

Finite element analysis of conjugate heat transfer and non-Newtonian blood flow in a stenosed bifurcated artery

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Abstract. In the present study, a detailed numerical analysis is conducted for conjugated heat transfer in non-Newtonian blood flow in a stenosed bifurcated artery. The purpose of the present study is to investigate the coupling mechanisms between the arterial geometry, flow behavior and heat transfer under physiologically meaningful situation. The blood is simulated as power-law fluid to portray the non-Newtonian flow characteristic and the stenosed region is treated as solid region with conductivity to analyze the conjugated heat transfer effect. Nonlinear momentum and energy equations are solved using FEM. The effect of different parameter, such as Re (100-500), stenosis level and n are analyzed. Results show that an increasing stenosis severity makes the flow separation stronger and a greater pressure drop is gained and the local Nusselt number drops to 18.4%, which represents degradation of the convection heat transfer. For a rise in Re the thermal transport gets increased, which results the improvement of average Nusselt number by approximately 27% due to larger inertial effect. Shear-thinning fluid $n < 1$ produces higher gradient of velocity, which increase the heat transfer performance; inversely, the increasing resistance of the shear-thickening fluid ($n > 1$) results in lower heat transfer performance. It can be inferred from the analysis of the present study that flow separation, artery morphology and fluid rheology play an important role in convective heat transfer; this analysis can be employed in design of artificial vascular graft and blood thermal therapies etc. In future, studies involving pulsatile flow, fluid-structure interaction and the dependence of viscosity on temperature need to be performed to have more physical relevance.

Keywords: bifurcation arteries; conjugate heat transfer; inflammation; power law fluid model; FEM; stenosis.

AMS Subject Classification: 76A05; 76D05; 76M10; 80A20.

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1 Introduction

Nomenclature

Symbol	Description (SI Unit)	Symbol	Description (SI Unit)
$\vec{u}$	Velocity vector (m s^{-1})	m	Consistency index (Pa s^n)
ρ	Density of blood (kg m^{-3})	n	Power-law index (-)
p	Pressure (Pa)	I	Second invariant of strain rate tensor (s^{-2})
τ	Stress tensor (Pa)	$n < 1$	Shear-thinning fluid (-)
f	Body force per unit volume (N m^{-3})	$n = 1$	Newtonian fluid (-)
Ω	Computational domain (-)	$n > 1$	Shear-thickening fluid (-)
α	Thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)	Pr	Prandtl number (-)
D	Strain rate tensor (s^{-1})	K_r	Thermal conductivity ratio (-)
K_f	Fluid thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)	K_s	Solid thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)

Bifurcated arteries are highly prone to disturbed flow due to geometrical changes, such as branching, and presence of stenosis. These geometries contribute to complex flow features like flow separation, recirculation and oscillation of wall shear stress (WSS), which have a strong correlation with development of various cardiovascular diseases. In this regard, fluctuation of WSS contributes to alter endothelial cell functionality and leads to acceleration of atherosclerosis (plaque accumulation and consequently more blockage) in artery. Therefore, modeling of such geometries becomes extremely important in studying vascular pathology. Apart from fluid dynamics, conjugated heat transfer is equally important to determine the thermal distribution within arteries. Variation of temperature affects the viscosity and rheological behavior of blood, which consequently alters the flow resistance and overall hemodynamic. Moreover, coupled heat transfer analysis of blood with arterial wall causes another complex physical situation, as conduction through the arterial wall affects the thermal boundary condition of blood flow. Thus coupled fluid flow and heat transfer analysis is needed to depict the behavior of such biological system. Blood shows non-Newtonian behavior in low shear rate regimes. This behavior needs to be modeled appropriately as the viscosity of the fluid varies with the rate of deformation. This non-Newtonian model needs to be properly incorporated in predicting flow pattern, wall shear stress and resultant thermal distribution within artery. A simplified yet realistic non-Newtonian model such as power-law can also be employed. Several studies have investigated the influence of non-Newtonian properties on blood flow in complex geometries. For instance, Jain et al. [29] presents a finite element approach that is stable and effective in the study of non-Newtonian fluid dynamics in blood with the two intersecting channels with overlapping biconstricted inlets. This paper has been produced by the effects that flow contraction and wall shear stress (WSS) have brought.

The Casson fluid model is applied to describe blood. They believe that the bifurcation in the blood's non-Newtonian characters is a critical factor that affects the flow regimes and the distribution of shear stress. As a continuation of their work, Jain and Ismail [27] also considered the influence of magneto-hydrodynamics (MHD) on the power-law of fluids in bifurcated arteries with overlapping stenosis. In their studies Magnetic fields are intended to enhance the shear-thinning and reduce the flow resistance of non-ferritic fluids. Blood from Newton can help. Finite element analysis of biomagnetic fluid flow in a channel with overlapping stenosis. Abdullah and Ismail [1]. In particular, one resulted found that an applied magnetic field can change both the flow patterns and wall shear stress such that blood clot formation in stenosed regions is less likely to occur. They also indicate that biomagnetic forces might have a future role in treating vessel flow under disease. Zain and Ismail [26] studied Newtonian blood flow in a bifurcated artery with stenosis under the influence of Hartmann and Reynolds numbers. Under the same condition, in another study where they varied the Hartmann number (i.e., magnetic field strength) it was shown that the flow stabilization was increased with larger Hartmann numbers leading to smaller oscillatory shear stress reducing therefore complications like plaque rupture. Shahzad et al. [22] demonstrated that vessel elasticity and non-Newtonian rheology significantly influence flow patterns and pressure drop in stenosed bifurcated arteries. Zain et al. [28] reported elevated wall shear stress near stenosis and bifurcation regions, while Anand et al. [2] showed that pulsatile flow enhances oscillatory shear stress and recirculation in post-stenotic zones. Advanced computational approaches, including variational multiscale, least-squares FEM, and smoothed FEM methods, have been developed to improve the accuracy and stability of non-Newtonian blood flow simulations in complex vascular geometries [4, 5, 17]. These studies highlight the importance of incorporating realistic rheology, vessel elasticity, and robust numerical techniques in hemodynamic modeling. Rabby et al. [19] reported that wall oscillations enhance flow separation and recirculation in stenosed arteries. Advanced numerical methods, including penalty, least-squares FEM, and Galerkin least-squares approaches, have been successfully applied to simulate complex non-Newtonian blood flows [6, 23, 31], while Machado et al. [18] highlighted the influence of Herschel–Bulkley rheology on flow behavior. Magnetic fields have been shown to suppress recirculation, reduce wall shear stress, and improve flow stability in stenosed arteries [14, 24]. In addition, studies on blood-based nanofluids revealed significant effects of viscous dissipation, solute transport, and MHD on flow and heat transfer characteristics [3, 15, 30]. These findings emphasize the importance of computational modeling in hemodynamic analysis and biomedical applications. Hussain et al. [7] investigated gold nanofluid drug delivery in stenosed vessels, highlighting the role of fluid rheology in enhancing therapeutic performance. Advanced non-Newtonian models have since been used to study blood flow behavior in stenosed and aneurysmal arteries, revealing significant effects on velocity, pressure, and wall shear stress distributions [8, 12]. Non-Newtonian nanofluids have also attracted considerable attention for heat transfer enhancement, with applications involving radiative, bioconvective, and pseudoplastic flows [10, 11].

Classical rheological models, such as the Eyring–Powell fluid, have been widely applied to engineering flows [20]. Recent studies further incorporated MHD, double-diffusive convection, and realistic arterial geometries to better capture complex transport phenomena [9, 13]. Despite these advances, studies combining non-Newtonian rheology, heat transfer, and complex geometries remain limited. Therefore, the present work aims to address this gap by investigating the coupled flow and thermal behavior in stenosed configurations.

Research gap

Although the dynamics of blood flow have been studied over a long period of time, the non-Newtonian behaviour of blood in conjugate heat transfer, in bifurcated arteries, is poorly understood. The available literature is mainly based on the Newtonian approximation or simplified heat transfer processes and tends to disregard the conduction and convection processes, which play a decisive role in physiological conditions. In addition, the effect on heat distribution and flow characteristics of stenosis conductivity, and inflammatory states has not been thoroughly studied.

2 Statement of the problem

Bifurcated stenotic arteries have a complicated hemodynamic and thermal response, which affects cardiovascular well-being. This paper shall examine how non-Newtonian blood rheology, conjugate heat transfer and the severity of stenosis affect the dynamics of the flow, temperature distribution and pressure changes. The artery is represented that is a bifurcated channel, the stenosis is in the form of an overlap as shown in Figure 1. Blood is described as a non-Newtonian fluid that is based on the power-law model, which adequately captures the shear-thinning behavior of blood, which has a significant impact on viscosity and resistance to flow in diseased arteries. The fluid is also assumed to be homogeneous, incompressible, and magnetically conducting as well as the variation in density caused by the fluctuations in temperature is taken into consideration by the Boussinesq approximation. Also, the effect of a solid cholesterol plaque with inflammation are introduced so as to include the physiological relevance in the context of stenosis as opposed to the simplified arterial geometry in the past studies. Also, bifurcated structure is incorporated in this research to provide a more realistic structure to capture the impact of stenosis on the separation of flow, turbulence as well as heat transfer. Although bio-nanofluids are not directly involved, the paper looks at the process of thermal transport in blood flow and specifically focuses on the functions of conduction and convection. It is known that arterial blood flow is pulsatile under physiological conditions, but, in the present study, limitation of steady-state formulation is adopted in our analysis. To achieve a proper modelling, the governing equations continuity, momentum and energy are solved subject to relevant boundary conditions. The geometry of the outer wall of the bifurcated artery with the stenosis at its outer wall is mathematically defined as in [29]. This study seeks to fill the gaps in the literature by examining stenosis

severity and its impact on hemodynamics and thermal performance and offer information on energy loss, turbulence, and defective heat incorporation of bifurcated arteries. Figure 1, which is the visual representation of the modeled geometry, shows the coordinate system and area of stenosis.

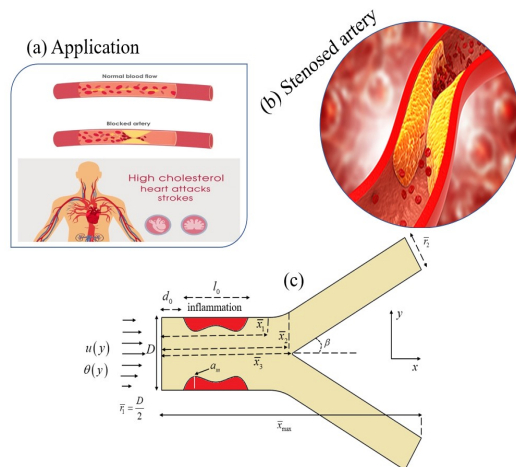


Figure 1. Schematic diagram of the problem.

2.1 Governing equations and boundary equations

We analyze the steady-state, incompressible, fully developed, and laminar flow of blood, formulated based on the principles of mass and momentum conservation. Physical sketches are drawn by using Equations (2.1) and (2.2).

$$\bar{R}_1 = \begin{cases} z_1, & 0 \leq \bar{x} \leq d_0 \text{ and } d_0 + l_0 \leq \bar{x} \leq \bar{x}_1, \\ z_1 - \frac{3a_m}{2l_0^4} \left(\begin{aligned} &11(\bar{x}-d_0)l_0^3 - 47(\bar{x}-d_0)^2l_0^2 \\ &+ 72(\bar{x}-d_0)^3l_0 - 36(\bar{x}-d_0)^4 \end{aligned} \right), & d_0 \leq \bar{x} \leq d_0 + l_0, \\ z_1 + \bar{r}_3 - \sqrt{\bar{r}_3^2 - (\bar{x} - \bar{x}_1)^2}, & \bar{x}_1 \leq \bar{x} \leq \bar{x}_2, \\ 2\bar{r}_2 \sec \beta + (\bar{x} - \bar{x}_2) \tan \beta, & \bar{x}_2 \leq \bar{x} \leq x_{\max} - z_2. \end{cases} \quad (2.1)$$

The following equations plot the inner walls.

$$\bar{R}_2 = \begin{cases} 0, & 0 \leq \bar{x} \leq \bar{x}_3, \\ \sqrt{\bar{r}_4 - (\bar{x} - (\bar{x}_3 + \bar{r}_4))^2}, & \bar{x}_3 \leq \bar{x} \leq \bar{x}_4, \\ \bar{r}_4 \cos \beta + (\bar{x} - \bar{x}_4) \tan \beta, & \bar{x}_4 \leq \bar{x} \leq x_{\max}. \end{cases} \quad (2.2)$$

The governing equations are constructed by incorporating these fundamental conservation laws.

$$\begin{aligned} \nabla \cdot \bar{\mathbf{u}} &= 0, \\ \rho \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} &= -\nabla p + \nabla \cdot (2\eta(\dot{\gamma})D) + \rho \mathbf{f}. \end{aligned} \quad (2.3)$$

Thus, this study gives notice of the fully developed laminar flow in blood which is steady and incompressible and to formulate a problem based on mass and momentum conservation. Here, (u) represents the velocity component, (ρ) is the density of blood, (τ) denotes the stress tensor, (p) corresponds to pressure and volumetric stress, (f) is the dimensional body force vector, and finally, (Ω) defines the computational domain. We adopt the generalized power-law model, a kind of generalized Newtonian fluid model, that is rheological applicable for the governing flow of blood. The significance of generalized Newtonian liquid (GNL) identified from the work of Zinani et al. [31] and Masud et al. [14], as they stated, is the expression of the stress tensor as the sum of volumetric and viscous stresses after the constitutive equation.

$$\begin{aligned}\bar{\mathbf{u}} \cdot \nabla \bar{T} &= \alpha \nabla^2 \bar{T}, \quad \text{Fluid region,} \\ 0 &= \nabla^2 \bar{T}, \quad \text{Solid region,} \\ D &= \frac{1}{2} \left(\nabla \bar{\mathbf{u}} + (\nabla \bar{\mathbf{u}})^t \right).\end{aligned}$$

The viscosity, η ($\dot{\gamma}$), is dependent on a scalar variable ($\dot{\gamma}$) representing the magnitude of the strain rate tensor. The Generalized Newtonian Liquid (GNL) model accounts for viscosity variations with shear rate, making it particularly useful for modeling shearing flows and analyzing the effects of shear-dominated behavior, including both shear-thinning and shear-thickening characteristics [31]. The viscosity function $\eta(\dot{\gamma})$, applied to Equation (2.3) based on the generalized power-law model, can be expressed as:

$$\eta(\dot{\gamma}) = -m|\langle \mathbf{I} \rangle|^{\frac{n-1}{2}},$$

where, (m) represents the parameter of consistency of the fluid, and the second invariant of the strain rate tensor, while (n) defines the power-law index. The condition under which the viscosity function, $\eta(\dot{\gamma})$, becomes constant: (μ), results in the model denoting a Newtonian fluid. The three cases of viscosity considered include shear thinning (with $n < 1$, diminishing viscosity with shear rate); Newtonian (with $n = 1$, constant viscosity); and shear thickening (with $n > 1$, increasing viscosity with shear rate). The representation of ($\mathbf{I}$) in the case of two-dimensional flow is given by.

$$\begin{aligned}\mathbf{I} &= 2 \left(\frac{\partial \bar{u}}{\partial \bar{x}} \right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{y}} \right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}} \right)^2, \\ x &= \frac{\bar{x}}{h}, \quad y = \frac{\bar{y}}{h}, \quad u = \frac{\bar{u}}{u}, \quad v = \frac{\bar{v}}{u}, \\ \theta &= \frac{T - T_c}{T_h - T_c} p = \frac{\bar{p}}{(\rho u_r)^{-2}}, \quad Re = \frac{\rho h^{-n}}{(mu)^{-n-2}}, \quad Pr = \frac{\nu}{\alpha}.\end{aligned}$$

Here, the characteristic length (h) is defined as the inlet length, whereas the reference velocity (u_r) stands for the average inflow velocity. Therefore, after

proper non-dimensionalization, the Reynolds number is obtained as:

$$\begin{aligned}\nabla \cdot \mathbf{u} &= 0, \\ \mathbf{u} \cdot \nabla \mathbf{u} - \frac{1}{Re} (\mathbf{I})^{\frac{n-1}{2}} \nabla \cdot \mathbf{D} + \nabla p &= \mathbf{f}, \\ \mathbf{u} \cdot \nabla \theta &= \frac{1}{RePr} \nabla^2 \theta, \quad 0 = \nabla^2 \theta,\end{aligned}$$

where $\mathbf{f} = \left(-Ha^2 \mathbf{v}/Re, \frac{Gr}{Re^2} \theta\right)$, representing body forces. Boundary conditions are illustrated below.

$$\begin{aligned}u = \theta &= (1 - y^2) \quad \text{and} \quad v = 0, \quad \text{at inlet,} \\ (-p\mathbf{I} + \boldsymbol{\tau})\mathbf{n} &= 0, \quad \text{at outlet,} \\ u = 0 \quad \text{and} \quad v &= 0, \quad \theta = 0 \quad \text{on } \Gamma_{\text{walls}}, \\ k_f \frac{\partial \theta}{\partial n} \Big|_{\text{fluid}} &= k_s \frac{\partial \theta}{\partial n} \Big|_{\text{solid}} \quad \text{on Stenosis,} \\ Nu &= Kr \sqrt{\left(\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2},\end{aligned}$$

where Kr is the effective thermal conductivity equations to k_s/k_f . The k_s and k_f are the thermal conductivities of solid and fluids, respectively. Thermophysical parameters of blood and plaque tissue are given in Table 1.

Table 1. Thermophysical parameters of blood and plaque tissue [16].

Parameter	Symbol	Unit	Blood	Plaque
Thermal conductivity	k	W/m.K	0.549	0.484
Specific heat capacity	C_p	J/kg.K	4390	4080
Dynamic viscosity	μ	Pa.s	0.0033	—

$$Nu_m = \int_{\Omega} Nu \partial \Omega,$$

where Ω represents inner side of stenosis of upper wall.

The viscosity and thermal conductivity of blood are treated as constant because the simulated temperature variations remain small ($\Delta T < 1^\circ\text{C}$) under physiological conditions. Experimental correlations Kim [16] show that blood viscosity changes by less than 5 % per $^\circ\text{C}$, which has a negligible effect compared with shear-rate-dependent variations captured by the power-law model. Thus, the temperature dependence of these properties is omitted without loss of generality, ensuring numerical stability while preserving physical accuracy for moderate temperature ranges.

3 Method of solution and code validation

The governing equations describing blood flow in the bifurcated channel are solved using COMSOL Multiphysics, which employs the Finite Element Method (FEM) for spatial discretization. The time-independent (steady-state) form of the governing partial differential equations (continuity, momentum, and energy equations) is discretized using the Galerkin finite element approach. A segregated solver strategy is employed to solve the coupled system of equations iteratively. Appropriate convergence criteria are set to ensure stability and accuracy.

Quantitative validation

To ensure the accuracy of the numerical model, a validation study is carried out by comparing the computed Nusselt number (Nu) with results from an established reference study. The absolute percentage error (APE) is calculated using the formula:

$$APE = \left| \frac{Nu_{\text{present}} - Nu_{\text{ref}}}{Nu_{\text{ref}}} \right| \times 100\%.$$

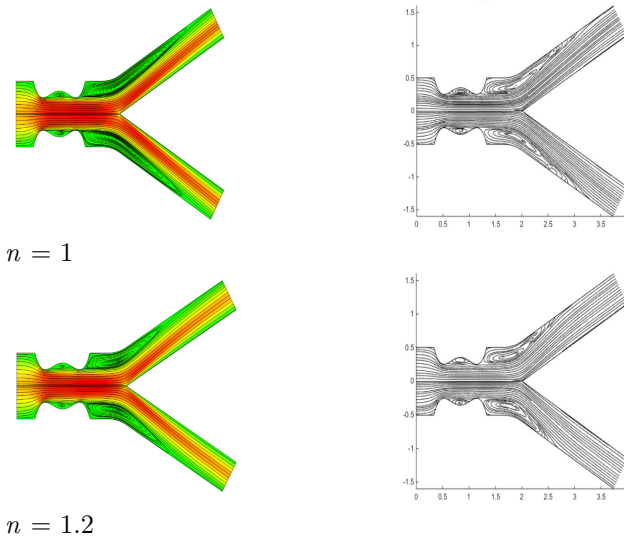


Figure 2. Comparison of stream lines at $a_m = 0.4\bar{r}_1$ between (a) present and (b) Zain et al. [29].

The model incorporates stenosis geometry, velocity gradients, viscosity, and non-Newtonian effects using a refined COMSOL mesh with local refinement near stenosis and bifurcation regions. Boundary conditions include no-slip walls, inlet velocity, and outlet pressure. Flow features such as separation and recirculation are captured via streamline and velocity contours, and results show good agreement with literature. As shown in Figures 2–3, present

results agree with Zain et al. [29] and Williams et al. [25], confirming similar flow patterns and Reynolds number effects.

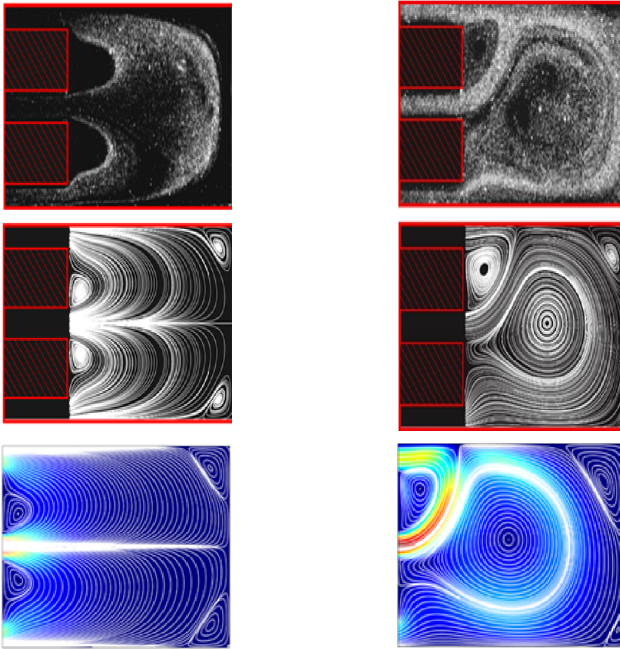
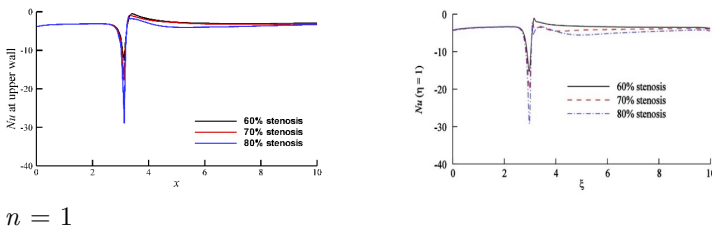


Figure 3. Code validation through streamlines graphs by comparing present and available data [25].

In Figure 4, Nusselt number trends for 60, 70 and 80 percent stenoses match published data, showing reduced Nu in the stenotic region and recovery downstream, with stronger effects at higher stenosis due to increased velocity gradients and recirculation. Table 2, presents the comparative data. The results reveal an **average APE below 3%** and an **RMSE of 0.051**, indicating minimal deviation. This strong agreement confirms that the present model accurately captures heat transfer characteristics in stenosed arterial geometries.



$$n = 1$$

Figure 4. Comparison of Nusselt number between present and published results [21].

Mesh distribution at two different levels coarse and finer are presented in Figure 5.

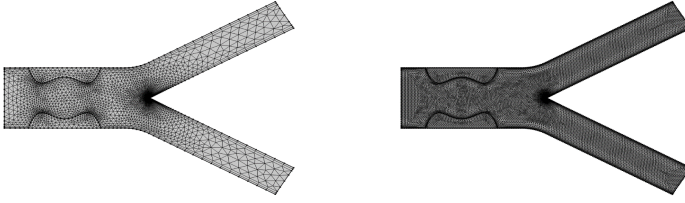


Figure 5. Mesh distribution at two different levels coarse (left) and finer (right).

Table 2. Comparison study for the Nusselt number published results [21].

X	Nu_m [25]	Nu_m (Present)	Absolute Error (%)
0.1	4.1284	4.1532	0.60 %
1	3.5654	3.4861	2.22 %
2	3.3811	3.2976	2.47%
3	13.9633	13.8934	0.50%
4	2.3984	2.3205	3.24%

3.1 Choice of numerical platform

The simulations are performed using COMSOL Multiphysics, a finite element-based software for solving coupled partial differential equations in complex geometries. It efficiently handles irregular, stenosed, and bifurcated domains with adaptive meshing and coupled thermo-fluid interactions. Compared to finite difference and finite volume methods, FEM in COMSOL offers higher flexibility and accuracy, making it suitable for biomedical flow problems with complex boundary conditions and strong geometry–flow coupling.

4 Mesh independence study

A mesh sensitivity analysis is performed to ensure numerical accuracy and eliminate discretization errors.

Table 3 presents results for different mesh densities in terms of elements, DoF, and average Nusselt number (Nu). The variation in Nu decreases with mesh refinement, confirming solution convergence. The coarse mesh gives unreliable results, while progressive refinement leads to stabilization of Nu. The solution becomes nearly mesh-independent at 13,961 elements, with negligible change upon further refinement. Based on accuracy and computational cost, this mesh is selected for all simulations. The mesh independence is further confirmed by a low APE ($< 3\%$) and RMSE (0.051), indicating strong numerical convergence.

Table 3. Mean Nusselt number (Nu) variation with n and Re .

n	$Re = 100$	$Re = 150$	$Re = 200$	$Re = 250$
0.5	1.562	1.5145	1.5163	1.5803
1	1.6407	1.6364	1.6402	1.6474
1.5	1.659	1.6901	1.7225	1.7526
2	1.6458	1.6916	1.7322	1.7856

5 Results and discussion

The following results distinctly illustrate the characteristics of flow properties, including stream function, temperature and pressure profiles. Note; n is the power law index, a_m is the stenosis peak, Kr is the thermal conductivity ratio plaque to blood.

The Figure 6, illustrates the flow dynamics, thermal behavior, shear stress and pressure distribution in a bifurcated artery under varying degrees of stenosis severity 50%, 60%, and 70% amplitude. As the severity increases, significant changes are observed in all physical characteristics of the flow. At 50% stenosis, the blood flow remains mostly streamlined with minimal flow separation, indicating low resistance and efficient convective heat transfer. The streamlines show stable laminar flow with no recirculation zones, while the isotherms remain nearly parallel, suggesting uniform heat distribution. The pressure profile follows a near-parabolic shape with a modest pressure drop across the stenosis, reflecting low energy loss. However, at 60% stenosis, the constricted geometry causes the development of recirculation zones downstream due to adverse pressure gradients and flow separation. The isothermal lines begin to distort, particularly downstream of the stenosis, indicating disrupted heat transfer caused by flow deceleration and recirculation. A significant pressure rise is noted in the upstream region, followed by a steep drop downstream, implying increased resistance and energy dissipation. At 70% stenosis, the narrowing becomes severe, producing a high-speed jet core and large recirculation regions, which intensify flow disturbances and mimic turbulence-like behavior. The isotherms become highly non-uniform, with thermal energy trapped near the stenosis and reduced convection in the core region. This leads to a substantial drop in local Nusselt number, indicating deterioration of heat transfer performance. As the Reynolds number increases, the wall shear stress also increases due to higher velocity gradients near the wall. The pressure gradient becomes extreme, with a pronounced upstream pressure buildup and a sharp downstream decline, signifying substantial energy loss and elevated stress on the arterial walls. Under 50%, 60%, and 70% stenosis severity, the deformation rate of a bifurcated artery increases progressively due to heightened flow disturbances and pressure gradients. The highest deformation rate occurs at 70% amplitude, where severe lumen narrowing maximizes wall shear stress and mechanical strain. Overall, increasing the amplitude of stenosis enhances flow instability, weakens heat transfer, and increases pressure buildup conditions that may lead to severe

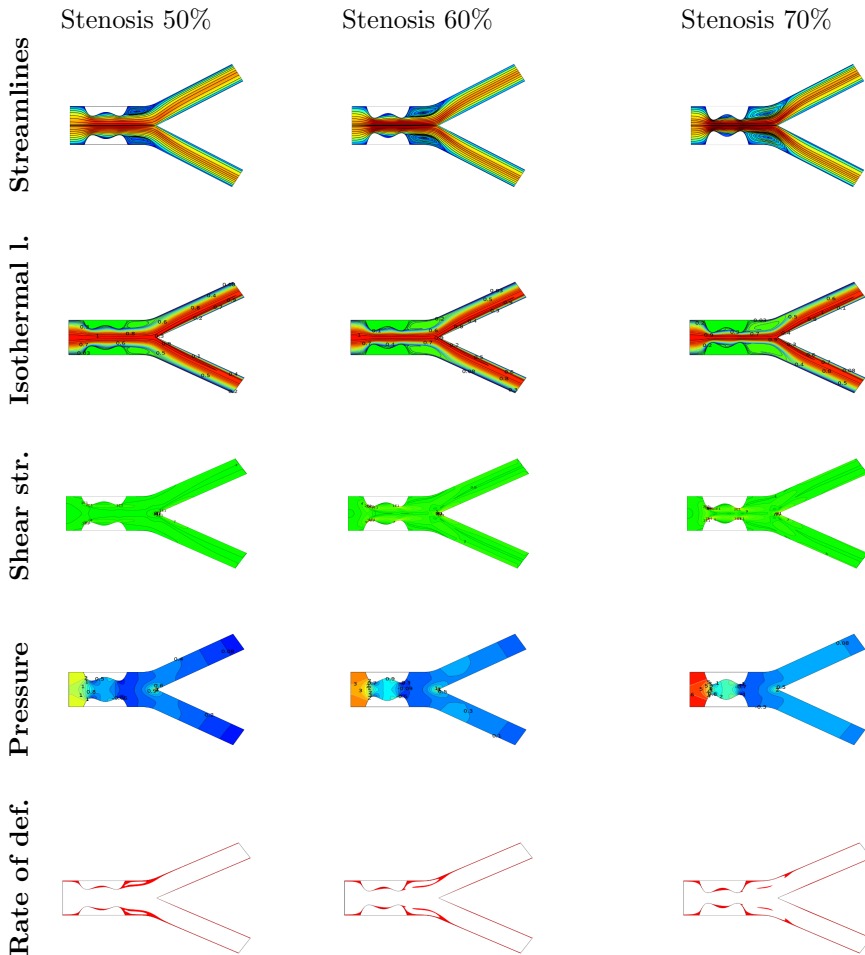


Figure 6. Streamline, isothermal, shear stress, pressure patterns and deformation rate for different amplitude of stenosis a_m and yield number of 0.08..

physiological consequences such as elevated shear stress, reduced perfusion efficiency, and impaired thermal regulation in blood vessels.

Figure 7, indicates the effect of Re ($Re=100$, 250 and 500) and severity of stenosis (50%, 60% and 70%) on the flow field, heat transfer, shear stress and pressure distribution in the bifurcated artery.

When $Re=100$, viscous effects are greater than inertial forces; therefore, the flow becomes smooth laminar and there is no flow separation around the stenosis. Velocity fluctuation is suppressed by the diffusion of momentum, thermal field is dominated by the conduction and isotherm become more or less parallel and convective transport become low. Pressure distribution is seen to have very low drop along the stenosed area; this is because the resistive forces

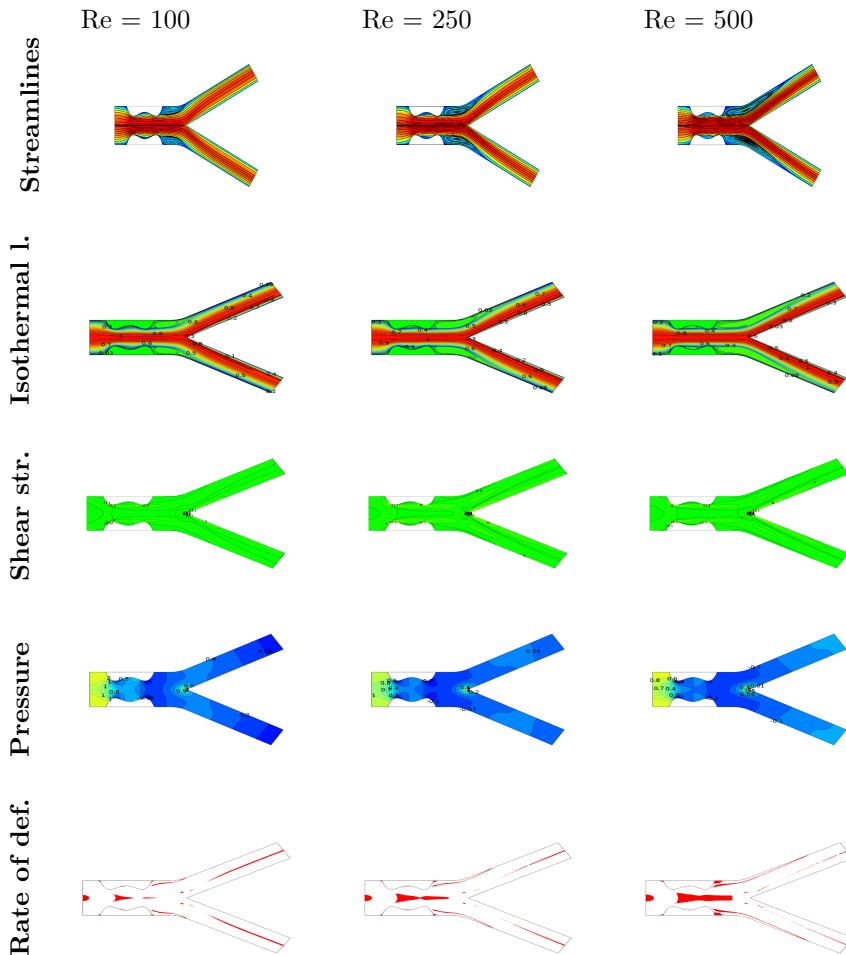


Figure 7. Streamline, isothermal, shear stress, pressure and deformation rate for different Re and yield number of 0.5.

are minimal and there are no circulating areas, so less energy is lost. When the Reynolds number is 250, inertial effects balance the viscous forces and the flow starts separating from the boundary wall after the stenosis; this occurs due to adverse pressure gradient at the post-stenotic region which leads to the separation of boundary layer to produce vortical structures which increase mixing; consequently the thermal boundary layer is thinned and there is enhanced convective heat transfer; so isotherms get distorted and tilted at the region of stenosis. Pressure has a greater build up at the pre-stenotic and higher drop at the post-stenotic region; This signifies more work (mechanical energy) is done on the flow.

When $Re=500$, inertial effects become more dominant compared to viscous

effects and high velocity jet-like structure is obtained from the stenosis throat which strikes the post-stenotic wall, leading to extensive region of recirculation and highly chaotic vortical motion signifying entry into unsteady or weakly turbulent regime; local convective heat transfer is largely improved by such increased mixing but there is thermal instability because isotherms are highly compressed and tilted; Due to high velocity gradient at the stenosis throat, wall shear stress becomes considerably larger at stenosis throat and reattachment points.

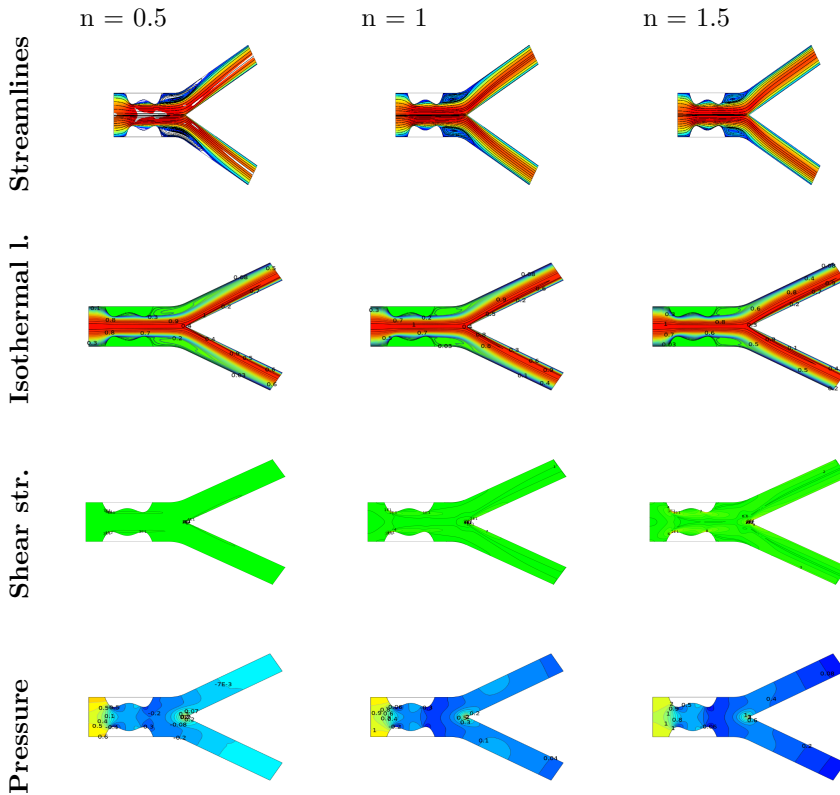


Figure 8. Streamline, isothermal, shear stress and pressure patterns for different n .

Figure 8, depicts the effect of the power-law index ($n = 0.5, 1, 1.5$) on flow structure, heat transfer, shear stress and pressure distributions of a stenosed bifurcated channel. In the case of shear-thinning fluids ($n = 0.5$), decreases in apparent viscosity weaken viscous damping. Consequently, the flow becomes unstable, and bigger recirculation regions form on the upstream and downstream sides of the stenosis. Strong velocity gradient is generated around the stenosis that contributes to increased mixing and convection for the thermal transport, thus causing highly distorted isotherms.

Furthermore, decreased viscous resistances may lower the pressure drop along the stenosis. In the Newtonian fluids ($n = 1$), a compromise is achieved between viscous and inertial force, and relatively small and stable recirculation zone is formed in front and rear of the stenosis. Heat transfer is obtained by the simultaneous effect of conduction and convection as the smooth variation of isotherms illustrates. Likewise, the pressure drop along the stenosis is relatively moderate.

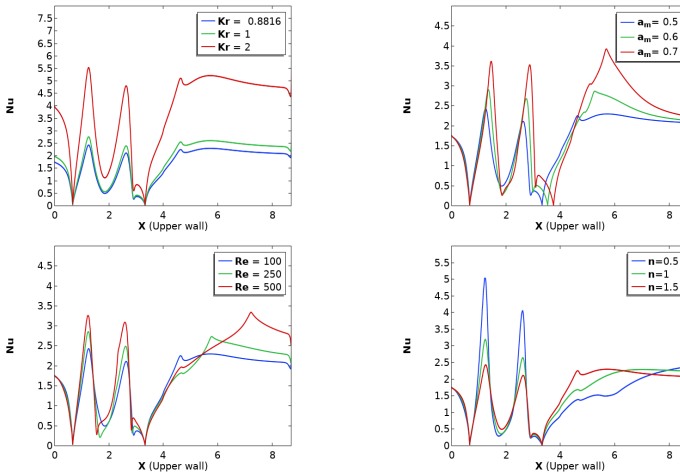


Figure 9. Variation in local Nusselt number with (a) Kr , (b) a_m , (c) Re and (d) n .

Shear-thickening fluid ($n = 1.5$) increases viscosity, suppresses convection, and enhances conduction, resulting in higher pressure drop and wall shear stress, shifting the flow toward viscosity-dominated behavior.

Figure 9, shows local Nusselt number variation along the upper wall. In Figure 9(a), higher (K_r) enhances heat transfer but Nu remains low near the inlet due to an undeveloped thermal layer. In Figure 9(b), increasing (a_m) strengthens constriction, velocity gradients, and produces dual Nu peaks at the throat. In Figure 9(c), higher (Re) increases inertia and convective heat transfer. In Figure 9(d), (n) enhances Nu for shear-thinning flow and reduces it for shear-thickening behavior.

From Figure 10, velocity is parabolic upstream, jet-like at the throat, and recovers downstream. Higher (Re) increases inertia, velocity gradients, and shear intensity across the stenosis.

Figure 11 shows Hartmann number effects on axial velocity at $(x = l_0)$, $(x = l_0 + d_0/2)$, and $(x = l_0 + d_0)$. Upstream flow is unaffected, indicating weak magnetic influence. At the throat, increasing Hartmann number reduces peak velocity and smooths gradients due to Lorentz force. Downstream, velocity decreases further and profiles flatten, as magnetic damping suppresses inertia and stabilizes the flow.

Figure 11. Effects of Ha on u -velocity at different positions along y -axis.

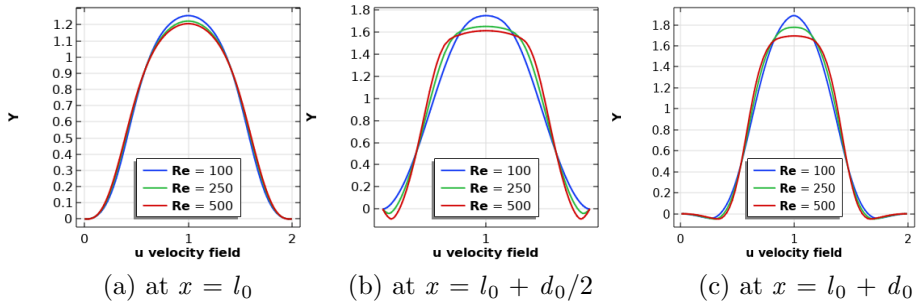


Figure 10. Effects of Re on u -velocity at different positions along y -axis.

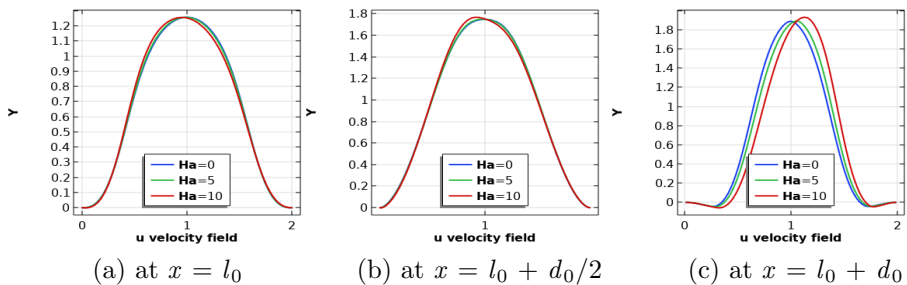


Figure 11. Effects of n on u -velocity at different positions along y -axis.

Table 3, shows variation of mean Nusselt number (Nu) with power-law index (n) and Reynolds number (Re). At fixed (Re), Nu increases as (n) decreases, indicating enhanced heat transfer under shear-thinning due to lower viscosity and stronger mixing. At fixed (n), Nu increases with (Re) because inertia strengthens flow velocity and thins the thermal boundary layer. Overall, reduced viscosity and higher inertia both enhance heat transfer.

For example, at $Re = 100$, the mean Nusselt number varies from 1.562 for $n = 0.5$ to 1.6458 for $n = 2.0$. At $n = 1.5$, an increase in mean Nusselt number occurs: from 1.659 at $Re = 100$ to 1.7526 at $Re = 250$. These trends show that the combination of rheological properties and flow dynamics has an effect on heat transfer performance, with higher n and Re being favorable for efficient convective heat transfer.

Table 4 shows the dependence of mean Nusselt number (Nu) on stenosis amplitude (a_m) and thermal conductivity ratio (K_r). At fixed (K_r), Nu increases with (a_m) due to enhanced constriction, which strengthens velocity gradients and convection. At fixed (a_m), Nu rises with (K_r) as higher thermal conductivity reduces thermal resistance and promotes heat transfer. Overall, increasing both (a_m) and (K_r) enhances heat transfer significantly.

Table 4. Effects of amplitude and thermal conductivity on N_m .

a_m	$Kr = 0.5$	$Kr = 1$	$Kr = 2$	$Kr = 4$
50%	0.94069	1.8817	3.7637	7.5275
60%	1.0181	2.0366	4.0735	8.1474
70%	1.1699	2.3405	4.6827	9.3656

6 Conclusions

This computational study looked at blood flow via a bifurcated artery with conjugate heat transfer and stenosis, using bio-nanofluids and analysing major governing parameters. The findings shed light on how stenosis severity, fluid rheology, and flow conditions affect hemodynamics and thermal behaviour. The paper contributes to the area by providing a comprehensive parametric analysis that can help optimism biomedical applications, particularly vascular stent design and disease modelling.

Key contributions:

1. This study shows that as stenosis severity increases, flow disruptions and pressure gradients worsen, shedding light on vascular disease progression.
2. This study examines how non-Newtonian features, measured by the power-law index, affect flow stability and heat transfer efficiency.
3. This study examines the relationship between Reynolds number, stenosis amplitude, and thermal characteristics to better understand convective heat transfer in bifurcated arteries.

Limitations

This study uses simplifying assumptions that limit physiological accuracy, including steady flow that neglects pulsatility and transient effects important in vascular dynamics. The arterial wall is assumed rigid, and thermal properties are taken as constant, while bio-nanofluids are not included in the present model. Future work will address pulsatile flow (e.g., 4D flow MRI), FSI, temperature-dependent properties, and nanofluid effects to improve realism and predictive capability.

7 Applications

The findings are applicable in biomedical engineering, particularly in designing arterial stents and optimizing surgical interventions for stenosed arteries. The insights into convective heat transfer also have implications in microchannel heat exchangers, energy systems, and cooling applications in industrial processes.

8 Future recommendations

Future research must include pulsatile dynamics of flow and fluid characteristics that depend on temperature so that it can be more relevant to physiology. To further validate the results through patient-specific model analysis would enhance the applicability of the results. Also, to increase the accuracy of the numerical predictions, the effects of arterial wall elasticity and biofouling may be included. It would be others to extend the research to the multi-phase nanofluid interactions in disturbed flow conditions to enhance the knowledge concerning the heat transfer in both biomedical and industrial applications.

References

- [1] N. Abdullah and Z. Ismail. Finite element analysis of biomagnetic fluid flow in a channel with an overlapping stenosis. *Malaysian J. Fundam. Appl. Sci.*, **18**:618–629, 2022. <https://doi.org/10.11113/mjfas.v18n6.2442>.
- [2] M. Anand, S. Aggarwal, H. Raj, D. Arora and D. Gajjar. CFD analysis of pulsatile non-Newtonian blood flow in a multi-staged stenosed bifurcated carotid artery. *J. Mines, Met. Fuels*, **71**:1601–1611, 2023. <https://doi.org/10.18311/jmmf/2023/35806>.
- [3] A. Ausaru, P. Nagarani and V.M. Job. Numerical investigation of solute dispersion in a non-Newtonian fluid flow through a catheterized artery with mild stenosis. *Chinese J. Phys.*, **89**:541–564, 2024. <https://doi.org/10.1016/j.cjph.2024.03.005>.
- [4] S. Averweg, A. Schwarz, C. Nisters and J. Schröder. A least-squares finite element formulation to solve incompressible non-Newtonian fluid flow. *PAMM*, **20**:e202000169, 2021. <https://doi.org/10.1002/pamm.202000169>.
- [5] T. He. A stabilized cell-based smoothed finite element method against severe mesh distortion in non-Newtonian fluid–structure interaction. *Int. J. Numer. Methods Eng.*, **123**:2162–2184, 2022. <https://doi.org/10.1002/nme.6930>.
- [6] J.C. Heinrich and C.A. Vionnet. The penalty method for the Navier-Stokes equations. *Arch. Comput. Methods Eng.*, **2**:51–65, 1995. <https://doi.org/10.1007/bf02904995>.
- [7] A. Hussain, M.N.R. Dar and A. Arif. Domination of fluid rheological parameters on gold nanofluid drug delivery in trapezoidal stenosed vessel. *Int. J. Mod. Phys. B*, **39**(20):2550180, 2025. <https://doi.org/10.1142/s0217979225501802>.
- [8] A. Hussain, M.N.R. Dar and A. Iftikhar. Investigating blood's shear-thinning effects while flowing through a post-stenotic dilated artery: A Cross-Williamson model approach. *Braz. J. Phys.*, **55**(5):193, 2025. <https://doi.org/10.1007/s13538-025-01809-7>.
- [9] A. Hussain, M.N.R. Dar, A. Rehman, F. Hussain and N. Farooq. Unconventional wall implications of intertwined stenosis in constrict artery. *Adv. Mech. Eng.*, **15**(3):16878132231162178, 2023. <https://doi.org/10.1177/16878132231162178>.
- [10] A. Hussain, M.N.R. Dar and E.M. Tag-Eldin. Effects of stenosis and aneurysm on blood flow in stenotic-aneurysmal artery. *Heliyon*, **9**(7):e17788, 2023. <https://doi.org/10.1016/j.heliyon.2023.e17788>.

- [11] A. Hussain, M.Y. Malik and F. Khan. Flow of an Eyring–Powell model fluid between coaxial cylinders with variable viscosity. *Chin. J. Eng.*, **2013**(1):808342, 2013. <https://doi.org/10.1155/2013/808342>.
- [12] A. Hussain, S. Raiz, A. Hassan, A.M. Hassan, H. Karamti and G. Bognár. Analysis of solet and dufour effects on radiative heat transfer in hybrid bioconvective flow of carbon nanotubes. *Sci. Rep.*, **14**(1):11970, 2024. <https://doi.org/10.1038/s41598-024-62647-2>.
- [13] A. Hussain, A. Rehman, N. Ahmed, A.S. El-Shafay, S.A. Najati, A.H. Almaliki and E.-S.M. Sherif. Heat transfer and flow characteristics of pseudoplastic nano-material liquid flowing over the slender cylinder with variable characteristics. *Crystals*, **12**(1):27, 2021. <https://doi.org/10.3390/cryst12010027>.
- [14] M.A. Ikbāl, S. Chakravarty, K.K.L. Wong, J. Mazumdar and P.K. Mandal. Unsteady response of non-Newtonian blood flow through a stenosed artery in magnetic field. *J. Comput. Appl. Math.*, **230**:243–259, 2009. <https://doi.org/10.1016/j.cam.2008.11.010>.
- [15] M.S. Khan, S. Ahmad, Z. Shah, M.H. Alshehri and E. Antonescu. Computational investigations of nanofluid blood flow in stenosed artery with effects of inflammation and viscous dissipation via finite element method. *Chinese J. Phys.*, **92**:453–469, 2024. <https://doi.org/10.1016/j.cjph.2024.09.013>.
- [16] T. Kim. Numerical analysis of the cooling effect of blood over inflamed atherosclerotic plaque. *J. Biomech. Eng.*, **130**, 2008. <https://doi.org/10.1115/1.2913236>.
- [17] B.V.R. Kumar and M. Chowdhury. Variational multiscale stabilized finite element analysis of non-Newtonian Casson fluid flow model fully coupled with transport equation with variable diffusion coefficients. *Comput. Methods Appl. Mech. Eng.*, **388**:114272, 2022. <https://doi.org/10.1016/j.cma.2021.114272>.
- [18] F. Machado, F. Zinani and S. Frey. Herschel-Bulkley fluid flows through a sudden axisymmetric expansion via Galerkin least-squares methodology. In *Proceedings of the 19th International Congress of Mechanical Engineering (COBEM 2007)*, Brasília, Brazil, 2007.
- [19] M.G. Rabby, R. Sultana, S.P. Shupti and M.M. Molla. Laminar blood flow through a model of arterial stenosis with oscillating wall. *Int. J. Fluid Mech. Res.*, **41**:417–429, 2014. <https://doi.org/10.1615/interjfluidmechres.v41.i5.30>.
- [20] M.B. Riaz, A. Hussain, A. Saddiqa and F. Jarad. Numerical evaluation for the peristaltic flow in the proximity of double-diffusive convection of non-Newtonian nanofluid under MHD. *Int. J. Thermofluids*, **23**:100791, 2024. <https://doi.org/10.1016/j.ijft.2024.100791>.
- [21] N.C. Roy, L.K. Saha, M.A. Hossain and R.S.R. Gorla. Blood flow in a stenotic artery with temperature-dependent viscosity. *Heat Transf.*, **50**:1590–1611, 2021. <https://doi.org/10.1002/htj.21943>.
- [22] H. Shahzad, X. Wang, A. Ghaffari, K. Iqbal, M.B. Hafeez, M. Krawczuk and W. Wojnicz. Fluid structure interaction study of non-Newtonian Casson fluid in a bifurcated channel having stenosis with elastic walls. *Sci. Rep.*, **12**:12219, 2022. <https://doi.org/10.1038/s41598-022-16213-3>.
- [23] J. Skála and M. Bárta. LSFEM implementation of MHD numerical solver. *Appl. Math.*, **3**(11A):1842–1850, 2012. <https://doi.org/10.4236/am.2012.331250>.
- [24] G. Varshney, V. Katiyar and S. Kumar. Effect of magnetic field on the blood flow in artery having multiple stenosis: a numerical study. *Int. J. Eng. Sci. Technol.*, **2**:967–982, 2010. <https://doi.org/10.4314/ijest.v2i2.59142>.

- [25] J.G. Williams, A.A. Castrejón-Pita, B.W. Turney, P.E. Farrell, S.J. Tavener, D.E. Moulton and S.L. Waters. Cavity flow characteristics and applications to kidney stone removal. *J. Fluid Mech.*, **902**:A16, 2020. <https://doi.org/10.1017/jfm.2020.583>.
- [26] N.M. Zain and Z. Ismail. Hartmann and Reynolds numbers effects in the Newtonian blood flow of a bifurcated artery with an overlapping stenosis. *Matematika*, pp. 213–227, 2019. <https://doi.org/10.11113/matematika.v35.n2.1177>.
- [27] N.M. Zain and Z. Ismail. Numerical solution of magnetohydrodynamics effects on a generalised power law fluid model of blood flow through a bifurcated artery with an overlapping shaped stenosis. *PLoS ONE*, **18**:e0276576, 2023. <https://doi.org/10.1371/journal.pone.0276576>.
- [28] N.M. Zain, Z. Ismail, M.S.A. Jamali and Y.J. Lim. Wall shear stress distribution of non-Newtonian blood flow in stenosed bifurcated artery. *Matematika*, pp. 75–86, 2023. <https://doi.org/10.11113/matematika.v39.n1.1452>.
- [29] N.M. Zain, Z. Ismail and P. Johnston. A stabilized finite element formulation of non-Newtonian fluid model of blood flow in a bifurcated channel with overlapping stenosis. *J. Adv. Res. Fluid Mech. Therm. Sci.*, **88**:126–139, 2021. <https://doi.org/10.37934/arfmts.88.1.126139>.
- [30] A. Zaman, A.A. Khan and N. Kousar. Simulation of magneto-hydrodynamics effects on cross fluid (blood) model with entropy generations in multiple stenosed (aneurysm) curved channel. *Chinese J. Phys.*, **85**:616–628, 2023. <https://doi.org/10.1016/j.cjph.2022.10.007>.
- [31] F. Zinani and S. Frey. Galerkin least-squares solutions for purely viscous flows of shear-thinning fluids and regularized yield stress fluids. *J. Brazilian Soc. Mech. Sci. Eng.*, **29**:432–443, 2007. <https://doi.org/10.1590/s1678-58782007000400012>.