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METHODS OF SYNTHESIS OF AUTOMATIC CONTROL SYSTEMS WITH DELAY

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Abstract. The paper investigates the procedure for introduction of systems containing delay elements. Shortcomings and difficulties in the synthesis of regulators and precompensators of control systems with delays in output and control channel where determined. The author focused on two approaches for the formation of promatrix and synthesis of control systems, considering the factor of delay.

Keywords: modal control, promatrix, embedding theory.

Introduction

In view of the current development of the automatic control theory and simulation of controlled objects, increasingly more notice is given to delays. The delay phenomenon exists because the output signal begins to change only after a certain period of time following the change in the input signal of a controlled object. The most common examples of delay can be found in such processes as drying and burning, calcination of a metal, belt conveyors, size reduction, and, in some cases, processes in chemical reactors.

Mathematical models of processes in controlled objects with delay are executed by means of differential equations with deviating argument. The difficulties in the mathematical solution of these equations stipulate problems pertaining to technical implementation of control systems with delays. Among numerous modern methods of total synthesis of modal control system regulators, the system embedding theory should be emphasised. The theory was developed by V. N. Bukov (2006). This method uses a system of matrix equations to solve problems of control and has shown good results when used in various industries. The method is based on “embedding” of the desired behaviour (both forced and free) in the image of a scalar system.

Let a linear stationary object with lumped delays be represented in the form of differential-difference equations:

$$\dot{x}(t) = \sum_{i=0}^{l}(\mathbf{A}_i x(t-\tau_i)) + \sum_{j=0}^{r}(\mathbf{B}_j u(t-\theta_j)),$$
$$y(t) = \sum_{i=0}^{l}(\mathbf{C}_i x(t-\tau_i)), \tag{1}$$

where: $\tau_0 = 0$, $0 < \tau_1, \tau_2, \ldots, \tau_l$ – fixed time delays in state and output channels; $\theta_0 = 0$, $0 < \theta_1, \theta_2, \ldots, \theta_r$ – fixed time delays in control channels, $i = 0, \ldots, l$, $j = 0, \ldots, r$, $u(t) \in \mathbf{R}^s$, $y(t) \in \mathbf{R}^m$, $x(t) \in \mathbf{R}^n$.

The task of synthesis of control systems using the system embedding theory in our case can be divided into two stages. The first stage involves forming a promatrix of the control system; while the second focuses on the construction of the system depending on the variant of its synthesis.

The use of the mathematics apparatuses of embedding theory assumes the representation of control system in the form of a block-matrix:

$$\Omega \begin{bmatrix} x \\ y \\ u \end{bmatrix} = \begin{bmatrix} \mathbf{x_0} \\ 0 \\ u \end{bmatrix}. \tag{2}$$

Square and always full (non-singular) matrix $\Omega(p)$ is called *promatrix* of control system in state space. Because of its completeness promatrix always has, regardless of the reversibility of the bilateral assignment matrixes **A**, **B**, **C** system. It follows from the uniqueness of the inverse of promatrix or *repromatrix*:

$$\Omega^{-1}(p) = \begin{bmatrix} \mathbf{E}_x^{\varphi_x}(p) & * & * & \mathbf{E}_x^{g}(p) \\ \mathbf{E}_y^{\varphi_x}(p) & * & * & \mathbf{E}_y^{g}(p) \\ \mathbf{E}_u^{\varphi_x}(p) & * & * & \mathbf{E}_u^{g}(p) \\ 0 & 0 & 0 & \mathbf{I}_s \end{bmatrix}, \tag{3}$$

where $\mathbf{E}_{ij}(p)$ – matrix transfer function from parameter i to parameter j.

Introduction of the system using a promatrix has exhaustive completeness. Systems that contain delays in the structure (promatrix) of the control system have the following form:

$$\Omega(p)=\begin{bmatrix} p\mathbf{I}_n-\sum\limits_{i=0}^{l}\mathbf{A}_i e^{\tau_i p} & 0 & -\sum\limits_{j=0}^{r}\mathbf{B}_j e^{\theta_i p} & 0 \\ -\sum\limits_{i=0}^{l}\mathbf{C}_i e^{\tau_i p} & I_m & 0 & 0 \\ 0 & \mathbf{K}(p) & \mathbf{I}_s & -\mathbf{G}(p) \\ 0 & 0 & 0 & \mathbf{I}_s \end{bmatrix}. \quad (4)$$

Following the embedding theory procedures, we can get the equation to calculate the MTF of precompensator $\mathbf{G}(p)$ and the controller $\mathbf{K}(p)$, for three cases: the synthesis of free and forced movements and, accordingly, as well as the joint synthesis of free and forced components of movements closed dynamical system.

After selecting the variant of synthetics and putting the desired matrix transfer function, they are equalled to the elements of repromatrix, containing combinations of the matrix system, precompensators and regulators.

The procedure for canonization of system matrices can be used to solve the matrix equations obtained.

The essence of the canonization of an arbitrary size of the matrix $\mathbf{M}$ $m\times n$ is in finding the four matrixes $(\mathbf{M})^{\tilde{}\mathbf{L}}_{r,m}$, $\overline{\mathbf{M}}^{\mathbf{L}}_{(m-r),n}$, $(\mathbf{M})^{\tilde{}\mathbf{R}}_{n,r}$, $\overline{\mathbf{M}}^{\mathbf{R}}_{n,(n-r)}$, which satisfy the equality:

$$\begin{bmatrix} (\mathbf{M})^{\tilde{}\mathbf{L}}_{r,m} \\ \overline{\mathbf{M}}^{\mathbf{L}}_{m-r,n} \end{bmatrix}\mathbf{M}\begin{bmatrix} (\mathbf{M})^{\tilde{}\mathbf{R}}_{n,r} & \overline{\mathbf{M}}^{\mathbf{R}}_{n,n-r} \end{bmatrix}=\begin{bmatrix} \mathbf{I}_r & \mathbf{0}_{r,n-r} \\ \mathbf{0}_{m-r,r} & \mathbf{0}_{m-r,n-r} \end{bmatrix}, \quad (5)$$

where: $(\mathbf{M})^{\tilde{}\mathbf{L}}_{r,m}$ ($(\mathbf{M})^{\tilde{}\mathbf{R}}_{n,r}$) – left (right) canonizator of matrix $\mathbf{M}$; $\overline{\mathbf{M}}^{\mathbf{L}}_{m-r,n}$ ($\overline{\mathbf{M}}^{\mathbf{R}}_{n,n-r}$) – left (right) zero divisor of the matrix $\mathbf{M}$; $r=\operatorname{rank}(\mathbf{M})$; $\mathbf{I}_r$ – identity matrix of size $r\times r$.

Summary canonizator $(\mathbf{M})^{\tilde{}}$ used for solving matrix equations, is given by

$$(\mathbf{M})^{\tilde{}}=(\mathbf{M})^{\tilde{}\mathbf{R}}(\mathbf{M})^{\tilde{}\mathbf{L}}. \quad (6)$$

These matrixes can be found either analytically or using the package *Matlab*.

Method

First, parameters of the controller and precompensator and down conditions required to solve these equations should be found.

1. *Synthesis of free movement of the closed part of a dynamic system.*

In this case, free motion of the system due to the initial conditions of the object does not depend on the choice of a precompensator, and the control law takes the following form:

$$u(p)=-\mathbf{K}(p)x(p). \quad (7)$$

The use of embedding theory in the synthesis of free movement results in the following equation to determine the controller $\mathbf{K}(p)$:

$$\mathbf{E}_y^{\varphi_x}(p)\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p}\mathbf{K}(p)=\sum_{i=0}^{l}\mathbf{C}_i e^{\tau_i p} -\mathbf{E}_y^{\varphi_x}(p)\left(p\mathbf{I}_n-\sum_{i=0}^{l}\mathbf{A}_i e^{\tau_i p}\right). \quad (8)$$

From this equation we can express many regulators:

$$\left\{\mathbf{K}(p)\right\}_{\mu}=(\mathbf{E}_y^{\varphi_x}(p)\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p})^{\tilde{}} \times\left(\sum_{i=0}^{l}\mathbf{C}_i e^{\tau_i p}-\mathbf{E}_y^{\varphi_x}(p)\left(p\mathbf{I}_n-\sum_{i=0}^{l}\mathbf{A}_i e^{\tau_i p}\right)\right) +\overline{\mathbf{E}_y^{\varphi_x}(p)\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p}}^{R}\boldsymbol{\mu}(p), \quad (9)$$

where $\boldsymbol{\mu}(p)$ – an arbitrary fractional polynomial matrix of appropriate dimensions.

2. *Synthesis of the forced motion of a closed part of a dynamic system.*

The use of technology of embedding theory in the synthesis of a forced movement gives the following equation for the desired transfer matrix $\mathbf{G}(p)$ and $\mathbf{K}(p)$:

$$\left\{\mathbf{K}(p)\right\}_{\mathbf{T},\boldsymbol{\lambda},\vartheta}=(\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p})^{\tilde{}} \times\left(\mathbf{T}^{-1}(p)\begin{bmatrix}(\sum\limits_{i=0}^{l}\mathbf{C}_i e^{\tau_i p})^{\tilde{}L}\sum\limits_{i=0}^{l}\mathbf{C}_i e^{\tau_i p} \\ \boldsymbol{\lambda}(p)\end{bmatrix}-\left(p\mathbf{I}_n-\sum_{i=0}^{l}\mathbf{A}_i e^{\tau_i p}\right)\right) +\overline{\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p}}^{R}\vartheta(p), \quad (10)$$

$$\left\{\mathbf{G}(p)\right\}_{\mathbf{N},\boldsymbol{\xi},\boldsymbol{\kappa}}=(\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p})^{\tilde{}}\mathbf{N}^{-1}(p)\begin{bmatrix}(\mathbf{E}_y^{g}(p))^{\tilde{}L}\mathbf{E}_y^{g}(p) \\ \boldsymbol{\kappa}(p)\end{bmatrix} +\overline{\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p}}^{R}\boldsymbol{\xi}(p), \quad (11)$$

where: $\mathbf{T}$, $\mathbf{N}$ are arbitrary invertible matrixes; $\boldsymbol{\lambda}$, $\boldsymbol{\kappa}$, $\boldsymbol{\xi}$ – arbitrary matrix, the corresponding complementary bases to rows dimensional state space n.

3. *Synthesis of free and forced motions of a closed part of a dynamic system.*

The use of embedding theory in the synthesis of free and forced movements results in the following equation for the desired transfer matrix $\mathbf{G}(p)$ and $\mathbf{K}(p)$:

$$\left\{\mathbf{K}(p)\right\}_{\boldsymbol{\mu}} = \left(\mathbf{E}_y^{\varphi_x}(p)\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p}\right)^{\sim} \times\left(\sum_{i=0}^{l}\mathbf{C}_i e^{\tau_i p} - \mathbf{E}_y^{\varphi_x}(p)\left(p\mathbf{I}_n - \left(\mathbf{A}_0+\mathbf{A}_1 e^{-\tau_1 p}+\dots+\mathbf{A}_l e^{-\tau_l p}\right)\right)\right) \overline{+\mathbf{E}_y^{\varphi_x}(p)\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p}}^{R}\ \boldsymbol{\mu}(p), \tag{12}$$

$$\left\{\mathbf{G}(p)\right\}_{\boldsymbol{\eta}} = \left(\mathbf{E}_y^{\varphi_x}(p)\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p}\right)^{\sim}\mathbf{E}_y^{g}(p) \overline{+\mathbf{E}_y^{\varphi_x}(p)\sum_{j=0}^{r}\mathbf{B}_j e^{\theta_i p}}^{R}\ \boldsymbol{\eta}(p). \tag{13}$$

As the equations for calculating the values of the controller and the compensator matrixes contain delays in the divisor of zeros and canonizators arise problems in their calculation and further synthesis become impossible.

The above calculations suggest the need to compensate delays when using the embedding technology. One possible way of implementation is the introduction of Smith compensator in output and control channel (Fig. 1).

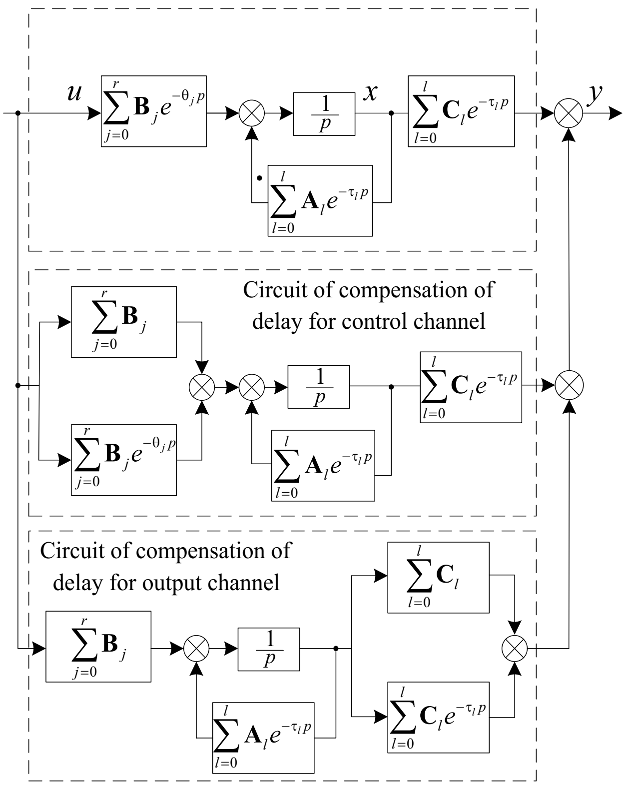

Fig. 1. The use of the Smith compensator for control and output channel

The circuit that compensates for the delay on the input channel (Fig. 1) can be represented as a matrix transfer function:

$$\mathbf{R}(p)=\sum_{i=0}^{l}\mathbf{C}_i e^{-\tau_i p}\left(p\mathbf{I}_n-\sum_{i=0}^{l}\mathbf{A}_i e^{-\tau_i p}\right)^{-1} \times\left(\sum_{j=0}^{r}\mathbf{B}_j-\sum_{j=0}^{r}\mathbf{B}_j e^{-\theta_j p}\right), \tag{14}$$

The circuit that compensates for the delay on the output channel (Fig. 2) can be represented as a matrix transfer function:

$$\mathbf{H}(p)=\left(\sum_{i=0}^{l}\mathbf{C}_i-\sum_{i=0}^{l}\mathbf{C}_i e^{-\tau_i p}\right)\left(p\mathbf{I}_n-\sum_{i=0}^{l}\mathbf{A}_i e^{-\tau_i p}\right)^{-1}\sum_{j=0}^{r}\mathbf{B}_j \tag{15}$$

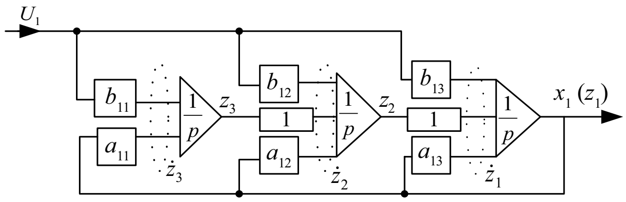

Fig. 2. Block diagram representation of a controlled object

However, this method may be used only where the model of the controlled object is calculated as accurately as possible, and, most importantly, does not change over time. Unfortunately, in most production processes the opposite situation (changing the raw materials, environment, etc.) is observed; namely, when parameters of the model of the controlled object as well as the values of Smith compensators for the control and output remain the same, and as a consequence, the phenomenon is not fully compensated for the delay. As a result, precipitous changes of values for the state space variables are exerted in terms of time point overlaps with delays. If the control object stays within stability bounds, these changes will significantly affect descriptive adjectives of the object.

On the other hand, the influence of delays takes into consideration the expansion of delays in different series. In automatic systems, series expansion of the Pade are most commonly used due to implementation simplicity.

$$W_\tau(p)=\frac{\sum_{k=1}^{n}\frac{(n+k)!}{k!(n-k)!}(-\tau p)^{n-k}}{\sum_{k=1}^{n}\frac{(n+k)!}{k!(n-k)!}(\tau p)^{n-k}}. \tag{16}$$

For the further application of the procedure it is necessary to submit an attachment object model in state space, which is most convenient to use the structural forms of transformation. A block diagram representation of the controlled object model called the canonical form of observability can be used.

The main advantage of representation of a controlled object in the state space is the conservation of the physical meaning for the state variables, because expected transfer functions have to be set at the stage of regulator and precompensator synthesis.

Expanding the delays in the Pade serial obtain a standard form of recording system model in state space:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=0}^{l} (\mathbf{A}_i x(t-\tau_i)) + \mathbf{B}^* u(t) \\ y(t) &= \mathbf{C}^* x(t), \end{aligned} \tag{17}$$

where $\mathbf{B}^*$, $\mathbf{C}^*$ – numerical matrixes of the state space obtained by the transition from the structural form of the system.

A form less complicated than in (9)–(13) of the procedure of embedding theory can be applied for the resulting control system.

For synthesis of free movement of the closed part of a dynamic system:

$$\begin{aligned} \left\{\mathbf{K}(p)\right\}_{\mu} &= (\mathbf{E}_y^{\varphi_x}(p)\mathbf{B}^*\tilde{)} \\ &\times\left(\mathbf{C}^* - \mathbf{E}_y^{\varphi_x}(p)\left(p\mathbf{I}_n - \sum_{i=0}^{l}\mathbf{A}_i e^{\tau_i p}\right)\right) + \overline{\mathbf{E}_y^{\varphi_x}(p)\mathbf{B}^*}^R \boldsymbol{\mu}(p). \end{aligned} \tag{18}$$

For synthesis of forced motion of the closed part of a dynamic system:

$$\begin{aligned} \left\{\mathbf{K}(p)\right\}_{\mathbf{T},\lambda,\vartheta} &= (\mathbf{B}^*\tilde{)}(\mathbf{T}^{-1}(p)\begin{bmatrix}(\mathbf{C}^*\tilde{)}^L\mathbf{C}^* \\ \boldsymbol{\lambda}(p)\end{bmatrix} \\ &-(p\mathbf{I}_n - \sum_{i=0}^{l}\mathbf{A}_i e^{\tau_i p})) + \mathbf{B}^{*R}\vartheta(p), \end{aligned} \tag{19}$$

$$\begin{aligned} \left\{\mathbf{G}(p)\right\}_{\mathbf{N},\xi,\boldsymbol{\kappa}} &= (\mathbf{B}^*\tilde{)}\mathbf{N}^{-1}(p) \\ &\times\begin{bmatrix}(\mathbf{E}_y^{g}(p)\tilde{)}^L\mathbf{E}_y^{g}(p) \\ \boldsymbol{\kappa}(p)\end{bmatrix} + \overline{\mathbf{B}^*}^R \boldsymbol{\xi}(p). \end{aligned} \tag{20}$$

For synthesis of free and forced motions of the closed part of a dynamic system:

$$\begin{aligned} \left\{\mathbf{K}(p)\right\}_{\boldsymbol{\mu}} &= (\mathbf{E}_y^{\varphi_x}(p)\mathbf{B}^*\tilde{)} \\ &\times\left(\mathbf{C}^* - \mathbf{E}_y^{\varphi_x}(p)\left(p\mathbf{I}_n - \left(\mathbf{A}_0 + \mathbf{A}_1 e^{-\tau_1 p} + \cdots + \mathbf{A}_l e^{-\tau_l p}\right)\right)\right) \\ &+\overline{\mathbf{E}_y^{\varphi_x}(p)\mathbf{B}^*}^R \boldsymbol{\mu}(p), \end{aligned} \tag{21}$$

$$\left\{\mathbf{G}(p)\right\}_{\boldsymbol{\eta}} = (\mathbf{E}_y^{\varphi_x}(p)\mathbf{B}^*\tilde{)}\mathbf{E}_y^{g}(p) + \overline{\mathbf{E}_y^{\varphi_x}(p)\mathbf{B}^*}^R \boldsymbol{\eta}(p). \tag{22}$$

After expansion, the equations for calculation of regulator and compensator delays are receded. As a result, it becomes possible to use the embedding theory for the synthesis of control systems.

Remarks

Approximation of delays by different types of series existing defective features can be eliminated by increasing the order of the expansion. It was demonstrated that structural forms can transition from differential-difference equations to the model in state space in terms of the model of a controlled object in the state space. In this case, the physical meaning of the state variables of the object is conserved, which facilitates the synthesis of the desired system behaviour as well as the construction of observers (if necessary) for systems with delays.

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AUTOMATINIO VALDYMO SISTEMŲ SU VĖLINIMO GRANDINĖMIS SINTEZĖS METODAI

A. Lapeta

Santrauka

Straipsnyje tiriama sistemų su vėlinimo elementais įterpimo procedūra. Išanalizuoti valdymo sistemų, reguliatorių ir kompensavimo grandžių su vėlinimo elementais išėjime ir valdymo kanale sintezės sudėtingumo aspektai ir trūkumai. Aptarti du promatricų formavimo ir valdymo sistemų sintezės, įvertinant valdomojo objekto vėlinimą, formavimo būdai.

Reikšminiai žodžiai: modalinis valdymas, promatrica, įterpimo teorija.