



## FINITE ELEMENT MODELLING OF BOND IN REINFORCED CONCRETE ELEMENTS

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**Abstract.** Numerical modelling of bond is a challenging due to a number of physical and mechanical phenomena arising from interaction of ribbed reinforcing bars and concrete. In present paper 2D and 3D approaches for bond analysis are compared in terms of global load-displacement behaviour of RC element, formation of microcracks and deterioration of concrete in front of the ribs. Nonlinear Finite element software ATENA was used for both 2D and 3D analysis. It was shown that differences between two approaches are minor and simpler 2D models may be used for the analysis of bond in RC structures.

**Keywords:** reinforced concrete, bond, finite element, cracking.

### Introduction

Reinforced concrete (RC) is a composite material comprising relatively brittle concrete and reinforcing bars which carries majority of tensile stresses. The interaction between these two materials, often referred as bond, is essential to develop the required performance of the RC structure.

Bond of reinforcement in concrete is accompanied by a number of physical and mechanical actions such as formation of secondary and splitting microcracks, slip of reinforcement, local crushing and shearing of concrete in front of the ribs. It was experimentally shown (Goto 1971) that cracks propagate around the deformed bar, forming specific pattern of inclined concrete studs separated by secondary (*Goto*) cracks (Fig. 1a). These studs transfer force to concrete by bearing between the face of the rib and uncracked concrete around the bar, as schematically shown in Figure 1c. Horizontal component of the force transmitted by the concrete stud is generally treated as bond force, however vertical components are also developed, creating radial (or bursting) forces on the uncracked concrete, as shown in Figure 1b. Bursting forces may result in formation of splitting cracks, protruding longitudinally along reinforcement.

Numerical modelling of these effects is a highly complex challenge. The existing approaches in numerical bond analysis may be separated in three groups:

1) One-dimensional (1D) models. In such models the assumption of plain sections is used and the displace-

ment field of concrete and reinforcement is assumed only along the axis of the element. One of the common one dimensional approach is also called stress-transfer models (Balazs 1993; Yankelevsky *et al.* 2008; Jakubovskis *et al.* 2014). Using the stress transfer algorithms it is possible to model basic behaviour of RC element: formation of primary cracks, reinforcement slippage, stress distribution of concrete and reinforcement and average deformation of an element. However, due to simplified representation of stress distribution (average concrete stresses in the section are assumed) it is impossible to model formation of secondary and splitting cracks, damage of concrete in front of the ribs, diffusion of tensile stresses from reinforcement to concrete.

2) Two-dimensional (2D) models. Generally these models are based on particular finite element software for bond analysis (Jendele, Cervenka 2006; Salem, Maekawa 2004). Finite element analysis also allows to model bond action in different scales: rib-scale, bar scale or member scale (Li 2010). In two-dimensional analysis it is possible to simulate stress diffusion from reinforcement to the concrete, stress concentrations in front of the ribs and formation of secondary cracks. From computational point of view, 2D models are efficient and can allow for realistic simulation of bond in different scales. The main drawback of 2D models is neglecting circumferential tensile stresses in concrete arising from

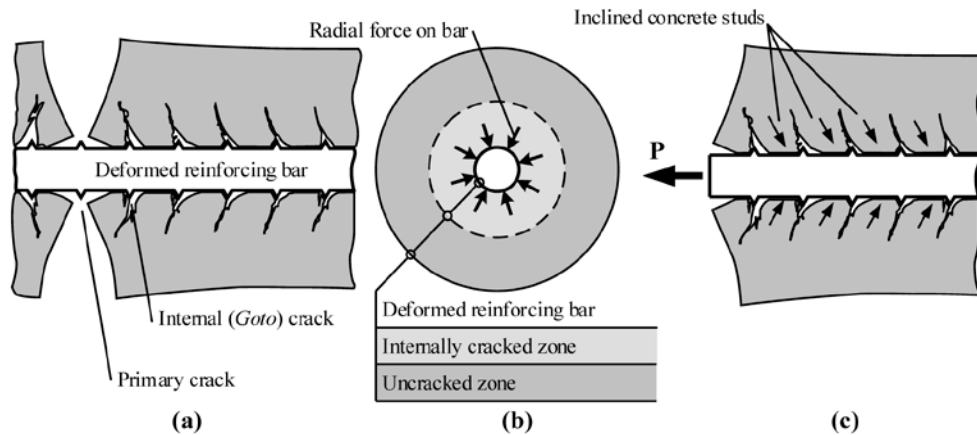


Fig. 1. Bond mechanics in reinforced concrete element:  
a) internal cracking of concrete around the deformed bar; b) development of radial forces;  
c) force transfer via inclined concrete studs

the bursting action and formation of splitting cracks.

3) Three-dimensional (3D) models. Finite element algorithms also are commonly used in such models. In three-dimensional approach it is possible to capture all the complexity of bond mechanics, however these models require huge computational resources and are only applicable to relatively simple elements (Bernardi *et al.* 2014; Michou *et al.* 2015).

The discussed approaches are used depending on the complexity of the problem and the required accuracy of the analysis. Finite element analysis, commonly used in aforementioned approaches, may serve as a powerful tool to investigate internal cracking and damage of concrete at the reinforcement interface, as it is highly complex to determine formation of secondary cracks experimentally.

In present paper 2D and 3D approaches for bond analysis are compared in terms of global load-displacement behaviour of RC element, formation of secondary and splitting cracks and deterioration of concrete in front of the ribs. Nonlinear Finite element software ATENA (Cervenka *et al.* 2005) was used for both 2D and 3D analysis of reinforcement and concrete interaction.

### Numerical modelling

To analyze the bond phenomenon in RC elements a simple tensile RC prism, reinforced with single bar was selected for modelling. The geometry of the prism along with the material properties are shown in Figure 2. The assumed height, pitch and width of the rib are also shown in Figure 2.

For 2D analysis a rib-scale model was generated as it represents a more fundamental approach to model bond zone in comparison to interface models (Li 2010). In rib

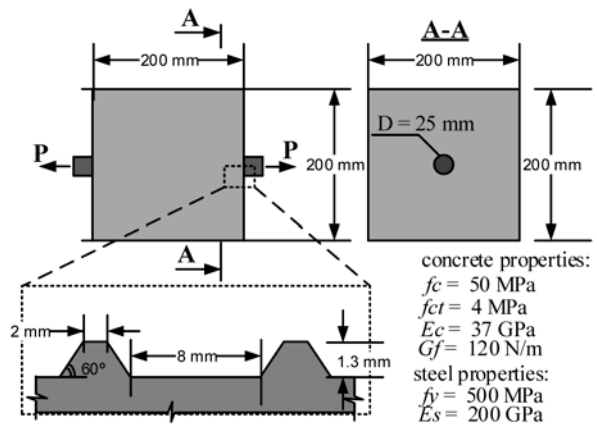


Fig. 2. Geometry and material properties of the modelled RC element

model, compression side displacement compatibility was assumed (only nodes that are in the compression side of the rib have the displacement compatibility with concrete). The generated 2D model is shown in Figure 3a. Due to symmetry conditions quarter of an element was modelled.

As the first approach, rib-scale model was also generated in 3D interface, however it appeared as extremely computationally expensive and simplified approach proposed by Michou *et al.* (2015) was further used in the present study. According to Michou *et al.* (2015), the interface of the concrete and reinforcement is modelled as periodic field of bond parameters: bond strength at the ribs is assumed  $\tau_{\max} = 200$  MPa, whereas between the ribs  $\tau_{\max} = 2.5$  MPa. Such alteration of the bond properties imitates the real ribs with the stress concentrations and secondary cracking in concrete as will be shown later. The created 3D model is shown in Figure 4a. Due to symmetry conditions, 1/8 of an element was modelled.

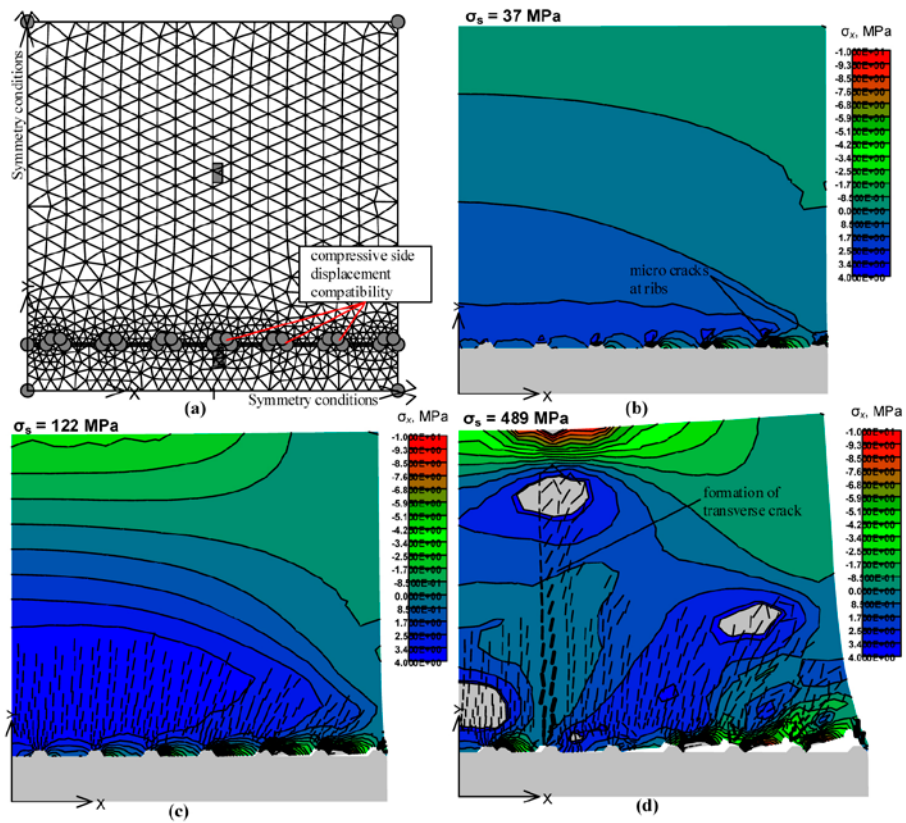


Fig. 3. 2D approach in bond modelling: a) FE model; b) – d) distribution of tensile stresses and cracks in concrete at different load levels

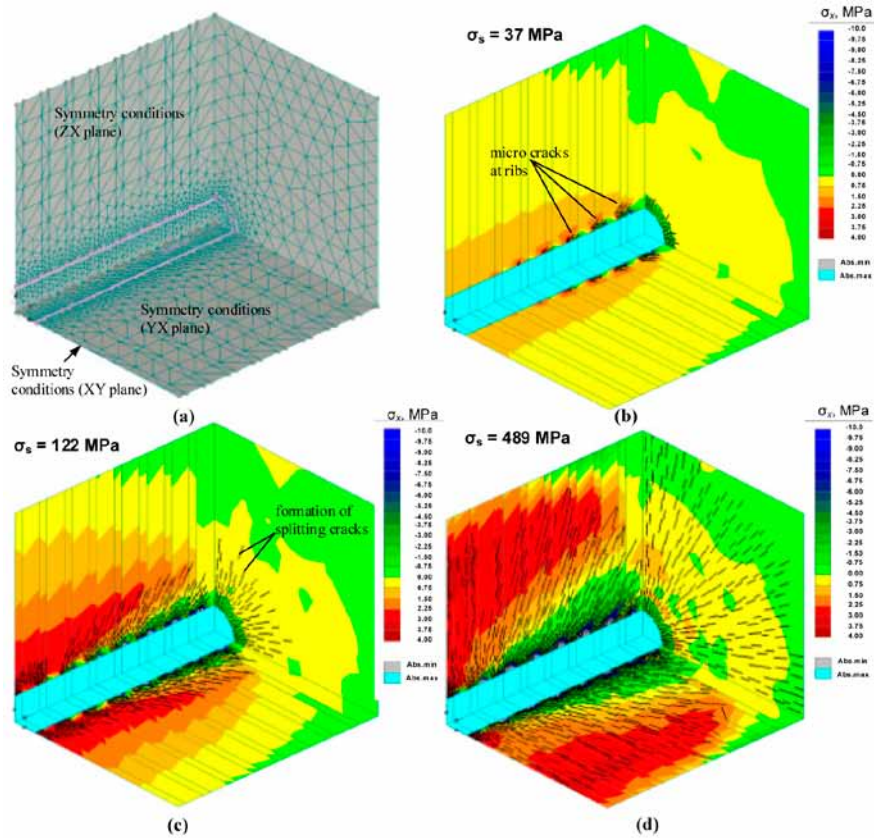


Fig. 4. 3D approach in bond modelling: a) FE model; b–d) distribution of tensile stresses and cracks in concrete at different load levels

## Results and discussion

Results of numerical simulation of bond behavior in 2D specimen are shown in Figure 3 b-d. The results are presented at three representative load levels:  $\sigma_s = 37, 122$  and  $489$  MPa, where  $\sigma_s$  is the stress in steel reinforcement. As can be seen from Figure 3b, even at relatively low load levels high stress concentrations occur in front of the ribs, resulting in formation of inclined micro-cracks. Stress from reinforcement to concrete is transmitted only to the limited area close to contact line. With increasing load micro cracks start to propagate towards the surface of concrete (Fig. 3c). At this stage a complex state of stresses in the element may be evident: high compressive stresses occur at the ribs whereas tensile stresses in concrete portages from the loaded end to the mid-section of the element, forming a particular cone-shaped area. Moreover, damage of concrete and debonding of the reinforcement starts at the loaded end, confirming the theory of damage zone in RC elements (Ruiz *et al.* 2007). At final loading stages (Fig. 3d), debonding zone propagates at about  $1.5 - 2D$  from the loaded end (where  $D$  is the bar diameter). Internal microcracks penetrate to the intact concrete and cover majority of concrete section. Inclined concrete studs around the ribs forms very similar to *Goto* microcracks shown in Figure 1. At final loading stage some microcracks near the mid section of the specimen coalesce and start to form transverse crack as may be seen from Figure 3d. The release in concrete stresses near the newly forming transverse crack may be noticed from the obtained tensile stress distribution.

In general, 2D rib scale model is able to simulate the main aspects of reinforcement and concrete interaction: stress concentration at the ribs, formation of microcracks, stress transfer via inclined concrete compressive studs and gradual stress diffusion from reinforcement to concrete. From computational point of view the generated 2D model required relatively low computational resources with analysis performed in several minutes. This makes the model reasonable for further calibration with experimental specimens. The main disadvantage of 2D model is neglecting the tensile ring stresses arising from radial bearing component of reinforcement ribs (Fig. 1). This effect was modeled in 3D model shown in Figure 4.

As may be seen from Figure 4, even at relatively low load levels (in 3D model three load levels are also analyzed:  $\sigma_s = 37, 122$  and  $489$  MPa, analogously to previous case) splitting cracks reaches the surface of the RC prism (Fig. 4b). At this stage stress concentrations and inclined microcracks also forms at the imitative ribs, similarly to 2D case. With increasing load both transverse and splitting

microcracks propagate and take almost all volume of the prism at final loading stages (Fig. 4 c, d). Stress diffusion in concrete is also similar to previous 2D case, forming a cone-shaped volume starting from the loaded end. Regarding computational resources of the 3D model, it took several hours to perform full analysis, comparing to several minutes in 2D case. This could partly impede the application of full 3D bond analysis in more complex RC members or structures.

The load-displacement diagrams of two modeled cases are compared in Figure 5. At low loading levels (up to 20 kN) the stiffness of the prisms is noticeably higher in comparison to the advanced load levels. This can be explained by the progressive development and growth of microcracks and accumulated damage of concrete. As was shown in Figure 3d and Figure 4d, at final loading stages almost all volume of concrete is internally cracked and unable to carry tensile forces. This resulted in gradually decreasing stiffness of the element with increasing load.

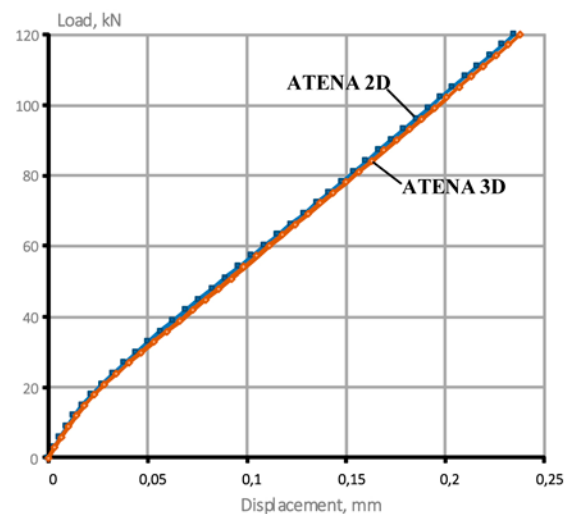


Fig. 5. Load-displacement diagrams of the modelled RC prisms

From the load displacement curves showed in Figure 5 it may be evident that only minor differences were obtained between 2D and 3D models. Formation of splitting cracks in case of 3D model resulted in slightly larger displacements (2–4%), especially at the higher loading levels. However, for practical cases such differences may be of minor importance and simpler properly adjusted 2D models may be used for analysis of RC structures.

## Conclusions

Numerical modeling of bond is a challenging due to a number of physical and mechanical phenomena arising from interaction of ribbed reinforcing bars and concrete. Different modeling approaches are used depending on the

complexity of the problem and required accuracy of the analysis. The following conclusions may be drawn from the performed numerical analysis:

1. The 2D rib scale model is able to simulate the main aspects of reinforcement and concrete interaction: stress concentration at the ribs, formation of microcracks, stress transfer via inclined concrete compressive studs and gradual stress diffusion from reinforcement to concrete. Being computationally efficient, 2D model is reasonable for further calibration with experimental specimens.
2. Using 3D model with imitative ribs it is possible to model both stress concentrations at the ribs, development of tensile ring stresses and splitting cracks. However such modeling is extremely computational expensive and this could partly impede the application of 3D modeling in more complex RC members or structures.
3. The differences in load-displacement behavior of 2D and 3D element were insignificant (2–4%). For practical cases such differences may be of minor importance and simpler properly adjusted 2D models may be used for the analysis of bond in RC structures.

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## ARMATŪROS IR BETONO SĄVEIKOS MODELIAVIMAS BAIGTINIŲ ELEMENTŲ METODU

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Santrauka

Skaitinis armatūros ir betono sąveikos modeliavimas yra įvairia-  
pusis ir sudėtingas uždavinys dėl sąlyčio zonoje vykstančių fiz-  
ikinių ir mechaninių procesų: betono glemžimo ties rumbeliais,  
armatūros slinkties, antrinių ir išilginių plyšių formavimosi. Šiame  
straipsnyje armatūros ir betono sąveika buvo modeliuojama  
baigtinių elementų metodu, sprendžiant dvimatį (2D) ir trimatį  
(3D) uždavinius. Gauti rezultatai parodė, kad, sąveiką idealizuo-  
jant dvimačiu modeliu, gaunamas charakteringas antrinių plyšių  
plitimas, betono pažeidimas greta plyšio ir įtempių persiskirstymas  
tarp armatūros ir betono. Dvimačio modelio rezultatai tik keliais  
procentais skyrėsi nuo trimačio modeliavimo, todėl skaičiavimo  
efektyvumo požiūriu 2D modeliai gali būti toliau tobulinami,  
analizuojant sudėtingesnių gelžbetoninių konstrukcijų elgseną.

**Reikšminiai žodžiai:** gelžbetonis, sukibimas, baigtiniai elemen-  
tai, pleišėjimas.