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APPLICATION OF MAJORITARIAN ELEMENT TO IMPROVE IOT COMMUNICATION

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Abstract. The work introduces the circle of the basic ideas and methods of error-free recovery of the binary signal to the multichannel digital technology based on the model of formal (artificial) neuron and aims to contribute to the further development of this theory of reliability. The work is useful for reservation or for exchange of a highly valuable information. During communication between machines, connection related problems could occur and this work will be of a great help for machines to choose right channel for connection. Work also is a successful attempt of figuring out binary signal error recovery probability of minimum highest value. Moreover, binary channels optimization problems addressed by using threshold model and exploring three different approaches. In the future, this work will be used in Internet of Things (IoT) for the exact communication between machines. Work will be used with threshold model for minimizing errors in the communication between things.

Keywords: Internet of Things, artificial neural network, multichannel, exchanging information, error probability, communication between machines, majoritarian element.

Introduction

The IoT can be used to improve our lives and businesses in many ways. It consists of three main components:

- the things themselves;
- the networks connecting the things;
- the computing systems that make use of the data flowing to and from things.

The network between things and information exchange are of primary importance of the IoT (Hui 2012). Mistakes in information that is sent or received between things needs to be avoided. For that purpose artificial neural network techniques can be used.

Artificial neuron is extremely simple abstraction of biological neuron, implemented as element in a program or perhaps as circuits made of silicon (Guo *et al.* 2011). Current networks of artificial neurons do not have a fraction of the power of the human's brain, but they can be trained to perform useful functions (Hagan, Demuth 2014).

USA scientists investigated majoritarian element in threshold model and showed that use of it minimizes mistakes between communicating objects. This technology nowadays is used in the army plains.

In the following, we will examine artificial neuron as implementation way of majoritarian element for the use of exact information exchange between IoT devices.

The exact expression for artificial neuron as the majoritarian element

Biological neuron consists of dendrites – collecting incoming information, synapses – weighting information from dendrites, and axon – processing information. Figure 1 presents McCulloch-Pitts *artificial neuron* functional scheme. It reassembles biological neuron and will be used in the following for exact communication development.

In the Figure 1 x is an input signal, b_i is the weight of the signal going through i-th channel and y is the neuron output signal. In the IoT, various different channels of communication (Wi-Fi, LAN, etc.) are usually used thus weighting of received information before the decision is a natural choice and fits well with neuron model.

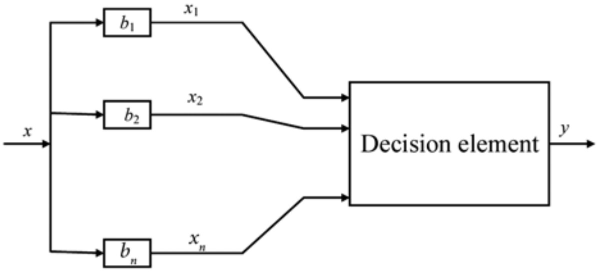

Fig. 1. McCulloch-Pitts artificial neuron functional scheme
1 pav. Dirbtinio neurono funkcinis McCulloch-Pitts modelis

Let's assume that the binary signal X, encoded by $+1$ and -1, is supplied to n data channels of the same type. Because of erroneous channels a transmitted variable X gets n representations, i. e., $x_1, x_2, \ldots, x_n$. This raises a problem of recognizing the true variable X from it's n versions. Of course, each quantities $x_i (i = \overline{1, n})$ also are binaries, that take values $+1$ and -1 (Gogiashvili *et al*. 2000).

The operation of restoring the body with the majority deciding element cannot be considered satisfactory if the probability of $q_1, q_2, \ldots, q_n$ binary channel errors $B_1, B_2, \ldots, B_n$ are different and, therefore, each of information x_i, coming from the binary channel B_i entering i input decisive element, it is necessary to ascribe the weight $a_i, \forall i = \overline{1, n}$, where a_i an arbitrary real number $(-\infty < a_i < +\infty)$. In this case, the decision on its output should be imposed as a result of weighted voting, according to the following:

$$y = \operatorname{sgn}\left(\sum_{i=1}^{n} a_i x_i - \underbrace{a_{n+1} x_{n+1}}_{\theta} \right). \tag{1}$$

Formally, assume that $\theta = a_{n+1}$, and $x_{n+1} \equiv -1$, meaning that there is some information channel B_{n+1}, always issuing -1 signal. Then (1) gets a short form:

$$y = \operatorname{sgn}\left(\sum_{i=1}^{n+1} a_i x_i \right). \tag{2}$$

This expression corresponds to a model casting body (threshold model), shown in Figure 2.

The probability of error recovery

The main purpose of this section is obtaining expression, for the probability Q, with the accepted threshold model shown in Figure 2, given that the solution y is incorrect, that means it does not coincide with the true value of the binary variable x.

Threshold main characteristics: $x_i B_i, x_i, y,$ are elements which are used in the algorithm for threshold model $i \in [1, n+1]$; It can be considered that that $B_{n+1} \to x_{n+1} \equiv -1;$ and after incoming of X on communication channel $B_i \left(i = \overline{1, n+1} \right)$, it takes values of $+1$ and -1. Here is: $B_{n+1} \to q_{n+1}, a_{n+1} \equiv \theta;$ and $x, x_i \left(i = \overline{1, n+1} \right)$, after this: $y \to q_{n+1}, a_{n+1} \equiv \theta;$

Based on the last identity follows the validity of the chain transformation:

$$\begin{aligned} Q_{n+1} &= \operatorname{Prob}\left(X \neq X_{n+1} \right) = \\ &\operatorname{Prob}\left(X \neq -1 \right) = \operatorname{Prob}\left(X = +1 \right); \\ Q &= \operatorname{Prob}\left(Y \neq X \right). \end{aligned} \tag{3}$$

Some experiments where done for this research (Table 1).

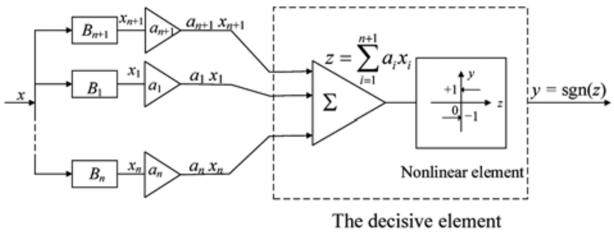

Fig. 2. The threshold model
2 pav. Šuolio modelis

Desired probability errors (3) in the output of the decision element take the form:

$$Q = \sum_{v<0} f(v) = \sum_{v<0} \mathop{*}_{i=1}^{n+1} f_i(v_i), \tag{4}$$

here summation is over all the negative values of discrete variable v, "$*$" represents convolution operation, while $f_i(v_i)$ is determined by:

$$f(v) = \mathop{*}_{i=1}^{n+1} f_i(v_i)\,.$$

Table 1. The calculation of error probability for Q^{maj}

1 lentelė. Q^{maj} klaidos tikimybės apskaičiavimas

j	v_1	v_2	v_3	v_4	V^{maj}	$Q_j^{\text{maj}} = \prod q_j$
1	$-a_1 = -1$	$-a_2 = -1$	$-a_3 = -1$	$-a_4 = 0$	–3	$q_1 q_2 q_3 q_4$
2	$-a_1 = -1$	$-a_2 = -1$	$-a_3 = -1$	$+a_4 = 0$	–3	$q_1 q_2 q_3 (1 - q_4)$
3	$-a_1 = -1$	$-a_2 = -1$	$+a_3 = +1$	$-a_4 = 0$	–1	$q_1 q_2 (1 - q_3) q_4$
4	$-a_1 = -1$	$-a_2 = -1$	$+a_3 = +1$	$+a_4 = 0$	–1	$q_1 q_2 (1 - q_3)(1 - q_4)$
5	$-a_1 = -1$	$+a_2 = +1$	$-a_3 = -1$	$-a_4 = 0$	–1	$q_1 (1 - q_2) q_3 q_4$
6	$-a_1 = -1$	$+a_2 = +1$	$-a_3 = -1$	$+a_4 = 0$	–1	$q_1 (1 - q_2) q_3 (1 - q_4)$
7	$-a_1 = -1$	$+a_2 = +1$	$+a_3 = +1$	$-a_4 = 0$	+1	–
8	$-a_1 = -1$	$+a_2 = +1$	$+a_3 = +1$	$+a_4 = 0$	+1	–
9	$+a_1 = +1$	$-a_2 = -1$	$-a_3 = -1$	$-a_4 = 0$	–1	$(1 - q_1) q_2 q_3 q_4$
10	$+a_1 = +1$	$-a_2 = -1$	$-a_3 = -1$	$+a_4 = 0$	–1	$(1 - q_1) q_2 q_3 (1 - q_4)$
11	$+a_1 = +1$	$-a_2 = -1$	$+a_3 = +1$	$-a_4 = 0$	+1	–
12	$+a_1 = +1$	$-a_2 = -1$	$+a_3 = +1$	$+a_4 = 0$	+1	–
13	$+a_1 = +1$	$+a_2 = +1$	$-a_3 = -1$	$-a_4 = 0$	+1	–
14	$+a_1 = +1$	$+a_2 = +1$	$-a_3 = -1$	$+a_4 = 0$	+1	–
15	$+a_1 = +1$	$+a_2 = +1$	$+a_3 = +1$	$-a_4 = 0$	+3	–
16	$+a_1 = +1$	$+a_2 = +1$	$+a_3 = +1$	$+a_4 = 0$	+3	–

Complete number of discrete values of v is 2^{n+1}, since $v = \tilde{a}_1 + \tilde{a}_2 + \cdots + \tilde{a}_n + \tilde{a}_{n+1}$ thus

$$V^{\text{maj}} = \tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3 + \tilde{a}_4, \quad Q_j^{\text{maj}} = \tilde{q}_1\tilde{q}_2\tilde{q}_3\tilde{q}_4,$$
$$\tilde{a}_k = \{+a_k \mid -a_k\}, \quad \text{for all } k = \overline{1,4}, \tag{5}$$
$$\tilde{q}_k = \begin{cases} 1-\tilde{q}_k, & \text{when } \tilde{a}_k = +a_k, \\ q_k, & \text{when } \tilde{a}_k = -a_k; \end{cases} \quad \text{for all } k = \overline{1,4}.$$

Then

$$Q = \sum_{v<0} Q_j = \sum_{v<0} \tilde{q}_1\tilde{q}_2 \cdots \tilde{q}_n\tilde{q}_{n+1} \ . \tag{6}$$

In order to demonstrate this algorithm, we consider the case of threshold majority element, when:

$$\begin{aligned} &n = 3;\ q_1 = q_2 = q_3 = q,\ q_4 = 0.5; \\ &a_1 = a_2 = a_3 = 1,\ a_4 \equiv \theta = 0. \end{aligned} \tag{7}$$

The results of calculations, of the probability of error Q threshold majority element, according to the algorithm (6) are presented in Table 1.

Here is threshold majoritarian element:

$$Q^{\text{maj}} = \sum_{k=]n/2+1[}^{n} C_n^k q^k (1-q)^{n-k}, \tag{8}$$

where C_n^k – the number of combinations of items, $]\bullet[$ – rounding towards zero. For the given example (n = 3), $k =]n/2+1[= 2$ and (8) yields $Q^{\text{maj}} = 3q^2(1-q) + q^3$.

If someone will add the expressions Q_j^{maj} in the last column of Table 1, corresponding to negative values of V^{maj}, that will result in the following algorithm:

$$\begin{aligned} Q^{\text{maj}} &= Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_9 + Q_{10} = \\ &3q^2(1-q) + q^3. \end{aligned} \tag{9}$$

Q^{+} can be taken as an upper bound for the probability of error Q in threshold element. The upper bound of this experiment minimum error probability is:

$$Q_{\min}^{+} = 2^{n+1} \times \prod_{i=1}^{n+1} \sqrt{q_i(1-q_i)}, \tag{10}$$

or

$$Q_{\min}^{+} = \exp\left(-\sum_{i=1}^{n+1} A(q_i)\right), \tag{11}$$

here

$$A(q_i) = \left|\ln\left(2\sqrt{q_i(1-q_i)}\right)\right|. \tag{12}$$

From (11), taking into account the non-negativity values $A(q_i)$, it is clear that: increasing the number of inputs in the threshold majority element (if only probability of errors on these inputs is not equal 0.5), leads to monotonous decrease on exhibitor even when it is the minimal upper bound probability of incorrect recognition.

The results are not likely to exceed the minimum at the top of the assessment, if:

$$a_i = \frac{1}{2|\ln S_0|} \ln \frac{1-q_i}{q_i}, \tag{13}$$

here $i = \overline{1, n+1}$, $k = \left(2|\ln S_0|\right)^{-1} > 0$ and $0 < S_0 < 1$.

Consequently, the optimal weights a_i ($i = \overline{1, n+1}$) providing minimum probability threshold authority errors are defined up to a common positive factor k.

After this work, Z was taken as random variable for the optimization of the weights, we have two Z because we use different X to Z. X is a weight and can take the meaning of $+1$ and -1,

$$Z = \sum_{i=1}^{n} a_i X_i, \text{ or } Z = \sum_{i=1}^{n} Z_i, \text{ where } Z_i = a_i X_i.$$

For the experiment, generalized (Mahalanobis) distance will be taken:

$$Z = \sum_{i=1}^{n+1} a_i X_i,\ X = +1,$$
$$Z = \sum_{i=1}^{n+1} a_i X_i,\ X = -1,\ \sigma_z^2 = \sum_{i=1}^{n} 4a_i^2 q_i (1-q_i), \tag{14}$$

where $p = \dfrac{(m_1 - m_2)^2}{\sigma_z^2}$, $m_1 = -m_2$; $m_1 = \sum_{i=1}^{n} a_i (1-2q_i)$ and $m_2 = \sum_{i=1}^{n} a_i (2q_i - 1)$.

Here are the weights, with which the maximum distance of the generalized (Mahalanobis) are determined:

$$a_i = k \frac{1-2q_i}{2q_i(1-q_i)}, \quad \forall i = \overline{1, n+1} \text{ and } k = \overline{0, \infty}. \tag{15}$$

Essence of Bayesian View:

$$\begin{aligned} P_1 &= \text{Prob}\left\{Z = \frac{z}{X} = +1\right\}\ y = +1, \\ P_2 &= \text{Prob}\left\{Z = \frac{z}{X} = -1\right\}\ y = -1, \end{aligned} \tag{16}$$

$$\begin{aligned} &\text{If } \frac{P_1}{P_2} > 1 \rightarrow y = +1, \\ &\text{If } \frac{P_2}{P_1} > 1 \rightarrow y = -1. \end{aligned} \tag{17}$$

The weights of a_i ($i = \overline{1, n+1}$), calculated using the Bayesian approach (by the criterion of maximum a posteriori probability), are given by:

$$a_i = k \ln \frac{1-q_i}{q_i}, \quad \forall i = \overline{1, n+1} \text{ and } k = \overline{0, \infty}. \tag{18}$$

If a set of $n+1$ data channels $B_1, B_2, \ldots, B_n, B_{n+1}$ regarded as a single source of binary information

$\vec{x} = (x_1, x_2, \ldots, x_{n+1})' = (1, 1, \ldots, -1)'$, the entropy of this source:

$$E = k\sum_{i=1}^{n+1}\left[-(1-q_i)\ln(1-q_i) - q_i \ln q_i\right]. \qquad (19)$$

Any change in entropy as a measure of the weight of the new channel should be considered in relation to the variation, of the channel error probability:

$$a_i = \frac{\partial E}{\partial q_i}, \forall i = \overline{1, n+1}. \qquad (20)$$

The weights calculated with entropy approach:

$$a_i = k\ln\frac{1-q_i}{q_i},\ \ k > 0,\ \overline{1, n+1}. \qquad (21)$$

Generalized (Mahalanobis) maximum a posteriori probability of the connection, weights and distances for awarding the maximum (entropy) of the criteria agreed with the weights: $a_{im} = \cot\left(\frac{1}{k} a_{ie}\right)$, where a_{im} is the weight and a_{ie} – the weights which is calculated based on the Bayesian approach, $a_{ie} = k\ln\left(\frac{1-q_i}{q_i}\right)$.

From all this, the absolute limit of the classification quality is determined by the Bayes classification rule. Therefore, despite the identical nature of the relations (15) and (20), we can expect that this strategy (16) and (17) minimize the probability of error in the threshold body. However, simply proceed from the fact that the probability of an error is determined by the threshold body.

For the majority decisive element (threshold element with the following information: $a_{n+1} \equiv 0$, $a_i = 1$, $q_i = q$ for all $i = \overline{1, n}$) the minimum upper bound for the error probability at the output is the following:

$$Q_a^+ = \exp\left[-nA(q)\right],\ \ n \to \infty, \qquad (22)$$

where $A(q) = \left|\ln 2\sqrt{q(1-q)}\right|$, if only $0 < q < q_0(n)$, and $q_0(n) = \left]n/2 + 1\right[/ (n+1)$, where greatest integer part of the value $(n/2+1)$, moreover $\lim_{n\to\infty} q_0(n) = 0.5$.

In this result there is a connection with Claude Shannon's theory: Number of messages in a given length *n* (duration *t*), which consist of separate symbols – in case of absence as well as presence of fixed and probability limits (in the latter case under the condition of source ergodicity) – with increase of *n* (or *t*) increases with asymptotic exponentially.

Conclusions

There has been a much advancement made in many standard areas, but more progress is needed, focusing main area such as communication, because communication is one of the most important part in IoT. In this paper is shown how to make exact communication between things with artificial neural network.

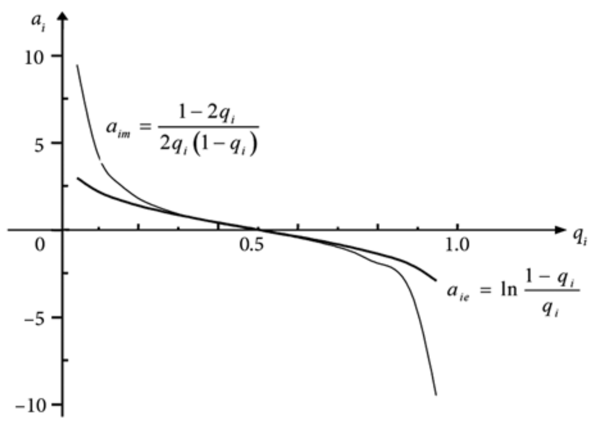

Fig. 3. Graphic interpretation (when $k = 1$) from the formulas (15) and (21).

3 pav. (15) ir (21) formulių grafinė iliustracija, kai $k = 1$.

The research is processed and developed in program with the algorithm which is calculating probability of errors in threshold model. It is established probability of error of the recovery binary signal channels, when the number of majoritarian incoming elements is infinitely growing.

This work is useful for the communication between the machines (M2M) or between the owner and machine, which will be used in the Internet of Things in my future works.

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References

Gogiashvili, J. G.; Namicheishvili, O. M.; Chonia, G. G. 2000. Optimization of weights for threshold redundancy of binary channels by the method of (Mahalanobis›) Generalized Distance, in *Second International Conference on Mathematical Methods in Reliability: Methodology, Practice and Interference*, 4–7 July 2000, Bordeaux, France. Université Victor Segalen Bordeaux 2, Abstracts' Book, Vol. 1, 463–466. ISBN: 978-960-474-088-8.

Guo, L. G.; Huang, Y. R.; Cai, J.; QU, L. G. 2011. Investigation of architecture, key technology and application strategy for the Internet of Things, in *Cross Strait Quad-Regional Radio Science and Wireless Technology Conference* (CSQRWC).

Hagan, M. T; Demuth, H. B. 2014. *Neural Network Design.* 2nd ed. Martin Hagan USA. ISBN 9780971732117.

Hui, S; Jiafu, WU; Cai Feng, Z; Jianqi, L. 2012. Security in the Internet of Things: a review, in *Computer Science and Electronics Engineering (ICCSEE),* 648–651.

DAUGUMOS ELEMENTO TAIKYMAI SKIRTI TOBULESNIAM RYŠIUI TARP INTERNETO DAIKTŲ

T. Dvali

Santrauka

Straipsnyje pristatomos idėjos ir metodai skirti daugiakanalių sistemų klaidoms skaitmeniniuose signaluose atstatyti. Nagrinėjama technologija grindžiama formaliuoju (dirbtiniu) neuronu, plėtoja patikimumo teoriją ir gali būti taikoma rezervavimo, siunčiant labai svarbią informaciją, sprendimams. Siekiant pagrįsti alternatyvaus kanalo tinkamą parinkimą, trimis skirtingais būdais nagrinėjamos dvejetainių signalų atstatymo tikimybės. Gauti rezultatai ateityje planuojami taikyti tiksliam (be klaidų) informacijos perdavimui ryšiuose tarp interneto daiktų.

Reikšminiai žodžiai: daiktų internetas, dirbtinių neuronų tinklai, daugiakanalis, informacijos mainai, klaidų tikimybės, daugumos elementas.