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2^n Positive Periodic Solutions to n Species Non-Autonomous Lotka-Volterra Unidirectional Food Chains with Harvesting Terms*

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Abstract. By using the Mawhin continuation theorem of coincidence degree theory and some results on inequalities, we establish the existence of 2^n positive periodic solutions for n species non-autonomous Lotka-Volterra unidirectional food chains with harvesting terms. Two examples are given to illustrate the effectiveness of our results.

Keywords: 2^n positive periodic solutions; non-autonomous Lotka-Volterra food chain; coincidence degree; harvesting term.

AMS Subject Classification: 34K13; 92D25.

1 Introduction

The food chain phenomenon is universal and interesting in ecosystems. It is one of important methods to investigate this phenomenon by establishing the model of differential equations. The three-level food chain was first studied by Rosenzweig in the famous paper on the paradox of enrichment [10], where he wrote "Man must be very careful in attempting to enrich an ecosystem in order to increase its food yield. There is a real chance that such activity may result in decimation of the food species that are wanted in greater abundance". Hereafter, A. Hastings and T. Powell [6] studied the chaos in a three-species food chain. K. McCann and P. Yodzis [9] investigated the biological conditions for chaos in a three-species food chain. P.A. Abrams and J.D. Roth [1] studied the effects of enrichment of three species food chains with non-linear functional

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responses. Yu.A. Kuznetsov and S. Rinaldi [8] studied the dynamics for a three species food chain, A. Gragnani and O. De Feo, S. Rinaldi [5] investigated the relationships between mean yield and complex dynamics for food chains in the chemostat. C.H. Chiu and S.B. Hsu [3] studied the extinction of top predator in a three level food chain model. Recently, L.M. Zhu, S.Y. Wang, X. Huang and M. Villasan [15] studied limit cycles of the three level food chain with inhibition responses. To the best of our knowledge, there are few results on n species food-chains in the literature. This motivates us to consider the following n species nonautonomous Lotka–Volterra food chain model with delays and harvesting terms

$$\begin{cases} \dot{x}_1(t) = x_1(t)\big(a_1(t) - b_1(t)x_1(t) - c_{12}(t)x_2(t-\tau_{12}(t))\big) - h_1(t), \\ \qquad\qquad \vdots \\ \dot{x}_i(t) = x_i(t)\big(a_i(t) - b_i(t)x_i(t) + c_{i,\,i-1}(t)x_{i-1}(t-\tau_{i,i-1}(t)) \\ \qquad\qquad - c_{i,\,i+1}(t)x_{i+1}(t-\tau_{i,i+1}(t))\big) - h_i(t), \\ \qquad\qquad \vdots \\ \dot{x}_n(t) = x_n(t)\big(a_n(t) - b_n(t)x_n(t) + c_{n,n-1}(t)x_{n-1}(t-\tau_{n,n-1}(t))\big) - h_n(t), \end{cases} \tag{1.1}$$

where $i = 2, 3, \ldots, n-1$, $x_i(t)\ (i = 1, 2, \ldots, n)$ is the ith species population density, $a_i(t)\ (i = 1, 2, \ldots, n), b_i(t) (i = 1, 2, \ldots, n)$ and $h_i(t)\ (i = 1, 2, \ldots, n)$ stand for the ith species growth rate, intra-specific competition rate and harvesting rate, respectively, $c_{i,i+1}(t)\ (i = 1, 2, \ldots, n-1)$ represents the $(i+1)$th species predation rate on the ith species, $c_{i,i-1}(t)\ (i = 2, 3, \ldots, n)$ stands for the transformation rate from the $(i-1)$th species to the i-th species. In addition, the effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Therefore, the assumptions of periodicity of the parameters are a way of incorporating the periodicity of the environment (e.g., seasonal effects of weather, food supplies, mating habits, etc.), which leads us to assume that $a_i(t), b_i(t), c_{ij}(t), \tau_{ij}(t)$ and $h_i(t)(i, j = 1, 2, \ldots, n)$ are all positive continuous ω-periodic functions.

The global existence and stability of a positive periodic solution is a very basic and important problem in the study of a population growth model with a periodic environment and it plays a similar role as a globally stable equilibrium does in an autonomous model. Though, only few results are found in the literature on the existence of positive periodic solutions to system (1.1). This motivates us to investigate the existence of a positive periodic or multiple positive periodic solutions for system (1.1). In fact, it is more likely for some biological species to take on multiple periodic change regulations and have multiple local stable periodic phenomena. Therefore, it is essential for us to investigate the existence of multiple positive periodic solutions for population models. Our main purpose of this paper is by using Mawhin's continuation theorem of coincidence degree theory [4], to establish the existence of 2^n positive periodic solutions for system (1.1). For the work concerning the multiple existence of periodic solutions of periodic population models which was done using coincidence degree theory, we refer to [2, 7, 11, 12, 13, 14].

The organization of the rest of this paper is as follows. In Section 2, by employing the continuation theorem of coincidence degree theory and the skills of inequalities, we establish the existence of 2^n positive periodic solutions of system (1.1). In Section 3, two examples are given to illustrate the effectiveness of our results. Final conclusions are given in Section 4.

2 Existence of 2^n Positive Periodic Solutions

In this section, by using Mawhin's continuation theorem and applying some inequalities, we shall show the existence of positive periodic solutions of (1.1). To do so, we need to make some preparations.

Let X and Z be real normed vector spaces. Let $L : \mathrm{Dom}\, L \subset X \to Z$ be a linear mapping and $N : X \times [0, 1] \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if dim Ker L = codim Im $L < \infty$ and Im L is closed in Z. If L is a Fredholm mapping of index zero, then there exists continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\mathrm{Im}\, P = \mathrm{Ker}\, L$ and $\mathrm{Ker}\, Q = \mathrm{Im}\, L = \mathrm{Im}\,(I - Q)$, and $X = \mathrm{Ker}\, L \bigoplus \mathrm{Ker}\, P, Z = \mathrm{Im}\, L \bigoplus \mathrm{Im}\, Q$. It follows that $L|_{\mathrm{Dom}\ L \cap \mathrm{Ker}\ P} : (I - P)X \to \mathrm{Im}\ L$ is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\bar{\Omega} \times [0, 1]$, if $QN(\bar{\Omega} \times [0, 1])$ is bounded and $K_P(I - Q)N : \bar{\Omega} \times [0, 1] \to X$ is compact. Because Im Q is isomorphic to Ker L, there exists an isomorphism $J : \mathrm{Im}\, Q \to \mathrm{Ker}\ L$.

The Mawhin's continuation theorem [4], p. 40 is given as follows:

Lemma 1. [4] *Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega} \times [0, 1]$. Assume*

(*a*) *for each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda N(x, \lambda)$ is such that $x \notin \partial\Omega \cap \mathrm{Dom}\ L$;*

(*b*) *$QN(x, 0)x \neq 0$ for each $x \in \partial\Omega \cap \mathrm{Ker}\ L$;*

(*c*) $\deg(JQN(x, 0), \Omega \cap \mathrm{Ker}\, L, 0) \neq 0$.

Then $Lx = N(x, 1)$ has at least one solution in $\overline{\Omega} \cap \mathrm{Dom}\, L$.

For the sake of convenience, we denote

$$f^l = \min_{t\in[0,\omega]} f(t), \quad f^M = \max_{t\in[0,\omega]} f(t), \quad \bar{f} = \frac{1}{\omega} \int_0^\omega f(t)\,\mathrm{d}t,$$

respectively, here $f(t)$ is a continuous ω-periodic function. We also need to introduce the following notations.

$$l_1^\pm = \frac{a_1^M \pm \sqrt{(a_1^M)^2 - 4b_1^l h_1^l}}{2b_1^l}, \quad A_n^\pm = \frac{a_n^l \pm \sqrt{(a_n^l)^2 - 4b_n^M h_n^M}}{2b_n^M},$$

$$l_i^\pm = \frac{(a_i^M + c_{i,\,i-1}^M l_{i-1}^+) \pm \sqrt{(a_i^M + c_{i,\,i-1}^M l_{i-1}^+)^2 - 4b_i^l h_i^l}}{2b_i^l}, \quad (i = 2, 3, \ldots, n),$$

$$A_i^{\pm}=\frac{(a_i^l-c_{i,i+1}^M l_{i+1}^+)\pm\sqrt{(a_i^l-c_{i,i+1}^M l_{i+1}^+)^2-4b_i^M h_i^M}}{2b_i^M},\quad (i=1,2,\dots,n-1).$$

Throughout this paper, we need the following assumptions:

(H_1) $a_i^l - c_{i,\,i+1}^M l_{i+1}^+ > 2\sqrt{b_i^M h_i^M}$, $i = 1, 2, \dots, n$;

(H_2) $a_n^l > 2\sqrt{b_n^M h_n^M}$.

Lemma 2. *Let $x > 0$, $y > 0$, $z > 0$ and $x > 2\sqrt{yz}$, then for the functions $f(x, y, z) = (x + \sqrt{x^2 - 4yz})/2z$ and $g(x, y, z) = (x - \sqrt{x^2 - 4yz})/2z$, the following assertions hold:*

1. *$f(x, y, z)$ and $g(x, y, z)$ are monotonically increasing and monotonically decreasing with respect to their first argument $x \in (0, \infty)$, respectively.*
2. *$f(x, y, z)$ and $g(x, y, z)$ are monotonically decreasing and monotonically increasing with respect to their second argument $y \in (0, \infty)$, respectively.*
3. *$f(x, y, z)$ and $g(x, y, z)$ are monotonically decreasing and monotonically increasing with respect to their third argument $z \in (0, \infty)$, respectively.*

Proof. In fact, for all $x > 0, y > 0, z > 0$, we have

$$\frac{\partial f}{\partial x}=\frac{x+\sqrt{x^2-4yz}}{2z\sqrt{x^2-4yz}}>0,\quad \frac{\partial g}{\partial x}=\frac{\sqrt{x^2-4yz}-x}{2z\sqrt{x^2-4yz}}<0,\quad \frac{\partial f}{\partial y}=\frac{-1}{\sqrt{x^2-4yz}}<0,$$

$$\frac{\partial g}{\partial y}=\frac{1}{\sqrt{x^2-4yz}}>0,\quad \frac{\partial f}{\partial z}=\frac{-x(x+\sqrt{x^2-4yz})}{2z^2\sqrt{x^2-4yz}}<0,\quad \frac{\partial g}{\partial z}=\frac{x(x-\sqrt{x^2-4yz})}{2z^2\sqrt{x^2-4yz}}>0.$$

By the relationship of the derivative and the monotonicity of differentiable functions, the above assertions obviously hold. ∎

Lemma 3. *Assume that (H_1) and (H_2) hold, then we have the following inequalities:*

$$0 < l_i^- < A_i^- < A_i^+ < l_i^+,\quad i = 1, 2, \dots, n.$$

Proof. It is easy to see that

$$\begin{aligned}
&a_1^M \ge a_1^l > a_1^l - c_{12}^M l_2^+ > 0,\quad 0 < b_1^l \le b_1^M,\quad 0 < h_1^l \le h_1^M,\\
&a_i^M + c_{i,\,i-1}^M l_{i-1}^+ > a_i^M \ge a_i^l > a_i^l - c_{i,\,i+1}^M l_{i+1}^+ > 0,\\
&0 < b_i^l \le b_i^M,\quad 0 < h_i^l \le h_i^M,\quad i = 2, 3, \dots, n-1,\\
&a_n^M + c_{n,\,n-1}^M l_{n-1}^+ > a_n^M \ge a_n^l,\quad 0 < b_n^l \le b_n^M,\quad 0 < h_n^l \le h_n^M.
\end{aligned}$$

By using assumptions $(H_1), (H_2)$, Lemma 2 and expressions of $A_i^{\pm}$ and $l_i^{\pm}$, we have

$$\begin{aligned}
0 < l_1^- &= g(a_1^M, b_1^l, h_1^l) < g(a_1^l - c_{12}^M l_2^+, b_1^M, h_1^M) = A_1^-\\
&< A_1^+ = f(a_1^l - c_{12}^M l_2^+, b_1^M, h_1^M) < f(a_1^M, b_1^l, h_1^l) = l_1^+,
\end{aligned}$$

$$0 < l_i^- = g(a_i^M + c_{i,\,i-1}^M l_{i-1}^+, b_i^l, h_i^l) < g(a_i^l - c_{i,\,i+1}^M l_{i+1}^+, b_i^M, h_i^M) = A_i^-$$
$$< A_i^+ = f(a_1^l - c_{i,\,i+1}^M l_{i+1}^+, b_i^M, h_i^M) < f(a_i^M + c_{i,\,i-1}^M l_{i-1}^+, b_i^l, h_i^l) = l_i^+,$$
$$0 < l_n^- = g(a_n^M + c_{n,\,n-1}^M l_{n-1}^+, b_n^l, h_n^l) < g(a_n^l, b_n^M, h_n^M) = A_n^-$$
$$< A_n^+ = f(a_n^l, b_n^M, h_n^M) < f(a_n^M + c_{n,\,n-1} l_{n-1}^+, b_n^l, h_n^l) = l_n^+,$$

where $i = 2, 3, \ldots, n-1$. Thus, we have $0<l_i^-<A_i^-<A_i^+<l_i^+, i=1, 2, \ldots, n$. The proof of Lemma 3 is complete. ∎

Theorem 1. *Assume that assumptions* (H) *hold. Then system* (1.1) *has at least* 2^n *positive* ω*-periodic solutions.*

Proof. By making the substitution

$$x_i(t) = \exp\big\{u_i(t)\big\}, \quad i = 1, 2, \ldots, n, \tag{2.1}$$

system (1.1) can be reformulated as

$$\begin{cases} \dot{u}_1(t) = a_1(t) - b_1(t)e^{u_1(t)} - c_{12}(t)e^{u_2(t-\tau_{12}(t))} - h_1(t)e^{-u_1(t)}, \\ \qquad \vdots \\ \dot{u}_i(t) = a_i(t) - b_i(t)e^{u_i(t)} + c_{i,\,i-1}(t)e^{u_{i-1}(t-\tau_{i,i-1}(t))} \\ \qquad -c_{i,\,i+1}(t)e^{u_{i+1}(t-\tau_{i,i+1}(t))} - h_i(t)e^{-u_i(t)}, \\ \qquad \vdots \\ \dot{u}_n(t) = a_n(t) - b_n(t)e^{u_n(t)} + c_{n,\,n-1}(t)e^{u_{n-1}(t-\tau_{n,n-1}(t))} - h_n(t)e^{-u_n(t)}, \end{cases} \tag{2.2}$$

where $i = 2, 3, \ldots, n-1$. Let

$$X = Z = \Big\{u = (u_1, u_2, \ldots, u_n)^T \in C(R, R^n) : u(t+\omega) = u(t)\Big\}$$

and define

$$\|u\| = \sum_{i=1}^{n} \max_{t\in[0,\omega]} |u_i(t)|, \quad u \in X \text{ or } Z.$$

Equipped with the above norm $\|\cdot\|$, X and Z are Banach spaces. Let

$$N(u,\lambda) = \begin{pmatrix} a_1(t) - b_1(t)e^{u_1(t)} - \lambda c_{12}(t)e^{u_2(t-\tau_{12}(t))} - h_1(t)e^{-u_1(t)} \\ \vdots \\ a_i(t) - b_i(t)e^{u_i(t)} + \lambda c_{i,i-1}(t)e^{u_{i-1}(t-\tau_{i,i-1}(t))} \\ -\lambda c_{i,i+1}(t)e^{u_{i+1}(t-\tau_{i,i+1}(t))} - h_i(t)e^{-u_i(t)} \\ \vdots \\ a_n(t) - b_n(t)e^{u_n(t)} + \lambda c_{n,n-1}(t)e^{u_{n-1}(t-\tau_{n,n-1}(t))} - h_n(t) \end{pmatrix}_{n\times 1},$$

where $i = 2, 3, \ldots, n-1$, $Lu = \dot{u} = \dfrac{du(t)}{dt}$. We put

$$Pu = \frac{1}{\omega}\int_0^\omega u(t)\,dt, \;\; u \in X, \quad Qz = \frac{1}{\omega}\int_0^\omega z(t)\,dt, \;\; z \in Z.$$

Thus, it follows that Ker $L = R^n$, Im $L = \{z \in Z : \int_0^\omega z(t)\,\mathrm{d}t = 0\}$ is closed in Z, $\dim \mathrm{Ker}\ L = n = \mathrm{codim}\ \mathrm{Im}\ L$, and P, Q are continuous projectors such that

$$\mathrm{Im}\ P = \mathrm{Ker}\ L, \quad \mathrm{Ker}\ Q = \mathrm{Im}\ L = \mathrm{Im}\ (I - Q).$$

Hence, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_P : \mathrm{Im}\ L \to \mathrm{Ker}\ P \bigcap \mathrm{Dom}\ L$ is given by

$$K_P(z) = \int_0^t z(s)\,ds - \frac{1}{\omega}\int_0^\omega \int_0^s z(s)\,ds.$$

Then

$$QN(u,\lambda) = \left(\frac{1}{\omega}\int_0^\omega F_1(s,\lambda)\,ds, \ \ \ldots, \ \ \frac{1}{\omega}\int_0^\omega F_n(s,\lambda)\,ds\right)^T_{1\times n}$$

and

$$K_P(I-Q)N(u,\lambda)= \begin{pmatrix} \int_0^t F_1(s,\lambda)\,ds - \frac{1}{\omega}\int_0^\omega\int_0^t F_1(s,\lambda)\,dsdt + (\frac{1}{2}-\frac{t}{\omega})\int_0^\omega F_1(s,\lambda)\,ds \\ \vdots \\ \int_0^t F_n(s,\lambda)\,ds - \frac{1}{\omega}\int_0^\omega\int_0^t F_n(s,\lambda)\,dsdt + (\frac{1}{2}-\frac{t}{\omega})\int_0^\omega F_n(s,\lambda)\,ds \end{pmatrix}_{n\times 1},$$

where

$$F(u,\lambda) = \begin{pmatrix} a_1(s) - b_1(s)e^{u_1(s)} - \lambda c_{12}(s)e^{u_2(s-\tau_{12}(s))} - h_1(s)e^{-u_1(s)} \\ \vdots \\ a_i(s) - b_i(s)e^{u_i(s)} + \lambda c_{i,\,i-1}(s)e^{u_{i-1}(s-\tau_{i,i-1}(s))} \\ -\lambda c_{i,i+1}(s)e^{u_{i+1}(s-\tau_{i,i+1}(s))} - h_i(s)e^{-u_i(s)} \\ \vdots \\ a_n(s) - b_n(s)e^{u_n(s)} + \lambda c_{n,n-1}(s)e^{u_{n-1}(s-\tau_{n,n-1}(s))} - h_n(s) \end{pmatrix}_{n\times 1}.$$

Obviously, QN and $K_P(I-Q)N$ are continuous. It is not difficult to show that $K_P(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega \subset X$ by using the Arzela-Ascoli theorem. Moreover, $QN(\overline{\Omega})$ is clearly bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

In order to use Lemma 1, we have to find at least 2^n appropriate open bounded subsets of X. Corresponding to the operator equation $Lu = \lambda N(u,\lambda)$,

$\lambda \in (0,1)$, we have

$$\begin{cases} \dot{u}_1(t) = \lambda\big(a_1(t) - b_1(t)e^{u_1(t)} - \lambda c_{12}(t)e^{u_2(t-\tau_{12}(t))} - h_1(t)e^{-u_1(t)}\big), \\ \quad \vdots \\ \dot{u}_i(t) = \lambda\big(a_i(t) - b_i(t)e^{u_i(t)} + \lambda c_{i,i-1}(t)e^{u_{i-1}(t-\tau_{i,i-1}(t)))} \\ \qquad - \lambda c_{i,i+1}(t)e^{u_{i+1}(t-\tau_{i,i+1}(t)))} - h_i(t)e^{-u_i(t)}\big), \\ \quad \vdots \\ \dot{u}_n(t) = \lambda\big(a_n(t) - b_n(t)e^{u_n(t)} + \lambda c_{n,n-1}(t)e^{u_{n-1}(t-\tau_{n,n-1}(t)))} - h_n(t)e^{-u_n(t)}\big), \end{cases} \tag{2.3}$$

where $i = 2, 3, \dots, n-1$. Assume that $u \in X$ is an ω-periodic solution of system (2.3) for some $\lambda \in (0,1)$. Then there exist $\xi_i, \eta_i \in [0,\omega]$ such that

$$u_i(\xi_i) = \max_{t\in[0,\omega]} u_i(t), u_i(\eta_i) = \min_{t\in[0,\omega]} u_i(t), \quad i = 1, 2, \dots, n.$$

It is clear that $\dot{u}_i(\xi_i) = 0$, $\dot{u}_i(\eta_i) = 0$, $i = 1, 2, \dots, n$. From this and (2.3), we have

$$\begin{cases} a_1(\xi_1) - b_1(\xi_1)e^{u_1(\xi_1)} - \lambda c_{12}(\xi_1)e^{u_2(\xi_1-\tau_{12}(\xi_1)))} - h_1(\xi_1)e^{-u_1(\xi_1)} = 0, \\ \quad \vdots \\ a_i(t) - b_i(\xi_i)e^{u_i(\xi_i)} + \lambda c_{i,i-1}(\xi_i)e^{u_{i-1}(\xi_i-\tau_{i,i-1}(\xi_i))} \\ - \lambda c_{i,i+1}(\xi_i)e^{u_{i+1}(\xi_i-\tau_{i,i+1}(\xi_i))} - h_i(\xi_i)e^{-u_i(\xi_i)} = 0, \\ \quad \vdots \\ a_n(t) - b_n(\xi_n)e^{u_n(\xi_n)} + \lambda c_{n,n-1}(\xi_n)e^{u_{n-1}(\xi_n)} - h_n(\xi_n)e^{-u_n(\xi_n-\tau_{n,n-1}(\xi_n))} = 0 \end{cases} \tag{2.4}$$

and

$$\begin{cases} a_1(\eta_1) - b_1(\eta_1)e^{u_1(\eta_1)} - \lambda c_{12}(\eta_1)e^{u_2(\eta_1-\tau_{12}(\eta_1))} - h_1(\eta_1)e^{-u_1(\eta_1)} = 0, \\ \quad \vdots \\ a_i(t) - b_i(\eta_i)e^{u_i(\eta_i)} + \lambda c_{i,i-1}(\eta_i)e^{u_{i-1}(\eta_i-\tau_{i,i-1}(\eta_i))} \\ - \lambda c_{i,\,i+1}(\eta_i)e^{u_{i+1}(\eta_i-\tau_{i,i+1}(\eta_i))} - h_i(\eta_i)e^{-u_i(\eta_i)} = 0, \\ \quad \vdots \\ a_n(t) - b_n(\eta_n)e^{u_n(\eta_n)} + \lambda c_{n,n-1}(\eta_n)e^{u_{n-1}(\eta_n-\tau_{n,n-1}(\eta_n))} - h_n(\eta_n)e^{-u_n(\eta_n)} = 0, \end{cases} \tag{2.5}$$

where $i = 2, 3, \dots, n-1$. On the one hand, according to (2.4), we have

$$\begin{aligned} b_1^l e^{2u_1(\xi_1)} - a_1^M e^{u_1(\xi_1)} + h_1^l &\le b_1(\xi_1)e^{2u_1(\xi_1)} - a_1(\xi_1)e^{u_1(\xi_1)} + h_1(\xi_1) \\ &= -\lambda c_{12}(\xi_1)e^{u_1(\xi_1)+u_2(\xi_1-\tau_{12}(\xi_1))} < 0, \end{aligned}$$

namely, $b_1^l e^{2u_1(\xi_1)} - a_1^M e^{u_1(\xi_1)} + h_1^l < 0$, which implies that

$$\ln l_1^- < u_1(\xi_1) < \ln l_1^+, \tag{2.6}$$

$$\begin{aligned} b_2^l e^{2u_2(\xi_2)} + h_2^l &< b_2(\xi_2)e^{2u_2(\xi_2)} + \lambda c_{23}(\xi_2)e^{u_2(\xi_2)+u_3(\xi_2-\tau_{23}(\xi_2))} + h_2(\xi_2) \\ &= a_2(t)e^{u_2(\xi_2)} + \lambda c_{21}(\xi_2)e^{u_2(\xi_2)+u_1(\xi_2-\tau_{21}(\xi_2))} < \big(a_2^M + c_{21}^M l_1^+\big)e^{u_2(\xi_2)}, \end{aligned}$$

that is $b_2^l e^{2u_2(\xi_2)} - \big(a_2^M + c_{21}^M l_1^+\big)e^{u_2(\xi_2)} + h_2^l < 0$, which implies that

$$\ln l_2^- < u_2(\xi_2) < \ln l_2^+. \tag{2.7}$$

By deducing for $i = 3, 4, \dots, n$, we obtain

$$\begin{aligned} b_i^l e^{2u_i(\xi_i)} + h_i^l &< b_i(\xi_i)e^{2u_i(\xi_i)} + \lambda c_{i,i+1}(\xi_i)e^{u_i(\xi_i)+u_{i+1}(\xi_i-\tau_{i,i+1}(\xi_i))} + h_i(t) \\ &= a_i(t)e^{u_i(\xi_i)} + \lambda c_{i,i-1}(\xi_i)e^{u_i(\xi_i)+u_{i-1}(\xi_i-\tau_{i,i-1}(\xi_i))} < \big(a_i^M + c_{i,i-1}^M l_{i-1}^+\big)e^{u_i(\xi_i)}, \end{aligned}$$

namely, $b_i^l e^{2u_i(\xi_i)} - \big(a_i^M + c_{i,\,i-1}^M l_{i-1}^+\big)e^{u_i(\xi_i)} + h_i^l < 0$, which implies that

$$\ln l_i^- < u_i(\xi_i) < \ln l_i^+, \quad i = 3, 4, \dots, n. \tag{2.8}$$

In view of (2.6), (2.7) and (2.8), we have

$$\ln l_i^- < u_i(\xi_i) < \ln l_i^+, \ \ i = 1, 2, \dots, n. \tag{2.9}$$

From (2.5), one can analogously obtain

$$\ln l_i^- < u_i(\eta_i) < \ln l_i^+, \ \ i = 1, 2, \dots, n. \tag{2.10}$$

By (2.9) and (2.10), we get

$$\ln l_i^- < u_i(\eta_i) < u_i(\xi_i) < \ln l_i^+, \ i = 1, 2, \dots, n. \tag{2.11}$$

On the other hand, in view of (2.4), we have

$$\begin{aligned} -b_1^M e^{2u_1(\xi_1)} + a_1^l e^{u_1(\xi_1)} - h_1^M &\le -b_1(\xi_1)e^{2u_1(\xi_1)} + a_1(\xi_1)e^{u_1(\xi_1)} - h_1(\xi_1) \\ &= \lambda c_{12}(\xi_1)e^{u_1(\xi_1)+u_2(\xi_1-\tau_{12}(\xi_1))} < c_{12}^M l_2^+ e^{u_1(\xi_1)}, \end{aligned}$$

namely, $b_1^M e^{2u_1(\xi_1)} - \big(a_1^l - c_{12}^M l_2^+\big)e^{u_1(\xi_1)} + h_1^M > 0$, which implies that

$$\ln A_1^+ < u_1(\xi_1) \ \text{ or } \ u_1(\xi_1) < \ln A_1^-, \tag{2.12}$$

$$\begin{aligned} &- b_2^M e^{2u_2(\xi_2)} + a_2^l e^{u_2(\xi_2)} - h_2^M < -b_2(\xi_2)e^{2u_2(\xi_2)} + a_2(\xi_2)e^{u_2(\xi_2)} + \lambda c_{21}(t) \\ &\times e^{u_1(\xi_2-\tau_{21}(\xi_2))+u_2(\xi_2)} - h_2(\xi_2)\lambda c_{23}(\xi_2)e^{u_2(\xi_2)+u_3(\xi_2-\tau_{23}(\xi_2))} < c_{23}^M l_3^+ e^{u_2(\xi_2)}, \end{aligned}$$

that is $b_2^M e^{2u_2(\xi_2)} - \big(a_2^l - c_{21}^M l_3^+\big)e^{u_2(\xi_2)} + h_2^M > 0$, which implies that

$$\ln A_2^+ < u_2(\xi_2) \ \text{ or } \ u_2(\xi_2) < \ln A_2^-. \tag{2.13}$$

By deducing for $i = 3, 4, \dots, n-1$, we obtain

$$\begin{aligned} - b_i^M e^{2u_i(\xi_i)} + a_i^l e^{u_i(\xi_i)} - h_i^M &< -b_i(\xi_i)e^{2u_i(\xi_i)} + a_i(\xi_i)e^{u_i(\xi_i)} \\ &= +\lambda c_{i,\,i-1}(t)e^{u_{i-1}(\xi_i-\tau_{i,i-1}(\xi_i))+u_i(\xi_i)} - h_i(\xi_i) \\ &= \lambda c_{i,i+1}(\xi_i)e^{u_i(\xi_i)+u_{i+1}(\xi_i-\tau_{i,i+1}(\xi_i))} < c_{i,\,i+1}^M l_{i,\,i+1}^+ e^{u_i(\xi_i)}, \end{aligned}$$

that is, $b_i^M e^{2u_i(\xi_i)} - \big(a_i^l - c_{i,\,i+1}^M l_{i+1}^+\big)e^{u_i(\xi_i)} + h_i^M > 0$, which implies that

$$\ln A_i^+ < u_i(\xi_i) \ \text{ or } \ u_i(\xi_i) < \ln A_i^-, \ i = 3, 4, \ldots, n-1, \tag{2.14}$$

$$0 = -b_n(\xi_n)e^{2u_n(\xi_n)} + a_n(\xi_n)e^{u_n(\xi_n)} + \lambda c_{n,n-1}(t)e^{u_{n-1}(\xi_n - \tau_{n,n-1}(\xi_n)) + u_n(\xi_n)}$$
$$- h_n(\xi_n) > -b_n^M e^{2u_n(\xi_n)} + a_n^l e^{u_n(\xi_n)} - h_n^M,$$

namely, $b_n^M e^{2u_n(\xi_n)} - a_n^l e^{u_n(\xi_n)} + h_n^M > 0$, which implies that

$$\ln A_n^+ < u_n(\xi_n) \ \text{ or } \ u_n(\xi_n) < \ln A_n^-. \tag{2.15}$$

In view of (2.12)–(2.15), we have

$$\ln A_i^+ < u_i(\xi_i) \ \text{ or } \ u_i(\xi_i) < \ln A_i^-, \ \ i = 1, 2, \ldots, n. \tag{2.16}$$

From (2.5), one can analogously obtain

$$\ln A_i^+ < u_i(\eta_i) \ \text{ or } \ u_i(\eta_i) < \ln A_i^-, \ \ i = 1, 2, \ldots, n. \tag{2.17}$$

By (2.11), (2.16), (2.17) and Lemma 3, we get for $i = 1, 2, \ldots, n$:

$$\ln A_i^+ < u_i(\eta_i) < u_i(\xi_i) < \ln l_i^+ \ \text{ or } \ \ln l_i^- < u_i(\eta_i) < u_i(\xi_i) < \ln A_i^-,$$

which imply that, for all $t \in R$,

$$\ln A_i^+ < u_i(t) < \ln l_i^+ \ \text{ or } \ \ln l_i^- < u_i(t) < \ln A_i^-, \ \ i = 1, 2, \ldots, n.$$

For convenience, we denote

$$G_i = \Big(\ln l_i^-,\ \ln A_i^-\Big),\ H_i = \Big(\ln A_i^+,\ \ln l_i^+\Big),\ i = 1, 2, \ldots, n.$$

Clearly, $l_i^\pm (i = 1, 2, \ldots, n)$ and $A_i^\pm (i = 1, 2, \ldots, n)$ are independent of λ. For each $i = 1, 2, \ldots, n$, we choose an interval between two intervals G_i and H_i, and denote it as Δ_i, then define the set

$$\Big\{u = (u_1, u_2, \ldots, u_n)^T \in X : u_i(t) \in \Delta_i, t \in R, i = 1, 2, \ldots, n\Big\}.$$

Obviously, the number of the above sets is 2^n. We denote these sets as $\Omega_k, k = 1, 2, \ldots, 2^n$. They are bounded open subsets of $X, \Omega_i \cap \Omega_j = \phi, i \neq j$. Thus $\Omega_k (k = 1, 2, \ldots, 2^n)$ satisfies the requirement (a) in Lemma 1.

Now we show that (b) of Lemma 1 holds, i.e., we prove that when $u \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap R^n$, then $QN(u, 0) \neq (0, 0, \ldots, 0)^T$, $k = 1, 2, \ldots, 2^n$. If it is not true, then $u = (u_1, u_2, \ldots, u_n)^T \in \partial\Omega_k \cap \operatorname{Ker} L = \partial\Omega_k \cap R^n, k = 1, 2, \ldots, 2^n$, satisfies

$$\int_0^\omega a_i(t)\,dt - \int_0^\omega b_i(t)e^{u_i}\,dt - \int_0^\omega h_i(t)e^{-u_i}\,dt = 0, \ \ i = 1, 2, \ldots, n.$$

It follows from the mean value theorem of calculus that there exist n points $t_i (i = 1, 2, \ldots, n)$ such that

$$a_i(t_i) - b_i(t_i)e^{u_i} - h_i(t_i)e^{-u_i} = 0, \ i = 1, 2, \ldots, n. \tag{2.18}$$

By (2.18), we have

$$u_i^{\pm} = \ln\Big[\frac{a_i(t_i) \pm \sqrt{(a_i(t_i))^2 - 4b_i(t_i)h_i(t_i)}}{2b_i(t_i)}\Big],\ i = 1, 2, \ldots, n.$$

Similar to the proof of Lemma 3, we obtain

$$\ln l_i^- < u_i^- < \ln A_i^- < \ln A_i^+ < u_i^+ < \ln l_i^+,\ i = 1, 2, \ldots, n.$$

Then u belongs to one of $\Omega_k \cap R^n, k = 1, 2, \ldots, 2^n$. This contradicts the fact that $u \in \partial\Omega_k \cap R^n, k = 1, 2, \ldots, 2^n$. This proves that (b) in Lemma 1 holds. Finally, we show that (c) in Lemma 1 holds. Note that the system of algebraic equations:

$$a_i(t_i) - b_i(t_i)e^{x_i^*} - h_i(t_i)e^{-x_i^*} = 0,\ i = 1, 2, \ldots, n$$

has 2^n distinct solutions since (H_1) and (H_2) hold,

$$(x_1^*, x_2^*, \ldots, x_n^*) = (\ln \hat{x}_1, \ln \hat{x}_2, \ldots, \ln \hat{x}_n),$$

where

$$x_i^{\pm} = \frac{a_i(t_i) \pm \sqrt{(a_i(t_i))^2 - 4b_i(t_i)h_i(t_i)}}{2b_i(t_i)}, \hat{x}_i = x_i^- \text{ or } \hat{x}_i = x_i^+, i = 1, 2, \ldots, n.$$

Similar to the proof of Lemma 3, it is easy to verify that

$$\ln l_i^- < \ln x_i^- < \ln A_i^- < \ln A_i^+ < \ln x_i^+ < \ln l_i^+,\ i = 1, 2, \ldots, n.$$

Therefore, $(x_1^*, x_2^*, \ldots, x_n^*)$ uniquely belongs to the corresponding Ω_k. Since $\operatorname{Ker} L = \operatorname{Im} Q$, we can take $J = I$. A direct computation gives, for $k = 1, 2, \ldots, 2^n$,

$$\deg\big\{JQN(u, 0), \Omega_k \cap \operatorname{Ker} L, (0, 0, \ldots, 0)^T\big\} = \operatorname{sign}\Big[\prod_{i=1}^{n}\Big(-b_i(t_i)x_i^* + \frac{h_i(t_i)}{x_i^*}\Big)\Big].$$

Since $a_i(t_i) - b_i(t_i)x_i^* - h_i(t_i)/x_i^* = 0$, $i = 1, 2, \ldots, n$, then for $k = 1, 2, \ldots, 2^n$,

$$\deg\big\{JQN(u, 0), \Omega_k \cap \operatorname{Ker} L, (0, 0, \ldots, 0)^T\big\} = \operatorname{sign}\Big[\prod_{i=1}^{n}\big(a_i(t_i) - 2b_i(t_i)x_i^*\big)\Big] = \pm 1.$$

So far, we have proved that $\Omega_k (k = 1, 2, \ldots, 2^n)$ satisfies all the assumptions in Lemma 1. Hence, system (2.2) has at least 2^n different ω-periodic solutions. Thus by (2.1), system (1.1) has at least 2^n different positive ω-periodic solutions. This completes the proof of Theorem 1. ∎

Remark 1. Set $c_{1,0}(t) \equiv 0$ and $c_{n,n+1}(t) \equiv 0$, then

$$l_1^{\pm} = \frac{a_1^M \pm \sqrt{(a_1^M)^2 - 4b_1^l h_1^l}}{2b_1^l}, \quad A_n^{\pm} = \frac{a_n^l \pm \sqrt{(a_n^l)^2 - 4b_1^l h_1^l}}{2b_1^l},$$

$$A_i^{\pm} = \frac{(a_i^l - c_{i,i+1}^M l_{i+1}^+) \pm \sqrt{(a_i^l - c_{i,i+1}^M l_{i+1}^+)^2 - 4b_i^M h_i^M}}{2b_i^M}$$

and

$$l_i^{\pm} = \frac{(a_i^M + c_{i,i-1}^M l_{i-1}^+) \pm \sqrt{(a_i^M + c_{i,i-1}^M l_{i-1}^+)^2 - 4b_i^l h_i^l}}{2b_i^l}$$

can be unified as

$$A_i^{\pm} = \frac{(a_i^l - c_{i,i+1}^M l_{i+1}^+) \pm \sqrt{(a_i^l - c_{i,i+1}^M l_{i+1}^+)^2 - 4b_i^M h_i^M}}{2b_i^M},$$
$$l_i^{\pm} = \frac{(a_i^M + c_{i,i-1}^M l_{i-1}^+) \pm \sqrt{(a_i^M + c_{i,i-1}^M l_{i-1}^+)^2 - 4b_i^l h_i^l}}{2b_i^l}, \; i = 1, 2, \ldots, n.$$

Remark 2. Two species Lotka-Volterra prey-predator system with harvesting terms is the special case of (1.1) when $n = 2$.

For system (1.1), let us assume that $c_{i,i-1}(t) \geq 0$, $(i = 2, 3, \ldots, n)$ and $c_{j,j+1}(t) \geq 0$, $(j = 1, 2, \ldots, n-1)$ and $a_i(t) > 0$, $b_i(t) > 0$, $h_i(t) > 0$ are continuous periodic functions, then similar to the proof of Theorem 1, one can prove the following theorem.

Theorem 2. *Assume that* (H) *holds. Then system* (1.1) *has at least* 2^n *positive* ω*-periodic solutions.*

Remark 3. In Theorem 2, $c_{i-1,\,i}(t) = 0$ means that the ith species does not prey the $(i-1)$-th species. That is to say, there is no relationship between the ith species and the $(i-1)$-th species.

3 Illustrative Examples

Example 1. Consider the following three species unidirectional food-chain with harvesting terms:

$$\begin{cases} \dot{x}(t) = x(t)\Big(3 + \sin t - \dfrac{4 + \sin t}{10} x(t) - c_{12}(t) y(t - \tau_{12}(t))\Big) - \dfrac{9 + \cos t}{20}, \\ \dot{y}(t) = y(t)\Big(3 + \cos t - \dfrac{5 + \cos t}{10} y(t) + c_{21}(t) x(t - \tau_{21}(t)) \\ \qquad - c_{23}(t) z(t - \tau_{23}(t))\Big) - \dfrac{2 + \cos t}{5}, \\ \dot{z}(t) = z(t)\Big(3 + \sin 2t - \dfrac{8 + \sin 2t}{10} z(t) + c_{32}(t) y(t - \tau_{32}(t))\Big) - \dfrac{8 + \cos 2t}{10}, \end{cases} \tag{3.1}$$

where $\tau_{12}, \tau_{21}, \tau_{23}, \tau_{32}$ are positive 2π-periodic functions. In this case,

$$a_1(t) = 3 + \sin t,\; b_1(t) = \frac{4 + \sin t}{10},\; h_1(t) = \frac{9 + \cos t}{20},$$
$$a_2(t) = 3 + \cos t,\; b_2(t) = \frac{5 + \cos t}{10},\; h_2(t) = \frac{2 + \cos t}{5},$$
$$a_3(t) = 3 + \sin 2t,\; b_3(t) = \frac{8 + \sin 2t}{10} \text{ and } h_3(t) = \frac{8 + \cos 2t}{10}.$$

Since

$$l_1^{\pm} = \frac{a_1^M \pm \sqrt{(a_1^M)^2 - 4b_1^l h_1^l}}{2b_1^l} = \frac{20 \pm 2\sqrt{97}}{3},$$

take $c_{21}(t) = c_{21}(t+2\pi) > 0$ such that $c_{21}^M = 1/l_1^+$, then we have

$$l_2^{\pm} = \frac{\left(a_2^M + c_{21}^M l_1^+\right) \pm \sqrt{\left(a_2^M + c_{21}^M l_1^+\right)^2 - 4b_2^l h_2^l}}{2b_2^l} = \frac{25 \pm \sqrt{617}}{4}.$$

Take $c_{32}(t) = c_{32}(t+2\pi) > 0$ such that $c_{32}^M = 1/l_2^+$, then

$$l_3^{\pm} = \frac{\left(a_3^M + c_{32}^M l_2^+\right) \pm \sqrt{\left(a_3^M + c_{32}^M l_2^+\right)^2 - 4b_3^l h_3^l}}{2b_3^l} = \frac{25 \pm 24}{7}.$$

Take $c_{12}(t) = c_{12}(t+2\pi) > 0$, $c_{23}(t) = c_{23}(t+2\pi) > 0$ such that $c_{12}^M = 1/2l_2^+$, $c_{23}^M = 1/5l_3^+$, then

$$\begin{aligned}
&a_1^l - c_{12}^M l_2^+ = \frac{3}{2} > 1 = 2\sqrt{b_1^M h_1^M}, \quad a_2^l - c_{23}^M l_3^+ = \frac{9}{5} > \frac{6}{5} = 2\sqrt{b_2^M h_2^M},\\
&a_3^l = 2 > \frac{18}{10} = 2\sqrt{b_3^M h_3^M}.
\end{aligned}$$

Therefore, all conditions of Theorem 1 are satisfied. By Theorem 1, system (3.1) has at least eight positive 2π-periodic solutions.

Example 2. Consider the following four species unidirectional food-chain with harvesting terms:

$$\begin{cases}
\dot{x}(t) = x(t)\Big(3 + \sin t - \dfrac{4+\sin t}{10}x(t) - c_{12}(t)y(t-\tau_{12}(t))\Big) - \dfrac{9+\cos t}{20},\\
\dot{y}(t) = y(t)\Big(3 + \cos t - \dfrac{5+\cos t}{10}y(t) + c_{21}(t)x(t-\tau_{21}(t))\Big) - \dfrac{2+\cos t}{5},\\
\dot{z}(t) = z(t)\Big(3 + \sin 2t - \dfrac{8+\sin 2t}{10}z(t) - c_{34}(t)u(t-\tau_{34}(t))\Big) - \dfrac{8+\cos 2t}{10},\\
\dot{u}(t) = u(t)\Big(3 + \cos 2t - \dfrac{4+\cos 2t}{15}u(t) + c_{43}(t)z(t-\tau_{43}(t))\Big) - \dfrac{4+\sin 2t}{15},
\end{cases} \tag{3.2}$$

where $\tau_{12}, \tau_{21}, \tau_{34}, \tau_{43}$ are positive 2π-periodic functions. In this case,

$$\begin{aligned}
&a_1(t)=3+\sin t,\ b_1(t)=\frac{4+\sin t}{10},\ h_1(t)=\frac{9+\cos t}{20},\\
&a_2(t)=3+\cos t,\ b_2(t)=\frac{5+\cos t}{10},\ c_{21}(t)=c_{21}(t+2\pi)>0,\ h_2(t)=\frac{2+\cos t}{5},\\
&a_3(t)=3+\sin 2t,\ b_3(t)=\frac{8+\sin 2t}{10},\ h_3(t)=\frac{8+\cos 2t}{10},\\
&a_4(t)=3+\cos 2t,\ b_4(t)=\frac{4+\cos 2t}{15},\ h_4(t)=\frac{4+\sin 2t}{15} \text{ and } c_{23}(t)=c_{32}(t)\equiv 0.
\end{aligned}$$

Since

$$l_1^{\pm} = \frac{a_1^M \pm \sqrt{(a_1^M)^2 - 4b_1^l h_1^l}}{2b_1^l} = \frac{20 \pm 2\sqrt{97}}{3},$$
$$l_3^{\pm} = \frac{a_3^M \pm \sqrt{(a_3^M)^2 - 4b_3^l h_3^l}}{2b_3^l} = \frac{20 \pm \sqrt{351}}{7},$$

take $c_{21}(t) = c_{21}(t+2\pi) > 0,\ c_{43}(t) = c_{43}(t+2\pi) > 0$ such that $c_{21}^M = 1/l_1^+$, $c_{43}^M = 1/l_3^+$, then we have

$$l_2^{\pm} = \frac{\left(a_2^M + c_{21}^M l_1^+\right) \pm \sqrt{\left(a_2^M + c_{21}^M l_1^+\right)^2 - 4b_2^l h_2^l}}{2b_2^l} = \frac{25 \pm \sqrt{617}}{4},$$
$$l_4^{\pm} = \frac{\left(a_4^M + c_{43}^M l_3^+\right) \pm \sqrt{\left(a_4^M + c_{43}^M l_1^+\right)^2 - 4b_4^l h_4^l}}{2b_4^l} = \frac{25 \pm 3\sqrt{69}}{2}.$$

Take $c_{12}(t) = c_{12}(t+2\pi) > 0$, $c_{34}(t) = c_{34}(t+2\pi) > 0$ such that $c_{12}^M = 1/2l_2^+$, $c_{34}^M = 1/6l_4^+$, then

$$a_1^l - c_{12}^M l_2^+ = \frac{3}{2} > 1 = 2\sqrt{b_1^M h_1^M}, \quad a_2^l = 2 > \frac{6}{5} = 2\sqrt{b_2^M h_2^M},$$
$$a_3^l - c_{34}^M l_4^+ = \frac{11}{6} > \frac{18}{10} = 2\sqrt{b_3^M h_3^M}, \quad a_4^l = 2 > \frac{2}{3} = 2\sqrt{b_4^M h_4^M}.$$

Therefore, all conditions of Theorem 2 are satisfied. By Theorem 2, system (3.2) has at least sixteen positive 2π-periodic solutions.

4 Discussions

In this paper, by using Mawhin's continuation theorem of coincidence degree theory, we establish the existence of 2^n positive periodic solutions for n species non-autonomous Lotka-Volterra unidirectional food chains with harvesting terms. From the proof of Theorem 1, we can see that if the harvesting terms $h_i(t) \equiv 0$, $i = 1, 2, \ldots, n$, system (1.1) has at least one positive periodic solution, but we could not conclude that system (1.1) has at least 2^n positive periodic solutions because we could not construct Ω_i, $i = 1, 2, \ldots, 2^n$ satisfying $\Omega_i \cap \Omega_j = \phi$. Therefore, adding the harvesting terms to population models can make biological species to take on multiple periodic change regulations and have multiple local stable periodic phenomena.

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