_r j_\phi. \tag{1.5}$$

From (1.5) it follows that

$$\begin{aligned} F_r(t,r,z) &= \Big(\frac{\mu j_0}{2\pi}\Big)^2 \sigma\omega \sum_{i,j=1}^{N} \alpha^r_{i,j}(r,z) cs(t) \\ F_z(t,r,z) &= \Big(\frac{\mu j_0}{2\pi}\Big)^2 \sigma\omega \sum_{i,j=1}^{N} \alpha^z_{i,j}(r,z) cs(t), \end{aligned} \tag{1.6}$$

where

$$\begin{aligned} &cs(t) = \cos(\omega t + (i-1)\theta)\sin(\omega t + (j-1)\theta), \\ &\alpha^r_{i,j}(r,z) = B^z_i A_j = \frac{A_i A_j}{r} + A_j \frac{\partial A_i}{\partial r} = \alpha^{rs}_{i,j} + \alpha^{ra}_{i,j}, \\ &\alpha^z_{i,j}(r,z) = -B^r_i A_j = A_j \frac{\partial A_i}{\partial z} = \alpha^{zs}_{i,j} + \alpha^{za}_{i,j}, \\ &\alpha^{zs}_{i,j} = 0.5 \frac{\partial (A_i A_j)}{\partial z}, \ \alpha^{za}_{i,j} = 0.5\Big(A_j \frac{\partial A_i}{\partial z} - A_i \frac{\partial A_j}{\partial z}\Big), \\ &\alpha^{rs}_{i,j} = 0.5 \frac{\partial (A_i A_j)}{\partial r} + \frac{A_i A_j}{r}, \ \alpha^{ra}_{i,j} = 0.5\Big(A_j \frac{\partial A_i}{\partial r} - A_i \frac{\partial A_j}{\partial r}\Big). \end{aligned}$$

Note, that symmetrical and anti-symmetrical parts of coefficients α are satisfied by the relationships

$$\alpha^{zs}_{i,j} = \alpha^{zs}_{j,i}, \quad \alpha^{rs}_{i,j} = \alpha^{rs}_{j,i}, \quad \alpha^{za}_{i,j} = -\alpha^{za}_{j,i}, \quad \alpha^{ra}_{i,j} = -\alpha^{ra}_{j,i}.$$

Since $cs(t) = 0.5\sin\big(2\omega t + (i+j-2)\theta\big) + 0.5\sin\big(\theta(j-i)\big)$, the average quantity in the time interval $2\pi/\omega$ of this expression is $0.5\sin\big((j-i)\theta\big)$. Therefore, the radial and axial components of averaged force vector can be written as

$$< F_r >= 0.5\Big(\frac{\mu j_0}{2\pi}\Big)^2 \sigma\omega S^r_N, ; \ < F_z >= 0.5\Big(\frac{\mu j_0}{2\pi}\Big)^2 \sigma\omega S^z_N, \tag{1.7}$$

where

$$\begin{aligned} S^r_N &= \sum_{i,j=1}^{N} \sin((j-i)\theta)\alpha^r_{i,j} = 2\sum_{k=1}^{N-1} \sin(k\theta) \sum_{i=1}^{N-k} \alpha^{ra}_{i,k+i}, \\ S^z_N &= \sum_{i,j=1}^{N} \sin((j-i)\theta)\alpha^z_{i,j} = 2\sum_{k=1}^{N-1} \sin(k\theta) \sum_{i=1}^{N-k} \alpha^{za}_{i,k+i}. \end{aligned}$$

Having calculated the azimuthal component of the rotor of force vector $\boldsymbol{F}$

$$\boldsymbol{f} = \operatorname{rot} \boldsymbol{F},\ f_\phi = \frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r} = \frac{\partial (B_z j_\phi)}{\partial z} + \frac{\partial (B_r j_\phi)}{\partial r}$$

we analogously obtain its average value as

$$< f_\phi >= 0.5\Big(\frac{\mu j_0}{2\pi}\Big)^2 \sigma\omega S_N^\phi, \tag{1.8}$$

where

$$S_N^\phi = \sum_{i,j=1}^{N} \sin\big((j-i)\theta\big)\alpha_{i,j}^\phi = 2\sum_{k=1}^{N-1}\sin(k\theta)\sum_{i=1}^{N-k}\alpha_{i,k+i}^{\phi a},$$
$$\alpha_{i,j}^\phi(r,z) = \frac{\partial (B_i^z A_j)}{\partial z} + \frac{\partial (B_i^r A_j)}{\partial r} = \alpha_{i,j}^{\phi s} + \alpha_{i,j}^{\phi a},$$
$$\alpha_{i,j}^{\phi s} = \alpha_{j,i}^{\phi s} = \frac{1}{r}\frac{\partial}{\partial z}(A_i A_j),$$
$$\alpha_{i,j}^{\phi a} = -\alpha_{j,i}^{\phi a} = \frac{\partial A_i}{\partial r}\frac{\partial A_j}{\partial z} - \frac{\partial A_i}{\partial z}\frac{\partial A_j}{\partial r}.$$

The function $\boldsymbol{f}$ is employed when we compute the motion of liquids according to the Navier-Stocks equation. It is readily seen that

$$\frac{\partial \alpha_{i,j}^{ra}}{\partial z} - \frac{\partial \alpha_{i,j}^{za}}{\partial r} = \alpha_{i,j}^{\phi a}.$$

2. THE HEAT TRANSFER EQUATION

The axially-symmetric stationary distribution of temperature field in a conducting cylinder Ω is described by the equation of heat conduction in cylindrical coordinates

$$\lambda\Big(\frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\partial T}{\partial r}\Big) + \frac{\partial^2 T}{\partial z^2}\Big) + \frac{1}{\sigma} < j_\phi^2 >= 0, \tag{2.1}$$

where $< j_\phi^2 >$ is averaging value of j_ϕ^2 (the source function), λ is the heat conductivity, $T = T(r,z)$ is the steady-state distribution of temperature in conductive medium.

Using (1.3)-(1.4) and the expression

$$\sin\big(\omega t + (j-1)\theta\big)\sin\big(\omega t + (i-1)\theta\big)$$
$$= -0.5\cos\big(2\omega t + (i+j-2)\theta\big) + 0.5\cos\big(\theta(j-i)\big)$$

we obtain

$$< j_\phi^2 >= 0.5\Big(\frac{\mu j_0 \sigma\omega}{2\pi}\Big)^2 S_N^Q, \tag{2.2}$$

where

$$S_N^Q = \sum_{i,j=1}^{N} \cos\big((j-i)\theta\big)\alpha_{i,j}^Q = 2\sum_{k=1}^{N-1}\cos(k\theta)\sum_{i=1}^{N-k}\alpha_{i,k+i}^Q + \sum_{i=1}^{N}\alpha_{i,i}^Q,$$

$$\alpha_{i,j}^Q = \alpha_{j,i}^Q = A_i A_j.$$

Calculating the derivatives in formulae (1.6) gives

$$\frac{\partial A_i}{\partial r} = \frac{1}{c_i}\Big[\Big(1-\frac{a^2+r^2+(z-z_i)^2}{r^2}\Big)K(k_i)+\Big(\frac{a^2-r^2-(z-z_i)^2}{(a-r)^2+(z-z_i)^2}+\frac{c_i^2}{r^2}\Big)E(k_i)\Big],$$

$$\frac{\partial A_i}{\partial z} = -B_i^r.$$

In order to formulate the boundary-value problem for equation (2.1), the boundary conditions are written in the form:

$$\lambda\frac{\partial T}{\partial r}\Big|_{r=a} = \alpha(T_a - T|_{r=a}),\ T|_{z=0} = T_a,\ \frac{\partial T}{\partial z}\Big|_{z=Z} = \frac{\partial T}{\partial r}\Big|_{r=0} = 0, \tag{2.3}$$

where T_a is the given constant external temperature, α is the heat exchange coefficient. Introducing dimensionless values

$$\tilde{T} = \frac{T-T_a}{T_a},\ \tilde{r} = \frac{r}{a},\ \tilde{z} = \frac{Z}{a},$$

we arrive at the following form of the boundary-value problem for Poisson's equation in cylindrical coordinates (the symbol "tilde" is omitted):

$$\begin{aligned} &\frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\partial T}{\partial r}\Big) + \frac{\partial^2 T}{\partial z^2} + K_T q = 0,\\ &\frac{\partial T}{\partial r}\Big|_{r=1} = -Bi\ T|_{r=1},\ T|_{z=0} = 0,\ \frac{\partial T}{\partial z}\Big|_{z=l} = \frac{\partial T}{\partial r}\Big|_{r=0} = 0, \end{aligned} \tag{2.4}$$

where $K_T = \dfrac{(\mu j_0 \omega a)^2\sigma}{(2\pi)^2\lambda T_a}$ is the heat sources parameter, $Bi = \dfrac{\alpha a}{\lambda}$ is the Biot number, $l = \dfrac{Z}{a}, q = 0.5 S_N^Q$.

3. THE FINITE-DIFFERENCE SCHEME

Considering a uniform grid $(n_r \times n_z)$:

$$\omega_h = \left\{ (r_i, z_j) | r_i = \frac{h_1}{2} + (i-1)h_1,\ z_j = (j-1)h_2,\ i = \overline{1, n_r},\ j = \overline{1, n_z} \right\},$$

with the steps h_1, h_2 we can approximate boundary-value problem (2.4) with the following finite-difference scheme of the second approximation order [4]:

$$\begin{aligned} &\frac{r_{i+0.5}(T_{i+1,j} - T_{i,j}) + r_{i-0.5}(T_{i,j} - T_{i-1,j})}{r_i h_1^2} \\ &\quad + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{h_2^2} + K_T q_{i,j} = 0, i = \overline{1, n_r},\ j = \overline{2, n_z}, \qquad (3.1) \\ &T_{i,1} = 0, \quad T_{i,n_z+1} = T_{i,n_z},\ i = \overline{1, n_r}, \\ &\frac{T_{n_r+1,j} - T_{n_r,j}}{h_1} = -Bi \frac{T_{n_r+1,j} + T_{n_r,j}}{2}, \quad j = \overline{2, n_z}, \end{aligned}$$

where $r_{i\pm 0.5} = r_i \pm 0.5h_1,\ T_{i,j} \approx T(r_i, z_j),\ q_{i,j} = q(r_i, z_j)$. Since $r_{0.5} = 0$ the symmetry condition $\frac{\partial T}{\partial r}\big|_{r=0} = 0$ is satisfied exactly.

4. THE NUMERICAL METHOD

The finite difference scheme (3.1) is solved with the over relaxation method in the following form:

$$T_{i,j}^{(n)} = \Omega_* T_{i,j}^z + (1 - \Omega_*) T_{i,j}^{(n-1)}, \qquad (4.1)$$

where $n = 1, 2, 3, \ldots$ is the iteration number, Ω_* is the relaxation parameter,

$$T_{i,j}^z = \Big[\frac{r_{i+0.5} T_{i+1,j}^{(n-1)} + r_{i-0.5} T_{i-1,j}^{(n)}}{r_i h_1^2} + h_2^2 \Big(T_{i,j+1}^{(n-1)} + T_{i,j-1}^{(n)} \Big) + K_T q_{i,j} \Big] \Big/ \Big[\frac{(r_{i+0.5} + r_{i-0.5})}{r_i h_1^2} + 2h_2^{-2} \Big]$$

is the n-th approximation by Seidel iteration technique,

$$\begin{aligned} &T_{n_r+1,j}^{(n-1)} = \frac{1 - Bih_1/2}{1 + Bih_1/2} T_{n_r,j}^{(n-1)}, \quad T_{i,n_z+1}^{(n-1)} = T_{i,n_z}^{(n-1)}, \quad T_{i,1}^{(n)} = 0, \\ &T_{i,j}^0 = 0, \quad i = \overline{1, n_r},\ j = \overline{2, n_z}. \end{aligned}$$

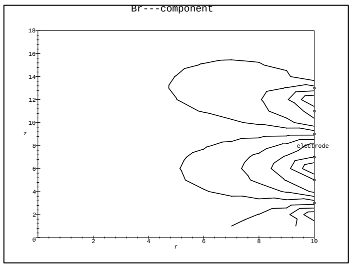

Figure 1. Momentary components $B_r \in (-10.7, 14.5)$.

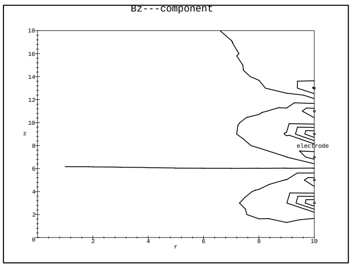

Figure 2. Momentary components $B_z \in (-22.9, 22.3)$.

It is difficult to find theoretically the optimal value of the relaxation parameter Ω_*^{opt} in cylindrical coordinates hence we apply the quantity taken from the approximation of Poisson's equation in the Descartes coordinates:

$$\Omega_*^{opt} = \frac{2}{1 + \sqrt{1 - \eta_0^2}}, \tag{4.2}$$

where $\eta_0 = 0.5\big(\cos(\pi/n_r) + \cos(\pi/n_z)\big)$. For instance, if $n_r = 10,\ n_z = 20,\ = 2$, then it follows from (4.2) that $\Omega_*^{opt} = 1.6$, but numerical experiments show that optimal value of ω in such a grid is $\Omega_* = 1.5$ (approximately 200 iterations) if $\Omega_* = 1$ (Seidel iterations), then we need $n = 2000$) iterations.

5. NUMERICAL RESULTS

As the basis for the calculations six circular conductors ($N = 6$) are chosen, which are arranged axially at the points $z_1 = 0.2,\ z_2 = 0.4,\ z_3 = 0.6,\ z_4 = 0.8,\ z_5 = 1.0,\ z_6 = 1.2$. The results of numerical experiments for $< F_r >$, $< F_z >, < f_\phi >, < j_\phi >$ in the dimensionless form $0.5S_N^r, 0.5S_N^z, 0.5S_N^\phi, 0.5S_N^Q$ and T were obtained with the help of the computer program MAPLE in the case of electrical current with different phases $\theta = \pi/3, 2\pi/3, \pi$, and $l = 2,\ h_1 = h_2 = 0.1$.

The approximate value of T is computed for $Bi = 0.1$. The numerical results essentially depend on the value of Biot number. For example, we have for $T_{max} = \max(T_{i,j})$ the following quantity: $T_{max} = 0.92 (Bi = 1)$, $T_{max} = 2.5 (Bi = 0.1)$, $T_{max} = 3.2 (Bi = 0)$. The distributions of temperature $T_{i,j}^1$ are

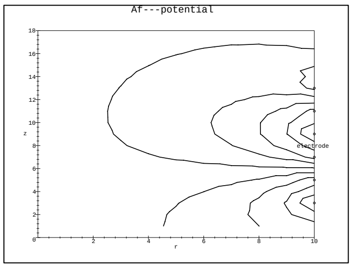

Figure 3. Momentary components $A_\phi \in (-3.0, 2.6)$.

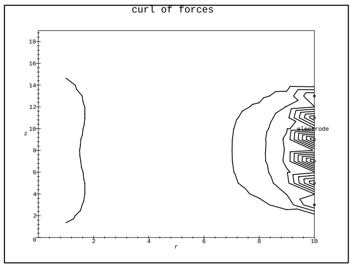

Figure 4. Distribution $< \mathrm{rot}_\phi \boldsymbol{F} > \in (-7.1, 236.0)$.

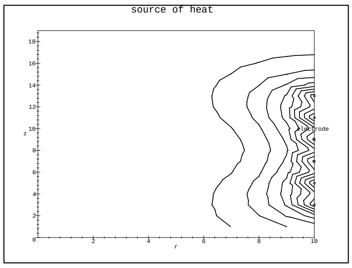

Figure 5. Distribution of source terms $q \in (0, 5.0)$.

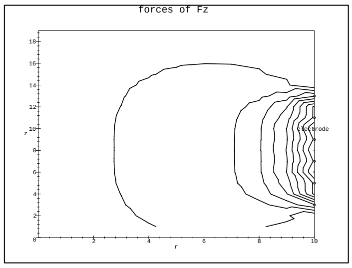

Figure 6. Distribution $< F_z > \in (-39.4, 3.7)$.

obtained for $K_T = 1$, but for every $K_T \neq 1$ we have $T_{i,j} = K_T T^1_{i,j}$.

The numerical results show that the force fields induced by alternating current as well as the heat source are concentrated on the cylinder's surface in the vicinity of the circular electrodes and that the conducting material is heated best on the cylindrical surface after the last electrode. The results depend on the arrangement of electrodes and on the phase shift.

Figs. 1-10 show typical results of calculations: the magnetic field and the distribution of heat sources rises to a maximum on the surface of the cylinder. Considering the momentary values of B_r, B_z, A_ϕ for different phases θ and distributions $zj = [z_1, z_2, z_3, z_4, z_5, z_6]$ of electrical current in the conductors,

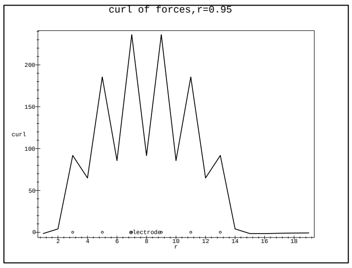

Figure 7. Distribution $\mathrm{rot}_\phi \boldsymbol{F}$ for $r = 0.95$.

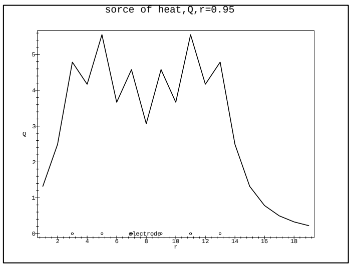

Figure 8. Distribution of source terms for $r = 0.95$.

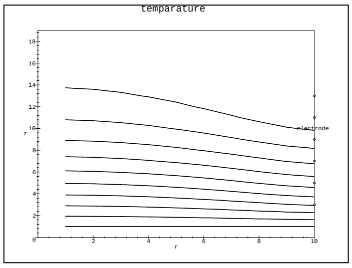

Figure 9. Distribution of temperature ($T_{max} = 1.2$).

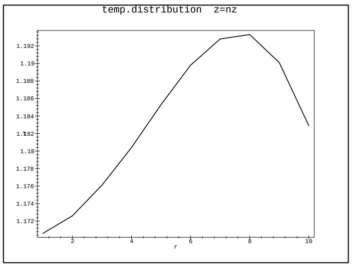

Figure 10. Distribution T for $z = 1.9$.

the following results are found valid:

1. $\theta = \pi/3, zj = [0.2, 0.4, 0.6, 0.8, 1.0, 1.2]$ (the conductors are connected normally one after another), $B_r \in (-10.7, 14.5)$, $B_z \in (-22.9, 22.3)$, $A_\phi \in (-3.0, 2.6)$;
2. $\theta = 2\pi/3, zj = [0.2, 0.6, 1.0, 0.4, 0.8, 1.2]$ (the conductors are connected to each other skipping one of them), $B_r \in (-9.2, 18.0)$, $B_z \in (-13.0, 23.5)$, $A_\phi \in (-2.5, 3.0)$;
3. $\theta = \pi, zj = [0.2, 0.8, 0.4, 1.0, 0.6, 1.2]$ (the conductors are connected to each other skipping two of them), $B_r \in (-10.6, 25.5)$, $B_z \in (-24.8, 24.8)$, $A_\phi \in (-4.3, 4.3)$.

Calculations of the dimensionless values of $< f_\phi >, q, < F_r >, < F_z >, T$ gave the following results $(Bi = 0.1)$:

1. $\theta = \pi/3, zj = [0.2, 0.4, 0.6, 0.8, 1.0, 1.2]$ (the conductors are connected nor-

mally one after another), $< f_\phi >\in (-7.1, 236.0)$, $q \in (0, 5.6)$, $< F_r >\in (-28.4, 28.4)$, $< F_z >\in (-39.4, 3.8)$, $T_{max} = 1.6$;

2. $\theta = \pi/3, zj = [0.2, 0.6, 1.0, 0.4, 0.8, 1.2]$ (the conductors are connected to each other skipping one of them), $< f_\phi >\in (-43.0, 20.0)$, $q \in (0, 1.3)$, $< F_r >\in (-12.5, 12.5)$, $< F_z >\in (-11.5, 10.8)$, $T_{max} = 0.2$;
3. $\theta = 2\pi/3, zj = [0.2, 0.4, 0.6, 0.8, 1.0, 1.2]$ (the conductors are connected normally one after another), $< f_\phi >\in (-0.3, 126.0)$, $q \in (0, 2.3)$, $< F_r >\in (-8.4, 11.3)$, $< F_z >\in (-16.7, 1.0)$, $T_{max} = 0.3$;
4. $\theta = \pi, zj = [0.2, 0.6, 0.4, 0.8, 1.0, 1.2]$ (the conductors are connected skipping one of them), $< f_\phi >= 0$, $q \in (0, 5.0)$, $< F_r >=< F_z >= 0$, $T_{max} = 0.5$;
5. $\theta = \pi, zj = [0.2, 1.2, 0.4, 1.0, 0.6, 0.8]$ (the conductors are connected symmetrically in pairs the first the sixth, the second with the fifth,the third with the fourth), $< f_\phi >= 0, q \in (0, 8.8)$, $< F_r >=< F_z >= 0, T_{max} = 2.0$;
6. $\theta = \pi, zj = [0.2, 0.4, 0.6, 0.8, 1.0, 1.2]$ (the conductors are connected normally one after another), $< f_\phi >= 0$, $q \in (0, 1.8)$, $< F_r >=< F_z >= 0$, $T_{max} = 0.2$.

As can be seen, in the 5^{th} variant (two-phase current) we obtain the highest temperature (four times higher than in the 4^{th} variant and ten times high than in the 6^{th} variant), although the averaged forces cancel.

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Elektromagnetinių laukų, jėgų ir temperatūros skaičiavimas baigtiniame cilindre

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Apskaičiuojamas elektromagnetinių laukų, jėgų ir temperatūrų pasiskirstymas trifazės elektros srovės ašinėje simetrinėje sistemoje baigtiniame cilindre. Panaudotas originalus metodas apskaičiuojant radialinę ir ašinę komponentes, vidurkines elektromagnetinių jėgų reikšmes, elektrinio lauko azimutinę komponentę ir elektromagnetinių jėgų rotorių. Temperatūros šaltinio nustatymui, naudojama šilumos laidumo uždavinio aproksimacija baigtinių skirtumų metodu. Tokia procedūra leidžia skaičiuoti temperatūros pasiskirstymą cilindro viduje, priklausomai nuo srovių jo paviršiuje.