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# Bright and Dark Soliton Solutions of the $(2 + 1)$-Dimensional Evolution Equations

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**Abstract.** In this paper, we obtained the 1-soliton solutions of the (2+1)-dimensional Boussinesq equation and the Camassa–Holm–KP equation. By using a solitary wave ansatz in the form of $\mathrm{sech}^p$ function, we obtain exact bright soliton solutions and another wave ansatz in the form of $\tanh^p$ function we obtain exact dark soliton solutions for these equations. The physical parameters in the soliton solutions are obtained nonlinear equations with constant coefficients.

**Keywords:** exact solutions, bright and dark solitons, Boussinesq equation, Camassa–Holm–KP equation.

**AMS Subject Classification:** 35C08; 35Q51; 35Q68; 37K40.

## 1 Introduction

The nonlinear evolution equations (NEEs) with a nonlinear source arise in many scientific applications such as mathematical biology, diffusion process, plasma physics, optical fibers, neural physics, solid state physics, chemical reactions and mechanics of porous media. It is well known that wave phenomena of optical fibers and nonlinear dispersive media are modeled by dark shaped $\tanh^p$ solutions or by bright shaped $\mathrm{sech}^p$ solutions. Nonlinear evolution equations are difficult to solve and give rise to interesting phenomena such as chaos. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are vital importance in nonlinear evolution equations. In the past decades, many methods were developed for finding exact solutions of NEEs as the tanh–sech method [15], extended tanh method [8, 9], sine-cosine method [22], first integral method [10], Jacobi elliptic function method [14], $(\frac{G'}{G})$-expansion method [21] and F-expansion method [1].

Much experimentation has been done using solitons in fiber optics applications. The theory of optical solitons has made spectacular progress in the past few decades. There have been many advances made in the area of nonlinear optics [13, 18]. Solitons in photonic crystal fibers as well as diffraction Bragg gratings have been studied. In addition, theories of dispersion managed solitons,quasi-linear pulses have also been developed [7]. Dark solitons are also known as topological optical solitons in the context of nonlinear optics media [6]. It is known that dark optical solitons are more stable in presence of noise and spreads more slowly in presence of loss, in the optical communication systems, as compared to bright solitons [3, 16].

The paper is organized as follows: in Section 2, we derived the bright and dark soliton solutions of nonlinear Boussinesq equation. In Section 3, we apply the ansatz method to the Camassa- Holm- KP equation and establish many solitons solutions. In last section, we briefly make a summary to the results that we have obtained.

## 2 Boussinesq equation

We consider nonlinear $(2+1)$-dimensional Boussinesq equation is given by [11]

$$u_{tt} - u_{xx} - u_{yy} - \left(u^2\right)_{xx} - u_{xxxx} = 0. \tag{2.1}$$

Tascan et al. [17] obtained some solitons solutions and periodic solutions by using the sine–cosine method. More new double periodic and multiple soliton solutions are obtained for the generalized $(2+1)$-dimensional Boussinesq equation in [4]. Chen et al. [5] study $(2+1)$-dimensional Boussinesq equation by using the new generalized transformation in homogeneous balance method.

**The bright (non-topological) soliton solution.** The solitary wave ansatz for the bright (non-topological) 1-soliton solution of (2.1) is taken to be given by the form [2, 19]

$$u(x,t) = \lambda \operatorname{sech}^p \tau, \quad \tau = \eta_1 x + \eta_2 y - vt. \tag{2.2}$$

Here $\lambda$ is the soliton amplitude, $v$ is the soliton velocity and $\eta_i$ $(i = 1, 2)$ are the inverse width of the soliton. The unknown $p$ will be determined during the course of derivation of the solutions of equation Eq. (2.1).

Therefore from(2.2), it is possible to get

$$\begin{aligned} u_{tt} &= p^2 \lambda v^2 \operatorname{sech}^p \tau - p(p+1)\lambda v^2 \operatorname{sech}^{p+2} \tau, \\ u_{xx} &= p^2 \lambda \eta_1^2 \operatorname{sech}^p \tau - p(p+1)\lambda \eta_1^2 \operatorname{sech}^{p+2} \tau, \\ u_{yy} &= p^2 \lambda \eta_2^2 \operatorname{sech}^p \tau - p(p+1)\lambda \eta_2^2 \operatorname{sech}^{p+2} \tau, \end{aligned}$$

$$\begin{aligned}\left(u^2\right)_{xx} &= 4p^2\lambda^2\eta_1^2 \operatorname{sech}^{2p}\tau - 2p(2p+1)\lambda^2\eta_1^2 \operatorname{sech}^{2p+2}\tau,\\ u_{xxxx} &= p^4\lambda\eta_1^4 \operatorname{sech}^p\tau - 2p(p+1)\left(p^2+2p+2\right)\lambda\eta_1^4 \operatorname{sech}^{2p+2}\tau\\ &\quad + p(p+1)(p+2)(p+3)\lambda\eta_1^4 \operatorname{sech}^{p+4}\tau,\end{aligned}$$

where $\tau = \eta_1 x + \eta_2 y - vt$. Thus, substituting this ansatz into (2.1), yields the relation

$$\begin{aligned}&p^2\lambda v^2 \operatorname{sech}^p\tau - p(p+1)\lambda v^2 \operatorname{sech}^{p+2}\tau - p^2\lambda\eta_1^2 \operatorname{sech}^p\tau + p(p+1)\lambda\eta_1^2 \operatorname{sech}^{p+2}\tau\\ &\quad - p^2\lambda\eta_2^2 \operatorname{sech}^p\tau + p(p+1)\lambda\eta_2^2 \operatorname{sech}^{p+2}\tau - 4p^2\lambda^2\eta_1^2 \operatorname{sech}^{2p}\tau\\ &\quad + 2p(2p+1)\lambda^2\eta_1^2 \operatorname{sech}^{2p+2}\tau + 2p(p+1)\left(p^2+2p+2\right)\lambda\eta_1^4 \operatorname{sech}^{p+2}\tau\\ &\quad - p^4\lambda\eta_1^4 \operatorname{sech}^p\tau - p(p+1)(p+2)(p+3)\lambda\eta_1^4 \operatorname{sech}^{p+4}\tau = 0. \qquad (2.3)\end{aligned}$$

Now, from (2.3), equating the exponents $2p+2$ and $p+4$ leads to

$$2p + 2 = p + 4$$

so that $p = 2$. From (2.3), setting the coefficients of $\operatorname{sech}^{2p+2}\tau$ and $\operatorname{sech}^{p+4}\tau$ terms to zero we obtain

$$20\lambda^2\eta_1^2 - 120\lambda\eta_1^4 = 0,$$

which gives after some calculations, we have $\lambda = 6\eta_1^2$. We find from setting the coefficients of $\operatorname{sech}^{2p}\tau$ and $\operatorname{sech}^{p+2}\tau$ terms in Eq. (2.3) to zero:

$$-6\lambda v^2 + 6\lambda\eta_1^2 + 6\lambda\eta_2^2 - 16\lambda^2\eta_1^2 = 0,$$

we get

$$\eta_2 = \pm\frac{1}{3}\sqrt{9v^2 - 9\eta_1^2 + 24\lambda\eta_1^2 - 180\eta_1^4}.$$

Similary, the soliton velocity $v$ is found from setting the cofficients of $\operatorname{sech}^p\tau$ terms to zero in Eq. (2.3) that

$$4\lambda v^2 - 4\lambda\eta_1^2 - 4\lambda\eta_2^2 - 16\lambda\eta_1^4 = 0,$$

also we get,

$$\eta_2 = \pm\sqrt{v^2 - \eta_1^2 - 4\eta_1^4}.$$

Thus, finally, the 1-soliton solution of (2.1) as follows:

$$u(x,y,t) = \lambda \operatorname{sech}^2(\eta_1 x + \eta_2 y - vt).$$

**The dark (topological) soliton solution.** In order to start off with the solution hypothesis, the following ansatz is assumed:

$$u(x,t) = \lambda \tanh^p\tau, \quad \tau = \eta_1 x + \eta_2 y - vt, \qquad (2.4)$$

where the parameters $\lambda$, $\eta_i$ $(i = 1, 2)$ are the free parameters and $v$ is the velocity of the soliton. The exponent $p$ is also unknown. These will be determined from (2.4) it is possible to obtain

$$\begin{aligned}
u_{tt} &= pv^2\lambda\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\},\\
u_{xx} &= \lambda p\eta_1^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\},\\
u_{yy} &= \lambda p\eta_2^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\},\\
\left(u^2\right)_{xx} &= 2p\lambda^2\eta_1^2\{(2p-1)\tanh^{2p-2}\tau - 4p\tanh^{2p}\tau + (2p+1)\tanh^{2p+2}\tau\},\\
u_{xxxx} &= p\eta_1^4\lambda\left\{\begin{array}{l}(p-1)(p-2)(p-3)\tanh^{p-4}\tau\\ -4(p-1)\left(p^2-2p+2\right)\tanh^{p-2}\tau\\ +2p\left(3p^2+5\right)\tanh^p\tau - 4\left(p+1\right)\left(p^2+2p+2\right)\tanh^{p+2}\tau\\ +(p+1)(p+2)(p+3)\tanh^{p+4}\tau\end{array}\right\},
\end{aligned}$$

where $\tau = \eta_1 x + \eta_2 y - vt$ . Substituting these equalities into (2.1), gives

$$\begin{aligned}
&pv^2\lambda\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}\\
&\quad - \lambda p\eta_1^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}\\
&\quad - \lambda p\eta_2^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}\\
&\quad - 2p\lambda^2\eta_1^2\{(2p-1)\tanh^{2p-2}\tau - 4p\tanh^{2p}\tau + (2p+1)\tanh^{2p+2}\tau\}\\
&\quad - p\eta_1^4\lambda\left\{\begin{array}{l}(p-1)(p-2)(p-3)\tanh^{p-4}\tau\\ \quad -4(p-1)\left(p^2-2p+2\right)\tanh^{p-2}\tau\\ \quad +2p\left(3p^2+5\right)\tanh^p\tau - 4(p+1)\left(p^2+2p+2\right)\tanh^{p+2}\tau\\ \quad +(p+1)(p+2)(p+3)\tanh^{p+4}\tau\end{array}\right\}\\
&= 0. \qquad (2.5)
\end{aligned}$$

Now, from (2.5) equating the exponents of $\tanh^{2p}\tau$ and $\tanh^{p+2}\tau$ gives

$$2p = p + 2,$$

which yields the following analytical condition:$p = 2$. Setting the coefficients of $\tanh^{2p+2}\tau$ and $\tanh^{p+4}\tau$ terms in Eq. (2.5) to zero, we have$\lambda = -6\eta_1^2$. Again, from (2.5) setting the coefficients of $\tanh^{p-2}\tau$ terms to zero, one obtains

$$\eta_1 = \pm\frac{1}{4}\sqrt{1 \pm \sqrt{1 - 32v^2 + 32\eta_2^2}}.$$

Setting the coefficients of $\tanh^{2p-2}\tau$ and $\tanh^p\tau$ terms in Eq. (2.5) to zero, thus the velocity of the soliton from (2.5) is given by

$$v = \pm\sqrt{-8\eta_1^4 + \eta_2^2 + \eta_1^2}.$$

Taking by the coefficients of $\tanh^{2p}\tau$ and $\tanh^{p+2}\tau$ terms in Eq. (2.5) to zero, we get

$$\eta_2 = \pm\frac{1}{3}\sqrt{9v^2 + 360\eta_1^4 - 9\eta_1^2 + 48\lambda\eta_1^2}.$$

Consequently, we can determine the dark soliton solution for the constant coefficient Boussinesq equation.as follows:

$$u(x,y,t) = \lambda \tanh^2(\eta_1 x + \eta_2 y - vt).$$

*Remark 1.* Stability of the following type of Boussinesq equations was studied by Kadomtsev and Petviashvili [12].

$$f''(\xi) + \frac{1}{2} f^2(\xi) - f(\xi) = 0.$$

Therefore, Eq. (2.1) is stable within the sense of Liapunov [25]

## 3 Camassa–Holm–KP equation

To understand the important role of dispersion in the formation of patterns in liquid drops, we considered the following water wave equations given by [23]

$$\left(u_t + 2ku_x - u_{xxt} - au^n u_x\right)_x + u_{yy} = 0, \tag{3.1}$$

which is a typical nonlinear evolution equation, where $a$ and $k$ are real constants and $n$ is called the strength of the nonlinearity. Wazwaz obtained the solitons, compactons, solitary patterns and periodic solutions for Eq. (3.1), and their analytic expressions in [23].

Now, the bright and dark soliton solution of this equation will be obtained.

**The bright (non topological) soliton solution.** To obtain the soliton solution of Eq. (3.1), the solitary wave ansatz admits the use of the assumption,

$$u(x,y,t) = \lambda \operatorname{sech}^p \tau, \tag{3.2}$$

where $\tau = \eta_1 x + \eta_2 y - vt$ which $\lambda$, $\eta$ and $v$ are constant coefficients. Respectively, here $\lambda$, $\eta$ and $v$ are the amplitude, the inverse width and the velocity of the soliton. The exponents p is unknown at this point and will be determined later. From the ansatz (3.2), we obtain:

$$\begin{aligned}
u_{xt} &= -p^2 \lambda \eta_1 v \operatorname{sech}^p \tau + p(p+1)\lambda \eta_1 v \operatorname{sech}^{p+2} \tau, \\
u_{xx} &= p^2 \lambda \eta_1^2 \operatorname{sech}^p \tau - p(p+1)\lambda \eta_1^2 \operatorname{sech}^{p+2} \tau, \\
(u_x)^2 &= \lambda^2 p^2 \eta_1^2 \operatorname{sech}^{2p} \tau - \lambda^2 p^2 \eta_1^2 \operatorname{sech}^{2p+2} \tau, \\
u_{yy} &= p^2 \lambda \eta_2^2 \operatorname{sech}^p \tau - p(p+1)\lambda \eta_2^2 \operatorname{sech}^{p+2} \tau, \\
u_{xxxt} &= -\lambda p^4 \eta_1^3 v \operatorname{sech}^p \tau + 2p(p+1)\left(p^2+2p+2\right)\lambda \eta_1^3 v \operatorname{sech}^{p+2} \tau \\
&\quad - p(p+1)(p+2)(p+3)\lambda \eta_1^3 v \operatorname{sech}^{p+4} \tau.
\end{aligned}$$

Substituting these equations into Eq. (3.1), we get

$$\begin{aligned}
&-p^2\lambda\eta_1 v\,\mathrm{sech}^p\,\tau + p(p+1)\lambda\eta_1 v\,\mathrm{sech}^{p+2}\,\tau\\
&\quad+2kp^2\lambda\eta_1^2\,\mathrm{sech}^p\,\tau - 2kp(p+1)\lambda\eta_1^2\,\mathrm{sech}^{p+2}\,\tau\\
&\quad+\lambda p^4\eta_1^3 v\,\mathrm{sech}^p\,\tau - 2p(p+1)\big(p^2+2p+2\big)\lambda\eta_1^3 v\,\mathrm{sech}^{p+2}\,\tau\\
&\quad+p(p+1)(p+2)(p+3)\lambda\eta_1^3 v\,\mathrm{sech}^{p+4}\,\tau\\
&\quad-anp^2\lambda^{n+1}\eta_1^2\,\mathrm{sech}^{pn+p}\,\tau + an\lambda^{n+1}p^2\eta_1^2\,\mathrm{sech}^{pn+p+2}\,\tau\\
&\quad-ap^2\lambda^{n+1}\eta_1^2\,\mathrm{sech}^{pn+p}\,\tau + a\lambda^{n+1}p(p+1)\eta_1^2\,\mathrm{sech}^{pn+p+2}\,\tau\\
&\quad+p^2\lambda\eta_2^2\,\mathrm{sech}^p\,\tau - p(p+1)\lambda\eta_2^2\,\mathrm{sech}^{p+2}\,\tau = 0.
\end{aligned}\tag{3.3}$$

Equating the exponents of $\mathrm{sech}^{pn+p}\,\tau$ and $\mathrm{sech}^{p+2}\,\tau$ term in Eq. (3.3), one obtains

$$pn + p = p + 2,$$

which implies $p{=}2/n$. By setting the corresponding coefficients of $\mathrm{sech}^{pn+p+2}\,\tau$ and $\mathrm{sech}^{p+4}\,\tau$ terms to zero one gets

$$24\lambda v\eta_1^3 + 4a\lambda^3\eta_1^2 = 0,$$

from which we obtain $v = -\frac{1}{6}\frac{a\lambda^2}{\eta_1}$. Setting the coefficients of $\mathrm{sech}^{pn+p}\,\tau$ and $\mathrm{sech}^{p+2}\,\tau$ terms in Eq. (3.3) to zero and we have

$$-20\lambda v\eta_1^3 - 2\lambda\eta_2^2 - 3a\lambda^3\eta_1^2 + 2\lambda\eta_1 v - 4k\lambda\eta_1^2 = 0,$$

which gives

$$\lambda = \pm\sqrt{6(\eta_2^2 + 2k\eta_1^2)/\big(a(\eta_1^2 - 1)\big)},$$

where $\lambda$ is an integration constant related to the initial pulse inverse width.

Finally, we get the bright (non topological) soliton solution for the constant coefficient Camassa–Holm–KP equation, when the above expressions of $p$, $v$ and $\lambda$ are substituted in (3.2) as:

$$u(x,y,t) = \lambda\,\mathrm{sech}^{\frac{2}{n}}(\eta_1 x + \eta_2 y - vt).$$

**The dark (topological) soliton solution.** In this section, we are interested in finding the dark solitary wave solution, as defined in [3] for the considered Camassa–Holm–KP equation (3.1). In order to construct dark soliton solutions for Eq. (3.1), we use an ansatz solution of the form [20]:

$$u(x,y,t) = \lambda\tanh^p\tau, \quad \tau = \eta_1 x + \eta_2 y - vt, \tag{3.4}$$

where $\lambda$, $\eta$ are unknown free parameters and $v$ is the velocity of the soliton, that will be determined. The exponent $p$ is also unknown. From Eq. (3.4), we have:

$$
\begin{aligned}
u_{xt} &= -p\lambda v\eta_1\big\{(p+1)\tanh^{p+2}\tau - 2p\tanh^{p}\tau + (p-1)\tanh^{p-2}\tau\big\},\\
u_{xx} &= \lambda p\eta_1^2\big\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\big\},\\
u_{xxxt} &= -\lambda p\eta_1^3 v\left\{\begin{array}{l}(p-1)(p-2)(p-3)\tanh^{p-4}\tau\\ \quad -4(p-1)\big(p^2-2p+2\big)\tanh^{p-2}\tau\\ \quad +2p\big(3p^2+5\big)\tanh^{p}\tau\\ \quad -4(p+1)\big(p^2+2p+2\big)\tanh^{p+2}\tau\\ \quad +(p+1)(p+2)(p+3)\tanh^{p+4}\tau\end{array}\right\},\\
u_x^2 &= \lambda^2p^2\eta_1^2\big\{\tanh^{2p-2}\tau - 2\tanh^{2p}\tau + \tanh^{2p+2}\tau\big\},\\
u_{yy} &= \lambda p\eta_2^2\big\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\big\},
\end{aligned}
$$

where $\tau = \eta_1 x + \eta_2 y - vt$. Substituting this equalities into Eq. (3.1), we obtain

$$
\begin{aligned}
&-p\lambda v\eta_1\big\{(p+1)\tanh^{p+2}\tau - 2p\tanh^{p}\tau + (p-1)\tanh^{p-2}\tau\big\}\\
&\quad + 2k\lambda p\eta_1^2\big\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\big\}\\
&\quad + \lambda p\eta_1^3 v\left\{\begin{array}{l}(p-1)(p-2)(p-3)\tanh^{p-4}\tau\\ \quad -4(p-1)\big(p^2-2p+2\big)\tanh^{p-2}\tau\\ \quad +2p\big(3p^2+5\big)\tanh^{p}\tau - 4(p+1)\big(p^2+2p+2\big)\tanh^{p+2}\tau\\ \quad +(p+1)(p+2)(p+3)\tanh^{p+4}\tau\end{array}\right\}\\
&\quad - an\lambda^{n+1}p^2\eta_1^2\big\{\tanh^{np+p-2}\tau - 2\tanh^{np+p}\tau + \tanh^{np+p+2}\tau\big\}\\
&-a\lambda^{n+1}p\eta_1^2\big\{(p-1)\tanh^{np+p-2}\tau - 2p\tanh^{np+p}\tau + (p+1)\tanh^{np+p+2}\tau\big\}\\
&\quad + \lambda p\eta_2^2\big\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\big\} = 0. \qquad (3.5)
\end{aligned}
$$

By equating the highest exponents of $\tanh^{np+p+2}\tau$ and $\tanh^{p+4}\tau$ terms in Eq. (3.5), one gets

$$np + p + 2 = p + 4,$$

which yields the following analytical condition:$p = 2/n$. By setting the corre-ponding coefficients of $\tanh^{pn+p+2}\tau$ and $\tanh^{p+4}\tau$ terms to zero one

$$24\lambda\eta_1^3 v - 4a\lambda^3\eta_1^2 = 0 \quad\Rightarrow\quad \lambda = \pm\sqrt{6\eta_1 v/a},$$

where $\lambda$ is an integration constant related to the initial pulse inverse width. Setting the coefficients of $\tanh^{pn+p-2}\tau$ and $\tanh^{p}\tau$ terms in Eq. (3.5) to zero we get

$$2\lambda\eta_1 v - 4k\lambda\eta_1^2 + 16\lambda\eta_1^3 v - 2a\lambda^3\eta_1^2 - 2\lambda\eta_2^2 = 0 \;\;\Rightarrow\;\; v = \frac{2k\eta_1^2 + \eta_2^2}{\eta_1(1+2\eta_1^2)}.$$

Setting the coefficients of $\tanh^{pn+p}\tau$ and $\tanh^{p+2}\tau$ terms in Eq. (3.5) to zero and

$$-2\lambda\eta_1 v + 4k\lambda\eta_1^2 - 40\lambda\eta_1^3 v + 6a\lambda^3\eta_1^2 + 2\lambda\eta_2^2 = 0, \qquad (3.6)$$

we have

$$\eta_2 = \pm\sqrt{\eta_1 v - 2k\eta_1^2 + 2\eta_1^3 v}.$$

Lastly, we can determine the dark (topological) soliton solution for the

$$u(x, y, t) = \lambda \tanh^{2/n}(\eta_1 x + \eta_2 y - vt).$$

*Remark 2.* Stability of this equation was studied by Zhang et al. [24].

## 4 Conclusions

In this paper, we have investigated the bright and dark soliton solutions of three variants of the Boussinesq and Camassa–Holm–KP equations by using the solitary wave ansatz method. Parametric conditions for the existence of the soliton solutions were found. We hope that the present solutions may be useful in further numerical analysis. Consequently, the method can be applied to nonlinear evolution equations with time-dependent coefficients and forcing term.

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